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**Pictures of Sounds:
Wittgenstein on Gramophone Records
and the Logic of Depiction**

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The year that Ludwig Wittgenstein was born in Vienna, 1889, nearby developments already underway portended two major changes of the coming century: the advent of controlled heavier-than-air flight and the mass production of musical sound recordings. Before they brought about major social changes, though, these innovations appeared in Europe in the form of children's toys. Both a rubber-band-powered model helicopter-like toy employing an ingenious solution to the problem of control, and a working toy gramophone with which music could be reproduced from hard discs, appeared in Europe in time for Ludwig's childhood. And, both reappear in his work as an adult. The relationship between the advent of flight and Wittgenstein's claim in Tractatus Logico-Philosophicus that a proposition is a picture, is a topic in its own right which has been treated elsewhere ¹ In this essay, I consider the way Wittgenstein employed the development of sound recordings in discussing logical form in the Tractatus. ²

The advent of the mass production of musical sound recordings was reflected in an event that took place the year Ludwig Wittgenstein was born, in the city in which he was born. On December 17th, 1889, Brahms recorded himself playing the piano. The recording was made on a wax cylinder in the apartment of his friend Dr. Fellingner in Vienna. Though extremely fragile, the recording has been preserved -- it was transferred from a wax cylinder to a gramophone disc in 1935 and is now available as an MP3 file on the internet. ³ That such care has been taken to preserve it reflects that it was regarded as something precious, both then and now.

The immediate family into which Ludwig Wittgenstein was born probably gave him an especially privileged vantage point from which to view these developments, for his father was Karl Wittgenstein, the immensely wealthy industrial magnate of the European steel and rail industry. Technological innovations would have been of interest in such a household. But so, too, were literature, art and music --- especially music. Leopoldine Wittgenstein, Ludwig's mother, was a pianist, and the Wittgenstein home in Vienna contained several grand pianos. One of Wittgenstein's biographers wrote of Leopoldine, or Poldi, that "Music was her chief means of real contact with her husband and children --- music and perhaps the stories that she told to children so beautifully" and that she "could play from sight the most elaborate pieces of music."⁴ At the time Ludwig Wittgenstein was born, listening to music meant hearing live performances, often at private gatherings, and the extravagant Wittgenstein home at 16 Allegasse in Vienna was the venue of many such "musical evenings". Johannes Brahms was a frequent guest in the household, as were Clara Schumann, Richard Strauss, Gustav Mahler, and many other composers.⁵

The possibility of having some sort of physical record of a live musical performance other than a musical score⁶ generated much interest in its own right, but the innovation of gramophone discs also introduced a possibility of great significance: a practical means of hearing music other than being present at a live performance. Up until then, the only way to distribute a musical composition other than by producing a live performance was by sheet music (i.e., the musical score), and the publication of sheet music was in fact a lively business. One visitor to the Wittgenstein household reported that "From time to time superb autograph manuscripts of the Viennese musical classics were to be seen lying around open as one wandered about . . ."⁷ The publication of sheet music of new compositions was greeted with great

enthusiasm and interest, somewhat as new releases of musical compact discs are today.

The fact that some kind of sound recording that could be played back at least once was possible in principle had been proved well before 1889, but in practice the recordings were fragile -- both the number of recordings of a single performance that could be produced, and the number of times each recording of it could be played back were, until just before Ludwig's birth, very limited. In addition, the quality of the recording was less than exact reproduction; in the early technologies, the reproduced sound was distorted and allowed only recognition of what was being said, not of who was speaking.

It was Emile Berliner's technology of hard gramophone discs that eventually beat out Thomas Edison's use of cylinder recordings in his phonograph, and in fact gramophones eventually came to be called phonographs in the U.S. Berliner eventually developed a method whereby the quality of the reproduced sound was so good that he described it as an "exact reproduction", and with which an unlimited number of discs of a single performance could be produced. Berliner was a German who had emigrated to the U.S. in 1870 at the age of nineteen. In 1888, ten days after he had invented the improved gramophone (but had not yet settled on rubber discs), he demonstrated it at a meeting of the Franklin Institute in Philadelphia, Pennsylvania, and remarked on the excitement of hearing recordings of voices of people from whom we are separated by time or distance. He closed his presentation of the improved gramophone with speculations about its practical applications: ". . . whole evenings will be spent at home going through a long list of interesting performances. Who will deny the beneficial influence which civilization will experience when the voices of dear

relatives and friends long ago departed, the utterances of the great men and women who lived centuries before, the radiant songs of Patti, Campanini, Nieman and others . . . can be heard and re-heard in every well-furnished parlor?"⁸ Although he lived in, loved, and developed his invention in America, in 1889 he travelled back to Germany to present his improved gramophone to the Electro-Technical Society of Berlin, at their invitation. While in Germany, he also arranged to have some single-sided gramophone discs produced there in late 1889, but sound quality was still an issue. A German toy manufacturer showed interest in the device, however, and the next year, in July 1890, the firm began to market a toy gramophone cranked by hand that was capable of reproducing music from hard 12.5 centimeter discs. In addition, they produced a "talking" doll that used a smaller, 8 centimeter disc. For a short period, these were also imported to England. Berliner returned to the United States the next year to further develop his invention and set up companies to produce it.

Thus, in 1889, the year Ludwig Wittgenstein was born, the first mass produced gramophone discs in the world were produced in Germany, and the next year, a working hand-cranked toy gramophone was sold in Germany. The advent of accurate, durable, mass produced sound recordings of musical performances should have been especially striking in the household into which Ludwig was born, since music played such a prominent role there. It is reported that, as an adult, Wittgenstein "when listening to music on the gramophone put the needle back repeatedly to some musical transition from which he wanted to extract everything."⁹ How far back this practice harked is hard to say, but it is true that it would have been during his early childhood that the gramophone first became available, and it is hard to imagine that the advent of the gramophone, which enabled anyone to conjure up great musical performances this way would not have been of great interest. McGuinness writes of

the attitude towards music in the household in which Ludwig grew up: “ All the emphasis was on the expression of the musical idea and it was this that was discussed with a minimum of technical terms and in the vocabulary of cultivated and perceptive participants in the long Allegasse analyses that followed each Vienna Philharmonic Concert.” ¹⁰

The invention was conceptually interesting as well as having a major practical impact, for, now, there was a way to represent a particular musical performance: by the little grooves in a rubber disc, from which sound could be reproduced by the motion of a needle. The existence of alternative durable representations of a musical composition -- i.e., the existence of both a written score consisting of marks on paper and an analogue gramophone record consisting of grooves in a rubber gramophone disc --- reappeared three decades later after the infant Wittgenstein present at the birth of the use of gramophone discs in Europe had grown into a young man concerned with solving problems in logic regarding the question of how a picture or model can depict something else. In particular, in considering the relationship between “the proposition -- one set out on the printed page, for example --” and “ the reality with which it is concerned”, he remarks that, although the proposition does not at first appear to be a picture of that reality, “neither do written notes seem at first sight to be a picture of a piece of music” ¹¹ Then, he reflected upon the relationship between four different things: a musical idea, the musical score, the sound waves made during a symphony performance, and a gramophone record of the symphony performance.

But gramophone record technology was not the first well-known example of sound phenomena being recorded graphically: Ernst Mach’s “schlieren” photographs of shock waves, by which invisible shock waves were ‘made visible’, were presented in a

lecture at the University of Vienna in 1887¹², had been featured in popular magazines and other media, and astounded general audiences as well as scientists. Besides being visually striking, the photographs were valuable in laboratory investigations. Regions of these photographs were darker or lighter, corresponding to geometrically similar regions of denser or lighter regions of air, so these photographs were what we would quite naturally regard as pictorial representations in the usual sense of 'pictorial'. They helped people visualize what was going on in the air when a bullet or other projectile shot through air faster than the speed of sound and produced the aural phenomenon of a sonic boom. In addition to the geometrical similarity between the photograph and the patterns of air density, there is also the fact that there is a causal connection between the shock waves in the air and the photograph. The photographs are quite directly caused by the phenomena they depict: Mach used an ingenious method of flash lighting to produce shadows of density patterns in the air.

Yet, Wittgenstein does not pick up on the knowledge of these well-known, beautiful, precise, striking photographs of otherwise invisible phenomena and use them to illustrate his points about the logic of depiction. Rather, he focuses on pairs of seemingly dissimilar representations: "At first glance the proposition -- one set out on the printed page, for example -- does not seem to be a picture of the reality with which it is concerned. But neither do written notes seem at first sight to be a picture of a piece of music, nor our phonetic notation (the alphabet) to be a picture of our speech."

¹³ Once we understand what logical form and internal similarity amount to for Wittgenstein, though, we see that it is in fact with good reason that he does not use examples in which similarity is based on visual or geometrical similarity.

Putting his view in a pithy aphorism, he writes: “A gramophone record, the musical idea, the written notes, and the sound-waves, all stand to one another in the same internal relation of depicting that holds between language and the world.” TLP 4.014
His answer: “They are all constructed according to a common logical pattern.”

But what logical structure amounts to is somewhat unexpected; he does not appeal directly to any features of any of these representations; the internal similarity is not established by referring to how the representations are actually constructed, how they can be decomposed, to the things of which they are composed, or to any structural or formal features. Rather, he appeals to the existence of rules by which one of them can be obtained from another:

There is a general rule by means of which the musician can obtain the symphony from the score, and which makes it possible to derive the symphony from the groove on the gramophone record, and using the first rule, to derive the score again. That is what constitutes the inner similarity between these things which seem to be constructed in such entirely different ways. (TLP 4.0141)

There is yet another kind of representation associated with the invention of the gramophone that was actually the springing-off point for the development of the gramophone record. It is a device for creating visible traces of sound patented in 1857 and called a phonautograph. Emile Berliner begins accounts of his own invention, the gramophone, by describing it. Phonautographs, or phonautograph records, produced by a machine also called a phonautograph seem to have been well-known at the time, for Berliner speaks of “Scott’s phonautograph” as if assuming audience familiarity with it, and another paper on the principles of the gramophone by a

Professor Houston refers to it as “the well-known phonautograph of Leon Scott.”¹⁴ Scott’s story was poignant: his family’s financial situation precluded them providing him an advanced education, and he was apprenticed to a printer as a boy. However, his work involved overseeing the printing of transactions of scientific societies, which he read in the course of copyediting. He came to know some of the scientists whose work he printed, and he corresponded with them -- about their work. These pursuits led to Scott inventing a machine that would produce a visual record of sound; these graphical objects could be printed. Scott’s illustration of his invention shows a person performing on a musical instrument in front of the machine, and the machine, built roughly on the model of the human eardrum, producing a series of wavy lines distinctive of the performance. The sound record is caused by the motion of a membrane whose motions are in turn similar to and caused by the sound waves produced by the musical instrument. The sound records were white wavy lines scratched in a black background formed by a smoky film on paper, but the point was that they were distinctive marks corresponding to sound waves, and that, like any other two-dimensional logo, they could be reproduced without limit by a printing process. The point was to have a method of recording sound, somewhat like present-day seismographs record waves travelling along the earth’s surface.

Some put special significance on the production of wavy lines that were geometrically similar to the sound waves that produced them. Berliner does seem to retain some of the attention to the visual aspect of a recording occasionally, as when he remarked of a hard zinc disc made during the process that it “becomes a picture of sound waves which, though slumbering in a bed of hard metal, is ready at any time, even centuries hence, to burst forth into the soft cadenzas of word and song, the ripple of laughter, the strains of martial music, as well as the melancholy and imploring drag of the organ-

grinder's tuneful melody."¹⁵ However, geometrical similarity to the actual sound waves was not essential to the goal of producing some sort of graphical or iconic representation of sound, so I think this reference to the visual aspects of the gramophone lines just reflects the general awareness of the visually distinctive lines that were produced by the processes used in making gramophone records. In March of 1857, Scott had been granted a patent for "a method of drawing or writing by sound, and for multiplying the result of this graphically with a view to industrial applications."¹⁶ The same kind of device was also called a logograph. Scott did not attempt to use the graphical representations to actually produce sound, but both Edison and Berliner saw the potential of such a complementary process, which led to the development of Edison's phonograph and Berliner's (far superior) gramophone.

Thus, for awhile, there were phonautographs, or visual records, of sound, that were valued and produced only as graphical representations, and these would have been well known before and during Ludwig's childhood. Emile Berliner remarks that Scott's phonautograph "is described in every book on physical science"¹⁷, and in fact, Berliner talks about using printed phonautographs as a means of conveying the gramophone sound recording easily via printed means, the recipient with the right kind of engraving equipment then being able to use the printed graphical wavy lines to produce an engraved disc which could then be played back on a gramophone player. The ability to produce sound from phonautographs, while still regarding them as visual representations, is illustrated in a particularly colorful way in Berliner's fanciful speculation that "We may then have dinner-sets, the dessert-plates of which have gramophone records pressed in them, and which furnish the after-dinner entertainment when the repast is over." which is immediately followed by the speculation that "Gramophone plaques with the voices of eminent people will adorn

our parlors and libraries.” Likewise, the dual aspect of a gramophone disc -- as like the written word in some ways and yet as able to be employed in mechanically reproducing the sound represented by it --- is evoked in Berliner’s statement that “I am carrying on a vocal correspondence with my friends in Europe, by means of small gramophone discs, which can be mailed in a good-sized letter envelope. . . . I could cite a number of instances where persons have been made happy by hearing and recognizing the voices of loved ones whom they had not seen in years, and the owners of which were thousands of miles away.”¹⁸

It is notable that, in his discussion in the Tractatus about the gramophone record, Wittgenstein does not include the kind of graphical record that a phonautograph is among the group of things that have the same logical structure as the musical score -- even though Scott was aiming especially at providing a graphical representation of sounds. That is, Wittgenstein does not mention these records as graphical objects; he mentions them only insofar as there is a process for generating sounds from them. It does not appear that these visual records were on a par with musical notation as far as musicians being able to “read” them as they were able to read a musical score. It makes sense, then, that Wittgenstein does not include Scott’s phonautograph, given his explanation there of what logical structure consists in, since there was no way to produce the musical score from a phonautograph unless there were some kind of playback mechanism. Of course by the time gramophones were in existence, he would have been aware that it was always theoretically possible to develop a machine to play back a phonautograph record, but the existence of a playback mechanism would essentially make the phonautograph record a gramophone record, which of course is what he does use to illustrate his points about logical structure and pictorial representation. It is striking that the crucial aspect that Wittgenstein cites as

accounting for internal similarity (and, hence, for common logical structure) in the philosophical treatise he writes as a young man is precisely the advance in sound recordings that was exhibited by the toy gramophone that premiered in nearby Germany just after his birth: “there is a rule by which one could reconstruct the symphony from the line on a gramophone record” (TLP 4.0141)

The key notion is translation, not interpretation in the sense used in formal logic or formalist approaches in mathematics. Translation is from a meaningful entity in one language to a meaningful entity in another language, whereas interpretation of an entity in a formal language is a matter of producing a meaningful entity in some language from an entity that needs completion or supplementation in order to have a meaning. In this Wittgenstein was following Frege, for Frege was against formalist approaches to mathematics and did not approve of what would now be called uninterpreted languages. Frege’s own recasting of Hilbert’s famous independence proofs were carried out in terms of mapping fully interpreted sentences to fully interpreted sentences.¹⁹ Wittgenstein’s statement about rules occurs in the section discussing internal similarity quoted earlier, TLP 4.0141.

Revisiting that passage, we see that it is the musician’s competence in translating a musical score into a symphony, and the fact that there is a process by which the symphony can be produced from a gramophone record, that makes it possible to translate from the language of the musical score into the language of the gramophone record. These rules are what provides a link between the two things that at first do not seem similar: a musical score (written notes) and grooves or lines on a gramophone record. In a passage just previous to the remark that at first a proposition as set out on the page does not seem to be a picture, Wittgenstein employed the notion of a model

as well as the notion of a picture, saying both that a proposition was a picture of reality, and that a proposition was a model of reality.²⁰ In this passage about the internal similarity of the lines on the gramophone record and the musical score, the role of these things as models is crucial, but the important upshot is the intertranslatability that can be obtained due to their being models of a symphony.

There is an asymmetry involved in the intermediate processes here that Wittgenstein does not discuss: whereas it is fairly straightforward to record a symphony performance as a musical score, there is a lot of room for artistic “interpretation” in performing a symphony from a musical score. So when he says that the musician is able to read the symphony out of the score, does he mean that the musician is able to read out of the score the criteria that a symphony must meet in order to count as a symphony of which that is the score? Or, does he mean simply that the musician is able to read out of the score some particular performance or performances that would count as a symphony to which that score corresponds? The same issue of asymmetry between depiction and depicted arose with recorded voices: in early technologies, the frequencies of the speaker’s performance were correct, but the overtones that reflected the unique peculiarities of the speaker and the performance, and allowed a hearer to recognize the speaker, were not recorded faithfully. Thus, some of the early technologies were suitable for use in dictation machines, but not for any use where the speaker’s voice should be recognizable. If Wittgenstein’s argument required that a particular symphony performance be read off the score, there would be more involved than simply inverting the rule of getting a musical score from a symphony. Yet he it seems he does not think it relevant, for he speaks of “a general rule by which the musician can obtain the symphony from the score” and then of using that same rule to construct the score from a (particular) symphony that has itself been reconstructed

from the (particular) lines on a gramophone record. It seems that all that matters to his explanation is that the musical score would, according to a musician, count as a musical scores of the symphony, not that the symphony imagined or performed by the musician and the one produced mechanically by a gramophone be precisely the same in every detail.

It seems to me that the only way to make sense of what he says here is to regard the “sound-waves” of the symphony performance as specified up to a level of description that specifies the notes used, i.e., at the level of description associated with the musical notation. This is interesting, for it has the consequence that there are some aspects of the symphony performance that are not captured in the language of the musical score. It is certainly not true that the lines of the gramophone do not capture further detail about the performance, for one striking thing about the gramophone was that not only could the hearer recognize what was said, but the hearer could recognize the speaker’s voice, and, analogously, the timbre of an instrument. This further detail would be exhibited in visual features of the lines on the gramophone disc. Yet, according to what he says in the Tractatus, such detail would not necessarily count as part of the logical structure of the gramophone lines.

To make sense of what Wittgenstein says, the features of the symphony from the standpoint of logical structure must be only those features of the symphony that are used in applying the rule by which the score is obtained from the symphony. This is in fact consistent with his remark that “that rule [the rule by which the musician is able to obtain the symphony from the score] is the law of projection which projects the symphony into the language of the musical notation.” TLP 4.0141 Thus the similarity of which he speaks is based upon those features of the symphony that can be

projected into that language. Thus only what is relevant to the mode of depiction belongs to the logical structure shared by the depiction and the depicted. This would mean that the peculiarities of an individual performance, unless they are captured in both the musical score and the gramophone lines, are not considered part of the logical structure of the symphony performance. The only logical structure discussed here is the logical structure that the depicted and the depiction have in common. On this account, then, whatever musical notation is in use --- and it would have to be one that permits a musician to “read” the symphony out of a musical score --- dictates what is and is not included in the logical form of the symphony.

What should we make of this consequence of Wittgenstein’s view: that logical structure of the depicted (the symphony) is not something independent of language and absolute, but is relative to the method of depiction (the musical notation used in musical scores) used to depict it ? Is such a relativism palatable? Perhaps neither the central place held by language (he refers to the language of musical notation and the language of the gramophone), nor the intimation that language is limited in some ways, should surprise us. And such a relativism is not as unsettling as it might seem at first, if we recall that , even in formal logic, often a sentence or argument in natural language can be formalized in more than one way in the formal language (though the propositions that are formalized in this way may not be identical), and that the formal language used may determine how the sentence or argument can be formalized. Wittgenstein remarks that from colloquial language “it is not humanly possible to gather immediately from [everyday language] what the logic of language is.” (TLP 4.002) It is one step further, of course, to talk about capturing the true logical form of a sentence or argument, and one could argue that it is not the task of a logic of depiction that it arbitrate between possible alternate depictions of reality. I think that what this

consequence does show is that logical structure, on his view, is rooted in the possibility of translating between two languages -- two languages that may be entirely different, even use different kinds of representations (notes on a musical score versus wavy lines on a gramophone disc), and may even have different expressive capabilities (the gramophone disc can capture more details about a particular symphony performance than the musical notation does).

Appreciating this point about translation helps us make sense of an otherwise puzzling statement Wittgenstein makes that even our phonetic alphabet has not lost its pictorial character. This is a wild claim, if seen in the context of the much-emphasized contrast drawn at the time between iconic symbols (such as hieroglyphs) and non-iconic symbols (such as letters of a phonetic alphabet). Although it may be slightly disingenuous to put the point as he does in saying that the phonetic alphabet has not lost its pictorial character, what Wittgenstein says about pictorial form does in fact support that statement. For, a claim that translation is possible is based on the existence of a skill or competence: although Wittgenstein speaks of “a rule” out of which such translatability ensues, that rule is really a matter of the ability of a trained musician to ‘read the symphony out of the score.’ Anyone who has learned to read will have the analogous skill for the phonetic alphabet, and thus, on his account, the phonetic alphabet and a hieroglyphic script could be said to share the same logical structure. Hence, as he says, ‘alphabetic script developed out of [hieroglyphic script] without losing what was essential to depiction.’ (TLP 4.016)

The emphasis on human competencies is actually indicated at the beginning of that section of the Tractatus; near the beginning of the set of statements whose number begins with “4” he remarks that : “The tacit conventions on which the understanding of

everyday language depends are enormously complicated.” TLP 4.022 The skill aspect is rather deep-seated, extending to the very capability for language itself: ‘Man possesses the capacity of constructing languages, in which every sense can be expressed, without having an idea how and what each word means --- just as one speaks without knowing how the single sounds are produced.’ TLP 4.002

For some people, the advent of the gramophone invited reflection upon the contrast between the analogue representation provided by a gramophone record and the discrete symbol notation of a musical score: a contrast, as some put it, between image and symbol. But, Wittgenstein instead used the occasion of the advent of the gramophone to reflect on the common logical structure of these two kinds of representations: i.e., the lines in the gramophone record and the musical score. The key notion turns out to be the ability to translate between the two, and this is done via intermediate steps involving a symphony --- though it is important to understand that the symphony generated by the gramophone player need not be identical in every detail to the symphony that the musician would produce from the musical score.

The musician’s ability to ‘read the symphony in the score’ is a complex human skill, but Wittgenstein regarded the ability to understand everyday language a complex skill, too, as indicated in the passage in the Tractatus: “The tacit conventions on which the understanding of everyday language depends are enormously complicated” (TLP 4.002) So another point that Wittgenstein’s analogy between musical notation, the phonetic alphabet, and hieroglyphics evokes is his earlier statement that, like visual images and musical ideas, “Everyday language is part of the human organism.” (TLP 4.022)

Endnotes

¹ Susan G. Sterrett, "Physical Pictures: Engineering Models circa 1914 and in Wittgenstein's Tractatus", in Michael Heidelberger and Friedrich Stadler, History of Philosophy of Science: New Trends and Perspectives (Kluwer Academic, February 2002). Earlier version also available on the Philosophy of Science Association's eprint archive at <http://philsci-archive.pitt.edu/archive/00000661/>. A book-length work on the subject entitled Wittgenstein Flies A Kite is in progress, forthcoming from Pi Press in 2005.

² Michael Biggs has also investigated the topic of musical recordings and the theme of depiction in the Tractatus in his paper 'Visualization and Wittgenstein's 'Tractatus'' (2002)

³ As of the time of this writing, the website <http://www.measure.demon.co.uk/sounds/Brahms.html> provides a link from which to download an MP3 version of the recording made by Brahms.

⁴ Brian McGuinness, Ludwig Wittgenstein: A Life (University of California Press, 1988), p. 20.

⁵ Ibid., p. 13

⁶ There was, in addition, the representation of a piano performance on a punched paper roll, which was a method specifically for compositions performed on a piano. A player piano roll from a live piano performance was produced which could then be played back mechanically on a player piano, reproducing the original performance to some extent.

⁷ E. Flindell, *Music Review* 32/1971. Quoted in Michael Nedo, op. cit., p. 13.

⁸ Emile Berliner, "The Gramophone: Etching the Human Voice", Journal of the Franklin Institute of the State of Pennsylvania, Vol. CXXV., No. 6, June 1888, p. 446.

⁹ Brian McGuinness, Wittgenstein: A Life / Young Ludwig (University of California Press, 1988), p. 34.

¹⁰ Ibid., p. 21.

¹¹ Tractatus Logico-Philosophicus, translated by Brian McGuinness and David Pears. (Routledge 1974).

Section 4.011

¹² Ernst Mach "Photographische Fixierung der durch Projektile in der Luft eingeleiteten Vorgänge", presented to the Academy of Sciences in Vienna in 1887

¹³ Tractatus Logico-Philosophicus, translated by Brian McGuinness and David Pears. (Routledge 1974)

¹⁴ Edwin J. Houston, "The Gramophone", in Journal of the Franklin Institute, January 1888.

¹⁵ Emile Berliner, "The Improved Gramophone", paper read at the 52nd meeting of the American Institute of Electrical Engineers, New York, December 16th, 1890, p. 21.

¹⁶ "The History of the Phonautograph" <http://www.phonautograph.com> , version updated January 15, 2004. This site contains many articles about, sketches of and photographs of, the machine, and explains its operation.

¹⁷ Emile Berliner, "The Improved Gramophone", paper read at the 52nd meeting of the American Institute of Electrical Engineers, New York, December 16th, 1890, p. 23.

¹⁸ Emile Berliner, "The Improved Gramophone", paper read at the 52nd meeting of the American Institute of Electrical Engineers, New York, December 16th, 1890, p. 28.

¹⁹ Susan G. Sterrett, 'Frege and Hilbert on the Foundations of Geometry', Talk presented at the University of Pittsburgh Graduate Student Colloquium, Fall 1994. Available on the Philosophy of Science Association e-print Archive at <http://philsci-archive.pitt.edu/archive/00000723/>

²⁰ '4.01 A proposition is a picture of reality. A proposition is a model of reality as we imagine it.' Tractatus Logico-Philosophicus, tr. C. K. Ogden. The same translation is given in B. F. McGuinness and David Pears (Routledge 1974) (The Ogden translation is available online a number of places; one provided by Barnes and Noble Bookstores is: <http://pd.sparknotes.com/philosophy/tractatus>)