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3t- 2 阶 Steiner 三连系构造的一种方法

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摘　要: 阐明了 v 阶 Steiner 三连系构造的基本思路, 给出边矩阵的定义. 提出 $3t-2$ 阶 Steiner 三连系构造的一种方法, 介绍 25 阶 Steiner 三连系构造的全过程, 最后讨论了 $3t-2$ 阶 Steiner 三连系不同构的个数问题.

关键词: Steiner 三连系; 构造; 完全图; 边矩阵; 阶
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区组设计理论是组合数学的一个重要分支, 它在试验设计、竞赛安排及数字通讯等许多领域中均有重要作用. 早在 1850 年, Kirkman 提出了一个有趣的"15 名女生"问题, 并于同年做出解答. 1971 年, D. R. Ray-Chaudhuri 与 R. M. Wilson 共同发表论文《Kirkman 女生问题的解》以阐明 $6n+3$ 阶 Kirkman 三连系的构造[1-6]. 百余年来, 就是否对每个 $n=0,1,2,3,\cdots$ 总是存在 $6n+3$ 阶 Steiner 三连系, 一直是个难题. 1961 年, 中国数学家陆家羲提出了 BIBD 设计可分解的充要条件.

1　基本思路

设 $G(V,E)$ 为一个 v 阶完全图 K_v, 若完全图 K_v 的阶数满足 $v=3t-2$. t 为已有阶 Steiner 三连系的阶数, 则 v 阶 Steiner 三连系的构造等价于一个完全图 K_v 的 $v(v-1)/6$ 个完全图 K_3 的分解. 但是, 当完全图 K_v 的阶数较高时, 无法将完全图 K_v 直接分解出 $v(v-1)/6$ 个完全图 K_3, 倘若将完全图 K_v 先分解出 3 个 t 阶完全图 $K_t^{(1)}, K_t^{(2)}, K_t^{(3)}$ 及 1 个完全三分图 $K_{t-1,t-1,t-1}$, 则 3 个 t 阶完全图 $K_t^{(i)}$ 中的 $3\times t\times(t-1)/6$ 个完全图 K_3 及 1 个完全三分图 $K_{t-1,t-1,t-1}$ 中的 $(t-1)\times(t-1)$ 个完全图 K_3 将构成 $v=3t-2$ 阶 Steiner 三连系中的 $v(v-1)/6$ 个完全图 K_3, 而将完全图 K_v 分解出 3 个 t 阶完全图 $K_t^{(i)}$ 和 1 个完全三分图 $K_{t-1,t-1,t-1}$ 的有力工具是完全图 K_v 的边矩阵 $\boldsymbol{K}_v'$.

定义 1　设 $G(V,E)$ 为一个完全图 K_v, 若将完全图 K_v 中的 $v(v-1)/2$ 个边按自然顺序排成上三角阵, 使得任意边 V_iV_j 分别与顶 V_i, V_j 相关联, 则所得到的上三角阵就称为完全图 K_v 的边矩阵, 并记为 $\boldsymbol{K}_v'$ [7-8],

$$\boldsymbol{K}_v' = \begin{array}{c} \begin{array}{ccccccc} v_2 & v_3 & v_4 & v_5 & v_6 & \cdots & v_n \end{array} \\ \left(\begin{array}{ccccccc} 1_2 & 1_3 & 1_4 & 1_5 & 1_6 & \cdots & 1_n \\ & 2_3 & 2_4 & 2_5 & 2_6 & \cdots & 2_n \\ & & 3_4 & 3_5 & 3_6 & \cdots & 3_n \\ & & & 4_5 & 4_6 & \cdots & 4_n \\ & & & & 5_6 & & 5_n \\ & & & & & \ddots & \vdots \\ & & & & & & (n-1)_n \end{array}\right) \end{array} \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ \vdots \\ v_{n-1} \end{array} .$$

2　Steiner 三连系的构造

设 Steiner 三连系的阶数满足 $v=3t-2$, 若令 $t=9$, 则 $v=3t-2=25$ 阶 Steiner 三连系的构造可被视为一个完全图

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K_{25} 的 $v(v-1)/6 = 25\times 24/6$ 个完全图 K_3 的分解. $v(v-1)/6$ 个完全图 K_3 的分解的具体步骤如下:

$$K_{25}' = \begin{array}{c} \begin{array}{cccccccccccccc} v_2 & v_3 & v_4 & \cdots & v_9 & v_{10} & v_{11} & \cdots & v_{17} & v_{18} & v_{19} & \cdots & v_{25} \end{array} \\ \left(\begin{array}{ccccccccccccc}
1_2 & 1_3 & 1_4 & \cdots & 1_9 & 1_{10} & 1_{11} & \cdots & 1_{17} & 1_{18} & 1_{19} & \cdots & 1_{25} \\
 & 2_3 & 2_4 & \cdots & 2_9 & 2_{10} & 2_{11} & \cdots & 2_{17} & 2_{18} & 2_{19} & \cdots & 2_{25} \\
 & & 3_4 & \cdots & 3_9 & 3_{10} & 3_{11} & \cdots & 3_{17} & 3_{18} & 3_{19} & \cdots & 3_{25} \\
 & & & \ddots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
 & & & & 8_9 & 8_{10} & 8_{11} & \cdots & 8_{17} & 8_{18} & 8_{19} & \cdots & 8_{25} \\
 & & & & & 9_{10} & 9_{11} & \cdots & 9_{17} & 9_{18} & 9_{19} & \cdots & 9_{25} \\
 & & & & & & 10_{11} & \cdots & 10_{17} & 10_{18} & 10_{19} & \cdots & 10_{25} \\
 & & & & & & & \ddots & \vdots & \vdots & \vdots & & 11_{25} \\
 & & & & & & & & 16_{17} & \vdots & \vdots & \cdots & \vdots \\
 & & & & & & & & & 17_{18} & 17_{19} & \cdots & 17_{25} \\
 & & & & & & & & & & 18_{19} & \cdots & 18_{25} \\
 & & & & & & & & & & & \ddots & 19_{25} \\
 & & & & & & & & & & & & \vdots \\
 & & & & & & & & & & & & 24_{25}
\end{array}\right) \end{array} \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_8 \\ v_9 \\ v_{10} \\ v_{11} \\ \vdots \\ v_{17} \\ v_{18} \\ v_{19} \\ \vdots \\ v_{24} \end{array} .$$

Step 1　将完全图 K_{25} 总的 $v(v-1)/2 = 25\times 24/2$ 个边按自然顺序排成上三角阵 —— 边矩阵 $\boldsymbol{K}_v'$.

Step 2　将边矩阵 $\boldsymbol{K}_v'$ 分解出 3 个完全图 $\boldsymbol{K}_9^{(i)}$ 及 1 个完全三分图 $K_{8\,8\,8}$ 或 3 个完全二分图 $\boldsymbol{K}_{8,8}^{(1,2)}$, $\boldsymbol{K}_{8,8}^{(1,3)}$ 和 $\boldsymbol{K}_{8,8}^{(2,3)}$ ($K_{8\,8\,8} = \boldsymbol{K}_{8,8}^{(1,2)} \cup \boldsymbol{K}_{8,8}^{(1,3)} \cup \boldsymbol{K}_{8,8}^{(2,3)}$) 有25种选择, 下面拟用25个方案当中的第9个方案进行完全图 K_{25} 的子图的分解, 所得到的3个完全图 $\boldsymbol{K}_9^{(1)}$, $\boldsymbol{K}_9^{(2)}$ 和 $\boldsymbol{K}_9^{(3)}$ 及 3 个完全二分图 $\boldsymbol{K}_{8,8}^{(1,2)}$, $\boldsymbol{K}_{8,8}^{(1,3)}$ 和 $\boldsymbol{K}_{8,8}^{(2,3)}$ 如下:

$$\boldsymbol{K}_9^{(1)} = \begin{array}{c} \begin{array}{cccccccc} v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 \end{array} \\ \left(\begin{array}{cccccccc}
1_2 & 1_3 & 1_4 & 1_5 & 1_6 & 1_7 & 1_8 & 1_9 \\
 & 2_3 & 2_4 & 2_5 & 2_6 & 2_7 & 2_8 & 2_9 \\
 & & 3_4 & 3_5 & 3_6 & 3_7 & 3_8 & 3_9 \\
 & & & 4_5 & 4_6 & 4_7 & 4_8 & 4_9 \\
 & & & & 5_6 & 5_7 & 5_8 & 5_9 \\
 & & & & & 6_7 & 6_8 & 6_9 \\
 & & & & & & 7_8 & 7_9 \\
 & & & & & & & 8_9
\end{array}\right) \end{array} \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array},$$

$$\boldsymbol{K}_9^{(2)} = \begin{array}{c} \begin{array}{cccccccc} v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} & v_{17} \end{array} \\ \left(\begin{array}{cccccccc}
9_{10} & 9_{11} & 9_{12} & 9_{13} & 9_{14} & 9_{15} & 9_{16} & 9_{17} \\
 & 10_{11} & 10_{12} & 10_{13} & 10_{14} & 10_{15} & 10_{16} & 10_{17} \\
 & & 11_{12} & 11_{13} & 11_{14} & 11_{15} & 11_{16} & 11_{17} \\
 & & & 12_{13} & 12_{14} & 12_{15} & 12_{16} & 12_{17} \\
 & & & & 13_{14} & 13_{15} & 13_{16} & 13_{17} \\
 & & & & & 14_{15} & 14_{16} & 14_{17} \\
 & & & & & & 15_{16} & 15_{17} \\
 & & & & & & & 16_{17}
\end{array}\right) \end{array} \begin{array}{c} v_9 \\ v_{10} \\ v_{11} \\ v_{12} \\ v_{13} \\ v_{14} \\ v_{15} \\ v_{16} \end{array},$$

$$\boldsymbol{K}_9^{(3)} = \begin{array}{c} \begin{array}{cccccccc} v_{18} & v_{19} & v_{20} & v_{21} & v_{22} & v_{23} & v_{24} & v_{25} \end{array} \\ \left(\begin{array}{cccccccc}
9_{18} & 9_{19} & 9_{20} & 9_{21} & 9_{22} & 9_{23} & 9_{24} & 9_{25} \\
 & 18_{19} & 18_{20} & 18_{21} & 18_{22} & 18_{23} & 18_{24} & 18_{25} \\
 & & 19_{20} & 19_{21} & 19_{22} & 19_{23} & 19_{24} & 19_{25} \\
 & & & 20_{21} & 20_{22} & 20_{23} & 20_{24} & 20_{25} \\
 & & & & 21_{22} & 21_{23} & 21_{24} & 21_{25} \\
 & & & & & 22_{23} & 22_{24} & 22_{25} \\
 & & & & & & 23_{24} & 23_{25} \\
 & & & & & & & 24_{25}
\end{array}\right) \end{array} \begin{array}{c} v_9 \\ v_{18} \\ v_{19} \\ v_{20} \\ v_{21} \\ v_{22} \\ v_{23} \\ v_{24} \end{array},$$

$$\boldsymbol{K}_{8,8}^{(1,2)} = \begin{array}{c} \begin{array}{cccccccc} v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} & v_{17} \end{array} \\ \left(\begin{array}{cccccccc}
1_{10} & 1_{11} & 1_{12} & 1_{13} & 1_{14} & 1_{15} & 1_{16} & 1_{17} \\
2_{10} & 2_{11} & 2_{12} & 2_{13} & 2_{14} & 2_{15} & 2_{16} & 2_{17} \\
3_{10} & 3_{11} & 3_{12} & 3_{13} & 3_{14} & 3_{15} & 3_{16} & 3_{17} \\
4_{10} & 4_{11} & 4_{12} & 4_{13} & 4_{14} & 4_{15} & 4_{16} & 4_{17} \\
5_{10} & 5_{11} & 5_{12} & 5_{13} & 5_{14} & 5_{15} & 5_{16} & 5_{17} \\
6_{10} & 6_{11} & 6_{12} & 6_{13} & 6_{14} & 6_{15} & 6_{16} & 6_{17} \\
7_{10} & 7_{11} & 7_{12} & 7_{13} & 7_{14} & 7_{15} & 7_{16} & 7_{17} \\
8_{10} & 8_{11} & 8_{12} & 8_{13} & 8_{14} & 8_{15} & 8_{16} & 8_{17}
\end{array}\right) \end{array} \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array},$$

$$
K_{8,8}^{(1,3)}=\begin{array}{c}
\begin{array}{cccccccc} v_{18} & v_{19} & v_{20} & v_{21} & v_{22} & v_{23} & v_{24} & v_{25}\end{array}\\
\left(\begin{array}{cccccccc}
1_{18} & 1_{19} & 1_{20} & 1_{21} & 1_{22} & 1_{23} & 1_{24} & 1_{25}\\
2_{18} & 2_{19} & 2_{20} & 2_{21} & 2_{22} & 2_{23} & 2_{24} & 2_{25}\\
3_{18} & 3_{19} & 3_{20} & 3_{21} & 3_{22} & 3_{23} & 3_{24} & 3_{25}\\
4_{18} & 4_{19} & 4_{20} & 4_{21} & 4_{22} & 4_{23} & 4_{24} & 4_{25}\\
5_{18} & 5_{19} & 5_{20} & 5_{21} & 5_{22} & 5_{23} & 5_{24} & 5_{25}\\
6_{18} & 6_{19} & 6_{20} & 6_{21} & 6_{22} & 6_{23} & 6_{24} & 6_{25}\\
7_{18} & 7_{19} & 7_{20} & 8_{21} & 7_{22} & 7_{23} & 7_{24} & 7_{25}\\
8_{18} & 8_{19} & 8_{20} & 9_{21} & 8_{22} & 8_{23} & 8_{24} & 8_{25}
\end{array}\right)
\end{array}
\begin{array}{c} \\ v_1\\ v_2\\ v_3\\ v_4\\ v_5\\ v_6\\ v_7\\ v_8\end{array},\quad
K_{8,8}^{(2,3)}=\begin{array}{c}
\begin{array}{cccccccc} v_{18} & v_{19} & v_{20} & v_{21} & v_{22} & v_{23} & v_{24} & v_{25}\end{array}\\
\left(\begin{array}{cccccccc}
10_{18} & 10_{19} & 10_{20} & 10_{21} & 10_{22} & 10_{23} & 10_{24} & 10_{25}\\
11_{18} & 11_{19} & 11_{20} & 11_{21} & 11_{22} & 11_{23} & 11_{24} & 11_{25}\\
12_{18} & 12_{19} & 12_{20} & 12_{21} & 12_{22} & 12_{23} & 12_{24} & 12_{25}\\
13_{18} & 13_{19} & 13_{20} & 13_{21} & 13_{22} & 13_{23} & 13_{24} & 13_{25}\\
14_{18} & 14_{19} & 14_{20} & 14_{21} & 14_{22} & 14_{23} & 14_{24} & 14_{25}\\
15_{18} & 15_{19} & 15_{20} & 15_{21} & 15_{22} & 15_{23} & 15_{24} & 15_{25}\\
16_{18} & 16_{19} & 16_{20} & 16_{21} & 16_{22} & 16_{23} & 16_{24} & 16_{25}\\
17_{18} & 17_{19} & 17_{20} & 17_{21} & 17_{22} & 17_{23} & 17_{24} & 17_{25}
\end{array}\right)
\end{array}
\begin{array}{c} \\ v_{10}\\ v_{11}\\ v_{12}\\ v_{13}\\ v_{14}\\ v_{15}\\ v_{16}\\ v_{17}\end{array},
$$

$$
K_{8,8}^{\prime(1,3)}=\left(\begin{array}{cccccccc}
1_{18} & 1_{19} & 1_{20} & 1_{21} & 1_{22} & 1_{23} & 1_{24} & 1_{25}\\
2_{19} & 2_{20} & 2_{21} & 2_{22} & 2_{23} & 2_{24} & 2_{25} & 2_{18}\\
3_{20} & 3_{21} & 3_{22} & 3_{23} & 3_{24} & 3_{25} & 3_{18} & 3_{19}\\
4_{21} & 4_{22} & 4_{23} & 4_{24} & 4_{25} & 4_{18} & 4_{19} & 4_{20}\\
5_{22} & 5_{23} & 5_{24} & 5_{25} & 5_{18} & 5_{19} & 5_{20} & 5_{21}\\
6_{23} & 6_{24} & 6_{25} & 6_{18} & 6_{19} & 6_{20} & 6_{21} & 6_{22}\\
7_{24} & 7_{25} & 7_{18} & 7_{19} & 7_{20} & 8_{21} & 7_{22} & 7_{23}\\
8_{25} & 8_{18} & 8_{19} & 8_{20} & 9_{21} & 8_{22} & 8_{23} & 8_{24}
\end{array}\right),\quad
K_{8,8}^{\prime(2,3)}=\left(\begin{array}{cccccccc}
10_{18} & 11_{19} & 12_{20} & 13_{21} & 14_{22} & 15_{23} & 16_{24} & 17_{25}\\
10_{19} & 11_{20} & 12_{21} & 13_{22} & 14_{23} & 15_{24} & 16_{25} & 17_{18}\\
10_{20} & 11_{21} & 12_{22} & 13_{23} & 14_{24} & 15_{25} & 16_{18} & 17_{19}\\
10_{21} & 11_{22} & 12_{23} & 13_{24} & 14_{25} & 15_{18} & 16_{19} & 17_{20}\\
10_{22} & 11_{23} & 12_{24} & 13_{25} & 14_{18} & 15_{19} & 16_{20} & 17_{21}\\
10_{23} & 11_{24} & 12_{25} & 13_{18} & 14_{19} & 15_{20} & 16_{21} & 17_{22}\\
10_{24} & 11_{25} & 12_{18} & 13_{19} & 14_{20} & 15_{21} & 16_{22} & 17_{23}\\
10_{25} & 11_{18} & 12_{19} & 13_{20} & 14_{21} & 15_{22} & 16_{23} & 17_{24}
\end{array}\right).
$$

Step 3　将 2 个完全二分图的边矩阵 $K_{8,8}^{(1,3)}$, $K_{8,8}^{(2,3)}$ 各自的 8×8 个边重新排列形成 2 个新的方阵 $K_{8,8}^{\prime(1,3)}$, $K_{8,8}^{\prime(2,3)}$, 再将 2 个新方阵 $K_{8,8}^{\prime(1,3)}$, $K_{8,8}^{\prime(2,3)}$ 与完全二分图的边矩阵 $K_{8,8}^{(1,2)}$ 相叠加, 得由 8×8 个完全图 K_3 构成的三连系矩阵 —— 完全三分图 $K_{8,8,8}^{(1,2,3)}$ 的边矩阵 $K_{8,8,8}^{\prime(1,2,3)}$,

$$
K_{8,8,8}^{\prime(1,2,3)}=\left(\begin{array}{cccccccc}
\{1,10,18\} & \{1,11,19\} & \{1,12,20\} & \{1,13,21\} & \{1,14,22\} & \{1,15,23\} & \{1,16,24\} & \{1,17,25\}\\
\{2,10,19\} & \{2,11,20\} & \{2,12,21\} & \{2,13,22\} & \{2,14,23\} & \{2,15,24\} & \{2,16,25\} & \{2,17,18\}\\
\{3,10,20\} & \{3,11,21\} & \{3,12,22\} & \{3,13,23\} & \{3,14,24\} & \{3,15,25\} & \{3,16,18\} & \{3,17,19\}\\
\{4,10,21\} & \{4,11,22\} & \{4,12,23\} & \{4,13,24\} & \{4,14,25\} & \{4,15,18\} & \{4,16,19\} & \{4,17,20\}\\
\{5,10,22\} & \{5,11,23\} & \{5,12,24\} & \{5,13,25\} & \{5,14,18\} & \{5,15,19\} & \{5,16,20\} & \{5,17,21\}\\
\{6,10,23\} & \{6,11,24\} & \{6,12,25\} & \{6,13,18\} & \{6,14,19\} & \{6,15,20\} & \{6,16,21\} & \{6,17,22\}\\
\{7,10,24\} & \{7,11,25\} & \{7,12,18\} & \{7,13,19\} & \{7,14,20\} & \{7,15,21\} & \{7,16,22\} & \{7,17,23\}\\
\{8,10,25\} & \{8,11,18\} & \{8,12,19\} & \{8,13,20\} & \{8,14,21\} & \{8,15,22\} & \{8,16,23\} & \{8,17,24\}
\end{array}\right).
$$

Step 4　将 3 个完全图 $K_9^{(i)}$ 各分解出 12 个完全图 K_3, 得 3 个 9 阶 Steiner 三连系 $ST^{(1)}(9)$, $ST^{(2)}(9)$ 和 $ST^{(3)}(9)$ 如下:

$ST^{(1)}(9) = \{\{1, 2, 3\}, \{1, 4, 7\}, \{1, 5, 6\}, \{1, 8, 9\}, \{2, 4, 5\}, \{2, 6, 8\}, \{2, 7, 9\}, \{3, 4, 8\}, \{3, 5, 9\}, \{3, 6, 7\}, \{4, 6, 9\}, \{5, 7, 8\}\}$;

$ST^{(2)}(9) = \{\{9, 10, 11\}, \{9, 12, 15\}, \{9, 13, 14\}, \{9, 16, 17\}, \{10, 12, 13\}, \{10, 14, 16\}, \{10, 15, 17\}, \{11, 12, 16\}, \{11, 13, 17\}, \{11, 14, 15\}, \{12, 14, 17\}, \{13, 15, 16\}\}$;

$ST^{(3)}(9) = \{\{9, 18, 19\}, \{9, 20, 23\}, \{9, 21, 22\}, \{9, 24, 25\}, \{18, 20, 21\}, \{18, 22, 24\}, \{18, 23, 25\}, \{19, 20, 24\}, \{19, 21, 25\}, \{19, 22, 23\}, \{20, 22, 25\}, \{21, 23, 24\}\}$.

再求 $ST^{(1)}(9)$, $ST^{(2)}(9)$, $ST^{(3)}(9)$ 和 $K_{8,8,8}^{\prime(1,2,3)}$ 之并, 即得 25 阶 Steiner 三连系 $ST^{(9)}(25)$,

$ST^{(9)}(25) = \{\{1, 2, 3\}, \{1, 4, 7\}, \{1, 5, 6\}, \{1, 8, 9\}, \{1, 10, 18\}, \{1, 11, 19\}, \{1, 12, 20\}, \{1, 13, 21\}, \{1, 14, 22\}, \{1, 15, 23\}, \{1, 16, 24\}, \{1, 17, 25\}, \{2, 4, 5\}, \{2, 6, 8\}, \{2, 7, 9\}, \{2, 10, 19\}, \{2, 11, 20\}, \{2, 12, 21\}, \{2, 13, 22\}, \{2, 14, 23\}, \{2, 15, 24\}, \{2, 16, 25\}, \{2, 17, 18\}, \{3, 4, 8\}, \{3, 5, 9\}, \{3, 6, 7\}, \{3, 10, 20\}, \{3, 11, 21\}, \{3, 12, 22\}, \{3, 13, 23\}, \{3, 14, 24\}, \{3, 15, 25\}, \{3, 16, 18\}, \{3, 17, 19\}, \{4, 6, 9\}, \{4, 10, 21\}, \{4, 11, 12\}, \{4, 12, 23\}, \{4, 13, 24\}, \{4, 14, 25\}, \{4, 15, 18\}, \{4, 16, 19\}, \{4, 17, 20\}, \{5, 7, 8\}, \{5, 10, 22\}, \{5, 11, 23\}, \{5, 12, 24\}, \{5, 13, 25\}, \{5, 14, 18\}, \{5, 15, 19\}, \{5, 16, 20\}$,

{5, 17, 21}, {6, 10, 23}, {6, 11, 24}, {6, 12, 25}, {6, 13, 18}, {6, 14, 19}, {6, 15, 20},
{6, 16, 21}, {6, 17, 22}, {7, 10, 24}, {7, 11, 25}, {7, 12, 18}, {7, 13, 19}, {7, 14, 20},
{7, 15, 21}, {7, 16, 22}, {7, 17, 23}, {8, 10, 15}, {8, 11, 18}, {8, 12, 19}, {8, 13, 20},
{8, 14, 21}, {8, 15, 22}, {8, 16, 23}, {8, 17, 24}, {9, 10, 11}, {9, 12, 15}, {9, 13, 14},
{9, 16, 17}, {9, 18, 19}, {9, 20, 23}, {9, 21, 22}, {9, 24, 25}, {10, 12, 13}, {10, 14, 16},
{10, 15, 17}, {11, 12, 16}, {11, 13, 17}, {11, 14, 15}, {12, 14, 17}, {13, 15, 17},
{18, 20, 21}, {18, 22, 24}, {18, 23, 25}, {19, 20, 24}, {19, 21, 25}, {19, 22, 23},
{20, 22, 25}, {21, 23, 24}}.

3　$3t-2$ 阶 Steiner 三连系的计数

$3t-2$ 阶 Steiner 三连系的个数 N 取决于 $3t-2$ 阶完全图 K_v 的 1 个完全三分图 $K_{t-1,t-1,t-1}$ 和 3 个完全图 $\boldsymbol{K}_t^{(i)}$ 的分解方案数 $N^{(1)}$, 以及完全三分图 $\boldsymbol{K}^{(1,2,3)}_{t-1,t-1,t-1}$ 的 $(t-1)\times(t-1)$ 个完全图 K_3 的分解方案 $N^{(2)} = t-1$, K_v 的子图分解方案数 $N^{(1)} = 3t-2$, $\boldsymbol{K}^{(1,2,3)}_{t-1,t-1,t-1}$ 的完全图 K_3 的分解方案数 $N^{(2)} = t-1$, 故 $3t-2$ 阶 Steiner 三连系的不同构的个数 $N = N^{(1)}\times N^{(2)} = (3t-2)(t-1)$, 25 阶 Steiner 三连系 $\boldsymbol{ST}(25)$ 的个数 $N = 25\times 8 = 200$ 个.

4　结语

(1) 给出了任意完全图 K_v 的边矩阵的定义, 使得边矩阵 $\boldsymbol{K}'_v$ 成为完全图 K_v 分解为 $v(v-1)/6$ 个完全图 K_3 的有效工具; (2) 提出了 $3t-2$ 阶 Steiner 三连系构造的一种方法; (3) 解决了 $3t-2$ 阶 Steiner 三连系的计数问题.

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A Method of Constructing Steiner Triple Systems of Order $3t-2$

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Abstract: The basic concept of constructing Steiner triple systems of order v is described. The definition of edges matrix is given. A method of constructing Steiner triple systems of order $3t-2$ is proposed. The entire procedure of constructing Steiner triple systems of order 25 is presented. The enumeration problem of disjoint Steiner triple systems of order 25 is discussed.

Key words: Steiner triple systems; construction; complete graph; edges matrix; order

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