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Distant Bottom Reverberation in Shallow Water

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Abstract: The method of coupled mode is introduced for investigation of bi-static distant bottom reverberation of impulsive source in shallow water, which will not contradict with principle of reciprocity in all cases. And the method of multi-pole for directional source is also introduced. It shows that in case of layered medium, intensity of bi-static bottom reverberation will decease according to the cubic power of receiving time *t*, and the transverse spatial correlation of bottom reverberation is a little greater than longitudinal correlation for equal separation of receivers, and both vary in form with the receiving time.

Keywords: shallow water; bi-static bottom reverberation; spatial correlation of reverberation
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1 Introduction

The channel effect of bottom reverberation had been investigated by many authors (Bucker and Morris, 1968; Holland, 2006; Mackenzie, 1962; Zhou and Zhang, 1977), but in most of these researches, bottom reverberation had been described as the sound field formed by distributed secondary sources on boundary, and results obtained in such way sometimes will contradict with principle of reciprocity in bi-static cases (Wang and Shang, 1981). It is desirable to give a method for computation of reverberation, which directly using the scattering effect of stochastic characteristics of water channel, and can give results obeying the principle of reciprocity in any case. In this paper, the method of coupled mode is used for evaluation of bottom reverberation field caused by roughness of bottom interface, and multi-pole method is introduced for consideration of directional source.

2 Description of the problem under consideration

In this paper the bottom reverberation of an almost two layered medium with plane sea surface at z=0, and rough bottom boundary with depth $H = H_0 + \zeta(x,y)$ $(\zeta << H_0)$ had been considered, and ρ_0, c_0 and ρ_1, c_1 are the density and sound speed of upper and lower medium respectively. Assume the source transmits sound impulse described by

$$f(t) = \begin{cases} e^{-i\omega_0 t} & |t| \le \tau \\ 0 & |t| > \tau \end{cases} \quad (1)$$

We can find the required solution from field of harmonic source by means of Fourier transform. Moreover, when wave solution of directional source should be considered, we could find the solution for point source at first, and then by means of differential calculation to get the required solution. For example, if the source is located at $(0,0,z_s)$ with principle axis in x-direction and directional pattern as shown in Fig.1, which can also be described by series of Legendre function as follows:

$$D(\vartheta) = a_0 P_0(\cos\vartheta) + a_1 P_1(\cos\vartheta) + a_2 P_2(\cos\vartheta) + a_3 P_3(\cos\vartheta) \quad (2)$$

where a_0 = − 2.5162, a_1 = 5.7531, a_2 = − 4.7757, a_3 =2.3525.

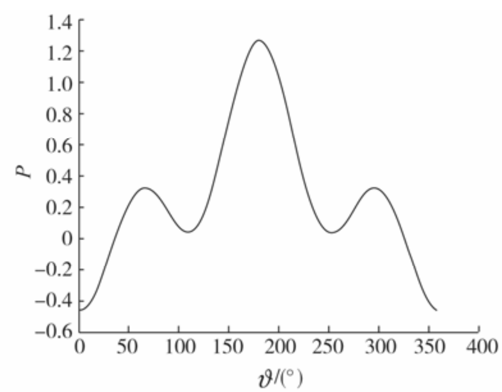

Fig.1The directional pattern of transmitter described by Eq.(2)

As each Legendre function corresponds to directional pattern of sound field for some multi-pole system (Yang, 1994), the required solution for given directional source can be obtained when the following differential operator on point source solution of sound field had been applied:

$$F(\psi) = \{a_0 + a_1 \frac{\partial}{\partial x} + a_2 \left[\frac{\partial^2}{\partial x^2} - \frac{1}{2}\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\right] + a_3 \frac{\partial}{\partial x}\left[\frac{\partial^2}{\partial x^2} - \frac{3}{2}\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\right]\}\psi \quad (3)$$

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3 Formal solution of bottom reverberation for point harmonic source

The potential function of sound field for point harmonic source is written in given case by method of coupled mode:

$$\varphi(\boldsymbol{r}) = \sum_n \psi_n(x,y) Z_n(z,\zeta) \tag{4}$$

Then

$$Z_n(z,\zeta) = \begin{cases} A_n \sin\beta_{0n} z & 0 \le z < H \\ A_n b \sin\beta_{0n} H \mathrm{e}^{\mathrm{i}\beta_{1n}(z-H)} & H < z \end{cases} \tag{5}$$

In Eq.(5)

$$\begin{cases} b = \dfrac{\rho_0}{\rho_1} \\ \beta_{0n} = \sqrt{k_0^2 - \xi_n^2} \\ \beta_{1n} = \sqrt{k_1^2 - \xi_n^2} \end{cases} \tag{6}$$

$$A_n = \frac{2\pi \mathrm{i} \sin\beta_{0n} z_s}{H + \dfrac{k_0^2 - k_1^2}{2\beta_{0n}\beta_{1n}^2}\sin 2\beta_{0n} H} = \frac{2\pi \mathrm{i}\sin\beta_{0n} z_s}{B_n} \tag{7}$$

and ξ_n is the *n*th root of the following dispersion equation.

$$\beta_0 \cos\beta_0 H - \mathrm{i} b \beta_1 \sin\beta_0 H = 0 \tag{8}$$

$$\begin{cases} B_n \xi_n \dfrac{\mathrm{d}\xi_n}{\mathrm{d}H} = \beta_{0n}^2 \\ \dfrac{\mathrm{d}\xi_n}{\mathrm{d}\omega} \approx \dfrac{k_0}{\xi_n c_0} \end{cases} \tag{9}$$

$$\int_0^\infty Z_n^2(z)\mathrm{d}z = \frac{1}{2}A_n^2\left\{B_n + \frac{(b-1)\beta_{0n}}{2\beta_{1n}^2}\sin 2\beta_{0n} H\right\} = \Gamma_n(x,y) \tag{10}$$

In deriving the second formula of Eq.(9), $\dfrac{c_0^2}{c_1^2} \approx 1$ had been used for simplification. When the terms with $\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right) Z_m$ be omitted as small quantity of second order, the differential equation of $\psi_n(x,y)$ can be written approximately as:

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \xi_n^2(x,y)\right]\psi_n \approx -\frac{1}{\Gamma_n}\sum_m \left(\frac{\partial\psi_m}{\partial x}\frac{\partial\zeta}{\partial x} + \frac{\partial\psi_m}{\partial y}\frac{\partial\zeta}{\partial y}\right)\int_0^\infty Z_n \frac{\partial Z_m}{\partial H}\mathrm{d}z \tag{11}$$

Notice that for plane bottom boundary:

$$\begin{cases} \psi_n^{(0)}(x,y) = H_0^{(1)}\left(\xi_n^{(0)} r\right) \\ r = \sqrt{x^2 + y^2} \end{cases} \tag{12}$$

and write for simplicity:

$$G_{mn} = \int_0^\infty Z_n \frac{\partial Z_m}{\partial H}\mathrm{d}z \tag{13}$$

Since the first order perturbation factor in Eq.(11) will correspond to the bottom reverberation caused by rough interface, the horizontal potential function of bottom reverberation for *n*th mode ψ_{rn} can be written approximately as:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \xi_n^2\right)\psi_{rn} \approx -\frac{2\beta_{0n}^2}{B_n} H_0^{(1)}(\xi_n r)\zeta + \sum_m \frac{\xi_m}{\Gamma_n} G_{mn} H_1^{(1)}(\xi_m r)\left(\frac{x}{r}\frac{\partial\zeta}{\partial x} + \frac{y}{r}\frac{\partial\zeta}{\partial y}\right) \tag{14}$$

where all the values of ξ_m are actually be $\xi_m^{(0)}$, and G_{mn} are independent of *r*. For the sake of simplicity, in the following expressions the shoulder index $\xi_m^{(0)}$ will be omitted everywhere, which obviously will not cause any confusion; and the required horizontal potential function ψ_{rn} of bottom reverberation will be the particular solution of above equation, or

$$\psi_{rn}(x,y) \approx \frac{\mathrm{i}}{4}\iint [-\frac{2\beta_{0n}^2}{B_n} H_0^{(1)}(\xi_n r')\zeta + \sum_m \frac{\xi_m}{\Gamma_n} G_{mn} H_1^{(1)}(\xi_m r')\left(\frac{x'}{r'}\frac{\partial\zeta}{\partial x'} + \frac{y'}{r'}\frac{\partial\zeta}{\partial y'}\right)]\times H_0^{(1)}\left(\xi_n\sqrt{(x-x')^2 + (y-y')^2}\right)\mathrm{d}x'\mathrm{d}y' \tag{15}$$

4 Bottom reverberation for impulsive point source

For impulsive source given by Eq.(1), the horizontal potential function Ψ_{rn} of bottom reverberation will be written as:

$$\begin{cases} \Psi_{rn}(x,y) \approx \dfrac{\mathrm{i}\tau}{4\pi}\displaystyle\int_{-\infty}^{\infty}\mathrm{d}\omega\iint [-\frac{2\beta_{0n}^2}{B_n} H_0^{(1)}(\xi_n r')\zeta + \\ \qquad \displaystyle\sum_m \frac{\xi_m}{\Gamma_n} G_{mn} H_1^{(1)}(\xi_m r')\left(\frac{x'}{r'}\frac{\partial\zeta}{\partial x'} + \frac{y'}{r'}\frac{\partial\zeta}{\partial y'}\right)]\times \\ \qquad \dfrac{\sin(\omega-\omega_0)\tau}{(\omega-\omega_0)\tau} H_0^{(1)}(\xi_n R)\mathrm{e}^{-\mathrm{i}\omega t}\mathrm{d}x'\mathrm{d}y' \\ R = \sqrt{(x-x')^2 + (y-y')^2} \end{cases} \tag{16}$$

Using asymptotic expressions of Hankel function, and calculate at first the corresponding Fourier transform by means of the method of stationary phase. Since the stochastic quantities $\zeta,\frac{\partial\zeta}{\partial x},\frac{\partial\zeta}{\partial y}$ are all independent with ω, the first part of integral in Eq.(16) can be written as:

$$I_{1n}\approx\frac{-\tau\zeta}{\pi^2B_n\sqrt{r'R}}\int_{-\infty}^{\infty}\frac{\beta_{0n}^2}{\xi_n}\frac{\sin(\omega-\omega_0)\tau}{(\omega-\omega_0)\tau}\exp\left\{\mathrm{i}\left[\xi_n(r'+R)-\omega t\right]\right\}\mathrm{d}\omega$$

The stationary point is determined by:

$$\begin{cases}\frac{\mathrm{d}\xi_n}{\mathrm{d}\omega}(r_n'+R_n)-(t\pm\tau)\approx\frac{k_0}{\xi_nc_0}(r_n'+R_n)-(t\pm\tau)=0\\ \omega=\omega_s\end{cases}\quad(17)$$

When the frequency band of sound impulse is not too wide, I_{1n} will have finite value only near the point $\omega_s\approx\omega_0$, therefore Eq.(17) expresses the relationship of R_n,r_n' for given values of x, y, t and τ. For the sake of simplicity write:

$$T_{n\pm}=\frac{\xi_nc_0}{k_0}(t\pm\tau)\quad(18)$$

For two receiving points $(r+L,D)$, $(r-L,-D)$ the corresponding scattering position on bottom boundary $(r'\cos\alpha,r'\sin\alpha)$ will be:

$$r_{n\pm}'=\frac{T_{n\pm}^2-(r\pm L)^2-D^2}{2\left[T_{n\pm}-(r\pm L)\cos\alpha\mp D\sin\alpha\right]}\quad(19)$$

$$R_{n\pm}=\frac{T_{n\pm}^2-2T_{n\pm}\left[(r\pm L)\cos\alpha\pm D\sin\alpha\right]+(r\pm L)^2+D^2}{2\left[T_{n\pm}-(r\pm L)\cos\alpha\mp D\sin\alpha\right]}\quad(20)$$

Also if $c_0t>>r$, $\tau<<t$ and τ is sufficiently small:

$$\mathrm{d}r_n'=r_{n+}'-r_{n-}'\approx\frac{2\xi_nc_0\tau R_n}{k_0\left[T_n-(r\pm L)\cos\alpha\mp D\sin\alpha\right]}\quad(21)$$

Eqs.(17) and (21) indicate that the scattering sound, received at certain moment by receiver, are coming from a region having the form of elliptical band with focuses at source and receiver, where the band width is proportional to τ. Using

$$\frac{\mathrm{d}^2\xi_n}{\mathrm{d}\omega^2}(r_n'+R_n)\approx-\frac{\beta_{0n}^2t}{\xi_n^2c_0k_0}$$

Then

$$I_{1n}\approx\frac{-\tau\beta_{0n}}{\pi^{3/2}B_n}\sqrt{\frac{2\omega_s}{t}}\mathrm{e}^{-\mathrm{i}\left[\omega t+\frac{\pi}{4}\right]}\iint_{\Omega}\frac{\zeta(r_n')}{\sqrt{r_n'R_n}}\mathrm{e}^{\mathrm{i}\xi_n(r_n'+R_n)}\mathrm{d}x'\mathrm{d}y'\quad(22)$$

where Ω is the effective region for scattering sound.

Similarly for the second part of integral in Eq.(16), the stationary point for the integration with respect to ω is determined by:

$$\frac{\xi_n}{\xi_m}r_{nm}'+R_{nm}=T_{n\pm}\quad(23)$$

Write $\varepsilon_{nm}=\frac{\xi_n}{\xi_m}$, since it can be proved that: in given case only local modes with neighboring orders will have noticeable value of coupling coefficient, therefore it will be reasonable to take $\varepsilon_{nm}\approx1$, and omit terms with higher power of $(\varepsilon_{nm}-1)$ in future calculation.

$$r_{nm}'\approx\frac{T_{n\pm}^2-\varepsilon_{nm}\left[(r\pm L)^2+D^2\right]-\frac{\varepsilon_{nm}-1}{2}\left(2T_{n\pm}r\cos\alpha-r^2\sin^2\alpha\right)}{(\varepsilon_{nm}+1)\left[T_{n\pm}-\varepsilon_{nm}(r\pm L)\cos\alpha\mp\varepsilon_{nm}D\sin\alpha\right]}\quad(24)$$

$$R_{nm}\approx\frac{T_{n\pm}^2-2T_{n\pm}\left[(\tilde r\pm\tilde L)\cos\alpha\pm\tilde D\sin\alpha\right]+(\tilde r\pm\tilde L)^2+\tilde D^2-\hat\varepsilon_{nm}\tilde r^2\sin^2\alpha}{(\varepsilon_{nm}+1)\left[T_{n\pm}-(\tilde r\pm\tilde L)\cos\alpha\mp\tilde D\sin\alpha\right]}\quad(25)$$

where $\tilde r=\varepsilon_{nm}r$, $\tilde L=\varepsilon_{nm}L$, $\tilde D=\varepsilon_{nm}D$, $\hat\varepsilon_{nm}=\frac{\varepsilon_{nm}-1}{2\varepsilon_{nm}}$.

Also:

$$\mathrm{d}r_{nm}'=r_{nm+}'-r_{nm-}'\approx\frac{2\xi_nc_0\tau R_{nm}}{k_0\left[T_n-(\tilde r\pm\tilde L)\cos\alpha\mp\tilde D\sin\alpha\right]}\quad(26)$$

Using the approximation

$$\begin{cases}\frac{\mathrm{d}^2\xi_m}{\mathrm{d}\omega^2}r_{nm}'+\frac{\mathrm{d}^2\xi_n}{\mathrm{d}\omega^2}R_{nm}\approx\frac{t}{k_0c_0}\left[1-k_0^2\left(\frac{1}{\xi_n^2}+\frac{1}{\xi_m^2}-\frac{1}{\xi_n\xi_m}\right)\right]\\ I_{2nm}\approx\frac{\tau G_{nm}}{\pi^2\Gamma_n\sqrt{2t}}\mathrm{e}^{-\mathrm{i}\left(\omega t+\frac{\pi}{2}\right)}\sqrt{\frac{\omega\xi_n\xi_m\pi}{\beta_{0n}^2+(\varepsilon_{nm}^2-\varepsilon_{nm})k_0^2}}\times\\ \iint_{\Omega'}\left(\frac{x'}{r'}\frac{\partial\zeta}{\partial x'}+\frac{y'}{r'}\frac{\partial\zeta}{\partial y'}\right)\frac{\mathrm{e}^{\mathrm{i}(\xi_mr_{nm}'+\xi_nR_{nm})}}{\sqrt{r_{nm}'R_{nm}}}\mathrm{d}x'\mathrm{d}y'\end{cases}\quad(27)$$

From Eq.(23) it can be shown that Ω' is a region having the form of some oval band, with band width proportional to τ, and origin at the center of transmitter and receiver.

5 Approximate analytic expression of intensity and spatial correlation of bi-static bottom reverberation

Assume all the random quantities $\zeta,\frac{\partial\zeta}{\partial x},\frac{\partial\zeta}{\partial y}$ are mutually independent with mean values equal to zero, and having respectively the following 2-D spatial correlation function:

$$\left\langle \zeta(x_1,y_1)\zeta(x_2,y_2)\right\rangle =$$
$$\zeta_0^2 \exp\left\{-\frac{(x_1-x_2)^2+2\gamma_0(x_1-x_2)(y_1-y_2)+(y_1-y_2)^2}{\sigma_0^2}\right\}$$

$$\left\langle \frac{\partial\zeta(x_1,y_1)}{\partial x}\frac{\partial\zeta(x_2,y_2)}{\partial x}\right\rangle =$$
$$\zeta_x^2 \exp\left\{-\frac{(x_1-x_2)^2+2\gamma_x(x_1-x_2)(y_1-y_2)+(y_1-y_2)^2}{\sigma_x^2}\right\}$$

$$\left\langle \frac{\partial\zeta(x_1,y_1)}{\partial y}\frac{\partial\zeta(x_2,y_2)}{\partial y}\right\rangle =$$
$$\zeta_y^2 \exp\left\{-\frac{(x_1-x_2)^2+2\gamma_y(x_1-x_2)(y_1-y_2)+(y_1-y_2)^2}{\sigma_y^2}\right\}$$

Then the statistical mean $\langle \Psi_{rn}\rangle$ would also be zero, and the spatial correlation function of bottom reverberation can be computed from

$$\left\langle \Psi_r(\boldsymbol{r}_1)\Psi_r^*(\boldsymbol{r}_2)\right\rangle = \left\langle \sum_n \Psi_{rn}(r_1)Z_n(z_1)\sum_{n'}\Psi_{rn'}^*(r_2)Z_{n'}^*(z_2)\right\rangle \quad (28)$$

Above expression contains terms of two kinds, one for $n=n'$, which gives the main value of correlation; the other for $n=n'$, which gives value vibrating around the mean value of correlation due to interference between different order modes and would be neglected, when only the average value of correlation has been taken into consideration. Also, though Z_n are functions having random part, but the random parts are only about the order ζ_0/H, therefore when only first order effects of boundary roughness are taken into consideration one could have

$$\left\langle \Psi_r(\boldsymbol{r}_1)\Psi_r^*(\boldsymbol{r}_2)\right\rangle \approx \sum_n\left\langle \Psi_{rn}(r_1)\Psi_{rn}^*(r_2)Z_n(z_1)Z_n^*(z_2)\right\rangle =$$
$$\sum_n\left[\left\langle I_{1n}(r_1)I_{1n}^*(r_2)\right\rangle+\sum_m\left\langle I_{2nm}(r_1)I_{2nm}^*(r_2)\right\rangle\right]Z_n(z_1)Z_n^*(z_2) \quad (29)$$

1) Intensity of bi-static bottom reverberation.

Using Eqs.(21), (22), (26), (27) and compute the integral by common method, finally we get successively:

$$\left\langle I_{1n}(r)I_{1n}^*(r)\right\rangle \approx \left(\frac{\sigma_0\zeta_0\beta_{0n}c_0\tau}{\pi B_n}\right)^2\frac{2\xi_n\tau}{t\sqrt{1-\gamma_0^2}}\int_0^{2\pi}\frac{\mathrm{d}\alpha}{T_n-r\cos\alpha} =$$
$$\left(\frac{\sigma_0\zeta_0\beta_{0n}c_0\tau}{\pi B_n}\right)^2\frac{4\pi\xi_n\tau}{t\sqrt{1-\gamma_0^2}\sqrt{T_n^2-r^2}} \quad (30)$$

$$\left\langle I_{2nm}(r)I_{2nm}^*(r)\right\rangle \approx S_{nm}\int_0^{2\pi}\left(\frac{\sigma_x^2\zeta_x^2\cos^2\alpha}{\sqrt{1-\gamma_x^2}}+\frac{\sigma_y^2\zeta_y^2\sin^2\alpha}{\sqrt{1-\gamma_y^2}}\right)\frac{\mathrm{d}\alpha}{T_n-\tilde r\cos\alpha} =$$
$$S_{nm}\left\{\frac{\sigma_x^2\zeta_x^2}{\sqrt{1-\gamma_x^2}}\frac{T_n}{\sqrt{T_n^2-\tilde r^2}}+\frac{\sigma_y^2\zeta_y^2}{\sqrt{1-\gamma_y^2}}\right\}\frac{T_n-\sqrt{T_n^2-\tilde r^2}}{\tilde r^2} \quad (31)$$

where

$$S_{nm}\approx\left(\frac{G_{nm}\xi_n c_0\tau}{\pi\Gamma_n}\right)^2\frac{\xi_m\tau}{\left[\beta_{0n}^2+\left(\varepsilon_{nm}^2-\varepsilon_{nm}\right)k_0^2\right]t} \quad (32)$$

Notice that for $T_n >> \tilde r$, $\frac{T_n-\sqrt{T_n^2-\tilde r^2}}{\tilde r^2}\approx\frac{1}{2T_n}$.

Eqs.(30)~(31) show that the intensity of bottom reverberation for each normal mode will be proportional to the length of sound pulse τ and inversely proportional to the square of time *t*. But since intensity of normal modes with complex eigen value decreases with propagating distance, when the sum of reverberation for all normal modes are taken into account, the intensity of bottom reverberation will be inversely proportional to the cube of *t*, just like the case of sound transmission loss in shallow water channel (Yang, 1994).

2) Spatial correlation of bi-static bottom reverberation.

When spatial correlation functions of different mode $Q_n^{(1)}, Q_{nm}^{(2)}$ are considered, the intensity factor of bottom reverberation can be omitted, and the scattering signal should come from the same point (r',α) of bottom boundary. Therefore:

$$2\pi Q_n^{(1)}\approx \mathrm{Re}\int_0^{2\pi}\mathrm{e}^{\mathrm{i}\xi_n(R_{n+}-R_{n-})}\mathrm{d}\alpha\approx\int_0^{2\pi}\left\{1-\frac{\xi_n^2}{2}\left[\left(1-\frac{r^2\sin^2\alpha}{(T_n-r\cos\alpha)^2}\right)(L\cos\alpha+D\sin\alpha)-\frac{2r\sin\alpha}{T_n-r\cos\alpha}(L\sin\alpha-D\cos\alpha)\right]^2\right\}\mathrm{d}\alpha =$$
$$2\pi\left\{1-\frac{\xi_n^2}{4}\left[L^2\left(\frac{T_n}{\sqrt{T_n^2-r^2}}\right)^3+D^2\left(1+\frac{T_n}{\sqrt{T_n^2-r^2}}\right)\frac{T_n\left(T_n-\sqrt{T_n^2-r^2}\right)}{r^2}\right]\right\} \quad (33)$$

$$2\pi Q_{nm}^{(2)} \approx \mathrm{Re}\int_0^{2\pi} \mathrm{e}^{\mathrm{i}\xi_n\left(R_{nm+}-R_{nm-}\right)}\mathrm{d}\alpha \approx \int_0^{2\pi}\left\{1-\frac{\xi_n^2}{2}\left[\left(1-\frac{\kappa\tilde{r}^2\sin^2\alpha}{\left(T_n-\tilde{r}\cos\alpha\right)^2}\right)\left(\tilde{L}\cos\alpha+\tilde{D}\sin\alpha\right)-\frac{2\tilde{r}\sin\alpha}{T_n-\tilde{r}\cos\alpha}\left(\tilde{L}\sin\alpha-\tilde{D}\cos\alpha\right)\right]^2\right\}\mathrm{d}\alpha =$$

$$2\pi\left\{1-\frac{\xi_n^2\tilde{L}^2}{4}\left[\left(7-3\kappa\right)\left(\kappa-1\right)+4\left(\kappa-1\right)\left(3\kappa-4-\frac{2\kappa T_n\left(T_n-\sqrt{T_n^2-\tilde{r}^2}\right)}{\tilde{r}^2}\right)\frac{T_n}{\sqrt{T_n^2-\tilde{r}^2}}+\right.\right. \tag{34}$$

$$\left.\left.\kappa^2\left(\frac{T_n}{\sqrt{T_n^2-\tilde{r}^2}}\right)^3\right]-\frac{\xi_n^2\tilde{D}^2}{4}\left[5\kappa^2-6\kappa+1-\left(15\kappa^2-24\kappa+8-\frac{\left(10\kappa^2-12\kappa+8\right)T_n}{\sqrt{T_n^2-\tilde{r}^2}}\right)\times\frac{T_n\left(T_n-\sqrt{T_n^2-\tilde{r}^2}\right)}{\tilde{r}^2}\right]\right\}$$

where $\kappa = \dfrac{\varepsilon_{nm}+1}{2\varepsilon_{nm}}$.

Since only the first order approximation of exponential function had been used in deriving Eqs.(33)~(34), the expressions are valid only if $\left(\xi_n L, \xi_n D\right) << 1$. But from these approximate expressions it still can be seen that: usually for spatial correlation in bi-static reverberation case, the transverse correlation will be greater than the longitudinal correlation, and the radius of spatial correlation will be gradually increase with receiving time. More detailed discussion will be published hereafter.

6 Conclusions

When using coupled mode method to estimate reverberation in ocean, which caused by the scattering effect of inhomogeneities of medium and roughness of boundaries, one can get results which never contradict with principle of reciprocity. Though the result of given example in this paper is only a special case, and very different result may be obtained when environment condition will be quite different. But it always will be the truth that not only boundary roughness but also the slope of deepness variation cause influence on intensity of reverberation.

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