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The Casimir effect in two dimensional black hole spacetime with a global monopole*

WANG Xiao-xia, YANG Shu-zheng
(Institute of Theoretical Physics, China West Normal University, Nanchong 637002, China)

Abstract: A thin layer of the event horizon vicinity under the two-dimension black hole spacetime with a global monopole is considered as a system of the Casimir type. The energy-momentum tensor is derived in Boulware vacuum, Hartle-Hawking vacuum and Unruh vacuum respectively. The values are derived in the massless scalar field which satisfies the Dirichlet boundary conditions. By the Wald's axioms, the result is the same with one by the usual regularized methods. Meanwhile, energy, energy density, and pressure acting on Dirichlet walls at the asymptotically flat background are also calculated. According to the energy, the Casimir force is derived.

Key words: Casimir effect; two dimensional black hole; global monopole
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Casimir effect is caused by the vacuum quantum fluctuation of the electromagnetic field. This effect is introduced by Casimir to calculate the vacuum energy appling the corresponding boundary conditions in the quantum field theory. He also predicted that the attractive force between the two infinitely large parallel conducting plates was related to the Planck parameter, the light velocity and the geometrical configuration[1,2], but was independent on charges. Afterward, Casimir and Poldtre also explained the attractive force between the two uncharged objects as the delay of Van der Waals effect. The phenomenon that uncharged parallel plates attract each other is called the Casimir effect, and the corresponding force is called the Casimir one. During the process of studying the Casimir effect, the Casimir energy, which is margin of the vacuum energy between all kinds of fields with various boundaries and the fields without the boundaries, should be researched. Therefore, the Casimir force can also be derived.

Casimir effect plays an important role in many fields such as quantum field theory, atomic and molecular physics, condensed matter physics, gravitation, cosmology and mathematical physics etc. Many factors can affect the Casimir effect, such as constrains, the space topology, temperature etc. Particulaily the flat background or the curved background is one of the important ones. In reference[3-5], the Casimir effect of the massless scalar field have been investigated in several different backgrounds. But the Casimir effect in the global monopole background has not been investigated up to now.

In this paper, considering a thin layer of the event horizon vicinity under the given black hole spacetime as a system of the Casimir style, we investigate the Casimir effect of the massless scalar field in the global monopole background. When we investigate the Casimir effect, we must calculate the renormalized energy-moment tensor. Since the energy-moment tensor of the constrained field can not be easily calculated, we only do our research in the two-dimension spacetime. This two-dimension black hole is asymptotically flat and satisfies

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Dirichlet boundary conditions. Although many regularized methods can be used[6−12], such as Green's function method, zeta function regularization, dimensional regularization, and point-splitting method etc., we apply the Wald's axioms[13, 14] and get the result derived by the usual regularized methods. Meanwhile the Casimir effect is discussed. The Wald's axioms are as follows:

(1) Expectation values of energy-momentum are conserved.

(2) In the Minkowski spacetime, the stand result should be obtained.

(3) For off-diagonal elements, the stand result should be obtained.

(4) Causality holds.

(5) The energy-momentum tensor contains no local curvature tensor depending on derivatives of the matrix higher than second order.

This paper is organized as follows: In section Ⅰ the geometrical property of the given black hole is described and the useful geometrical quantities are also calculated. And the most general form of the energy momentum tensor is obtained. In section Ⅱ, The expectation values of the renormalized energy momentum tensors for the massless scalar field in the given spacetime are calculated in Boulware vacuum, Hartle-Hawking vacuum and Unruh vacuum. Meanwhile we calculate the energy, energy density, and pressure acting on Dirichlet walls at the asymptotically flat background, and derive the Casimir force. Finally, in section Ⅲ, the results are discussed.

1 The background spacetime and the general form of the energy momentum tensor

As the background of this paper, the line element of the two-dimension black hole with a global monopole is[15]

$$ds^2 = -\left(1 - 8\pi\eta^2 - \frac{2m}{r} + \frac{Q^2}{r^2}\right)dt^2 + \left(1 - 8\pi\eta^2 - \frac{2m}{r} + \frac{Q^2}{r^2}\right)^{-1}dr^2, \tag{1}$$

where m, η and Q are the mass, the scale of the symmetry breaking and the charge of the black hole respectively. The even horizon of the black hole is

$$r_H = \frac{m + \sqrt{m^2 - (1 - 8\pi\eta^2)Q^2}}{1 - 8\pi\eta^2}. \tag{2}$$

The Hawking temperature T_H of the event horizon is

$$T_H = \frac{(1 - 8\pi\eta^2)^2\sqrt{P}}{2\pi}, \tag{3}$$

where $P = \dfrac{m^2 - (1 - 8\pi\eta^2)Q^2}{\left(m + \sqrt{M^2 - (1 - 8\pi\eta^2)Q^2}\right)^4}$, in order to reach the best conclusions in the Minkowski spacetime, we make the conformal transformation to the line element (1), and obtain

$$ds^2 = f(r)(-dt^2 + dR^2), \tag{4}$$

where

$$f(r) = 1 - 8\pi\eta^2 - \frac{2m}{r},\quad \frac{dr}{dR} = f(r). \tag{5}$$

The non-zero Christoffel symbols of the line element (4) are

$$\Gamma^R_{tt} = \Gamma^t_{tR} = \Gamma^t_{Rt} = \Gamma^R_{RR} = \frac{1}{2}\frac{\partial f(r)}{\partial r} = \frac{mr - Q^2}{r^3}. \tag{6}$$

The Ricci scalar of the line element (4) is

$$R(r) = \frac{-2(2mr - 3Q^2)}{r^4}. \tag{7}$$

In the process of regularization, the trace is non-zero, and is given as follows[16−19]

$$T^{\alpha}_{\alpha}(r) = \frac{R(r)}{24\pi}. \tag{8}$$

For the two-dimension black hole, the trace of the energy-momentum tensor is

$$T^{\alpha}_{\alpha}(r) = \frac{-(2mr - 3Q^2)}{12\pi r^4}. \tag{9}$$

Based on Wald's first axiom, the renormalized energy-momentum tensor satisfies the conservation equation

$$T^{\mu}_{\nu;\mu} = 0, \tag{10}$$

and can split into equations[3-5]

$$\frac{\mathrm{d}T^{R}_{t}}{\mathrm{d}R} + \Gamma^{R}_{RR}T^{R}_{t} - \Gamma^{R}_{tt}T^{t}_{R} = 0, \tag{11}$$

$$\frac{\mathrm{d}T^{R}_{R}}{\mathrm{d}R} + \Gamma^{t}_{tR}T^{R}_{R} - \Gamma^{t}_{Rt}T^{t}_{t} = 0. \tag{12}$$

Since $T^{t}_{R} = -T^{R}_{t}$ and $T^{t}_{t} = T^{\alpha}_{\alpha} - T^{R}_{R}$, we get

$$\frac{\mathrm{d}T^{R}_{t}}{\mathrm{d}R} + 2\Gamma^{t}_{tR}T^{R}_{t} = 0, \tag{13}$$

$$\frac{\mathrm{d}T^{R}_{R}}{\mathrm{d}R} + 2\Gamma^{t}_{tR}T^{R}_{R} = \Gamma^{t}_{tR}T^{R}_{R}. \tag{14}$$

Substituting Eq.(6) into Eq.(13), we can get

$$\frac{\mathrm{d}}{\mathrm{d}r}(f(r)T^{R}_{t}) = 0. \tag{15}$$

The solution of Eq.(15) is

$$T^{R}_{t} = \alpha f^{-1}(r), \tag{16}$$

where α is a constant of integration. In the same way, Eq.(14) becomes

$$\frac{\mathrm{d}}{\mathrm{d}r}(f(r)T^{R}_{R}) = \frac{1}{2}\frac{\partial f(r)}{\partial r}T^{\alpha}_{\alpha}. \tag{17}$$

The solution of Eq.(17) is

$$T^{R}_{R} = \frac{1}{f(r)}[H(r) + \beta], \tag{18}$$

where

$$H(r) = \frac{1}{2}\int_{H}^{r}\frac{\mathrm{d}f(r')}{\mathrm{d}r'}T^{\alpha}_{\alpha}(r')\mathrm{d}r', \tag{19}$$

and β is a constant of integration, while the point r_H the position of the event horizon. From Equations (2), (6) and (9), Eq.(19) becomes

$$H(r) = \frac{1}{24\pi}\left(\frac{m^2}{r^4} - \frac{2mQ^2}{r^5} + \frac{Q^4}{r^6}\right) - D, \tag{20}$$

where $D = \frac{1}{24\pi}\left(\frac{m^2}{r_+^4} - \frac{2mQ^2}{r_+^5} + \frac{Q^4}{r_+^6}\right)$ and r_+ is the outer event horizon of the black hole. The limiting values of $H(r)$ are follows as

if $r \to -\infty (r \to r_H)$ then $H(r) = 0$,

if $r \to +\infty$ then $H(r) = -\frac{1}{24\pi}\left(\frac{m^2}{r_+^4} - \frac{2mQ^2}{r_+^5} + \frac{Q^4}{r_+^6}\right) = -D$.

In any two-dimension background, the most general form of the energy-momentum tensor is given by

$$T^{\mu}_{\nu} = \begin{bmatrix} T^{\alpha}_{\alpha}(r) - f^{-1}(r)H(r) & 0 \\ 0 & f^{-1}(r)H(r) \end{bmatrix} + f^{-1}(r)\begin{bmatrix} -\beta & -\alpha \\ \alpha & \beta \end{bmatrix}. \tag{21}$$

Where α and β are two unknown parameters. We will apply the second and the third Wald's axioms to determine their values.

We put two "parallel plates" at r_H and r'_H ($r'_H = r_H + L$) in the massless scalar field. The massless scalar field satisfies Dirichlet boundary condition at r_H and r'_H. Next we will investigate the renormalized energy-momentum tensor in different vacua.

2 The energy-momentum tensor in different vacua

2.1 Boulware vacuum In Boulware[20], is no particles exists at infinity. The renormalized energy-momentum tensor should coincide at infinity with the standard energy-momentum tensor in the Minkowski spacetime[21,22], e.g.

$$T^{\mu}_{\nu} = \frac{\pi}{24L^2}\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{22}$$

Since the black hole has asymptotically flat spacetime, e.g. Minkowski spacetime at inifnity, for $r \to +\infty$, the Eq.(21) should coincide with the Eq.(22). Thus we can get the parameter α and β as

$$\alpha = 0, \quad \beta = \frac{\pi(1 - 8\pi\eta^2)}{24L^2} + D. \tag{23}$$

Therefore, the renormalized energy-momentum tensor in the Boulware vacuum is

$$T^{(\eta)\mu}_{\nu} = \begin{bmatrix} T^{\alpha}_{\alpha}(r) - f^{-1}(r)H(r) & 0 \\ 0 & f^{-1}(r)H(r) \end{bmatrix} + f^{-1}(r)\left[\frac{\pi(1 - 8\pi\eta^2)}{24L^2} + D\right]\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{24}$$

Eq.(24) can be written as

$$T^{(\eta)\mu}_{\nu} = T^{\mu}_{\nu(\mathrm{gravitational})} + T^{\mu}_{\nu(\mathrm{boundary})}, \tag{25}$$

where η denotes the renormalized energy-momentum tensor in the Boulware vacuum[14]. The first term denotes the contribution due to the gravitation background, and the second term denotes the contribution due to the boundary.

For $r \to +\infty$, the energy density, pressure, and energy are given by

$$\rho = T^{(\eta)t}_{t} = -\frac{\pi}{24L^2}, \tag{26}$$

$$p = -T^{(\eta)R}_{R} = -\frac{\pi}{24L^2}, \tag{27}$$

$$E(L) = \int_{r_1}^{r_1+L} \rho \, \mathrm{d}R = -\frac{\pi}{24L}. \tag{28}$$

The corresponding Casimir force between the boundaries is

$$F(L) = -\frac{\partial E(L)}{\partial L} = -\frac{\pi}{24L^2} < 0, \tag{29}$$

and is an attractive force.

2.2 Hartle-Hawking vacuum In Hartle-Hawking vacuum[23], the black hole is in thermal equilibrium state with an infinite reservoir of black body radiation with temperature T, and the standard energy-momentum tensor (22) will be modified by an additional term for the thermal equilibrium with temperature T.

$$T^{\mu}_{\nu} = \frac{\pi T^2}{12}\begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix} = \frac{\pi T^2}{6}\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \tag{30}$$

where T is the Hawking temperature of the black hole, the standard energy-momentum tensor becomes

$$T^{\mu}_{\nu} = \frac{\pi}{24L^2}\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{(1 - 8\pi\eta^2)^4 P}{24\pi}\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{31}$$

For $r \to +\infty$, the Eq.(21) should coincide with the Eq.(31). Thus we can get the parameter α and β as

$$\alpha = 0, \tag{32}$$

$$\beta = \frac{\pi(1 - 8\pi\eta^2)}{24L^2} + \frac{(1 - 8\pi\eta^2)^5 P}{24\pi} + D. \tag{33}$$

The renormalized energy-momentum tensor is

$$T_{\nu}^{(\upsilon)\mu} = \begin{bmatrix} T_{\alpha}^{\alpha}(r) - f^{-1}(r)H(r) & 0 \\ 0 & f^{-1}(r)H(r) \end{bmatrix} + f^{-1}(r)$$

$$\left[\frac{\pi(1-8\pi\eta^2)}{24L^2} + \frac{(1-8\pi\eta^2)^5 P}{24\pi} + D\right]\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \tag{34}$$

where υ denotes the renormalized energy-momentum tensor in Hartle-Hawking vacuum[14]. Thus the energy-momentum tensor $T_{\nu}^{(\upsilon)\mu}$ is

$$T_{\nu}^{(\upsilon)\mu} = T_{\nu(\text{gravitational})}^{\mu} + T_{\nu(\text{boundary})}^{\mu} + T_{\nu(\text{bath})}^{\mu}, \tag{35}$$

where the last term denotes the contribution due to thermal bath with temperature $T_{\rm H}$.

For $r \to +\infty$, the energy density, pressure, and energy are given as follows

$$\rho = T_{\rm t}^{(\upsilon)\rm t} = -\left[\frac{\pi}{24L^2} + \frac{(1-8\pi\rho^2)^4 P}{24\pi}\right], \tag{36}$$

$$p = -T_{\rm R}^{(\upsilon)\rm R} = -\left[\frac{\pi}{24L^2} + \frac{(1-8\pi\eta^2)^4}{24\pi}P\right], \tag{37}$$

$$E(L, T_{\rm H}) = \int_{r_1}^{r_1+L} \rho \mathrm{d}R = -\left[\frac{\pi}{24L} + \frac{(1-8\pi\eta^2)^4 LP}{24\pi}\right] = -\left(\frac{\pi}{24L} + \frac{\pi L}{6}T_{\rm H}^2\right). \tag{38}$$

The corresponding Casimir force F between the boundaries is

$$F(L, T_{\rm H}) = -\left(\frac{\partial E(L, T_{\rm H})}{\partial L}\right)_{T_{\rm H}} = -\frac{\pi}{24L^2} + \frac{(1-8\pi\eta^2)^4 P}{24\pi} = -\frac{\pi}{24L^2} + \frac{\pi}{6}T_{\rm H}^2 \tag{39}$$

and is not always an attractive force. It is clear that the Casimir force has the following properties:

(a) attractive,

$$L < \frac{1}{2T_{\rm H}} = \frac{\pi}{(1-8\pi\eta^2)^2\sqrt{P}}; \tag{40}$$

(b) zero,

$$L = \frac{1}{2T_{\rm H}} = \frac{\pi}{(1-8\pi\eta^2)^2\sqrt{P}}; \tag{41}$$

(c) repulsive,

$$L > \frac{1}{2T_{\rm H}} = \frac{\pi}{(1-8\pi\eta^2)^2\sqrt{P}}. \tag{42}$$

Therefore the distance L dominates property of the Casimir force.

2.3 Unruh vacuum In unruh vacuum[24], the two-dimensional black hole is in a thermal state with Hawking temperature $T_{\rm H}$[25, 26]. Because of Hawking radiation, an outward flux can be detected at infinity in the vacuum. The standard energy-momentum tensor (22) will be modified by an additional term

$$T_{\nu}^{\mu} = \frac{\pi T_{\rm H}^2}{12}\begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} = \frac{(1-8\pi\eta^2)^4 P}{48\pi}\begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}. \tag{43}$$

The renormalized energy-momentum tensor (21) should now coincide at infinity with the following stress tensor

$$T_{\nu}^{\mu} = \frac{\pi}{24L^2}\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{(1-8\pi\eta^2)^4 P}{48\pi}\begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}. \tag{44}$$

Therefore we get

$$\alpha = \frac{(1-8\pi\eta^2)^4 P}{48\pi}, \tag{45}$$

$$\beta = \frac{\pi(1-8\pi\eta^2)}{24L^2} + \frac{(1-8\pi\eta^2)^5 P}{48\pi} + D. \tag{46}$$

The renormalized energy-momentum tensor $T_{\nu}^{(\xi)\mu}$ becomes

$$T_{\nu}^{(\xi)\mu}=\begin{bmatrix} T_{\alpha}^{\alpha}(r)-f^{-1}(r)H(r) & 0 \\ 0 & f^{-1}(r)H(r)\end{bmatrix}+f^{-1}(r)$$
$$\begin{bmatrix} -\frac{\pi(1-8\pi\eta^2)}{24L^2}-\frac{(1-8\pi\eta^2)^5P}{48\pi}-D & -\frac{(1-8\pi\eta^2)^4P}{48\pi} \\ \frac{(1-8\pi\eta^2)^4P}{48\pi} & \frac{\pi(1-8\pi\eta^2)}{24L^2}+\frac{(1-8\pi\eta^2)^5P}{48\pi}+D\end{bmatrix}, \tag{47}$$

where ξ denotes the renormalized energy-momentum tensor in Unruh vacuum[14]. The renormalized energy-momentum tensor $T_{\nu}^{(\xi)\mu}$ is

$$T_{\nu}^{(\xi)\mu}=T_{\nu(\text{gravitational})}^{\mu}+T_{\nu(\text{boundary})}^{\mu}+T_{\nu(\text{radiation})}^{\mu}, \tag{48}$$

where the last term represents the contribution due to Hawking radiation.

In the vacuum, for $r\to\infty$, the detected energy density, pressure, and energy at infinity are given as follows

$$\rho=T_{t}^{(\xi)t}=-\left[\frac{\pi}{24L^2}+\frac{(1-8\pi\eta^2)^4P}{48\pi}\right], \tag{49}$$

$$p=-T_{R}^{(\xi)R}=-\left[\frac{\pi}{24L^2}+\frac{(1-8\pi\eta^2)^4}{48\pi}P\right], \tag{50}$$

$$E(L,T_H)=\int_{r_1}^{r_1+L}\rho\,dR=-\left[\frac{\pi}{24L}+\frac{(1-8\pi\eta^2)^4LP}{48\pi}\right]=-\left(\frac{\pi}{24L}+\frac{\pi L}{12}T_H^2\right), \tag{51}$$

The Casimir force F between the boundaries is

$$F(L,T_H)=-\left(\frac{\partial E(L,T_H)}{\partial L}\right)_{T_H}=-\frac{\pi}{24L^2}+\frac{(1-8\pi\eta^2)^4P}{48\pi}=-\frac{\pi}{24L^2}+\frac{\pi}{12}T_H^2, \tag{52}$$

and has following properties:

(a) attractive,

$$L<\frac{1}{\sqrt{2}T_H}=\frac{\sqrt{2}\pi}{(1-8\pi\eta^2)^2\sqrt{P}}; \tag{53}$$

(b) zero,

$$L=\frac{1}{\sqrt{2}T_H}=\frac{\sqrt{2}\pi}{(1-8\pi\eta^2)^2\sqrt{P}}; \tag{54}$$

(c) repulsive,

$$L>\frac{1}{\sqrt{2}T_H}=\frac{\sqrt{2}\pi}{(1-8\pi\eta^2)^2\sqrt{P}}. \tag{55}$$

The property of the Casimir force is also decided by the distance L.

Removing the last term in Eqs.(39) and (52) respectively, we can derive the net force. The reason is that in both vacua (Hartle-Hawking and Unruuh vacua) the forces acting on both sides of each Dirichlet wall are the same, and their total contribution to the net force is zero[13]. Therefore the net force acting on the Dirichlet walls is

$$F_{\text{net}}=-\frac{\pi}{24L^2}. \tag{56}$$

Obviously the net force is always negative.

3 Conclusions

From Ref.[27] we can know that if the energy-momentum tensor of a certain field with one exterior boundary in the Minkowski spacetime is obtained, it can also be obtained for the same field with the same boundary in curved spacetime.

In this paper, we explicitly calculate the renormalized energy-momentum tensor of a massless scalar field satisflying Dirichlet boundary conditions in the black hole spacetime with a global monopole. The renormalized energy momentum tensor is treated in the Boulware, Hartle-Hawking, and Unruh vacua separately. In all these vacua, for $r \to +\infty$, the energy density, pressure and energy acting on Dirichlet walls are obtained. The values of the above-mentioned quantities are all negative. We also calculate the Casimir force and find it to be negative in Boulware vacuum. But in Hartle-Hawking and Unruh vacua it can be attractive, repulsive, and zero depending on distance L between the Dirichlet walls. Otherwise, the Casimir force is independent of the black hole change. We also evaluate the net force exerted on the Dirichlet walls and find it to be always negative. When $\eta = 0$, the results are coincide with ones in the two-dimension Reissner-Nordström spacetime. When $\eta = 0$ and $Q = 0$, the results are the same with ones in the two-dimension Schwarzschild spacetime[27].

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value of α is no longer a constant but is changeable. The value of $\Delta Q_{e反}$ can be calculated from $\Delta m_{e反}$ and that of the anomalous magnetic moment can also be done from both $\Delta m_{e反}$ and $\Delta Q_{e反}$, which are near the experimental Values. And finally , the quantization conditions about Planck scale is discussed.

Key words: dirac large number; Planck large number; fine structure constant α; “anomalous electric charge”; “anomalous magnetic moment”

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两维整体单极黑洞时空中的 Casimir 效应*

王晓霞, 杨树政
(西华师范大学 理论物理研究所, 四川 南充　637002)

摘要: 两维整体单极黑洞事件视界附近的一个薄层作为 Casimir 型系统, 得到了在 Boulware 真空、Hartle-Hawking 真空和 Unruh 真空的能动张量. 这些值都是在满足 Dirichlet 边界条件的无质量标量场中得到. 应用 Wald 公理得到了用通常的正规化方法得到的结果. 同时, 计算了在渐进平直时空下的能量、能量密度和作用在 Dirichlet 边界上的压强, 并由能量得到了 Casimir 力.

关键词: Casimir 效应; 两维黑洞; 整体单极

* 作者简介: 王晓霞(1979-), 女, 山东人, 硕士生, 主要从事引力理论和黑洞物理方面的研究.