# Carnap, Popper, Gödel: can Unity be Refuted by Incompleteness?

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**Abstract:** In this paper all the "acting" philosophers play their classical role: Gödel is present with his incompleteness theorems. Carnap is present with the positivist view of unity of science, and specifically with the thesis about a universal language. Finally, Popper tries to refute Carnap's thesis with the help of Gödel's. Unfortunately this debate did not take place in real, only one claim and reponse was made in Shilpp's volume. I attempt to clarify this question in the present paper.
The main focus is on Carnap's view. I will show that it is possible to hold a thesis about a possible universal language if this is meant in a weaker sense: as a syntactical framework. The concept of "language" in Carnap's view is also examined, and I come to the conclusion that it was used both in a wider and both in a narrower meaning. I also try to clarify this conceptual issue.

## 1. How the problem came into my sight

The incompleteness theorems of Gödel are probably the most significant achievements of 20th century mathematics from the perspective of philosophy. On the other hand it is apparent that they are not understood precisely or are interpreted in a wrong way by some philosophers and by a lot of laymen. The corollaries of Gödel's theorems became clear quite early in mathematics, but in my opinion the consequences are not all clear yet for philosophy. There is a good review on the latter by Raatinainen[1].
The problem I will discuss in this paper is if Gödel's theorems do refute the possibility of a universal language, which is formal and appropriate for the unified empirical sciences. According to my studies this question has not been analyzed comprehensively till now.
The problem appeared for me first - after some informal conversations - when I was reading Popper's „Conjectures and refutations"[2] (in the following Conjectures). Chapter 11 of this book is titled „The Demarcation between Science and Metaphysics", where Popper analyzes and mostly criticizes some aspects of Carnap's philosophy. As its title shows the chapter deals with the demarcation of metaphysics and science and with the neo-positivist thesis of the meaninglessness of metaphysics. This is an enormous topic with a lot of interesting questions for me, but currently I only consider one question, which Popper discusses in subchapter 4 (b), the question of the universal language and Gödel's theorems.
I sum up the idea of Popper with the following quote:

> "Now, the queer thing about the thesis of the *one* [italics in the original M.B.] universal language is that before it was ever published (on the 30th of December 1932) it had been refuted by one of Carnap's colleagues in the Vienna Circle. For Gödel, by his two famous incompleteness theorems, had proved that one unified language would not be sufficiently universal for even the purposes of elementary number theory: although we may construct a language in which all assertions of this theory can be *expressed*, no such language suffices for formalizing all the proofs of those assertions which (in some other language) can be *proved*." (Conjectures, p. 362 highlighting in the original. I will refer to this quote as "P1".)

This sounds at "first sight" if Popper accused Carnap not knowing about Gödel's result. But Popper himself did clarify that this is not his claim:

> "Carnap was the first philosopher who recognized the immense importance of Gödel's discoveries, and he did his best to make them known to the philosophical world." (Conjectures, p. 365)

And:

[1] Panu Raatikainen: „On the Philosophical Relevance of Gödel's Incompleteness Theorems", forthcoming in the Revue Internationale de Philosophie.
[2] Karl Popper: „Conjectures and refutations", Routledge, 2002, (first edition Routledge, 1963).

> "I do not of course, suggest that Carnap did not know all of this; but that he did not see its devastating effect upon the doctrine of the unified science in the unified language." (Conjectures, p. 364)

Indeed, Gödel was a member of the Vienna Circle, although his views were not genuinely neo-positivist, and Carnap had extensive conversations with him especially in these times. Carnap knew and admired sincerely the results of Gödel[3] and, according to his own account[4], he took into account Gödel's findings in his work already before their publication, thanks to their co-work.

Chapter 11 of Conjectures - as it is obvious from the book - was published first in „The Philosophy of Rudolf Carnap"[5] edited by Schilpp (from now on Schilpp-volume). In this book Carnap also reflected on the essays, among them on Popper's:

> "These results [i.e. the results of Gödel and Tarsky referred by Popper (M.B.)]are certainly of the greatest importance. But they show only that no fixed language can be logically and semantically complete; every language can be further strengthened by the addition of new logical forms of expression and new logical means of deduction. The thesis of the unity of science, as Neurath and I maintained it, has nothing to do with the question of logical completeness." (Schilpp-volume, p. 880)

In the following Carnap explains that their goal was the unification of natural and social sciences on a uniform basis, i.e. they were opposing a kind of dualism. Indeed this was a main goal of the Unity of Science Movement of Vienna Circle from the beginning. Popper also mentions that Neurath's goal was this[6]. But Neurath was in favor of a universal slang, which was criticized by Popper from another viewpoint: the slang is not appropriate for the demarcation of metaphysics. I agree with this critique.

So, the concept of unity of science included many different theses and views. Schlick has a system in mind with a clear reduction of all sciences to physics[7]. Neurath[8] conceived of a mosaic of sciences and of the encyclopedia, which is a loose system with a universal slang[9]. Carnap had also more different ideas on this topic, which resembles to some of Schlick's ideas and some of Neurath's. For a summary about the earlier views of Carnap on the unity of science see Pincock's work[10], where the main issue about unity is also this anti-dualism.

The views of Carnap however enclosed also the possibility of a formal unified language, and therefore we indeed have to examine the effect of Gödel's results. Carnap's answer quoted from the Schilpp-volume contains a short indication concerning this, but it is not explained in detail. On the other hand Popper's critique is not detailed enough either: he does not state precisely how the results of Gödel refute the possibility of a universal language in his view.

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[3] For a more detailed description of Carnap's reception of the theorems of Gödel see J.W. Dawson, Jr: „The Reception of Gödel's Incompleteness Theorems" in S.G. Shanker (ed.): "Gödel's Theorem in Focus", Routledge, 1991 (first edition 1988), see especially page 76.

[4] R. Carnap: „Mein Weg in die Philosophie", Reclam, 1999 (first edition 1963), p. 83. This autobiography can also be found as the first part of the Schilpp volume.

[5] P. A. Schilpp (ed.): „The Philosophy of Rudolf Carnap", Open Court, 1991. (first edition 1963.)

[6] I cannot deal with Neurath's position here and it is not needed, since there are excelent papers on this. George Reisch: „Planning Science: Otto Neurath and the International Encyclopediaof Unified Science", Brittish Journal for History o Science, 27, 153-75, 1994, George Reisch: „Pluralism, Logical Empirism, and the Problem of Pseudoscience", Philosophy of Science, 65, 333-348, 1998, Nyíri Kristóf: "From Texts to Pictures: The New Unity of Science", in: Kristóf Nyíri, ed., Mobile Learning: Essays on Philosophy, Psychology and Education, pp. 45-67, Vienna: Passagen Verlag, 2003.

[7] M. Schlick: "Die Probleme der Philosophie in ihrem Zusammengahng", Suhrkamp, 1986. See the chapter titled: "Das Sytem der Wissenschaften".

[8] O. Neurath: "Wissenschaftliche Weltauffasung, Sozialismus und Logischer Empirismus", Suhrkamp, 1979. See the chapter titled: "Die Neue Enzyklopädie". N. Cartwright, J. Cat, L. Fleck, T.E. Uebel: "Otto Neurath: Philosophy Between Science and Politics", Cambridge University Press, 1996. See chapter: "Unity without the pyramid".

[9] O. Neurath, N. Bohr, J. Dewey, B. Russell, R. Carnap, Ch. W. Morris: "Foundation of the Unified Science", International Encyclopedia of Unified Science, Vol 1. No. 1, The University of Chicago Press, 1938. Beside Neurath's chapter see also Carnap's contribution.

[10] Chris Pincock: "Carnap and the Unity of Science: 1921-1928", in T. Bonk (ed.), Language, Truth and Knowledge: Contributions to the Philosophy of Rudolf Carnap, Vienna Circle Institute Library, Volume 2, pp. 87-96, Kluwer, 2003. You can also find in this paper about the views of Schlick and Reichenbach on the unity of science.

Footnote 41 in Conjectures, page 362 is: "Erkentnis, 3, 1932, p. 108". Unfortunately Popper does not specifies the title, but from the Schilpp-volume it is possible to trace back that it is the paper „Psychologie in physikalischer Sprache", which starts at page 108. (I did not think it important to find out the reason of the one page difference.) I have read the paper in English version[11]. Carnap has a small book specifically on the unity of science titled: „The Unity of Science"[12]. However, the two publications of Carnap mentioned above do not contain a description of a formal language. So the application of Gödel's theorems would need a formal reconstruction for these.

Nevertheless, Carnap has a publication which is suitable for this - and the continuation of the critique of Popper considers Carnap's „The logical syntax of language" (In the following "Syntax")[13], so I will also take this into account as a basis. In Syntax Carnap writes the following statement:

> "For everyone who takes the point of view of Physicalism, it follows that our Language II forms a complete syntactical framework for science." (Syntax, p. 151)

This is probably the most unambiguous thesis in this book for our means.

Despite the critique of Popper it is surprising to find out that in the motivation and content of Syntax the results of Gödel play a big role. The arithmetisation technique of Gödel is used, and a goal of Syntax was to solve the problems, which arise from the incompleteness theorems. In „Questions of Form"[14] Joëlle Proust describes Syntax as a mostly successful account inspired by the methods of Gödel and by the problems arising from his achievements:

> "The Logical Syntax will attempt to save analycity from the blind alley into which the theorems of incompleteness led it." (ibid. p. 213)

Richard Creath in „Languages Without Logic"[15] explains a similar view: he writes about the reconstruction of a kind of logical completeness after the incompleteness-theorems.

Without going into details about this I just mention that Gödel himself had also tried to find some foundation for mathematics after his incompleteness theorems. See for example about this Feferman's analyzis about the Dialectica paper of Gödel.[16]

In the foreword of 1934 in Prague Carnap mentions that Gödel had read the manuscript of 1932, and had some constructive comments. The results of Gödel are referred to in the references, the topic of irresolubility is a separate chapter in Syntax (§36.). It is clear from this that Carnap did not only know about Gödel's result, but they had a central significance in his book. In these circumstances the critique of Popper is highly improbable, but since it may be true, we have to examine it.

If we examine the writings related to Syntax in the literature, we find a lot of analysis about Gödel's theorems, but they are dealing with other related questions, like analicity, translatability[17], the semantic turn, the concept of truth and so on. I did not find an analysis about the relation of the Incompleteness Theorems and the possibility of a universal language. It seems that this question has not been cleared until now. I will try this in the following.

If we go back to the quote P1 of Popper, then the following questions arise:

(i) What is the limit of considering a language the same language?

(ii) The theorems of Gödel are valid for deductive systems. What is the relation of a deductive system and a language?

(iii) Does a Gödel-undecidable empirical question exist?

(iv) If it exists then does this refute the possibility of a universal language in the empirical sciences?

These questions are inexact now, but they will become clearer.

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[11] R. Carnap: „Phsychology in Physical Language", in. A. J. Ayer (ed.) „Logical Positivism", The Free Press, 1959.

[12] R. Carnap: "The Unity of Science", Thoemes Press, 1995, (first edition 1934)

[13] Rudolf Carnap: „The logical syntax of language", Open Court, 2002. (first edition 1937)

[14] Joëlle Proust: „Questions of Form, Logic and the Analytic Proposition from Kant to Carnap", University of Minnesotta Press, 1986.

[15] Richard Creath: „Languages Without Logic ", in „Origins of Logical Empiricism", ed. R. Giere, A.W. Richardson, University of Minnesota Press, 1996. p. 256.

[16] S. Feferman: "Gödel's Dialectica interpretation and its two-way stretch", Computational Logic and Proof Theory (Proc. 3d Kurt Gödel Colloquium, Brno Aug. 1993), G. Gottlob et at. eds., LNCS, 713, 23–40, 1993.

[17] R. Ciera: "Carnap and the Vienna Circle", Edition Rodopi, 1994.

## 2. The question of language-identity[18]

In the case of natural languages the question of "to what extent are certain dialects one language", – which I call the question of language-identity – is not decidable (definable) exactly. The decision is influenced by political, sociological, geographical and of course also linguistical aspects. The linguistical aspects are also of different kinds: lexical, phonetical, grammatical similarity. Just to mention some interesting examples: usually we regard Chinese as one language, despite that Cantonese and Mandarin differ almost as much as Italian and French do. If we consider them as one language, this may have a reason in national unity, or that these Chinese understand each other in writing, the big difference being in the spoken language. The opposites are Serbs and Croats: they understand each other in words, but they differ in the Latin and Cyrillic writing (at least until recently), and there is a national separation. Hindi and Urdu are a similar example. Norwegian, Danish and Swedish people understand each other in spoken and written language as well. Arab people usually understand each other in neighboring countries, but not if there is a larger geographical distance; on the other hand they may understand each other completely in classical Arabic.

It is often difficult to decide if we are facing two different dialects or two different languages. The papers I found about this topic dealt with the comparison of languages or dialects from lexical, phonetical, grammatical aspects[19]. They discussed questions of language change, language development, history of languages, and a lot of comparisons of genetic and linguistic development. There are very detailed examinations of Dutch, Norwegian and other dialects, and the methods of numerical comparison[20]. The methods used include some borrowed from genetics, e.g. cladistical categorization algorithms.

It seems that the criteria of language-identity is usually – except of such cases like those ones mentioned –that people understand each other. But because of the lack of transitivity there is also a mathematical problem with this, since the mutual intelligibility relation is not an equivalence-relation, so it does not construct classes that could be regarded as the languages. There are also a good example for this: if we travel from Barcelona through Nice to Rome we can observe that in each village they understand the language of the next one. It is probably surprising that there is no discontinuity even on the border. This is the so called west romance dialect continuum[21].

Intelligibility depends obviously on phonetic, lexical, and grammatical similarity and integrates them. Since this is not the topic of this paper I do not go into details in this issue. However I found a good book, „Dialectology" from Chambers and Trudgill[22], in which our philosophical question is raised, so instead of analyzing further I close this issue with referring to that book. The authors explain similarly that the best approximation for the definition of language identity is mutual understanding, but it is not perfect. They mention the same counter examples. Their conclusion is:

> "It seems then, that while the criterion of mutual intelligibility may have some relevance, it is not especially useful in helping us to decide what is and is not a language. In fact, our decision of the Scandinavian languages and German suggests that (unless we want to change radically our everyday assumptions about what a language is) we have to recognize that, paradoxically enough, a 'language' is not a particularly linguistic notion at all." (ibid. p. 4)

However for formal languages and especially for our topic we have to have an exact definition for language identity. The cultural, geographical and phonetic aspects are excluded. If the question is if it is possible to describe science in one language, then it is necessary to define how much change do we accept before we regard

[18] In this chapter I wish to thank László Kálmán for consultation.

[19] P. Heggarty: „Quantifying change over time in phonetics." In Colin Renfrew, April McMahon and Larry Trask (eds.) Time Depth in Historical Linguistics, Volume 2: 531-62. Cambridge: McDonald Institute for Archaeological Research 2000.

[20] W. Heeringa: „Measuring Dialect Pronunciation Differences using Levenshtein Distance", Ph.D. Dissertation, Rijksuniversiteit Groningen, Groningen, 2004.
http://www.let.rug.nl/~heeringa/dialectology/thesis/thesis.pdf

[21] This does not mean the official language of course, but the local dialect, which is spoken for example on the market.

[22] J. K. Chambers, Peter Trudgill: „Dialectology", Cambridge [Eng.] ; Cambridge University Press, 1980.

the language changed to another language. Unfortunately Popper did not clear this question[23], so we have to start with this.
As an outline, and referring back to the previous ideas, we can say that lexical and semantic rules are relevant. When writing Syntax, neither Carnap nor other scholars did yet make a clear distinction between semantics and syntax - these concepts were being formed by the works of Carnap and Tarsky, - so he only wrote about syntactical rules, but the construction of syntax includes some, which we would regard as semantics. In the following I will take the construction and concepts of syntax, not the modern linguistic vocabulary. Nor do I have the possibility here to investigate their relation.
Carnap explicates on p. 180 of syntax that - when defining a language- we can chose freely whether we include, and if so, how many so called P-rules (physical rules) we include in the syntactical rules of our language besides the so called L-rules (logical rules). For example he brings up the possibility to include natural laws in the syntactical rules, but also the extreme case, that all the statements, which are considered true are included in the syntactical rules of the language.

> "Whether in the construction of a language S we formulate only L-rules or include also P-rules, and, if so, to what extent, is not a logico-philosophical problem, but a matter of convention and hence, at most, a question of expedience. If P-rules are stated, we may frequently be placed in the position of having to alter the language; and if we go so far as to adopt all acknowledged sentences as valid, then we must be continuously expanding it." (Syntax, p. 180, I refer to this quote as C1)

So the decision is practical, and a question of consensus. We may define the language in the extent that we like, and we will regard something to be outside this language if it is in contradiction with these rules. To contradict those rules which were not stated as syntactical rules does not mean a language change. Rickets emphasizes the same in his paper titled „Carnap, From Logical Syntax to Semantics"[24], but unfortunately he does not deal with our topic.
What I described so far is a free framework, so let us now examine, what is possible for a universal language for empirical sciences!

## 3. A universal language for suitable for research

If we consider the process of scientific research, we can conclude the following. First, we acquire a lot of new experiences, we do a lot of experiments. It is obviously not practical to take these empirical propositions as syntactical rules of the universal language, since we would have to change the language frequently. On the other hand they are singular, so not much worth as a rule.
On the other hand the theories of science are also synthetic. Carnap thought it more realistic to take them as syntactical P-rules, but since we also have theory changes this would be neither practical for the universal language. It is possible that we have some theoretical statements, which are eternal truths, but we do not know this for sure. Accordingly it may be that we could have theoretical statements as P-rules, but we would not know when we would have to reject them and create language change. So it seems the safe solution is not to take any synthetic (contingent) statement as syntactical rule, since we would have to change language in scientific research, as Carnap already pointed out in quote C1.
As a conclusion we could consider as a hypothesis that a possible unified language of universal empirical sciences could be Language II of Syntax with logical syntactical rules, but no physical syntactical rule at all. Carnap denotes such a language as L-language, so let us denote this language as L-Language II. The synthetic empirical and the theoretical statements are not included in the syntactical rules defining the language, but are only a part of the statements of the language.
I wrote "one possible", because from the Principle of Tolerance - introduced in Syntax - it follows that there can be more possible languages and also more possible universal languages of science. Carnap himself states also that more languages could be suitable as the universal language and the decision about them is a practical choice. Moreover, if we have a universal language, it may be practical to use different language for the different sciences, especially if we know their equivalency.

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[23] Similarly Greg Retall faces the question of language identity in: „Carnap's Tolerance, Language Change and Logical Pluralism", but concerning the choice of logical axioms. It is apparent that this question is rised later by Quine, and there is still no agreement about it.
[24] Thomas Ricketts: „Carnap, From Logical Syntax to Semantics", in „Origins of Logical Empiricism", ed. R. Giere, A.W. Richardson, p. 238, University of Minnesota Press, 1996..

L-Language II is quite close to the concept of language identity in everyday life. Since we do not say that somebody changes his/her language just because he/she has a new experience or just because his/her opinion changed about the truth of a theoretical statement. We can also understand somebody who makes statement, with "the same logic" as us, even if we regard that statement to be false. However we have to ask for an explanation, if somebody changes "the logic" of his speaking. I think, the set of L-rules is quite close to the meaning of "logic" in the former sentences.
There is also a difference however: in everyday language we have also some basics synthetic knowledge included in the meaning of words and through this in the rules of the language. In our universal language we can not afford this, since it would be a risk.
This harmonizes with some of the statements of Carnap as well, for example with the description of page 921 in the Schilpp-volume.

## 4. Deductive systems in the construction of Syntax

The incompleteness theorems apply to deductive systems. Now, a language is not just a deductive system. A deductive system is embedded in a language, for example the first theorem of Gödel is about statements expressible in the language, but not resoluble in the deductive system. Popper also writes about expresibility in the language. However it is not clear what the relation of the language and the deductive system is. So, the first question is, what is the deductive system Popper's critique could be related to?
In Syntax Carnap borrowed the idea from Tarsky to distinguish formally the object and the metalanguage. The intended universal language of science is the object language II. The metalanguage is some other kind of mathematical language, which is defined in plain English (German). Later Carnap shows that the metalanguage can be described by the object language with the arithmethization method.
First, Popper could have referred to the system defining the metalanguage. If there are irresoluble propositions in this system, then they are in the metalanguage, and they are about the object language, so they are not a part of empirical science. Carnap intended the object language to be the language of science, so this case is not relevant to our question.
For empirical sciences an irresoluble proposition may only make trouble if it is in the object language. So we have to concern the deductive system in the object language. These are the axioms which define the language and eventually other theoretical statements of science.
The second problem is that Carnap uses two types of deductions which he calls derivation and consequence. I will use these names in this paper. The theorems of Gödel apply to the former, and not for the latter. Gödel himself tried to find a foundation for mathematics with non finitist methods, which is similar to Carnap's notion of "consequence"[25]. However this direction seems also to result in an infinite hierarchy of systems. I do not want to go to details in this direction, so let us consider from this point only "derivation".

## 5. The character of irresoluble propositions

Let us consider a Gödelian irresoluble proposition in the derivative system in the object language, Let us denote it by γ. For the further examination we should distinguish between two cases, namely when γ is analytic and when it is synthetic. The two concepts are defined in language II, and consist a disjunct and complete partition of possible Gödelian irresoluble propositions (Inconsistent statements can not be irresoluble.). The famous critique of Quine[26] is not relevant for our case here.
If γ is analytic, then it does not concern empirical sciences. The statement constructed by Gödel was analytic, and the corresponding statement in language II is also analytic (see Syntax p. 133). We know the consequences for this case: every mathematical axiomatic system, which is equivalent to or stronger than the Peano-axiom system, is inconsistent or incomplete. This means that the axiom system of language II can be extended by γ or its opposite. So language II can be extended by an infinite structure of mathematical axiom system. Carnap admits this:

> "In other words, everything mathematical can be formalized, but mathematics can not be exhausted by one system; it requires an infinite series of ever richer languages." (Syntax, p.222)

But this does not concern empirical sciences, because we do not necessarily need these extensions for empirical sciences. So we may extend the system, but it is not necessary. Moreover, the infinite structure of mathematical axiomatic systems does not need to be all included in the syntactical rules. It is enough to include a minimum of

[25] S. Feferman (1993).
[26] Quine accepts, that analyic ans synthetic is formally well defined in Language II. W.O.V. Quine: "Two Dogmas of Empiricism", Philosophical Review, 60, 20-43, 1951.

them. For example, classical logic may be the basis of the syntactical rules and we may include as an instrument a modal logic defined in the language, but not as a part of its syntactical rules.
The situation is different if γ is synthetic. This seems to be a more serious problem, but only for the first sight. What do we have here? We have a synthetic statement, which is not derivable or refutable from our theory. This is quite like a normal situation of scientific research. It often occurs that an experience cannot be explained in a theory by derivation from the theoretical statements. We may include γ or its opposite in our theory and try to test it. What happens after this is a methodological question, for which Popper had his answer, so it should not be a problem for him.
Carnap writes about his current methodological view on p. 317 of Syntax. This is similar to Popper's methodology, but we also know their differences in this question. Nonetheless this is irrelevant for our question and out of scope of my current paper. We can conclude that we may have changed our theories in some cases, but this does not mean to change our language.

## 6. Do irresoluble Gödelian empirical statements exist?

As I mentioned, Gödel could prove irresubility for mathematical, i.e. analytic statements. There is no consensus whether there exists a synthetic irresoluble statement. There are some papers on this, but it seems there is not a clear conclusion. One early review is the paper of Wolfram[27], another similar and later paper is that of Svozil titled „Undecidability everywhere?" [28]
The paper uses the achievements of Church and Turing which are closely related to the Gödel theorems and deal with the halting problem of Turing machines. Turing machines are the abstraction of computers, so their physical interpretation is obvious.
According to Turing's thesis there cannot exist a universal algorithm which could decide for every algorithm if it halts in finite time. In my opinion this is not a good example.
First, the difference between a real computer and a Turing machine is that a real computer has finite memory, while the Turing machine has an infinite one. Turing's thesis applies only for the abstract case. For the real case we also know that a computer would not run for infinite time, since we can not supply it with power for ever, and it would break down after a while.
Second, it is not stated that for a particular algorithm and machine we could not decide if it halts in finite time, only that there is no universal algorithm for this[29]. In particular every halting problem can be resoluble.
Third, if for a particular algorithm we could prove that its halting problem is irresoluble (which is not the case), this would not mean a problem for the empirical sciences.
Because if we have such a theoretically irresoluble question, we can make experiments and create a hypothesis about it. It would not be certain, just as any other hypothesis in science. We cannot prove any universal statement in empirical sciences, we can only test them. Popper was who pointed this out, so this should not be a problem for him.
We can measure and derive the state of the computer at any given time, so we have a complete physical description of it. The halting problem does not mean an empirical problem. All these objections aply also to Wolfram's example about dynamical systems, which are in a certain way equivalent to the halting problem.
I also refer to Svozil who states on p. 5. that Gödel did not believe in the physical application of his theorems.
Another group of research in this topic seems to have had a center in the Santa Fe Institute in the middle 90's. There was even a teleconference titled: "The Limits to Scentific Knowledge", and a report about it titled "On Limits"[30]. A lot of problems, limitating factors were discussed on this conference, which I cannot address here. Several scholars agreed that the theorems of Gödel do not imply a limit of empirical science, or it is at least not proven to imply. F.A. Doria remarks in the comment titled "" that a stronger concept of proof or analog computation, may help to resolve incompleteness. Also a more finitary system could help: in Gödel's famous Dialectica paper from 1958, he proves a finitary version of arithmetic to be consistent. John L. Casti remarks that: "But for the most part these limits are not the kind envisioned by Gödel…", E. Attlee Jackson agrees that finiteness is a relevant difference between mathematics and physics of reality. Attle Jackson rounds up next year these thoughts in a paper titled: "No provable Limits to Scientific Knowledge"[31].

[27] S. Wolfram: „Undecidability and Intractability in Theoretical Physics", Physical Review Letters 54, 735-738, 1985.
[28] K. Svozil: „Undecidability everywhere?", in „Boundaries and Barriers. On the Limits to Scientific Knowledge", ed. by J. L. Casti and A. Karlquist (Addison-Wesley, Reading, MA, 1996), pp. 215-237.
[29] Note that it is not the same to state that "there exist a single algorithm to decide for every algorithm the halting problem", and " for every algorithm there exist a proof to decide the halting problem".
[30] J.L. Casti, J.F. Traub (ed.): "On Limits", Technical Report# 94-10-056, Santa Fe Institute, 1994.
[31] E. Atlee Jackson: "No Provable Limits to 'Scientific Knowledge'", Complexity, Vol. 1, No. 2, 14-17, 1995.

I also found some additional paper of Joseph F. Traub on the topic. The most relevant seems to be "The Unknown And The Unknowable"[32], which has a strong focus on Gödel's theorem. Traub writes:

> "We cannot prove scientific unknowability. That can only be done in mathematics. This is sometimes not understood, even by professionals. I expressed my interest in the unknowable to a very senior European scientist. He immediately responded that this had been, of course, settled by Gödel's theorem. Not so; Godel's theorem limits the power of mathematics and does not establish that certain scientific questions are unanswerable." (ibid. p.3.)

## 7. Proof of consistency

The other argument of Popper was related to the impossibility of proof of inconsistency:

> "It would have been best, therefore, to scrap forthwith this doctrine of the one universal language of the one universal science (especially in view of Gödel's second theorem which showed that it was pointless to try to discuss the consistency of a language in that language itself)." (Conjectures, p. 362)

It could be puzzling, that in §34i of Syntax Carnap proves the consistency of language II. This is however not done in the language itself, but in the metalanguage. On p. 134 of Syntax Carnap emphasizes this, and refers to the same theorem of Gödel as Popper does. So, the consistency is indeed not derivable in an absolute manner, since the proof in the metalanguage needs the consistency of the metalanguage, which is a stronger language, therefore the Gödel theorem applies again, so this results in an infinite regress.
In mathematics there are similar theorems about consistency. They prove the consistency of one axiom-system based on another. They are important, but they are not absolute proofs.
Only the consistency of a few axiom-systems is proved in an absolute manner, these are of course weaker than the Peano axiom-system[33].
Popper however commits the mistake of mixing up that for the usability of a language the consistency is needed, but not the proof of the consistency. Language II may be consistent, while this is not provable in the language itself. We may accept the proof from the metalanguage, or just accept it until it is not refuted. This is again according to the scheme of Popper, so he should not have problem with it.
The mistake in Popper's argument is the same as in one of Gödel's which was not published, only reconstructed in „How Carnap could Have Replied to Gödel"[34] by Awodey and Carus. My argument is similar than theirs for a similar case.
There is a similar argument as Poppers in Seldin's paper[35], but ironically directed against Popper. In my opinion this argument is also false for similar reasons, but I can not give a detailed explanation here.

## 8. Carnap's concept of language

It seems that if we define it wide enough, a language could be suitable as a universal language of empirical sciences despite of the theorems of Gödel. In the following I try to examine the views of Carnap from and after Syntax about language reconstruct what Carnap's concept about language was, and examine if this is compatible with my result described just above.
Concerning Carnap's concept of language I already quoted C1, a part of Syntax where Carnap states that the „range" of the definition of the language is not decidable by logical analysis, it is a pragmatic choice. In the later works we unfortunately only have a few remarks on this question. For example in „Meaning and Necessity"

> "The acceptance or rejection of abstract linguistic forms, just as the acceptance or rejection of any other linguistic forms in any branch of science, will finally be decided by their efficiency as instruments, the ratio of the results achieved to the amount of

[32] J. F. Traub: "The Unknown And The Unknowable", Technical Report# CUCS-014-97, Santa Fe Institute, 1997.
[33] For example Gödel prooved in his dissertation the consistency of first order predicatum calculus inside the system. Kurt Gödel: „Über die Vollständigkeit der Axiome des Logikkalkuls", Dissertation, Wien, 1929.
[34] S. Awodey, A.W. Carus: „How Carnap could Have Replied to Gödel", in S. Adowey, C. Klein (eds.).: „Canrap Brought Home", Open Court, 2004, page 207.
[35] See J. P. Seldin: "Gödel, Kuhn, Popper, and Feyerabend", „Karl Popper 2002", Vienna 3-7 July 2002.

> complexity of the efforts required. (Meaning and Necessity, p. 221)[36]

This is also a tolerant view consistent with my previous results. On the other hand when Reisch[37] compares the linguistical frameworks of Carnap to the paradigms of Kuhn, he considers the former to be something similar to a paradigm. So this would mean that scientific revolutions would involve language change. He quotes Carnap from „Philosophy and Logical Syntax"[38]:

> "… a change in language, and a mere change in or addition of, a truth-value ascribed to an indeterminate statement. … A change of the first kind constitutes a radical alteration, sometimes a revolution, and it only occurs at certain historically decessive points in the development of science. … A change of the first kind constitutes, stricktly speaking, a transition from a language $L_n$ to a new language $L_{n+1}$.(Philosophy and Logical Syntax).

Irzik[39] has the same view. Now, if scientific revolution would mean language change, then this is inconsistent with the possibility of one universal language if we presuppose that there will always be scientific revolutions. Carnap would probably agree with this presupposition. So the quotation of „Philosophy and Logical Syntax" can only be consistent with our view and with Carnap's view in Syntax if there was a change in between in the concept of „language". It seems therefore that the concept of „language" is used by Carnap in at least two different sense.

We have to assume that when Carnap was writing about methodological issues, he used a narrower concept of „language", and a wider one when he wrote about a unified language of science. On page 880 of the Schilpp-volume he calls the narrower „fixed language". The universal language on the other hand is called "syntactic framework" on page 151 of Syntax. I will propose to distinguish these two concepts, and I will use the term "syntactical framework" (this is quite the same as an L-Language) and "fixed language" in the following.

These two concepts are in harmony with quote C1 where Carnap states that the range of the definition of language is a pragmatic choice. It is confusing that Carnap changes the content of the "language" concept without an explicit indication.

If we replace the word "language" with the other expressions, we get a harmonic of Carnap's ideas. In the development of science the big theoretical changes cause - because of holism - a change of meaning of words, which is already a linguistic phenomenon. This can be considered as the change of the "fixed language". On the other hand any theory of global science can be expressed in a common syntactic framework, in which the rules of logical inference are the same, and the syntax of statements is the same. This constitutes a unity of science.

In conclusion we can say about the "language" concept - especially in the case of formal ones Carnap was implementing - that there is a gradual scale. According to the topic we may use a narrower or a wider concept, whichever is fruitful for our purposes. It is better to make this explicit or not to speak about language at all, only about the syntactical rules.

The situation is more difficult, since for example in "An introduction to the Philosophy of Science"[40], when Carnap discusses special relativity, he writes about a Euclidean and a non-Euclidean language, which are equivalent (ibid. p. 153). This means that in Carnap's view the revolution of Einstein was a change in the language (in the narrower sense), but it was a change to an equivalent language, with a new theory, which could be described in the old language as well. This means that the change of the particular language happened, but was not necessary in Carnap's opinion. It was a practically motivated change.

There is one more question, which comes to my mind here. On page 151 of Syntax Carnap connects physicalism with language II of syntax. This is a hint that Carnap may consider physical language to be the same as language-II. The physical language probably includes also P-rules, so that some basic physical knowledge becomes a set of syntactic rules of speaking. For example it has no meaning at all to speak about physical bodies without position in space-time. Carnap made a big deal about showing that the entire science can be translated to physical language. On the other hand it is obvious that this language is not necessarily a possible unified language of science forever, since the physical assumptions may be rejected. It is possible that they are true, or

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[36] Rudolf Carnap: „Empiricism Semantics and Ontology", in „Meaning and Neccesity: A study in Semantics and Modal Logic", The University of Chicago Press, Midway Reprint, second edition, 1988.

[37] G. Reisch: „Did Kuhn Kill Logical Empiricism?", Philosophy of Science, 58, 264-277, 1991.

[38] Carnap: „Philosophy and Logical Syntax", Thoemmes Press, 1996 (first edition 1935)

[39] For example Gürol Irzik: „Changing conceptions of Rationality from Logical Empiricism to Potspositivism", in Logical Empiricism, University of Pittsburg Press, 325-348, 2003.

[40] R. Carnap, M. Gardner (ed.): "An introduction to the Philosophy of Science", Dover Publications, 1995 (first edition 1966).

we will always know them as true, in which case physicalism could be a universal language forever. We do not know about this.
Physicalism is a contingent, i.e. uncertain, true thesis of science as we know it today, but the statement that the syntactic framework (L-Language II) is a universal framework for science is not depending on this uncertainty.

## 9. Conclusion

It seems that although the theorems of Gödel have a great significance in mathematics, in the empirical sciences their consequence does not have such a big effect, and in particular they do not refute the possibility of the universal language.
First, because Carnap uses also consequence, not just derivation, and Gödel's theorem holds for derivation only. Second, because analytical irresoluble statements are not a part of empirical sciences. Third, because empirical irresoluble statements do not pose a problem for empirical science, since we can handle them empirically. And fourth, because it is not sure if there exist an empirical irresoluble statement at all. The syntactic framework of Carnap without P-rules (L-Language II) is a possible universal language of unified sciences.
So I think, the idea of Popper was a good question, but not a right critique. On the other hand both Popper and Carnap missed to explain their views in more detail and to make a thorough analysis on this question. I tried to reconstruct a possible answer for Carnap to Popper in this paper. I can not claim that Carnap would have replied in this way. In fact he did not, but let us assume that he would have spent more time for the answer. My answer is the right answer in my opinion and it is more or less consistent with Carnap's view.
I also analyzed some questions about language. Carnap seems to have used at least two concepts of "language": in the context of universal language a wider concept, which means a syntactical framework, on the other hand in some methodological questions a narrower one. This way of use is consistent with the principle of tolerance, which he also applied to the definition of language itself. However, it would be less confusing if Carnap had made it explicit that these concepts are different.
I noted that the wider and narrower concepts of language constitute a gradual scale, and that it may be practical to define a language at any point of this scale.
In my opinion when speaking about physicalism, there was an aspect in Carnap's view, which he shared with Neurath and this is some kind of anti-dualistic view of science. The physical language is probably a narrower language and not the same as the pure syntactic framework. It may be a universal language of science forever, but this is not certain. The syntactical framework on the other hand is a possible universal language of science for ever, or at least Gödel's theorems do not refute this.
It is an interesting question, why Carnap did not reply to Popper's critique in more detail. If not in the Schilpp-volume then in a separate paper. Or why did philosophers since then pay so little attention to this question. Especially considering that the theorems of Gödel are so popular in philosophy.
Carnap probably did not regard it to be very important if science can have one constant unified language. He liked to build big construction, like the one in Syntax and Aufbau as instruments. The possibility and usability was important, but to stick to one construction forever was not his style and therefore not so important a question for him. According to the Principle of Tolerance he would propose and support to create many languages anyway, and use more of them in different fields as practical. Here translatability is an important issue. Maybe that is why the one and constant language was not so crucial for him.
From the view of the chapter of "Conjectures" it is also an open question if the possible universal language, the syntactic framework is suitable for demarcating metaphysic. I guess it is only suitable together with some methodological framework.

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