

Momentum measurement by the Multiple Coulomb Scattering method in the OPERA lead emulsion target

OPERA COLLABORATION

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Abstract

A new method of momentum measurement of charged particles through Multiple Coulomb Scattering (MCS) in the OPERA lead emulsion target is presented. It is based on precise measurements of track angular deviations performed thanks to the very high resolution of nuclear emulsions. The algorithm has been tested with Monte Carlo (MC) pions. The results are found to describe within the expected uncertainties the data obtained from test beams. We also report a comparison of muon momenta evaluated through MCS in the OPERA lead emulsion target with those determined by the electronic detectors for neutrino charged current interaction events. The two independent measurements agree within the experimental uncertainties, and the results validate the algorithm developed for the emulsion detector of OPERA.

1 Introduction

The momentum of charged particles can be measured in Emulsion Cloud Chambers (ECC) [1] made of massive material plates, used as target, interleaved with nuclear emulsion films acting as high resolution tracking devices. This technique was exploited by the DONUT experiment [2] and is currently used in the OPERA experiment searching for $\nu_\mu \rightarrow \nu_\tau$ oscillations in the CNGS neutrino beam [3]. The study shown in this paper uses the geometry and the characteristics of the OPERA neutrino target ECC elements called “bricks”. They have dimensions of $12.7 \times 10.2 \times 7.5 \text{ cm}^3$ and are composed of a sequence of 56 lead plates (1 mm thick) and 57 emulsion films (44 μm thick emulsion layers on each side of a 205 μm thick plastic base). The total length of a brick corresponds to about $10X_0$.

Charged particles crossing the emulsions ionise silver bromide crystals, and clusters of silver grains, appearing as black dots, are formed along their paths after film processing. Automatic microscopes [4] are used to reconstruct 3D particle track segments. Micro-track segments are reconstructed in single emulsion layers as sequences of aligned grains. Two matching micro-tracks in a film define a base-track, obtained as the straight line connecting the grains closest to the plastic base in the two emulsion layers.

A track reconstructed through connecting segments in two or more films is called a volume-track.

The momentum measurement by Multiple Coulomb Scattering (MCS) can be performed by using either the track position (coordinate method) [5] or the track angle (angular method) [6] measured in each emulsion film. The two methods determine the deviations of the trajectory from a straight line on the basis of position or angle measurements, respectively. The use of one method rather than the other depends on the required accuracy, and on the achievable spatial and angular resolutions. In OPERA ECC bricks, base-track directions are measured with a precision of a few mrad. Moreover, the angular method does not depend on a precise knowledge of the relative alignment of the different emulsion films. The evolution of slopes of consecutive base-tracks forming a volume-track can thus be used to compute the mean Coulomb scattering angle in a given lead thickness, which is directly related to the particle momentum. The angular resolution of the emulsions allows the determination of charged particle momentum from several hundreds of MeV/c to a few GeV/c, which corresponds to the momentum range of secondary hadrons produced in neutrino interactions in the OPERA experiment. Several approaches to perform the angular deviation measurements in lead have been tested and compared in previous studies. The method presented in this paper is based on the work detailed in [7] and is used in the analysis of the neutrino events observed in OPERA [8].

The first part describes the method and the special treatment used for the large angle tracks. Results from Monte Carlo and from data analysis with pions from 1 to 8 GeV/c momentum for various track lengths are summarised in sections 3 and 4. In the last section the results from the application of the algorithm to muon tracks reconstructed independently in the OPERA electronic detectors with momenta below 6 GeV/c are presented.

2 Measurement method

2.1 Scattering angle dependence on the lead thickness

The main ingredient for the angular method measurement is the availability of several angular measurements along a volume-track. The present approach uses the angle differences measured in pairs of emulsion films separated by lead. In the following, one cell corresponds to one lead plate and one film. Figure 1 provides a schematic view of a volume-track and its associated base-tracks in the XZ projection plane. Let θ_i be the angle of a given base-track in the i^{th} emulsion film, $i \in \{1, \dots, 57\}$ in the XZ or YZ projection plane. Defining $\theta_{ik} = \theta_{i+k} - \theta_i$ as the scattering angle after crossing a number k of cells, its distribution is peaked at zero and has a shape that can be approximated by a Gaussian with a standard deviation given by [9]

$$\theta_0 = \frac{13.6}{(pc\beta)} \times \sqrt{\frac{x}{X_0}} \times \left[1 + 0.038 \ln \left(\frac{x}{X_0} \right) \right] \quad (1)$$

where p is the particle momentum in MeV/c, βc its velocity, x is the distance traversed and X_0 is the radiation length in the material. The accuracy of this approximation of Moliere’s theory of scattering is

better than 11% in any material, with $0.001 < x/X_0 < 100$ [10] for single charged particles with $\beta \approx 1$. The scattering is dominated by the lead since the radiation length in the emulsion layers and the plastic base is larger by more than one order of magnitude. For this reason, the value $X_0 = 5.6$ mm will be assumed in the analysis and a thickness of 1mm will be used for each cell, neglecting the emulsion films. By denoting the number of cells crossed by a particle track as N_{cell} , the above-mentioned expression becomes:

$$\theta_0 \approx \frac{13.6}{(pc\beta)} \times \sqrt{\frac{N_{cell}}{5.6}} \times \left[1 + 0.038 \ln \left(\frac{N_{cell}}{5.6} \right) \right]. \quad (2)$$

The variance of the scattering angle distribution for a given cell depth $N_{cell} = k$ is given by

$$\langle \theta_{meas}^2 \rangle_k = \sum_i (\theta_{ik})^2 / N_{meas} = \theta_0^2 + \delta\theta^2, \quad (3)$$

where N_{meas} is the number of scattering angle measurements and $\delta\theta$ is an additional term corresponding to the base-track angular resolution ¹.

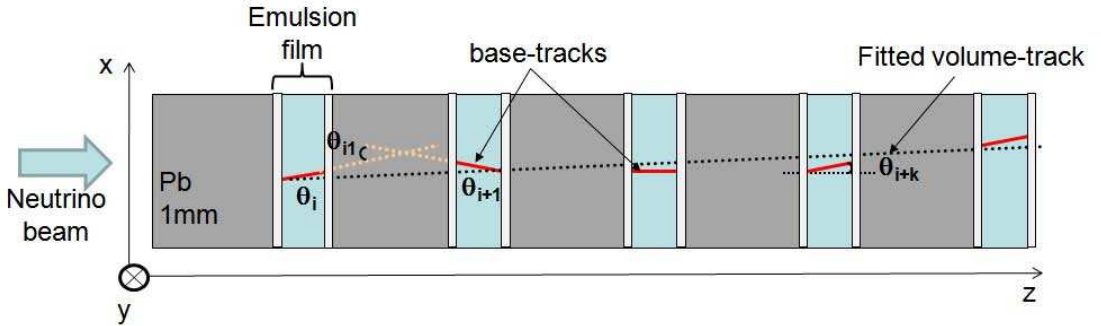


Figure 1: Sketch of 5 lead cells in a target brick, where a volume-track and its base tracks are represented in the XZ projection.

The current experimental value of $\delta\theta$ is about 2.1 mrad. In order to determine p up to a few GeV/c through the scattering angle, a fit of the dependence of θ_{meas} on the number of crossed cells is performed, treating p as a free parameter and fixing the angular resolution. With increasing p , the MCS starts dominating over $\delta\theta$ at larger values of N_{cell} , where the number of available measurements decreases, thus increasing the statistical error. In order to improve the sensitivity to high-momentum tracks, it is important to reduce the statistical uncertainty at large crossed thicknesses.

The method is illustrated in Figure 2. It consists of using the differences between all combinations of pairs of angles separated by N_{cell} cells. For a given cell depth N_{cell} and a total track span N_{pl} measured as the number of lead plates traversed by the particle, the number of available measurements N_{meas} is given by:

$$N_{meas} = \sum_{i=1}^{N_{cell}} \text{int} \left[\frac{N_{pl} - i + 1}{N_{cell}} \right] \quad (4)$$

2.2 Track angle-dependence

For large-angle tracks the following effects have to be taken into account. First, the crossed lead thickness varies as $1/\cos\theta$, with θ being the track angle measured with respect to the normal to the emulsion plane (Z coordinate). Second, also the angular resolution $\delta\theta$ depends on θ , as the longitudinal uncertainty affects the measured grain positions along the optical Z axis. This effect is dominated by the vertical resolution of the scanning system, and is about $2.5 \mu\text{m}$ [4]. For angles above 200 mrad, this uncertainty is one order of magnitude larger than that in the transverse X and Y coordinates.

¹ $\delta\theta$ is the angular resolution between 2 base-tracks. The single base-track angular resolution is $\delta\theta_s = \frac{\delta\theta}{\sqrt{2}}$.

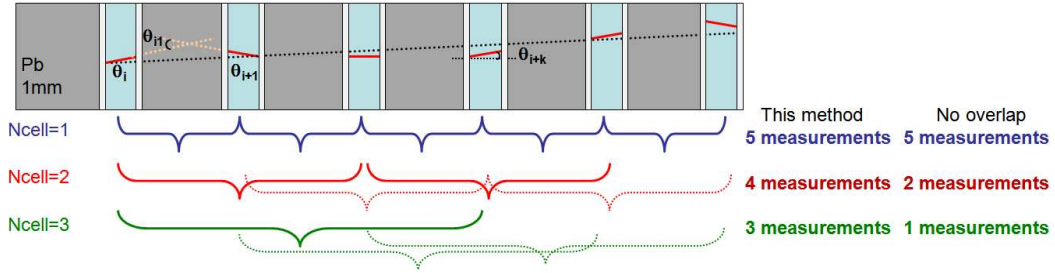


Figure 2: Representation of the number of possible measurements available when applying the MCS method up to $N_{cell} = 3$.

In order to decouple the intrinsic angular resolution from the slope-dependent contribution, the algorithm is constructed in a new reference coordinate system. It uses transverse and longitudinal coordinates (denoted respectively as T and L) as defined in [4], projected on θ_T and θ_L axes of the reference frame schematically shown in Figure 3. The T and L coordinates are obtained from X and Y by applying a rotation:

$$\theta_T = \theta_X \cos(\phi) + \theta_Y \sin(\phi) \quad (5)$$

$$\theta_L = -\theta_X \sin(\phi) + \theta_Y \cos(\phi) \quad (6)$$

where $\phi = \arctan(\frac{\theta_Y}{\theta_X})$. The 3-dimensional (3D) space angle can be written as

$$\tan(\theta_{3D}) = \sqrt{\tan^2(\theta_X) + \tan^2(\theta_Y)} = \sqrt{\tan^2(\theta_T) + \tan^2(\theta_L)}. \quad (7)$$

As can be seen in Figure 3, the T coordinate gives an angular spread which remains the same for any

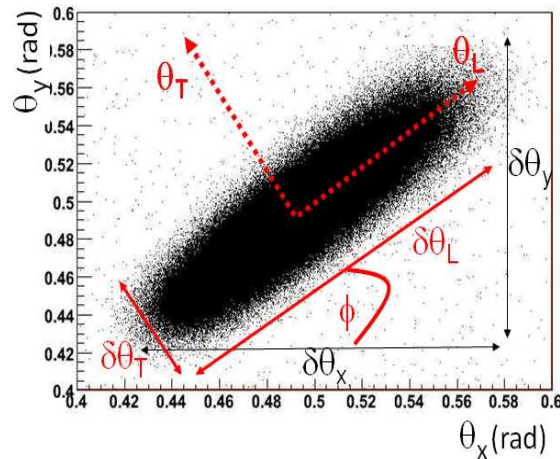


Figure 3: Schematic view of T-L coordinate reference frame, superimposed on the θ_y versus θ_x plot for the base tracks of 10 GeV/c MC muons at large angle ($\theta_X = \theta_Y = 500$ mrad).

track angle. The angular dependence of the resolution can be parameterised as [11]:

$$\delta\theta_T(\theta) = \delta\theta_T(0) = \delta\theta_{3D}(0), \quad (8)$$

and

$$\delta\theta_L(\theta) = \delta\theta_L(0) + \epsilon_z \tan\theta \quad (9)$$

where ϵ_z is a parameter that linearly depends on the longitudinal uncertainty.

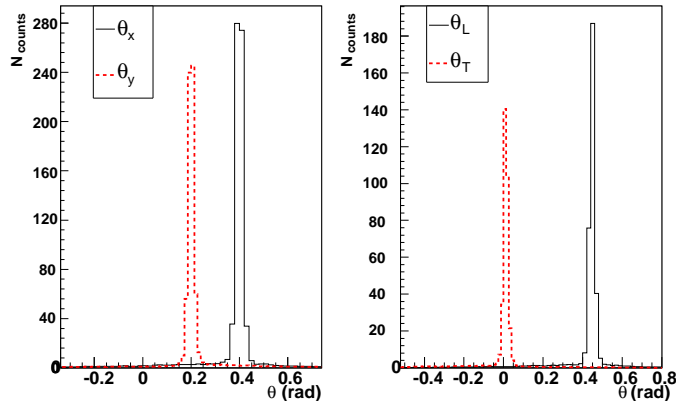


Figure 4: Angular distributions of base tracks from 4 GeV/c MC pions simulated with $\theta_X = 400$ mrad and $\theta_Y = 200$ mrad in XY (left) and TL projection planes (right).

This transformation allows keeping θ_T centred around 0 mrad as shown in Figure 4. As discussed in section 3.2, an unbiased algorithm would use the 3D coordinate (both T and L or X and Y measurements) for small angles, and only the T coordinate at large angles. However, the latter choice results in only half the statistics, even though it is angle-independent and free of bias. In the following, all the results are obtained using the T-L coordinate system.

2.3 Momentum and resolution estimate

In order to estimate the momentum resolution, samples of same-momentum tracks can be analysed. Assuming a Gaussian distribution for θ_0 , the shape of the momentum distribution can be approximated by the function

$$f(p) = \frac{p_0}{p^2} \times \exp\left(-\frac{(1/p - 1/p_1)^2}{p_2^2}\right) \quad (10)$$

where p_0 , p_1 , and p_2 are free parameters. The parameter p_1 corresponds to the average of the reconstructed momenta p_{mean} . Figure 5 (left) shows an example of this fitted distribution for tracks of 4 GeV/c Monte Carlo pions passing through 56 cells. In order to take into account possible uncertainties coming from the Gaussian analytic approximation of $1/p$, the mean reconstructed momentum is obtained from the average fits of the distributions of both momentum and inverted momentum (Figure 5 (right)). The first one has sensitivity to the high reconstructed momentum tail while the second is more sensitive to the lower reconstructed momentum values. The difference of the two results is the systematic uncertainty of the average fitted momentum determination. In the previous example, the results give $\langle p \rangle = 3.97 \pm 0.01$ (stat) ± 0.08 (syst) GeV/c.

Since the inverted momentum distribution $1/p$ has a Gaussian shape, the width of the Gaussian divided by $1/p_{mean}$ directly gives the momentum resolution estimate $\frac{\Delta(1/p)}{(1/p)}$. Its uncertainty can be obtained by propagating the errors on the two components which are the width of the distribution and the reconstructed momentum. Therefore, the momentum resolution of the 4 GeV/c MC sample of pions passing through an entire OPERA target brick is 20.1 ± 0.6 %.

3 Monte Carlo results

In this section, the results obtained from Monte Carlo simulations are reported. The MC data correspond to 2, 4, 6 and 8 GeV/c pion samples of 1000 events each that have been generated with the simulation tool ORFEO, based on GEANT and developed in the OPERA framework [11]. It simulates particle interactions inside a brick and includes the main experimental effects such as the track efficiency and spatial resolutions.

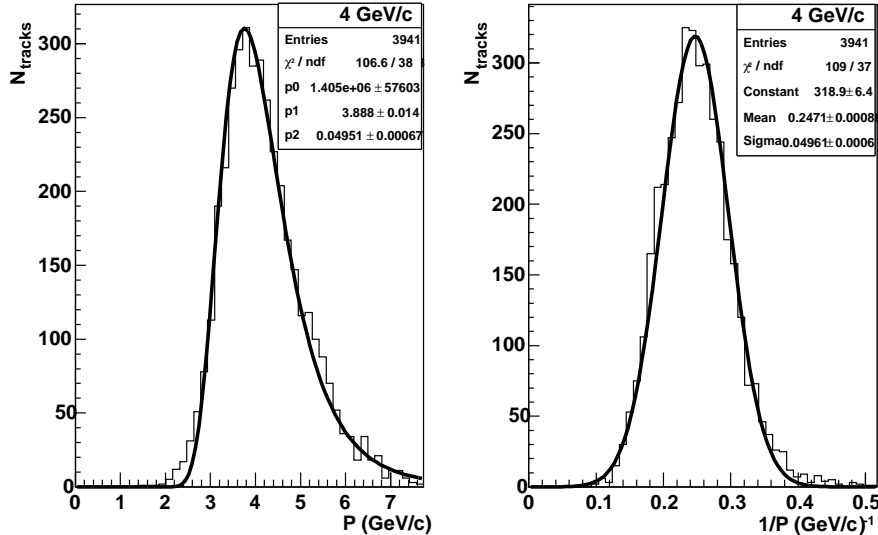


Figure 5: Momentum distribution (left) and inverted momentum distribution (right) for about 4000 tracks of 4 GeV/c MC pions reconstructed in an ECC brick.

This section is divided into two parts: the first gives the results for small incident angles ($\theta < 200$ mrad), the second for large incident angles ($\theta > 200$ mrad).

3.1 Tracks at small incident angles

Figure 6 shows the dependence of the scattering angle on N_{cell} for different momenta from 1 to 8 GeV/c². Since the MC samples contain a large number of tracks with the same momentum, the single base-track angular resolution $\delta\theta_s$ can be directly determined together with the particle momentum from the fits of Figure 6. Results are summarised in Table 1.

p_{MC} (GeV/c)	$\delta\theta_s$ (mrad)	$\langle p \rangle$ (GeV/c)	$\frac{\Delta(1/p)}{(1/p)}$ (%)
1	1.8 ± 0.2	1.03 ± 0.01	14.2 ± 0.3
2	1.76 ± 0.05	2.04 ± 0.03	15.4 ± 0.3
3	1.67 ± 0.02	3.01 ± 0.05	17.6 ± 0.5
4	1.68 ± 0.01	3.97 ± 0.09	20.1 ± 0.6
6	1.66 ± 0.01	5.99 ± 0.17	22.0 ± 0.7
8	1.66 ± 0.01	8.13 ± 0.30	26.0 ± 1.0

Table 1: Reconstructed values of the single base-track resolution $\delta\theta_s$, the average momentum $\langle p \rangle$, and the momentum resolution $\frac{\Delta(1/p)}{(1/p)}$ for MC samples of tracks crossing an entire brick and for different energies simulated with $\delta\theta_s^{MC} = 1.67$ mrad.

The values of $\langle p \rangle$ and $\frac{\Delta(1/p)}{(1/p)}$ have been obtained with the method described in Section 2. It appears, that the linearity between reconstructed and MC momenta is kept over the whole range, and that the momentum resolution worsens with the momentum, as expected. The linearity of the MC reconstructed momentum and the evolution of the momentum resolution show the consistency of the method. They also demonstrate that the approximation of lead as the main scattering element is well-suited for the OPERA ECC configuration.

²The MC samples have been tuned in order to reproduce the measured $\delta\theta_s$, obtained in the scanning of the test beam data samples with same momenta.

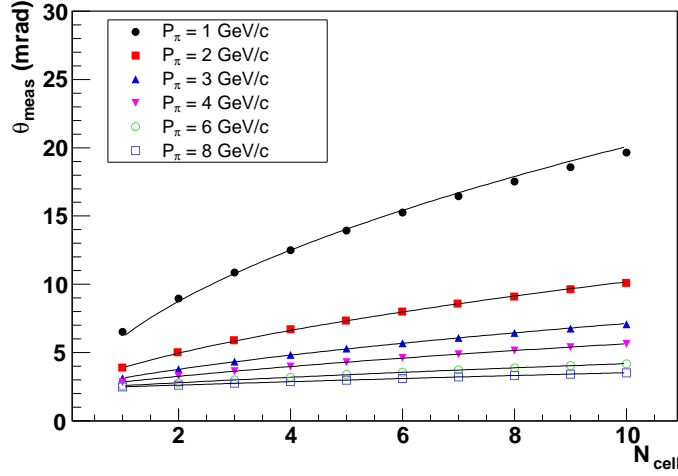


Figure 6: θ_{meas} dependence on N_{cell} for MC pions of different energies, where $\delta\theta_s$ has been simulated at a value of $\delta\theta_s^{MC} = 1.67$ mrad. The solid curves correspond to the fitted expectations.

These results were obtained for tracks passing through 56 cells of an ECC brick. Figure 7 shows how the resolution worsens with increasing tracks momentum and with decreasing track span.

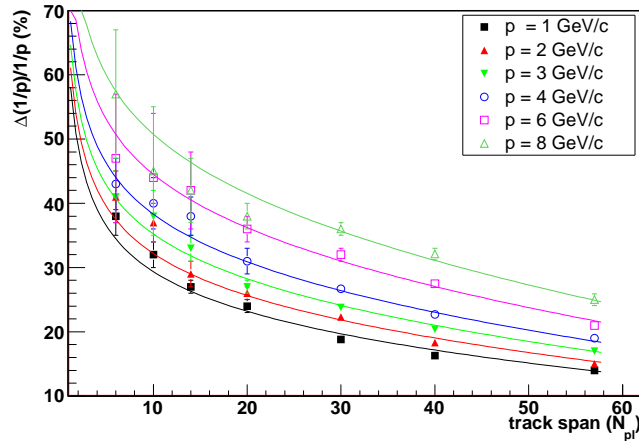


Figure 7: Momentum resolution dependence on track span N_{pl} for MC pions with $\delta\theta_s^{MC} = 1.67$ mrad. The solid lines correspond to the fitted parameterised resolution function of Eq. 11.

Using all the MC results for different track spans and momenta values, the momentum resolution $\frac{\Delta(1/p)}{(1/p)}$ has been parameterised in terms of the momentum p and track span N_{pl} as:

$$\frac{\Delta(1/p)}{(1/p)} = (0.397 + 0.019 \times p) / \sqrt{N_{pl}} + (0.176 + 0.042 \times p) + (-0.014 - 0.003 \times p) \times \sqrt{N_{pl}}. \quad (11)$$

The fitted function well describes all momentum measurements from 1 to 8 GeV/c for various track lengths.

3.2 Tracks at large incident angles

A first sample of 2, 4, and 6 GeV/c MC pions has been generated at angles of $\theta_X = 200$ and 400 mrad and $\theta_Y = 0$ mrad. The $\delta\theta_T$ and $\delta\theta_L$ angular dependences have been parameterised according to Equations 8 and 9 using the resolution parameters measured with a special brick consisting of a sequence of emulsion films, without lead exposed to 7 GeV/c pions at several incident angles. The track resolution parameters are measured to be $\delta\theta_L(0) = \delta\theta_T(0) = 2.1$ mrad, and $\epsilon_z = 9.3$.

The different MC samples have been simulated using this parameterisation of the angular resolution. For the MC event samples at $\theta_X = 200$ mrad, the measured values of $\langle p \rangle$ in 3D and in 2D projections T, L are compatible with the expected values. The values of $\langle p \rangle$ in the 3D and L projections for 4 GeV/c and 6 GeV/c pions at $\theta_X = 400$ mrad are respectively 10% and 20% lower than the true momentum while there is agreement in the T projection. This is explained by the angular dependence of the longitudinal resolution which increases linearly with the angle reaching already a factor of two for track angles of 200 mrad. Note, that the T projection is not affected since it is angle-independent.

For the same reason, the momentum resolution is stable, as can be observed in Figure 8 depicting the dependence of $\frac{\Delta(1/p)}{(1/p)}$ on the angle in the T projection. This plot also shows that the momentum resolution in the T projection is worse than in the 3D case for tracks at 0 mrad, due to the 50% reduced statistics when using only one projection.

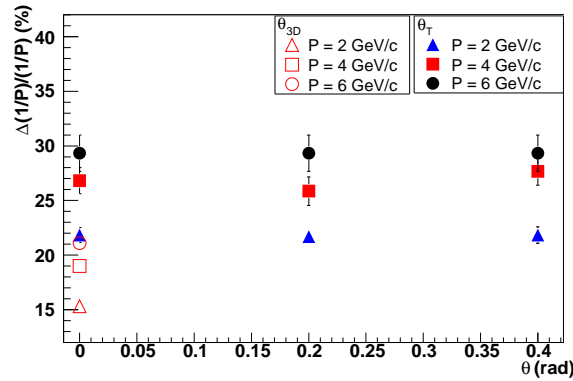


Figure 8: Momentum resolution with respect to the 3D angle θ for different MC pion momenta, obtained using only the T projection. As a reference, the values obtained at 0 mrad in 3D are indicated by the open symbols.

All the previous considerations lead to the conclusion that at large angles, the optimal method for estimating the momentum is to use the T projection, which is not biased and not angle-dependent. However, at small angles, the 3D calculation remains statistically more accurate, resulting in improved momentum resolution. In the algorithm, the threshold for large angles is set to 200 mrad, such that the angular resolution $\delta\theta$ is kept independent and always equal to the value for $\theta = 0$.

A second sample of 2 and 4 GeV/c MC pions has been generated with $\theta_X = 400$ mrad and $\theta_Y = 200$ mrad. Since the 3D angle is above 200 mrad, we report only the results obtained with the T projection. The value of $\delta\theta_T$ is fixed at 2.1 mrad. The measured values of $\langle p \rangle$ and $\frac{\Delta(1/p)}{(1/p)}$ are given in Table 2. The momentum measurements are compatible with the input values, and the momentum resolutions agree with previous estimates.

The method using the T projection for angles larger than 200 mrad is thus validated. Similarly to Equation 11 for small angles, it is now possible to parameterise this dependence at large angles as well, using the T projection and a similar analytic formula. It gives:

$$\frac{\Delta(1/p)}{(1/p)} = (1.400 - 0.022 \times p) / \sqrt{N_{pl}} + (-0.040 + 0.051 \times p) + (0.003 - 0.004 \times p) \times \sqrt{N_{pl}}. \quad (12)$$

This single function, shown in Figure 9, describes all the MC results from 1 to 8 GeV/c for various

p_{MC} (GeV/c)	$\langle p \rangle$ (GeV/c)	$\frac{\Delta(1/p)}{(1/p)}$ (%)
2	1.9 ± 0.1	22 ± 1
4	3.9 ± 0.2	26 ± 1

Table 2: Results on $\langle p \rangle$ and $\frac{\Delta(1/p)}{(1/p)}$ with the T projection for 2 and 4 GeV/c pions, with $\theta_X = 400$ mrad and $\theta_Y = 200$ mrad.

p and N_{pl} values. As in Equation 11, it is used to assign the confidence level-ranges of single-track momentum measurement.

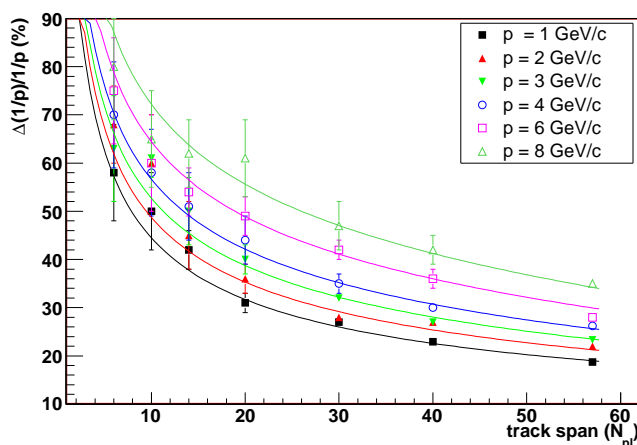


Figure 9: Momentum resolution dependence on track span N_{pl} for MC pions using the T projection with $\delta\theta_s^{MC} = 1.67$ mrad. The solid lines correspond to the fitted parameterised resolution function of Eq. 12.

3.3 Comments on method comparisons and systematics

Various MC studies concerning systematic errors and comparisons with other methods have been carried out in [7]:

- As explained in Section 2, using the differences between all combinations of pairs of angles separated by N_{cell} cells increases the number of measurements. It has been established that when using only the differences between successive pairs of angles, the fit of the momentum distributions diverges above 4 GeV/c. Moreover, the momentum resolutions are 1.5 times worse.
- A track momentum p can be measured with both $\delta\theta$ (angular base-track resolution) and p as free parameters in the fit procedure. However, an error of a few % on $\delta\theta$ can affect the momentum reconstruction by more than 10% for high-energy tracks (above 4 GeV/c). The best results are obtained with the proposed method keeping $\delta\theta$ fixed. The physical value of the base-track angular resolution is usually between 1 and 2 mrad and depends mainly on the experimental conditions. The value of $\delta\theta$ can be determined or verified with reference measurements of angular deviations in emulsion films, without scattering in heavy materials.
- Effects from the correlations among the θ_{meas} values measured at the different N_{cell} cells have been estimated by building covariance matrices at different energies and track lengths with MC samples. Fits of the track scattering angle dependence on N_{cell} have been repeated by incorporating the covariance matrix in the minimization function used to compute the track momentum. The difference with the uncorrelated fit is found to be less than a few percent for the absolute momentum value determination for pion momenta ranging from 2 to 8 GeV/c and the resolution stays unchanged.

4 Analysis of pion test beam data

We report here the results obtained with real data collected in a test beam-exposure of OPERA bricks to 2, 4, 6, and 8 GeV/c pions produced by the CERN PS accelerator. Figures 10 and 11 compare the momentum distributions of MC (red solid line) to real data (black crosses) for pions crossing the entire brick with momenta of 2 and 6 GeV/c, respectively. Table 3 summarises the values of the single base-track resolution $\delta\theta_s$, the average momentum $\langle p \rangle$, and the resolution $\frac{\Delta(1/p)}{(1/p)}$ obtained for each data sample.

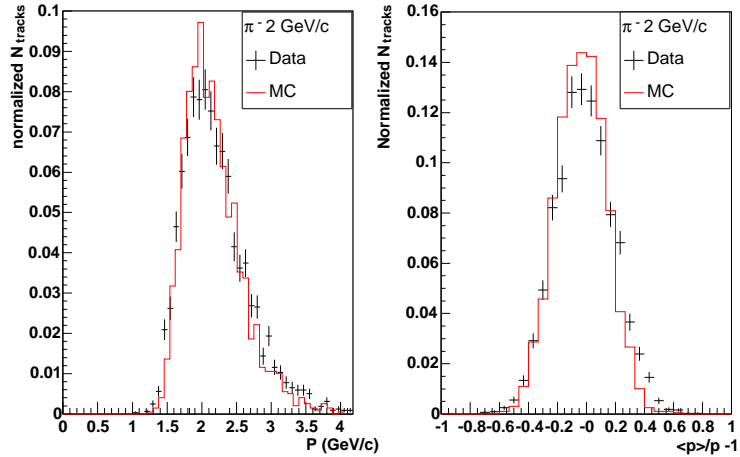


Figure 10: Data/MC comparison for 2 GeV/c pions. Left: momentum distribution. Right: inverted momentum distribution ($\langle p \rangle / p - 1$).

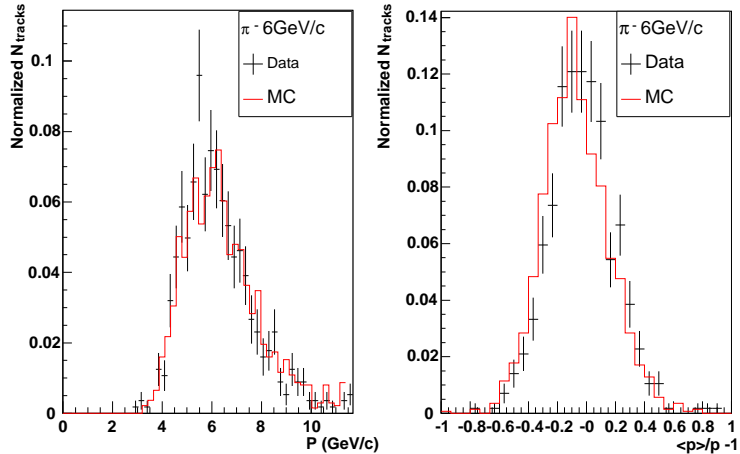


Figure 11: Data/MC comparison for 6 GeV/c pions. Left: momentum distribution. Right: inverted momentum distribution ($\langle p \rangle / p - 1$).

The results for real and simulated data are consistent within 11%. Concerning additional systematic uncertainties coming from beam composition, it appears that while the 4 GeV/c and 6 GeV/c data sample have the expected p resolution, the momentum resolution for 2 and 8 GeV/c data is measured to be respectively 4% and 10% worse than for the MC expectations. The discrepancy at 8 GeV/c has been understood to come from a higher muon contamination produced after the momentum selection

p_π (GeV/c)	$\delta\theta_s$ (mrad)	$\langle p \rangle$ (GeV/c)	$\frac{\Delta(1/p)}{(1/p)}$ (%)
2	2.26 ± 0.01	2.08 ± 0.05	19.6 ± 0.4
4	1.72 ± 0.01	4.32 ± 0.08	19.4 ± 0.4
6	1.90 ± 0.01	5.9 ± 0.2	21.0 ± 3.0
8	1.48 ± 0.01	7.2 ± 0.5	32.0 ± 2.0

Table 3: Reconstructed values of $\delta\theta_s$, $\langle p \rangle$, and $\frac{\Delta(1/p)}{(1/p)}$ obtained with pion test beam data.

collimators in the pion beam, which was not taken into account in the simulation. In the case of the 2 GeV/c sample, the resolution is slightly worse due to scattering on different materials placed along the beam line in front of the bricks.

Data at large angles from test beam pions of several energies and different incident angles recorded in one OPERA brick have also been analysed. Figure 12 shows the angular distribution in θ_X for reconstructed tracks with length (N_{pl}) ranging from 25 to 30 plates. The different peaks correspond to:

- 2 GeV/c pions at 200 and 400 mrad,
- 4 GeV/c pions at -200 and -400 mrad,
- 6 GeV/c pions at 100, 300 and 600 mrad,
- 8 GeV/c pions at 50 mrad, used as reference data at small angles.

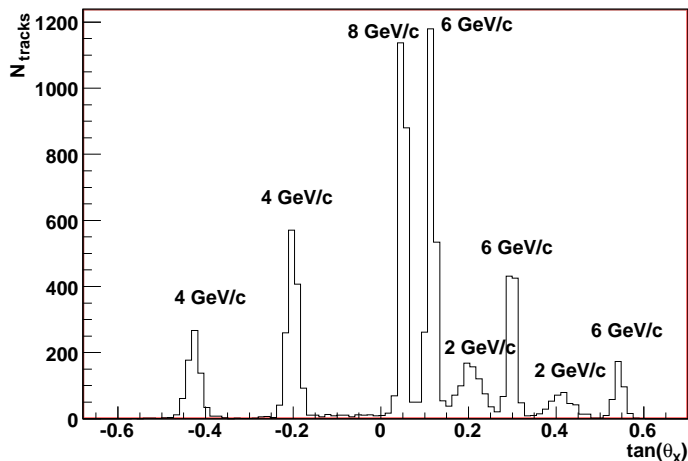


Figure 12: Angular distribution in θ_X for pion tracks reconstructed in the brick with a span between 25 and 30 plates.

The results for large angles are summarised in Table 4. The values of $\langle p \rangle$ are compatible with the expected pion beam momentum. The reconstructed momentum resolution can be compared to the one parameterised in Equation 12, also given in Table 4. The measured values are compatible with the expectations except for the 2 GeV/c samples, where the measured value of $\frac{\Delta(1/p)}{(1/p)}$ is over-estimated by 25 to 40 %. This effect is also due to the scattering on different materials, placed along the beam line in front of the brick during the test beam. The 2 GeV/c pions have been particularly affected, as can be seen from the broad peaks in Figure 12: the interactions on materials lead to a dispersion in angle and in energy, which deteriorates the results on momentum resolution at low energies.

Taking into account these effects, one can conclude that MC and test beam data are compatible and give consistent results at both small and large angles.

p_{true} (GeV/c)	N_{pl}	θ_{3D} (rad)	$\langle p \rangle$ (GeV/c)	$\frac{\Delta(1/p)}{(1/p)}$	$\frac{\Delta(1/p)}{(1/p)}_{expected}$
2	36	0.2	2.2 ± 0.2	$37 \pm 5\%$	26%
2	28	0.4	2.1 ± 0.1	$38 \pm 3\%$	30%
4	36	0.2	4.3 ± 0.3	$32 \pm 2\%$	32%
4	28	0.4	4.0 ± 0.5	$42 \pm 6\%$	37%
6	36	0.1	6.3 ± 0.6	$44 \pm 5\%$	38%
6	36	0.3	6.1 ± 0.6	$38 \pm 4\%$	38%
6	28	0.5	5.7 ± 0.5	$45 \pm 4\%$	44%

Table 4: Results of momentum measurements obtained with the pion test beam at different angles and energies. The calculation was done in the T projection, with $\delta\theta_T$ fixed at 2.1 mrad.

5 Soft muon momentum measurement in OPERA

In order to validate the algorithm with charged particles produced in neutrino interactions, a sample of muons recorded in the 2008 run originating from charged current interactions (ν_{μ}^{CC}) in the OPERA target bricks was selected. Details on the detector characteristics, data acquisition, event reconstruction, and analysis procedures are described in [12]. More details on the performance of the OPERA electronic detectors can be found in [13].

The muon momentum in the electronic detectors was obtained either from the range of the particle in the OPERA target tracker, or in the spectrometer yoke, or from the magnetic spectrometer measurement. The corresponding momentum resolution $\Delta p/p$ is estimated at about 10% for the analysed sample. In order to match the momentum range accessible with the MCS algorithm, charged current interactions where a muon was reconstructed in the electronic detectors with a momentum below 6 GeV/c were selected. The corresponding neutrino interaction vertices were located in the emulsion target, and one emulsion track per event was matched to the muon track predicted by the electronic detectors. Additional selection criteria have been applied on track quality and length. The required minimum track span is 10 cells. The final sample corresponds to 43 events. Figure 13 shows the dependence of the angular deviation on the thickness of lead traversed in the 3D (left) and the T (right) coordinates for two different muon tracks. The dashed line shows the expected angular dependence obtained with the momentum measured

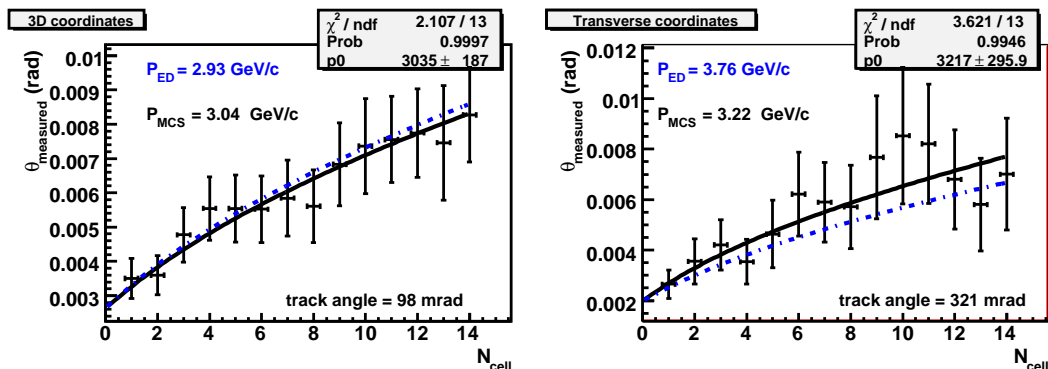


Figure 13: Angular deviation dependence on the thickness of the lead traversed by a muon track with an incident angle of 98 mrad using the 3D coordinates (left) and for a muon track with an incident angle of 321 mrad using the T coordinate (right). The dashed line shows the expected angular dependence obtained with the momentum measured by the electronic detectors while the solid line corresponds to the momentum measured by the MCS algorithm in emulsion.

by the electronic detectors while the solid line is the result of the fit of the momentum by the MCS method described in this paper. The two momentum measurements from MCS and electronic detectors

are fully compatible.

The muon momenta in the selected sample range from 2 to 6 GeV/c, as can be seen in the left plot of Figure 14, which shows the correlation between the two measurements: the right plot shows the relative difference with respect to the electronic detector value.

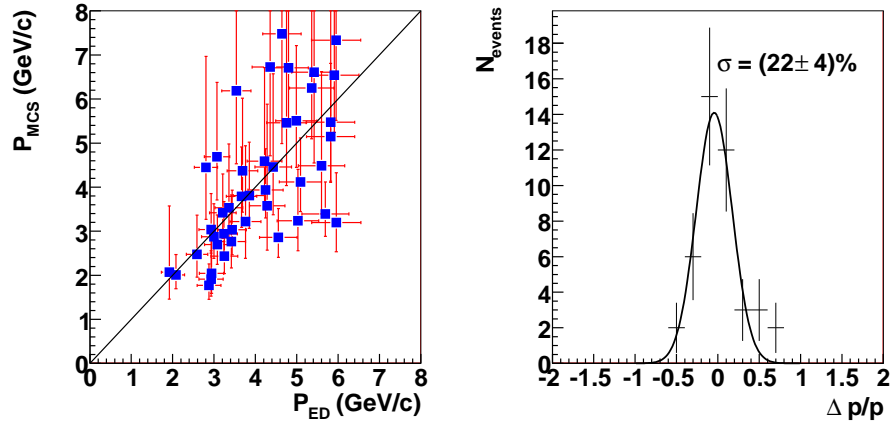


Figure 14: Left: Muon momenta measured by MCS (P_{MCS}) as a function of the momenta obtained from the electronic detectors (P_{ED}). The error bars correspond to the 68% confidence level range. Right: The relative difference between the two measurements with respect to the electronic detector measurement.

The distribution is a Gaussian centered at zero. The width gives an average resolution of $(22 \pm 4)\%$, compatible with the expectation obtained by folding the track sample characteristics with the parameterised resolution functions. The width includes also a contribution from the electronic detector resolution. In order to cross-check the estimate of the experimental uncertainty, the differences of the measured inverted momenta have been normalised to the uncertainty estimates on $1/p$, given by Equations 11 and 12 for the different track spans and angles. The resulting Gaussian distribution has a standard deviation of 1.10 ± 0.24 , compatible with unity. This shows that the uncertainty for each track is properly estimated.

6 Conclusions

An improved angular method has been developed to exploit Multiple Coulomb Scattering for the momentum measurement of charged particles in Emulsion Cloud Chamber detectors. The results of Monte Carlo studies and pion test beam data show that momenta up to 8 GeV/c can be measured with a resolution better than 30%. The approach has been optimized for small incident angles, as well as for large-angle tracks entering the OPERA lead-emulsion target elements and is well-suited for the neutrino interaction analysis. The results obtained with muons measured with the OPERA electronic detectors have confirmed the validity of the approach and assessed the performance of the algorithm.

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