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Application of the generalized quasi-complementary energy principle to the fluid-solid coupling problem

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Abstract: The fluid-solid coupling theory, an interdisciplinary science between hydrodynamics and solid mechanics, is an important tool for response analysis and direct design of structures in naval architecture and ocean engineering. By applying the corresponding relations between generalized forces and generalized displacements, convolutions were performed between the basic equations of elasto-dynamics in the primary space and corresponding virtual quantities. The results were integrated and then added algebraically. In light of the fact that body forces and surface forces are both follower forces, the generalized quasi-complementary energy principle with two kinds of variables for an initial value problem is established in non-conservative systems. Using the generalized quasi-complementary energy principle to deal with the fluid-solid coupling problem and to analyze the dynamic response of structures, a method for using two kinds of variables simultaneously for calculation of force and displacement was derived.

Keywords: fluid-solid coupling; elasto-dynamics; generalized quasi-complementary energy principle; dynamic response

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1 Introduction

Wu Yousheng, a famous expert in ship mechanics, has indicated that ship mechanics is the key of ship science and technology and that the developing history of modern ship is also a history of the development of ship mechanics. Ship mechanics includes ship structural mechanics (corresponding to the solid mechanics in the field of mechanics) and ship hydrodynamics (namely hydrodynamics). The fluid-solid coupling theory, which is an interdisciplinary science between ship hydrodynamics and ship structural mechanics, will become an important tool of the response analysis and the direct design for the structure of naval architecture and ocean engineering. This is one of the general development trends of ship mechanics in the 21st Century[1].

Generalized quasi-complementary energy principle of non-conservative systems is applied to the study of the fluid-solid coupling problem. The research of generalized quasi-variational principles of non-conservative systems is a very important field in many subjects. As an overseas representative, Leipholz introduced the concept of generalized self-conjugacy and established the generalized Hamilton principle, that led to the well-known Leipholz rod model[2-3]. But Leipholz only studied potential energy principle of non-conservative systems. Under the follower forces systems, the scholars of China established complementary energy principle of non-conservative systems, and then established the generalized variational principles of non-conservative systems of elastic theory in the study of Ref.[4]. The first and second generalized quasi-variational principles of non-conservative systems with two kinds of variables were established and stability behavior of a fluid-solid coupling systems was illuminated in Ref.[5].

However, there have been few published works in this field due to the level of difficulty in establishing the generalized variational principles of non-conservative systems and using them to solve practical scientific and engineering problems. According to the corresponding relations between generalized forces and generalized displacements, convolutions are performed between the basic equations of elasto-dynamics in the primary space and corresponding virtual quantities, integrated and then added algebraically. Considering that the body forces and the surface forces are both follower forces, the generalized quasi-complementary energy principle of non-conservative systems with two kinds of variables for initial value problems is established in convolutional variational integral method[6-8]. Using the generalized quasi-complementary energy principle to deal with the fluid-solid coupling problem and analyze the dynamic response of structures, the method of two kinds of

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variables for calculation of force and displacement simultaneously is derived.

2 The generalized quasi-complementary energy principle with two kinds of variables

Basic equations of linear elasto-dynamics:

$$\sigma_{ij,j}+\bar{F}_i-\dot{p}_i=0\,, \quad \text{in } V\,. \tag{1}$$

$$\sigma_{ij}n_j-\bar{T}_i=0\,, \quad \text{on } S_\sigma\,. \tag{2}$$

$$\varepsilon_{ij}-\frac{1}{2}u_{i,j}-\frac{1}{2}u_{j,i}=0\,, \quad \text{in } V\,. \tag{3}$$

$$u_i-\bar{u}_i=0\,, \quad \text{on } S_u\,. \tag{4}$$

$$\sigma_{ij}-a_{ijkl}\varepsilon_{kl}=0\,, \quad \text{in } V\,; \tag{5}$$

or

$$\varepsilon_{ij}-b_{ijkl}\sigma_{kl}=0\,, \quad \text{in } V\,. \tag{6}$$

$$\dot{u}_i-v_i=0\,, \quad \text{in } V\,. \tag{7}$$

$$p_i-\rho v_i=0\,, \quad \text{in } V\,; \tag{8}$$

or

$$v_i-\frac{1}{\rho}p_i=0\,, \quad \text{in } V\,. \tag{9}$$

Initial conditions

$$u_i(0)=\bar{u}_i(0)\,, \tag{10}$$

$$p_i(0)=\bar{p}_i(0)\,, \tag{11}$$

where, σ_{ij} is stress, ε_{ij} is strain, u_i is displacement, a_{ijkl} is stiffness coefficient, b_{ijkl} is flexibility coefficient, $\bar{F}_i$ is body force, $\bar{T}_i$ is surface force, ρ is mass density, v_i is velocity and p_i is momentum.

According to the corresponding relations between generalized forces and generalized displacements, convolutions are performed between Eqs.(1)~(4), (7) in the primary space and corresponding virtual quantities, integrated with volume and area, and then added algebraically, deriving

$$\begin{aligned}&\iiint_V[(\sigma_{ij,j}+\bar{F}_i-\dot{p}_i)*\delta u_i-(\dot{u}_i-v_i)*\delta p_i+\\&(\varepsilon_{ij}-\frac{1}{2}u_{i,j}-\frac{1}{2}u_{j,i})*\delta\sigma_{ij}]\mathrm{d}V-\\&\iint_{S_\sigma}(\sigma_{ij}n_j-\bar{T}_i)*\delta u_i\mathrm{d}S+\\&\iint_{S_u}(u_i-\bar{u}_i)*\delta\sigma_{ij}n_j\mathrm{d}S=0.\end{aligned} \tag{12}$$

Using Green's theorem and formula of convolutional integration by parts

$$-\iiint_V u_{i,j}*\delta\sigma_{ij}\mathrm{d}V=-\iint_{S_\sigma+S_u}u_i*\delta\sigma_{ij}n_j\mathrm{d}S+\iiint_V u_i*\delta\sigma_{ij,j}\mathrm{d}V. \tag{13}$$

$$-\dot{u}_i*\delta p_i=-u_i*\delta\dot{p}_i+u_i(0)\delta p_i\,. \tag{14}$$

Substituting Eqs.(13) and (14) into Eq.(12) derives

$$\begin{aligned}&\iiint_V[(\sigma_{ij,j}+\bar{F}_i-\dot{p}_i)*\delta u_i+\varepsilon_{ij}*\delta\sigma_{ij}+\\&u_i*\delta\sigma_{ij,j}-u_i*\delta\dot{p}_i+u_i(0)\delta p_i+v_i*\delta p_i]\mathrm{d}V-\\&\iint_{S_\sigma}(\sigma_{ij}n_j*\delta u_i+u_i*\delta\sigma_{ij}n_j-\bar{T}_i*\delta u_i)\mathrm{d}S-\\&\iint_{S_u}\bar{u}_i*\delta\sigma_{ij}n_j\mathrm{d}S=0.\end{aligned} \tag{15}$$

Substituting material's constitutive relations (6) and (9) into Eq.(15) derives

$$\begin{aligned}&\iiint_V[(\sigma_{ij,j}+\bar{F}_i-\dot{p}_i)*\delta u_i+b_{ijkl}\sigma_{kl}*\delta\sigma_{ij}+\\&u_i*\delta\sigma_{ij,j}-u_i*\delta\dot{p}_i+u_i(0)\delta p_i+\frac{1}{\rho}p_i*\delta p_i]\mathrm{d}V-\\&\iint_{S_\sigma}(\sigma_{ij}n_j*\delta u_i+u_i*\delta\sigma_{ij}n_j-\bar{T}_i*\delta u_i)\mathrm{d}S-\\&\iint_{S_u}\bar{u}_i*\delta\sigma_{ij}n_j\mathrm{d}S=0.\end{aligned} \tag{16}$$

Eq.(16) can be further shown as

$$\begin{aligned}&\delta\{\iiint_V[(\sigma_{ij,j}+\bar{F}_i-\dot{p}_i)*u_i+\frac{1}{2}b_{ijkl}\sigma_{ij}*\sigma_{kl}+\\&u_i(0)p_i+\frac{1}{2\rho}p_i*p_i]\mathrm{d}V-\\&\iint_{S_\sigma}(\sigma_{ij}n_j-\bar{T}_i)*u_i\mathrm{d}S-\iint_{S_u}\bar{u}_i*\sigma_{ij}n_j\mathrm{d}S\}-\\&\iiint_V u_i*\delta\bar{F}_i\mathrm{d}V-\iint_{S_\sigma}u_i*\delta\bar{T}_i\mathrm{d}S=0.\end{aligned} \tag{17}$$

Eq.(17) can be simplified as

$$\delta\Gamma_{21}-\delta Q-\delta P=0\,, \tag{18}$$

where,

$$\begin{aligned}\Gamma_{21}=&\iiint_V[\frac{1}{2\rho}p_i*p_i+\frac{1}{2}b_{ijkl}\sigma_{ij}*\sigma_{kl}+(\sigma_{ij,j}+\bar{F}_i-\dot{p}_i)*u_i+\\&u_i(0)p_i]\mathrm{d}V-\iint_{S_\sigma}(\sigma_{ij}n_j-\bar{T}_i)*u_i\mathrm{d}S-\iint_{S_u}\bar{u}_i*\sigma_{ij}n_j\mathrm{d}S\end{aligned}$$

$$\delta Q=\iiint_V u_i*\delta\bar{F}_i\mathrm{d}V\,,\quad \delta P=\iint_{S_\sigma}u_i*\delta\bar{T}_i\mathrm{d}S\,.$$

This is convolutional generalized quasi-complementary energy principle with two kinds of variables. When the body force $\bar{F}_i$ and surface force $\bar{T}_i$ are not follower forces, $\delta Q=0$, $\delta P=0$, then this theorem is degenerated into a usual convolutional complementary energy principle with two kinds of variables of elasto-dynamics.

3 Using the generalized quasi-complementary energy principle to deal with the fluid-solid coupling problem

With the development of marine air-hydrofoils theory, marine air-hydrofoils can be widely applied to warships in the ship structural design and study. Marine air-hydrofoils consist of not only hydrofoil, rudder, oar, submarine, fin-wing of mine, rotary vane of water-jet propulsion and so on, but also the whole underwater ship (such as submarine) that can be regarded as a small aspect ratio hydrofoil for the ship on the maneuvering motion. The differences between hydrofoil and aerial wing are mainly free surface and cavitation phenomenon, which have much influence on the fluid dynamic characteristic of hydrofoils. It will be complex[9-10]. Considering the dissertation of cavitation phenomenon that will be shown in another paper, the influence of free surface and cavitation phenomenon will not be considered.

Taking a hydrofoil of hydrofoil ship as an example to study some problems, it is often simplified as closed multi-cell thin walled cantilever beam. The extended part of the hydrofoil can be simplified as a model of free bending and torsion problem. Hydrodynamic forces on the hydrofoil can make it turn an angle. Conversely, the torsional angle can also change the hydrodynamic forces. Thus, this is a dynamic problem of non-conservative system. The shear force Q_y of section that is led by hydrodynamic forces is a follower force. The problem is solved by convolutional generalized quasi-complementary energy principle with two kinds of variables of elasto-dynamics.

Considering the section of three closed cells shown in Fig.1, the shear force of section is Q_y. Suppose that x_a is the coordinate of hydrodynamic center. Suppose that the mass center coincides with rigid center and the hydrofoil section is a symmetrical section. For this reason, bending is not coupled with torsion. Here just torsion deformation is considered.

Suppose that x_r is the coordinate of rigid center. By "cutting" each closed cell of the section, the bending shearing flow of "open" section is zero. The unknown shear flows of the three sections are q_1, q_2 and q_3 respectively (shown in Fig.2).

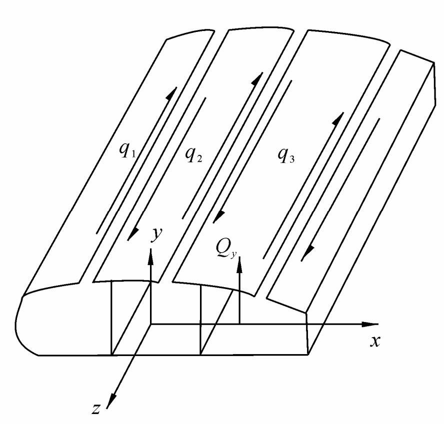

Fig.1 Section of three closed cells

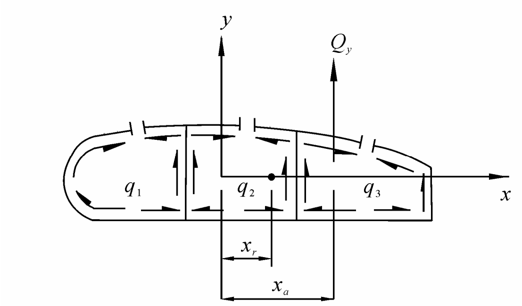

Fig.2 Unknown shear flow on section of three closed cells

Disregarding the effect of damping, study hydrofoil section per unit length, the function Γ_{21} can be expressed as

$$\Gamma_{21} = \frac{1}{2I_c} L * L + \varphi(0)L + \oint_{1-3} [\frac{(q_1+q_2+q_3)*(q_1+q_2+q_3)}{2Gt} - \rho\varphi*(q_1+q_2+q_3)]\mathrm{d}s + \varphi * M_z - \dot{L}*\varphi, \tag{19}$$

where I_c is moment of inertia about centroidal axis, G is shear modulus, t is thickness of wall, φ is relative torsional angle per unit length of section of the hydrofoil, ρ is vertical distance from infinitesimal shear force $q_i \mathrm{d}S$ about moment center (i =1,2,3), q_1, q_2 and q_3 are shear flow of three closed sections respectively, L is moment of momentum about rigid center, and $M_z = Q_y(x_a - x_r)$ is external moment (the moment about rigid center). The direction of Q_y has two kinds of expressional methods: the equivalent method, and the balancing method. The equivalent method is used in this paper.

Considering $\delta Q = 0$ (body force has no effect on this concrete problem), the convolutional generalized quasi-complementary energy principle with two kinds of variables is expressed as

$$\begin{aligned}
&\delta\Gamma_{21}-\delta P=\frac{1}{I_c}L*\delta L+(\oint_1 \frac{q_1}{Gt}\mathrm{d}s-\int_{1-2}\frac{q_2}{Gt}\mathrm{d}s-\varphi\Omega_1)*\delta q_1+\\
&(-\int_{1-2}\frac{q_1}{Gt}\mathrm{d}s+\oint_2\frac{q_2}{Gt}\mathrm{d}s-\int_{2-3}\frac{q_3}{Gt}\mathrm{d}s-\varphi\Omega_2)*\delta q_2+\\
&(-\int_{2-3}\frac{q_2}{Gt}\mathrm{d}s+\oint_3\frac{q_3}{Gt}\mathrm{d}s-\varphi\Omega_3)*\delta q_3-\\
&[\Omega_1 q_1+\Omega_2 q_2+\Omega_3 q_3-(x_a-x_r)Q_y+\dot{L}]*\delta\varphi+\\
&\varphi(0)\delta L-\varphi*\delta\dot{L}=0,
\end{aligned} \tag{20}$$

where $\Omega_1=\oint_1 \rho\mathrm{d}s$, $\Omega_2=\oint_2 \rho\mathrm{d}s$, $\Omega_3=\oint_3 \rho\mathrm{d}s$, $\oint_{1-2}$ is the closed contour integral of the first and second closed cells, $\int_{1-2}$ is the boundary integral of the first and second closed cells, $\oint_{2-3}$ is the closed contour integral of the second and third closed cells, $\int_{2-3}$ is the boundary integral of the second and third closed cells.

Using formula of convolutional integration by parts

$$-\varphi*\delta\dot{L}=-\dot{\varphi}*\delta L-\varphi(0)\delta L\,. \tag{21}$$

Substituting Eq.(21) into Eq.(20) derives

$$\begin{aligned}
&\delta\Gamma_{21}-\delta P=\frac{1}{I_c}(L-\dot{\varphi})*\delta L+\\
&(\oint_1 \frac{q_1}{Gt}\mathrm{d}s-\int_{1-2}\frac{q_2}{Gt}\mathrm{d}s-\varphi\Omega_1)*\delta q_1+\\
&(-\int_{1-2}\frac{q_1}{Gt}\mathrm{d}s+\oint_2\frac{q_2}{Gt}\mathrm{d}s-\int_{2-3}\frac{q_3}{Gt}\mathrm{d}s-\varphi\Omega_2)*\delta q_2+\\
&(-\int_{2-3}\frac{q_2}{Gt}\mathrm{d}s+\oint_3\frac{q_3}{Gt}\mathrm{d}s-\varphi\Omega_3)*\delta q_3-\\
&[\Omega_1 q_1+\Omega_2 q_2+\Omega_3 q_3-(x_a-x_r)Q_y+\dot{L}]*\delta\varphi=0.
\end{aligned} \tag{22}$$

Because of the arbitrariness of δq_0, $\delta\varphi$ and δL, take q_1, q_2 and q_3 out of integral sign with considering the basic assumption of thin-wall structure (the basic assumption of thin-wall tube torsion theory) , the following equations can be derived from Eq.(22):

$$\begin{cases}
\frac{1}{I_c}L-\dot{\varphi}=0,\\
q_1\oint_1\frac{\mathrm{d}s}{Gt}-q_2\int_{1-2}\frac{\mathrm{d}s}{Gt}=\Omega_1\varphi,\\
-q_1\int_{1-2}\frac{\mathrm{d}s}{Gt}+q_2\oint_2\frac{\mathrm{d}s}{Gt}-q_3\int_{2-3}\frac{\mathrm{d}s}{Gt}=\Omega_2\varphi,\\
-q_2\int_{2-3}\frac{\mathrm{d}s}{Gt}+q_3\oint_3\frac{\mathrm{d}s}{Gt}=\Omega_3\varphi,\\
\dot{L}+\Omega_1 q_1+\Omega_2 q_2+\Omega_3 q_3=Q_y(x_a-x_r).
\end{cases} \tag{23}$$

Solving the above equation group derives

$$q_1=\frac{\begin{vmatrix}\varphi\Omega_1 & -\int_{1-2}\frac{\mathrm{d}s}{Gt} & 0\\ \varphi\Omega_2 & \oint_2\frac{\mathrm{d}s}{Gt} & -\int_{2-3}\frac{\mathrm{d}s}{Gt}\\ \varphi\Omega_3 & -\int_{2-3}\frac{\mathrm{d}s}{Gt} & \oint_3\frac{\mathrm{d}s}{Gt}\end{vmatrix}}{\begin{vmatrix}\oint_1\frac{\mathrm{d}s}{Gt} & -\int_{1-2}\frac{\mathrm{d}s}{Gt} & 0\\ -\int_{1-2}\frac{\mathrm{d}s}{Gt} & \oint_2\frac{\mathrm{d}s}{Gt} & -\int_{2-3}\frac{\mathrm{d}s}{Gt}\\ 0 & -\int_{2-3}\frac{\mathrm{d}s}{Gt} & \oint_3\frac{\mathrm{d}s}{Gt}\end{vmatrix}}=$$

$$\frac{\varphi\Omega_1[\oint_2\frac{\mathrm{d}s}{Gt}\oint_3\frac{\mathrm{d}s}{Gt}-(\int_{2-3}\frac{\mathrm{d}s}{Gt})^2]+\varphi\Omega_2\int_{1-2}\frac{\mathrm{d}s}{Gt}\oint_3\frac{\mathrm{d}s}{Gt}+\varphi\Omega_3\int_{1-2}\frac{\mathrm{d}s}{Gt}\int_{2-3}\frac{\mathrm{d}s}{Gt}}{\oint_1\frac{\mathrm{d}s}{Gt}\oint_2\frac{\mathrm{d}s}{Gt}\oint_3\frac{\mathrm{d}s}{Gt}-\oint_1\frac{\mathrm{d}s}{Gt}(\int_{2-3}\frac{\mathrm{d}s}{Gt})^2-(\int_{1-2}\frac{\mathrm{d}s}{Gt})^2\oint_3\frac{\mathrm{d}s}{Gt}}. \tag{24}$$

$$q_2=\frac{\begin{vmatrix}\oint_1\frac{\mathrm{d}s}{Gt} & \varphi\Omega_1 & 0\\ -\int_{1-2}\frac{\mathrm{d}s}{Gt} & \varphi\Omega_2 & -\int_{2-3}\frac{\mathrm{d}s}{Gt}\\ 0 & \varphi\Omega_3 & \oint_3\frac{\mathrm{d}s}{Gt}\end{vmatrix}}{\begin{vmatrix}\oint_1\frac{\mathrm{d}s}{Gt} & -\int_{1-2}\frac{\mathrm{d}s}{Gt} & 0\\ -\int_{1-2}\frac{\mathrm{d}s}{Gt} & \oint_2\frac{\mathrm{d}s}{Gt} & -\int_{2-3}\frac{\mathrm{d}s}{Gt}\\ 0 & -\int_{2-3}\frac{\mathrm{d}s}{Gt} & \oint_3\frac{\mathrm{d}s}{Gt}\end{vmatrix}}= \tag{25}$$

$$\frac{\varphi\Omega_1\int_{1-2}\frac{\mathrm{d}s}{Gt}\oint_3\frac{\mathrm{d}s}{Gt}+\varphi\Omega_2\oint_1\frac{\mathrm{d}s}{Gt}\oint_3\frac{\mathrm{d}s}{Gt}+\varphi\Omega_3\oint_1\frac{\mathrm{d}s}{Gt}\int_{2-3}\frac{\mathrm{d}s}{Gt}}{\oint_1\frac{\mathrm{d}s}{Gt}\oint_2\frac{\mathrm{d}s}{Gt}\oint_3\frac{\mathrm{d}s}{Gt}-\oint_1\frac{\mathrm{d}s}{Gt}(\int_{2-3}\frac{\mathrm{d}s}{Gt})^2-(\int_{1-2}\frac{\mathrm{d}s}{Gt})^2\oint_3\frac{\mathrm{d}s}{Gt}}.$$

$$q_3=\frac{\begin{vmatrix}\oint_{1}\frac{\mathrm{d}s}{Gt} & -\int_{1-2}\frac{\mathrm{d}s}{Gt} & \varphi\Omega_1\\ -\int_{1-2}\frac{\mathrm{d}s}{Gt} & \oint_{2}\frac{\mathrm{d}s}{Gt} & \varphi\Omega_2\\ 0 & -\int_{2-3}\frac{\mathrm{d}s}{Gt} & \varphi\Omega_3\end{vmatrix}}{\begin{vmatrix}\oint_{1}\frac{\mathrm{d}s}{Gt} & -\int_{1-2}\frac{\mathrm{d}s}{Gt} & 0\\ -\int_{1-2}\frac{\mathrm{d}s}{Gt} & \oint_{2}\frac{\mathrm{d}s}{Gt} & -\int_{2-3}\frac{\mathrm{d}s}{Gt}\\ 0 & -\int_{2-3}\frac{\mathrm{d}s}{Gt} & \oint_{3}\frac{\mathrm{d}s}{Gt}\end{vmatrix}}=$$

$$\frac{\varphi\Omega_1\int_{1-2}\frac{\mathrm{d}s}{Gt}\int_{2-3}\frac{\mathrm{d}s}{Gt}+\varphi\Omega_2\oint_{1}\frac{\mathrm{d}s}{Gt}\int_{2-3}\frac{\mathrm{d}s}{Gt}+\varphi\Omega_3[\oint_{1}\frac{\mathrm{d}s}{Gt}\oint_{2}\frac{\mathrm{d}s}{Gt}-(\int_{1-2}\frac{\mathrm{d}s}{Gt})^2]}{\oint_{1}\frac{\mathrm{d}s}{Gt}\oint_{2}\frac{\mathrm{d}s}{Gt}\oint_{3}\frac{\mathrm{d}s}{Gt}-\oint_{1}\frac{\mathrm{d}s}{Gt}(\int_{2-3}\frac{\mathrm{d}s}{Gt})^2-(\int_{1-2}\frac{\mathrm{d}s}{Gt})^2\oint_{3}\frac{\mathrm{d}s}{Gt}}. \tag{26}$$

$$L=I_c\dot{\varphi}. \tag{27}$$

Let's convert the shear force Q_y into related form with torsional angle φ, i.e. $Q_y=qAC_y^{\alpha}(\alpha-\varphi)$, where q is velocity pressure, A is area of hydrofoil section, taking different hydrofoil section parameters to different concrete expressions of A, C_y^{α} is lift coefficient derivative, α is attack angle of hydrofoil section, φ is torsional angle of hydrofoil section.

Substituting the above equation into the fifth equation of Eq.(23) derives

$$\ddot{\varphi}+\frac{1}{I_c}\{\frac{\Omega_1[\oint_{2}\frac{\mathrm{d}s}{Gt}\oint_{3}\frac{\mathrm{d}s}{Gt}-(\int_{2-3}\frac{\mathrm{d}s}{Gt})^2]+\Omega_2\int_{1-2}\frac{\mathrm{d}s}{Gt}\oint_{3}\frac{\mathrm{d}s}{Gt}+\Omega_3\int_{1-2}\frac{\mathrm{d}s}{Gt}\int_{2-3}\frac{\mathrm{d}s}{Gt}}{\oint_{1}\frac{\mathrm{d}s}{Gt}\oint_{2}\frac{\mathrm{d}s}{Gt}\oint_{3}\frac{\mathrm{d}s}{Gt}-\oint_{1}\frac{\mathrm{d}s}{Gt}(\int_{2-3}\frac{\mathrm{d}s}{Gt})^2-(\int_{1-2}\frac{\mathrm{d}s}{Gt})^2\oint_{3}\frac{\mathrm{d}s}{Gt}}\Omega_1$$
$$+\frac{\Omega_1\int_{1-2}\frac{\mathrm{d}s}{Gt}\oint_{3}\frac{\mathrm{d}s}{Gt}+\Omega_2\oint_{1}\frac{\mathrm{d}s}{Gt}\oint_{3}\frac{\mathrm{d}s}{Gt}+\Omega_3\oint_{1}\frac{\mathrm{d}s}{Gt}\int_{2-3}\frac{\mathrm{d}s}{Gt}}{\oint_{1}\frac{\mathrm{d}s}{Gt}\oint_{2}\frac{\mathrm{d}s}{Gt}\oint_{3}\frac{\mathrm{d}s}{Gt}-\oint_{1}\frac{\mathrm{d}s}{Gt}(\int_{2-3}\frac{\mathrm{d}s}{Gt})^2-(\int_{1-2}\frac{\mathrm{d}s}{Gt})^2\oint_{3}\frac{\mathrm{d}s}{Gt}}\Omega_2$$
$$+\frac{\Omega_1\int_{1-2}\frac{\mathrm{d}s}{Gt}\int_{2-3}\frac{\mathrm{d}s}{Gt}+\Omega_2\oint_{1}\frac{\mathrm{d}s}{Gt}\int_{2-3}\frac{\mathrm{d}s}{Gt}+\Omega_3[\oint_{1}\frac{\mathrm{d}s}{Gt}\oint_{2}\frac{\mathrm{d}s}{Gt}-(\int_{1-2}\frac{\mathrm{d}s}{Gt})^2]}{\oint_{1}\frac{\mathrm{d}s}{Gt}\oint_{2}\frac{\mathrm{d}s}{Gt}\oint_{3}\frac{\mathrm{d}s}{Gt}-\oint_{1}\frac{\mathrm{d}s}{Gt}(\int_{2-3}\frac{\mathrm{d}s}{Gt})^2-(\int_{1-2}\frac{\mathrm{d}s}{Gt})^2\oint_{3}\frac{\mathrm{d}s}{Gt}}\Omega_3$$
$$+(x_a-x_r)qAC_y^{\alpha}\}\varphi=\frac{1}{I_c}(x_a-x_r)qAC_y^{\alpha}\alpha. \tag{28}$$

The natural frequency of system is written as

$$\omega_n=\frac{1}{I_c^2}\{\frac{\Omega_1[\oint_{2}\frac{\mathrm{d}s}{Gt}\oint_{3}\frac{\mathrm{d}s}{Gt}-(\int_{2-3}\frac{\mathrm{d}s}{Gt})^2]+\Omega_2\int_{1-2}\frac{\mathrm{d}s}{Gt}\oint_{3}\frac{\mathrm{d}s}{Gt}+\Omega_3\int_{1-2}\frac{\mathrm{d}s}{Gt}\int_{2-3}\frac{\mathrm{d}s}{Gt}}{\oint_{1}\frac{\mathrm{d}s}{Gt}\oint_{2}\frac{\mathrm{d}s}{Gt}\oint_{3}\frac{\mathrm{d}s}{Gt}-\oint_{1}\frac{\mathrm{d}s}{Gt}(\int_{2-3}\frac{\mathrm{d}s}{Gt})^2-(\int_{1-2}\frac{\mathrm{d}s}{Gt})^2\oint_{3}\frac{\mathrm{d}s}{Gt}}\Omega_1+$$
$$\frac{\Omega_1\int_{1-2}\frac{\mathrm{d}s}{Gt}\oint_{3}\frac{\mathrm{d}s}{Gt}+\Omega_2\oint_{1}\frac{\mathrm{d}s}{Gt}\oint_{3}\frac{\mathrm{d}s}{Gt}+\Omega_3\oint_{1}\frac{\mathrm{d}s}{Gt}\int_{2-3}\frac{\mathrm{d}s}{Gt}}{\oint_{1}\frac{\mathrm{d}s}{Gt}\oint_{2}\frac{\mathrm{d}s}{Gt}\oint_{3}\frac{\mathrm{d}s}{Gt}-\oint_{1}\frac{\mathrm{d}s}{Gt}(\int_{2-3}\frac{\mathrm{d}s}{Gt})^2-(\int_{1-2}\frac{\mathrm{d}s}{Gt})^2\oint_{3}\frac{\mathrm{d}s}{Gt}}\Omega_2+$$
$$\frac{\Omega_1\int_{1-2}\frac{\mathrm{d}s}{Gt}\int_{2-3}\frac{\mathrm{d}s}{Gt}+\Omega_2\oint_{1}\frac{\mathrm{d}s}{Gt}\int_{2-3}\frac{\mathrm{d}s}{Gt}+\Omega_3[\oint_{1}\frac{\mathrm{d}s}{Gt}\oint_{2}\frac{\mathrm{d}s}{Gt}-(\int_{1-2}\frac{\mathrm{d}s}{Gt})^2]}{\oint_{1}\frac{\mathrm{d}s}{Gt}\oint_{2}\frac{\mathrm{d}s}{Gt}\oint_{3}\frac{\mathrm{d}s}{Gt}-\oint_{1}\frac{\mathrm{d}s}{Gt}(\int_{2-3}\frac{\mathrm{d}s}{Gt})^2-(\int_{1-2}\frac{\mathrm{d}s}{Gt})^2\oint_{3}\frac{\mathrm{d}s}{Gt}}\Omega_3+$$
$$(x_a-x_r)qAC_y^{\alpha}\}^{\frac{1}{2}}. \tag{29}$$

The quantity group is written as

$$F_0=\frac{(x_a-x_r)qAC_{\alpha}^{y}\alpha}{I_c}. \tag{30}$$

Then Eq.(28) is rewritten as

$$\ddot{\varphi}+\omega_n^2\varphi-F_0=0. \tag{31}$$

From Eq.(31) and initial conditions at $t=0:\varphi=\varphi(0)$, $\dot{\varphi}=\dot{\varphi}(0)$, the following equation can be derived:

$$\varphi=[\varphi(0)-\frac{F_0}{\omega_n^2}]\cos\omega_n t+\frac{\dot{\varphi}(0)}{\omega_n}\sin\omega_n t+\frac{F_0}{\omega_n^2}. \tag{32}$$

Further on,

$$q_1=\frac{\Omega_1[\oint_{2}\frac{\mathrm{d}s}{Gt}\oint_{3}\frac{\mathrm{d}s}{Gt}-(\int_{2-3}\frac{\mathrm{d}s}{Gt})^2]+\Omega_2\int_{1-2}\frac{\mathrm{d}s}{Gt}\oint_{3}\frac{\mathrm{d}s}{Gt}+\Omega_3\int_{1-2}\frac{\mathrm{d}s}{Gt}\int_{2-3}\frac{\mathrm{d}s}{Gt}}{\oint_{1}\frac{\mathrm{d}s}{Gt}\oint_{2}\frac{\mathrm{d}s}{Gt}\oint_{3}\frac{\mathrm{d}s}{Gt}-\oint_{1}\frac{\mathrm{d}s}{Gt}(\int_{2-3}\frac{\mathrm{d}s}{Gt})^2-(\int_{1-2}\frac{\mathrm{d}s}{Gt})^2\oint_{3}\frac{\mathrm{d}s}{Gt}}$$
$$\{[\varphi(0)-\frac{F_0}{\omega_n^2}]\cos\omega_n t+\frac{\dot{\varphi}(0)}{\omega_n}\sin\omega_n t+\frac{F_0}{\omega_n^2}\}. \tag{33}$$

$$q_2=\frac{\Omega_1\int_{1-2}\frac{\mathrm{d}s}{Gt}\oint_{3}\frac{\mathrm{d}s}{Gt}+\Omega_2\oint_{1}\frac{\mathrm{d}s}{Gt}\oint_{3}\frac{\mathrm{d}s}{Gt}+\Omega_3\oint_{1}\frac{\mathrm{d}s}{Gt}\int_{2-3}\frac{\mathrm{d}s}{Gt}}{\oint_{1}\frac{\mathrm{d}s}{Gt}\oint_{2}\frac{\mathrm{d}s}{Gt}\oint_{3}\frac{\mathrm{d}s}{Gt}-\oint_{1}\frac{\mathrm{d}s}{Gt}(\int_{2-3}\frac{\mathrm{d}s}{Gt})^2-(\int_{1-2}\frac{\mathrm{d}s}{Gt})^2\oint_{3}\frac{\mathrm{d}s}{Gt}}$$
$$\{[\varphi(0)-\frac{F_0}{\omega_n^2}]\cos\omega_n t+\frac{\dot{\varphi}(0)}{\omega_n}\sin\omega_n t+\frac{F_0}{\omega_n^2}\}. \tag{34}$$

$$q_3=\frac{\Omega_1\int_{1-2}\frac{\mathrm{d}s}{Gt}\int_{2-3}\frac{\mathrm{d}s}{Gt}+\Omega_2\oint_{1}\frac{\mathrm{d}s}{Gt}\int_{2-3}\frac{\mathrm{d}s}{Gt}+\Omega_3[\oint_{1}\frac{\mathrm{d}s}{Gt}\oint_{2}\frac{\mathrm{d}s}{Gt}-(\int_{1-2}\frac{\mathrm{d}s}{Gt})^2]}{\oint_{1}\frac{\mathrm{d}s}{Gt}\oint_{2}\frac{\mathrm{d}s}{Gt}\oint_{3}\frac{\mathrm{d}s}{Gt}-\oint_{1}\frac{\mathrm{d}s}{Gt}(\int_{2-3}\frac{\mathrm{d}s}{Gt})^2-(\int_{1-2}\frac{\mathrm{d}s}{Gt})^2\oint_{3}\frac{\mathrm{d}s}{Gt}}$$
$$\{[\varphi(0)-\frac{F_0}{\omega_n^2}]\cos\omega_n t+\frac{\dot{\varphi}(0)}{\omega_n}\sin\omega_n t+\frac{F_0}{\omega_n^2}\}. \tag{35}$$

4 Discussion

1) Investigators researching on non-conservative systems found out that, although non-conservative systems cover many fields, it's not easy to give some proper examples of follower force systems. Forces which act on conservative systems will lead to the effect that consists of intensity, rigidity and stability. Forces which act on non-conservative systems will lead to the effect that can change the forces. Thus, these forces are follower forces. The follower force is associated with the effect that is led by follower force. Since such characteristic of the follower force is known, it can be found easily.

2) Above ideas are to use the generalized quasi-complementary energy principle to obtain the analytical solution. How is the approximating solution obtained with the generalized quasi-complementary energy principle? According to the ideas of this paper, convolutional generalized quasi-complementary energy principle with two kinds of variables is deduced. Then approximate calculation is done by convolutional spline function[11,12]. Analytical solution and approximating solution are obtained by the generalized quasi-complementary energy principle. The approximating solution agrees well with the analytical solution. Due to the limit of space, they are not described in full details.

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应用广义拟余能原理研究流固耦合问题

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摘　要：水动力学与固体力学交叉的流固耦合理论是船舶与海洋工程结构响应分析与直接设计的重要工具．本文应用卷变积方法，按照广义力和广义位移之间的对应关系，将弹性动力学的基本方程卷乘上相应的虚量，然后积分，代数相加，并考虑到体积力和面积力均为伴生力，建立了非保守系统初值问题的两类变量的广义拟余能原理．应用广义拟余能原理研究流固耦合问题，分析了结构的动力响应，给出同时求解力类量和位移类量两类变量的计算方法．

关键词：流固耦合；弹性动力学；广义拟余能原理；动力响应