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质子结构函数 F_2 对小 x 小 q^2 双对数依赖的初步解释*

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摘要: 在小 x, 小 q^2 的限制条件下, 推出了与实验拟合结果定性吻合的质子结构函数 F_2 与 x, q^2 的双对数依赖关系.

关键词: 小 x 区域; 结构函数 F_2; 截面
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在 HERA 的 H1 和 ZEUS 合作组通过 eP 深度非弹性散射在 1995~1996 年测出了 $x < 0.01$, $q^2 > 0.1\text{GeV}^2$ 的质子结构函数的 F_2 的一系列值之后, 对实验数据进行拟合, W. Buchmüler 和 D. Haidt 给出了 F_2 与 x, q^2 的双对数依赖, 但目前理论方面尚未对此做出成功的解释, 本文借助于 QCD 及高等量子力学给出了定性符合实验拟合结果的 F_2 与 x, q^2 的依赖关系.

1　理论部分

1.1　核子结构函数与虚光子吸收截面之间的关系[1]　质子吸收虚光子(包括横光子, 纵光子) 的总截面

$$\sigma_{\text{tot}}^{\gamma^* N} = \frac{4\pi^2\alpha_{\text{em}}}{k}\frac{q^2 + v^2}{q^2 v}F_2(x, q^2), \qquad (1)$$

其中 $k = v - q^2/2M_{\text{N}}$, M_{N} 为核子静质量, v 为电子传递给核子的能量, q^2 为虚光子质量的平方, $x = \frac{q^2}{2M_{\text{N}}v}$, $\alpha_{\text{em}} = \frac{1}{137}$. 当 x, q^2 都很小时有

$$\sigma_{\text{tot}}^{\gamma^* p} = \frac{4\pi^2\alpha_{\text{em}}}{q^2}F_2(x, q^2). \qquad (2)$$

1.2　微扰量子力学(**PQCD**)对散射总截面的计算[2]　N. K. Nikolaev 和 B. G. Zakharov 给出核子对虚光子散射的总截面为

$$\sigma_{\text{tot}}^{\gamma^* N} = 2\pi\iint \text{d}\rho\,\text{d}\alpha\,|\varphi_{\gamma^*}|^2\rho\sigma_{q\bar{q}P}(\rho q^2), \qquad (3)$$

其中 $|\varphi_{\gamma^*}|^2 = \Phi_T^2(\varepsilon\rho) + \Phi_L^2(\varepsilon\rho)$.

$$\Phi_T^2(\varepsilon\rho) = \frac{6\alpha_{\text{em}}}{(2\pi)^2}\sum_f^{N_f} Z_f^2(1 - 2\alpha(1 - \alpha))\varepsilon^2 K_1^2(\varepsilon\rho) + m_f^2 K_0^2(\varepsilon\rho),$$

$$\Phi_L^2(\varepsilon\rho) = \frac{24\alpha_{\text{em}}}{(2\pi)^2}\sum_f^{N_f} Z_f^2 q^2\alpha^2(1 - \alpha)^2 K_0^2(\varepsilon\rho).$$

α 为正反夸克对所带的光锥动量分数, $\alpha: 0 \sim x$; ρ 为正反夸克对之间的横向距离. 正反夸克对是由虚光子 γ^* 涨落形成的, 虚光子涨落成正反夸克对的同时还可能辐射胶子, 而 γ^* 所具有的光锥分数动量为 x, γ^* 的光锥分数动量在夸克和胶子之间分配, 所以 α 的最大取值为 x.

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ρ: $0 \sim \frac{q_0 x^{-3/2}}{q^2}$. ρ的最大值取为$\frac{q_0 x^{-3/2}}{q^2}$, 原因有 3 点: ①作为 ρ 的上限的$\frac{q_0 x^{-3/2}}{q^2}$ 具有长度的量纲. ②ρ的取值范围为$0 \to \infty$, 事实上 ρ的上限只要取得不太小(1 fm 左右或大于 1 fm) 就没有原则上的错误; 而当 $x \leqslant 0.01$ 时, 不妨取

$x = 0.01$, $q_0^2 = 0.5\ \mathrm{GeV}^2$, $q^2 = 5\ \mathrm{GeV}^2$ 代入$\frac{q_0 x^{-3/2}}{q^2}$, 得 $\rho_{max} = \frac{\sqrt{0.5}}{5 \times 5 \times (10^{-2})^{\frac{3}{2}}} = 28.28(\mathrm{fm})$,

若取 $x = 0.001$, 则 $\rho_{max} = 8\,944.3(\mathrm{fm})$, 可见当 $x \leqslant 0.01$ 时, $\frac{q_0 x^{-3/2}}{q^2}$ 能满足 ρ的上限所要求的条件. ③由于 α的上限为x 的限制, 要得到与实验相吻合的结果, ρ的上限的表达式中必须包含因子$x^{\frac{3}{2}}$, 原因是: 关于 α 的积分其结果中含有x^3 项, 而质子结构函数 $F_2(x, q^2)$ 中不含 x.

$\sum_f^{N_f} Z_f^2 = \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 = \frac{2}{3}$, 在此只考虑了 u, d, s 3 种夸克; $\sigma_{q\bar{q}P}$ 为正反夸克对与质子的反应截面, $\sigma_{q\bar{q}P} = C(1 - e^{-\rho^2/\rho_0^2})$; C 与ρ, α 无关; q^2 为虚光子质量的平方, m_f 为夸克的质量, 在此取 $m_f = 0$; 式中 $K_0(\varepsilon\rho)$, $K_1(\varepsilon\rho)$ 分别为零阶, 一阶变形Besel 函数. 当 $\varepsilon\rho$很小时(即 q^2 很小时) 有 $K_0(\varepsilon\rho) \approx -\ln\frac{\varepsilon\rho}{2}$, $K_1(\varepsilon\rho) \approx \frac{1}{\varepsilon\rho}$. 将上述诸式代入(3) 式得

$$\sigma_{tot}^{\gamma^* P} = \frac{2\alpha_{em} C}{\pi}\left[\int_0^x (1 - 2\alpha(1-\alpha))\,\mathrm{d}\alpha \int_0^{\left(q_0 x^{-3/2}\right)/q^2} \frac{1 - e^{-\rho^2/\rho_0^2}}{\rho}\,\mathrm{d}\rho + 4q^2 \int_0^x \alpha^2 (1-\alpha)^2\,\mathrm{d}\alpha \cdot \int_0^{\left(q_0 x^{-3/2}\right)/q^2} (1 - e^{-\rho^2/\rho_0^2})\,\rho\left(\ln\frac{\sqrt{\alpha(1-\alpha)}\,q\rho}{2}\right)\mathrm{d}\rho\right].$$

将上式中方括号内 2 项分别表示为 Ⅰ, Ⅱ 有: $\sigma_{tot}^{\gamma^* P} = \frac{2\alpha_{em} C}{\pi}(\mathrm{I} + 4q^2\,\mathrm{II}) = \sigma_T^{\gamma^* P} + \sigma_L^{\gamma^* P}$,

$$\mathrm{I} = \int_0^x (1 - 2\alpha(1-\alpha))\,\mathrm{d}\alpha \int_0^{\left(q_0 x^{-3/2}\right)/q^2} \frac{1 - e^{-\rho^2/\rho_0^2}}{\rho}\,\mathrm{d}\rho \approx x\,\frac{1 - e^{-\xi_1^2 q_0^2 x^{-3}/q^4\rho_0^2}}{\xi_1}\,\frac{2\alpha_{em} M}{\pi},$$

其中 $0 < \xi_1 < 1$, 由于 $x < 10^{-2}$, $\alpha \leqslant x$, 故将含 α 的相对高次项略去, 取 $1 - \alpha \approx 1$(下同). 令 $\mathrm{II} = A - B$.

$$A = \int_0^x \int_0^{\left(q_0 x^{-3/2}\right)/q^2} \rho\left(\ln\frac{\sqrt{\alpha(1-\alpha)}\,q\rho}{2}\right)^2 \mathrm{d}\rho\,\alpha^2 (1-\alpha)^2\,\mathrm{d}\alpha,$$

$$B = \int_0^x \int_0^{\left(q_0 x^{-3/2}\right)/q^2} \rho e^{-\rho^2/\rho_0^2}\left(\ln\frac{\sqrt{\alpha(1-\alpha)}\,q\rho}{2}\right)^2 \mathrm{d}\rho\,\alpha^2 (1-\alpha)^2\,\mathrm{d}\alpha,$$

$$\mathrm{II} \approx \frac{q_0^2}{24\,q^4}\left[3.878\ln^2\frac{x_0}{x} + 10.428\ln\frac{x_0}{x} - 3.878\ln\frac{x_0}{x}\ln\frac{q^2}{q_0^2} + 0.969\ln^2\frac{q^2}{q_0^2} - 5.214\ln\frac{q^2}{q_0^2} + 7.827\right],$$

$\sigma_L^{\gamma^* P} = \frac{q_0^2 \alpha_{em} C}{3q^2\pi}\mathrm{II}$, 与 $\sigma_L^{\gamma^* P}$ 相比, $\sigma_T^{\gamma^* P}$ 可略去.

因此 $\sigma_{tot}^{\gamma^* N} \approx \frac{q_0^2 \alpha_{em} C}{3q^2\pi}\mathrm{II} = \sigma_L^{\gamma^* N}$. (4)

将(4) 式代入(2) 式得:

$$F_2^p(x, q^2) \approx \frac{C q_0^2}{12\pi^3}\left[3.878\ln^2\frac{x_0}{x} + 10.428\ln\frac{x_0}{x} - 3.878\ln\frac{x_0}{x}\ln\frac{q^2}{q_0^2} + 0.969\ln^2\frac{q^2}{q_0^2} - 5.214\ln\frac{q^2}{q_0^2} + 7.872\right], \quad (5)$$

2 实验分析[3]

通过对在运动学范围 $x < 0.01$, $q^2 > 5\,\mathrm{GeV}^2$ 质子结构函数 F_2 的实验数据进行拟合, W. Buchmüler 和

D.Haidt 给出了 F_2 与 x, q^2 有如下的双对数依赖关系

$$F_2(x, q^2) \approx \alpha + m\ln\frac{q^2}{q_0^2}\ln\frac{x_0}{x}, \tag{6}$$

其中 $\alpha = 0.074$, $m = 0.364$, $x_0 = 0.074$, $q_0^2 = 0.5\ \mathrm{GeV}^2$.

3　讨　论

3.1　给定 x, 讨论 F_2 与 q^2 的依赖关系　当 $x \leqslant 0.01$ 时, 为确定起见, 不妨取 $x = 0.001$, 由(5) 式可知只要

$$\left|-3.878\ln\frac{x_0}{x} - 5.241\right| \gg 0.969\ln\frac{q^2}{q_0^2},$$

在此取 $\left|-3.878\ln\frac{x_0}{x} - 5.241\right| = (5 \to 50)0.969\ln\frac{q^2}{q_0^2}$, (7)

从而 $q^2 = (0.97 \to 45.97)\mathrm{GeV}^2$(基本满足小 q^2 条件), 此时可将(5) 式中 $0.969\ln^2\frac{q^2}{q_0^2}$ 项略去不计.

事实上, 当 $x < 0.001$ 时, q^2 值范围还可增大, 譬如: 当 $x = 0.0001$ 时, 满足的(7) 式的 $q^2 = (0.95 \to 290.34)\ \mathrm{GeV}^2$, 基本能够满足小 q^2 的条件.

可见, 当 $x \leqslant 0.001$ 时, 与之相应, q^2 也基本能够满足小 q^2 的条件, 在这样的情况下, 由(5) 式可见

$$F_2 \approx \alpha + b\ln\frac{q^2}{q_0^2}\ln\frac{x_0}{x} = \alpha + b\ln\frac{q^2}{q_0^2}\quad . \tag{8}$$

譬如当 $x = 0.001, 0.0001$ 时, 分别有

$$F_2^p(x, q^2) \approx \frac{Cq_0^2}{12\pi^3}\left(-21.951\ln\frac{q^2}{q_0^2} + 124.550\right),\ q^2 = [0.79 \to 45.97]\ \mathrm{GeV}^2$$ 和

$$F_2^p(x, q^2) \approx \frac{Cq_0^2}{12\pi^3}\left(-30.835\ln\frac{q^2}{q_0^2} + 245.987\right),\ q^2 = [0.93 \to 238.36]\ \mathrm{GeV}^2.$$

3.2　给定 q^2, 讨论 F_2 与 x 的依赖关系　为确定起见, 不妨取 $q^2 = 5\ \mathrm{GeV}^2$, 由(5) 式可知只要

$$\left|-3.878\ln\frac{q^2}{q_0^2} + 10.428\right| \gg 3.878\ln\frac{x_0}{x}$$

在此取 $\left|-3.878\ln\frac{q^2}{q_0^2} + 10.428\right| = [5 \to 50]3.878\ln\frac{x_0}{x}$,

从而 $x = [0.0685 \to 0.0740]$(基本上保持了小 x 区域的条件).

在这样的条件下可将(5) 式中 $3.878\ln^2\frac{x_0}{x}$ 项略去不计. 事实上, 当 $q^2 > 5\mathrm{GeV}^2$ 时, x 值的范围还可增大, 譬如当 $q^2 = 50\ \mathrm{GeV}^2$ 时: $x = [0.050 \to 0.0740]$.

可见, 当 $q^2 \geqslant 5\ \mathrm{GeV}^2$ 时, 与之相应, x 值基本也能满足小x 的条件; 在这样的情况下, 一旦 q^2 值给定, 由(5) 式可见

$$F_2 \approx \alpha' + m'\ln\frac{q^2}{q_0^2}\ln\frac{x_0}{x} = \alpha' + b'\ln\frac{q^2}{q_0^2}, \tag{9}$$

譬如, 当 $q^2 = 5\ \mathrm{GeV}^2, 50\ \mathrm{GeV}^2$ 时, 分别有

$$F_2^p(x, q^2) = \frac{Cq_0^2}{12\pi^3}\left(1.489\ln\frac{x_0}{x} + 0.959\right)\qquad x = [0.0685 \to 0.0740],$$

$$F_2^p(x, q^2) = \frac{Cq_0^2}{12\pi^3}\left(7.431\ln\frac{x_0}{x} + 4.366\right)\qquad x = [0.050 \to 0.071].$$

综上所述可见, (8)(9) 式在小 x 条件下, 定性与(1) 式吻合, PQCD 理论定性地对小 x, 小 q^2 条件下关于质子结构函数 $F_2^p(x, q^2)$ 的实验结果作出了初步的解释.

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An approach to gain the proton structure function F_2 of double-logarithmic dependence in the small- x, small- q^2 region

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Abstract: The proton structure function F_2 is deduced in the small- x, small- q^2 region, which depends on the double- logarithmic x and q^2 and is similar to the result of the fit to the HERA experiment data by W. Buchmüler, Haidt.

Key words: small- x region; structure function F_2; cross section.

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Non-embedded wavelet color image coding scheme based on Context modeling

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Abstract: A novel wavelet coding scheme of color image based on Context modeling is presented. the three components of original color image are transformed into another three components. Then, these three components are processed separately: The discrete wavelet transformed coefficients are first selected by a threshold. The significant coefficients are then quantized with a uniform quantizer and then decomposed into two parts: the most significant bit and the residual bits for entropy encoding. Simple but effective Context modeling schemes are proposed for better squeezing of the redundancy lying in the significant map symbol stream determined by the threshold operation and the MSB symbol stream decomposed from the quantized significant coefficients. With these innovations, the proposed coding scheme is competitive with other best coding algorithms reported in the literature.

Key words: wavelet transform; zero tree; contert modeeing; adaptive arithnetic coding