Article ID: 1007－2985(2011)04－0015－11

On Brunn-Minkowski Inequality for the Quermassintegrals and Dual Quermassintegrals of L_p-Mixed Centroid Bodies*

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Abstract: The notions of new geometric body $\Gamma_{-p,i}K$ and L_p-mixed harmonic Blaschke add $k+p_L$ are defined. The Brunn-Minkowski inequalities for the quermassintegrals and dual quermassintegrals of L_p-mixed centroid body $\Gamma_{p,i}K$ and its polar body associated with L_p-mixed harmonic Blaschke add are established, and the monotonicity of operator $\Gamma_{p,i}$ and $\Gamma_{-p,i}$ is proved.

Key words: L_p-centroid body; L_p-mixed centroid body; L_p-mixed projection body; quermassintegrals; dual quermassintegrals; L_p-mixed harmonic Blaschke add

CLC number: O184 **Document code**: A

1 Introduction

Let $\mathcal{K}^n$ denote the set of convex bodies (compact, convex subsets with non-empty interiors) in Euclidean space $\mathbf{R}^n$, $\mathcal{K}_o^n$ and $\mathcal{K}_s^n$ denote the set of convex bodies containing origin in their interiors and the set of origin-symmetric convex bodies in $\mathbf{R}^n$, respectively. Let S^{n-1} denote the unit sphere in $\mathbf{R}^n$, $V(K)$ denote the n-dimensional volume of body K. If K is the standard unit ball B in $\mathbf{R}^n$, it is denoted as $\omega_n=V(B)$.

In 1997, ref. [1－2] first posed the notion of L_p-centroid body as follows: let K be the compact star-shaped about the origin in $\mathbf{R}^n$, and let $p\geqslant 1$ be arbitrary real number, and then the L_p-centroid body Γ_pK of K is the origin-symmetric conves body whose support function is given by

$$h^p_{\Gamma_pK}(u)=\frac{1}{c_{n,p}V(K)}\int_K |u\cdot x|^p\mathrm{d}x, \tag{1}$$

where $u\in S^{n-1}$, and $c_{n,p}=\dfrac{\omega_{n,p}}{\omega_2\omega_n\omega_{p-1}}$, $\omega_n=\pi^{\frac{n}{2}}/\Gamma(1+\dfrac{n}{2})$.

By using polar coordinate transformation and (1), we easily obtain

$$h^p_{\Gamma_pK}(u)=\frac{1}{(n+p)c_{n,p}V(K)}\int_{S^{n-1}} |u\cdot v|^p\rho_K^{n+p}(v)\mathrm{d}S(v). \tag{2}$$

* **Received date**: 2011－04－30

Foundation item: National Natural Science Foundation of China (10971128); Foundation of the Education Department of Gansu Province (2009B—09)

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In respect of the L_p-centroid body, ref. [1-3] recently made a series of studies, where many important results were proven. Recently, together with (2), ref. [4] defined a new geometric body as follows.

Let $K \subset \mathbf{R}^n$ be compact star-shaped about the origin $K \subseteq \mathbf{R}^n$, and let $p \geqslant 1$ and i be arbitrary real numbers, and then the L_p-mixed centroid body $\Gamma_{p,i}K$ of K is the origin-symmetric convex body whose support function is given by

$$h^p_{\Gamma_{p,i}K}(u) = \frac{1}{(n+p)c_{n,p}V(K)}\int_{S^{n-1}} |u \cdot v|^p \rho_K^{n+p-i}(v)\mathrm{d}S(v), \tag{3}$$

where $u \in S^{n-1}$.

Obviously, from definition (2) and (3), we have, if $i=0$, $\Gamma_{p,0}K = \Gamma_p K$.

In this paper, we will propose the notion of new geometric body $\Gamma_{-p,i}K$ and L_p-mixed harmonic Blaschke add $K \ddot{+}_p L$, and establish the Brunn-Minkowski inequalities for L_p-mixed centroid body $\Gamma_{p,i}K$ and its polar body, and prove the monotonicity of operator $\Gamma_{p,i}$ and $\Gamma_{-p,i}$.

2 Preliminaries

2.1 Support Function, Radial Function and Polar of Convex

If $K \in \mathrm{K}^n$, its support function $h_k := h(K, \cdot): \mathbf{R}^n \longrightarrow (0, +\infty)$ is defined by (see ref. [5]) $h(K,x) = \max\{x \cdot y : y \in K\}$, $x \in \mathbf{R}^n$, where $x \cdot y$ denotes the standard inner product of x and y.

If K is a compact star-shaped (about the origin) in $\mathbf{R}^n$, its radial function $\rho_K := \rho(K, \cdot): \mathbf{R}^n \backslash \{0\} \longrightarrow [0, +\infty)$ is defined by (see ref. [5]) $\rho(K,x) = \max\{\lambda \geqslant 0 : \lambda x \in K\}$, $x \in \mathbf{R}^n \backslash \{0\}$, when ρ_K is positive and continuous, and K is called a star body (about the origin). Let S^n_o denote the set of star bodies (about the origin) in $\mathbf{R}^n$. Two star bodied K and L are said to be dilates each other if $\rho_K(u)/\rho_L(u)$ is independent on $u \in S^{n-1}$.

From the definition of radial function, we know that for $\lambda > 0$, $\rho_K(u) \leqslant \lambda \rho_L(u)$ for any $u \in S^{n-1}$ if and only if $K \subseteq \lambda L$.

For $K \in \mathrm{K}^n_o$, the polar body K^* of K is defined by (see ref. [5])

$$K^* = \{x \in \mathbf{R}^n : x \cdot y \leqslant 1, y \in K\}. \tag{4}$$

Obviously, we have $(K^*)^* = K$.

From the definition (4), we also know that, if $K \in \mathrm{K}^n_o$, the support and radial function of K^* and the polar body of K are defined respectively by

$$h_{K^*} = \frac{1}{\rho_K}, \rho_{K^*} = \frac{1}{h_K}. \tag{5}$$

2.2 Mean Value Integral, L_p-Mixed Mean Value Integral and L_p-Mixed Volume

For $K \in \mathrm{K}^n$, the mean value integral $W_i(K)$ $(i=0,1,\cdots,n-1)$ are defined by (see ref. [5-6])

$$W_i(K) = \frac{1}{n}\int_{S^{n-1}} h(K,u)\mathrm{d}S_i(K,u), \tag{6}$$

where $S_i(K, \cdot)$ is a classical positive Borel measure on S^{n-1}.

From definition (6), we easily see that

$$W_0(K) = V(K). \tag{7}$$

For $p \geqslant 1$, $K, L \in \mathrm{K}^n_o$ and $\varepsilon > 0$, the Firey L_p-combination $K +_p \varepsilon \cdot L \in \mathrm{K}^n_o$ is defined by (see ref. [7])

$$h(K +_p \varepsilon \cdot L, \cdot)^p = h(K, \cdot)^p + \varepsilon h(L, \cdot)^p,$$

where "$\cdot$" in $\varepsilon \cdot L$ denotes the Firey scalar multiplication. Firey L_p-combination of convex bodies were defined and studied by Firey (see ref. [7]).

Associated with the Firey L_p-combination, ref. [7] defined L_p-mixed quermassintegrals (also called mixed p-Quermassintegrals) as follows.

For $K, L \in \mathcal{K}_o^n$, $\varepsilon > 0$ and real number $p \geqslant 1$, the L_p-mixed quermassintegrals $W_{p,i}(K,L)$ $(i=0,1,\cdots,n-1)$ are defined by

$$\frac{n-i}{p} W_{p,i}(K,L) = \lim_{\varepsilon \to 0^+} \frac{W_i(K +_p \varepsilon \cdot L) - W_i(K)}{\varepsilon}. \tag{8}$$

Obviously, for $i=0$, by (7) and (8), the L_p-mixed quermassiontegrals $W_{p,0}(K,L)$ is just the L_p-mixed volume $V_p(K,L)$, namely

$$W_{p,0}(K,L) = V_p(K,L). \tag{9}$$

Furthermore, ref. [7] has shown that, for $p \geqslant 1$, $i=0,1,\cdots,n-1$ and each $K \in \mathcal{K}_o^n$, there exists a positive Borel measure $S_{p,i}(K,\cdot)$ on S^{n-1}, such that the L_p-mixed quermassintegral $W_{p,i}(K,L)$ has the following integral representation:

$$W_{p,i}(K,L) = \frac{1}{n}\int_{S^{n-1}} h_L^p(v)\,dS_{p,i}(K,v), \tag{10}$$

for all $K \in \mathcal{K}_o^n$. It turns out that the measure $S_{p,i}(K,\cdot)$ $(i=0,1,\cdots,n-1)$ on S^{n-1} is absolutely continuous with respect to $S_i(K,\cdot)$, and has the Radon-Nikodym derivative

$$\frac{dS_{p,i}(K,\cdot)}{dS_i(K,\cdot)} = h^{1-p}(K,\cdot), \tag{11}$$

where $S_i(K,\cdot)$ is a classical positive Borel measure on S^{n-1}. The case $i=0$, $S_{p,0}(K,\cdot)$ is just the L_p-surface area measure $S_p(K,\cdot)$ of K, together with (9) and (10), then the integral representation of L_p-mixed volume $V_p(K,L)$ is obtained by $V_p(K,L) = \frac{1}{n}\int_{S^{n-1}} h_L^p(v)\,dS_p(K,v)$.

From the definition of the L_p-mixed quermassiontegrals, it follows immediately that, for each $K \in \mathcal{K}_0^n$,

$$W_{p,i}(K,K) = W_i(K), \tag{12}$$

for all $p \geqslant 1$.

We shall require the Minkowski inequality for the L_p-mixed quermassiontegrals $W_{p,i}$ as follows: For $K, L \in \mathcal{K}_o^n$, and $p \geqslant 1$, $0 \leqslant i < n$, then (see ref. [7])

$$W_{p,i}(K,L)^{n-i} \geqslant W_i(K)^{n-i-p} W_i(L)^p, \tag{13}$$

with equality if only if K and L are dilations each other.

2.3 Dual Quermassiontegrals and L_p-Dual Mixed Quermassiontegrals

For $K \in \mathcal{S}_o^n$ and any real number i, the dual quermassiontegrals $\widetilde{W}_i(K)$, of K are defined by (see ref. [6-7])

$$\widetilde{W}_i(K) = \frac{1}{n}\int_{S^{n-1}} \rho_K^{n-i}(u)\,dS(u). \tag{14}$$

Obviously,

$$\widetilde{W}_0(K) = V(K). \tag{15}$$

For $K, L \in \mathcal{S}_o^n$, and $\varepsilon > 0$, then for $p \geqslant 1$, the L_p-harmonic radial combination $K +_p \varepsilon \cdot L \in \mathcal{S}_o^n$ is defined by (see ref. [8])

$$\rho(K +_{-p} \varepsilon \cdot L,\cdot)^{-p} = \rho(K,\cdot)^{-p} + \varepsilon \rho(L,\cdot)^{-p}.$$

Note that here "$\varepsilon \cdot L$" is different from "$\varepsilon \cdot L$" in L_p-combination.

For $K, L \in \mathcal{S}_o^n$, $\varepsilon > 0$, $p \geqslant 1$ and real number $i \neq n$, the L_p-dual mixed combination quermassiontegrals, $\widetilde{W}_{-p,i}(K,L)$ of K and L are defined by

$$\frac{n-i}{-p}\widetilde{W}_{-p,i}(K,L) = \lim_{\varepsilon \to 0^+} \frac{\widetilde{W}_i(K +_{-p} \varepsilon \cdot L) - \widetilde{W}_i(K)}{\varepsilon}. \tag{16}$$

If $i=0$, using (15), we easily see that definition (16) is just definition of L_p-dual mixed volume, namely

$$\widetilde{W}_{-p,0}(K,L)=V_{-p}(K,L). \quad 17)$$

From this, the L_p-dual mixed quermassiontegrals is the extension of L_p-dual mixed volume.

Furthermore, from definition (16), the integral representation of the L_p-dual mixed quermassiontegrals is given by (see ref. [9]): if $K,L\in S_0^n$, $p\geqslant 1$, and real number $i\neq n$,

$$\widetilde{W}_{-p,i}(K,L)=\frac{1}{n}\int_{S^{n-1}}\rho_K^{n+p-i}(u)\rho_L^{-p}(u)\mathrm{d}S(u), \quad (18)$$

where the integration is with respect to spherical Lebesgue measure on S^{n-1}.

Together with (14) and (18), for $K\in S_0^n$, $p\geqslant 1$, and $i\neq n$, we get

$$\widetilde{W}_{-p,i}(K,K)=\widetilde{W}_i(K). \quad (19)$$

Furthermore, ref. [9] proved the following analog of the Minkowski inequality for L_p-dual mixed quermassiontegrals: if $K,L\in S_o^n$, $p\geqslant 1$, for $i<n$ or $i>n+p$,

$$\widetilde{W}_{-p,i}(K,L)^{n-i}\geqslant \widetilde{W}_i(K)^{n+p-i}\widetilde{W}_i(L)^{-p}; \quad (20)$$

for $n<i<n+p$, $\widetilde{W}_{-p,i}(K,L)^{n-i}\leqslant \widetilde{W}_i(K)^{n+p-i}\widetilde{W}_i(L)^{-p}$, with equality in every inequality if and only if K and L dilate each other.

2.4 L_p-Mixed Projection Bodies

In 2000, ref. [2] posed the notion of L_p-mixed projection body as follows.

For each $K\in \mathcal{K}^n$ and $p\geqslant 1$, the L_p-mixed projection body $\Pi_p K$ of K is an origin-symmetric convex body whose support function is given by

$$h^p_{\Pi_p K}(u)=\frac{1}{n\omega_n c_{n-2,p}}\int_{S^{n-1}}|u\cdot v|^p\mathrm{d}S_p(K,v). \quad (21)$$

For all $u\in S^{n-1}$, where $u\cdot v$ denotes the standard inner product of u and v, $S_p(K,\cdot)$ is a positive Borel measure on S^{n-1}, it is called the L_p-surface area measure of K. The unusual normalization of the definition (21) is chosen so that for the unit ball B, we have $\Pi_p B=B$. In particular, for $p=1$, the convex body $\Pi_1 K$ is the classical projection body ΠK of K under the normalization of the definition (11) (see ref. [2]). Regard to the studies of the L_p-projection body, we can refer to ref. [10－14].

Furthermore, ref. [13] shows the notion of L_p-mixed projection body as follows: for each $K\in\mathcal{K}_o^n$, real number $p\geqslant 1$ and $i=0,1,\cdots,n-1$, the L_p-mixed projection body $\Pi_{p,i}K$ of K is an the origin-symmetric convex body whose support function is given by

$$h^p_{\Pi_{p,i} K}(u)=\frac{1}{n\omega_n c_{n-2,p}}\int_{S^{n-1}}|u\cdot v|^p\mathrm{d}S_{p,i}(K,v), \quad (22)$$

for all $u,v\in S^{n-1}$, $S_{p,i}(K,\cdot)(i=0,1,\cdots,n-1)$ is a positive Borel measure on S^{n-1}. By using (21) and the case $i=0$ in (22), we have $\Pi_{p,0}K=\Pi_p K$.

2.5 Convex Body $\Gamma_{-p}K$ and New Geometric Body $\Gamma_{-p,i}K$

The notion of geometric body $\Gamma_{-p}K$ is shown by in ref. [15]. If $K\in\mathcal{K}_0^n$ and $p\geqslant 1$, geometric body $\Gamma_{-p}K$ is an origin-symmetric body whose radial function is defined by

$$\rho^{-p}_{\Gamma_{-p}K}(u)=\frac{1}{V(K)}\int_{S^{n-1}}|u\cdot v|^p\mathrm{d}S_p(K,v), \quad (23)$$

for all $u\in S^{n-1}$. Note for $p\geqslant 1$, the geometric body $\Gamma_{-p}K$ is an origin-symmetric body (see ref. [15]).

Together with the notion (23), we also show the notion of new geometric body $\Gamma_{-p,i}K$ as follows: if $K\in\mathcal{K}_o^n$ and $p\geqslant 1$, body $\Gamma_{-p,i}K\,(i=0,1,\cdots,n-1)$ is an origin-symmetric body whose radial function is given by

$$\rho^{-p}_{\Gamma_{-p,i}K}(u)=\frac{1}{V(K)}\int_{S^{n-1}}|u\cdot v|^p\mathrm{d}S_{p,i}(K,v), \quad (24)$$

where $S_{p,i}(K,\cdot)(i=0,1,\cdots,n-1)$ are Borel measure on S^{n-1}. By using (23) and the case $i=0$ in (24), we have $\Gamma_{-p,0}K=\Gamma_{-p}K$.

3 Brunn-Minkowski Inequality for L_p-Mixed Centroid Bodies and Their Polers

In this section, we prove the Brunn-Minkowski inequality for the quermassintegrals and dual quermassintegrals of L_p-mixed centroid bodies and their polers associated with L_p-mixed harmonic Blaschke add.

Let $K, L \in S_o^n$ and $p \geqslant 1$. For each real number $i \neq n, n+p$, we will define the new notion of L_p-mixed harmonic Blaschke add $K \ddot{+}_p L$ of K and L. We define $\xi > 0$,

$$\xi^{\frac{p-i}{n+p-i}} = \frac{1}{n}\int_{S^{n-1}} [V(K)^{-1}\rho(K,u)^{n+p-i} + V(L)^{-1}\rho(L,u)^{n+p-i}]^{\frac{n}{n+p-i}} \mathrm{d}S(u), \tag{25}$$

and then the radial function of star body $K \ddot{+}_p L$ is defined by

$$\xi^{-1}\rho(K \ddot{+}_p L, \cdot)^{n+p-i} = V(K)^{-1}\rho(K, \cdot)^{n+p-i} + V(L)^{-1}\rho(L, \cdot)^{n+p-i}. \tag{26}$$

Obviously, for $i=0$, the $K \ddot{+}_p L$ is just the L_p-harmonic Blaschke add $K \mp_p L$ of K and L (see ref. [17]), for $i=0$ and $p=1$, the $K \ddot{+}_p L$ is just harmonic add $K \hat{+} L$ of K and L.

Theorem 1 If $K, L \in S_o^n$, for $p \geqslant 1, j=0,1,\cdots,n-1$ and each real number $i \neq p, n+p$,

$$W_j(\Gamma_{p,i}(K \ddot{+}_p L))^{\frac{p}{n-j}} \geqslant W_j(\Gamma_{p,i}K)^{\frac{p}{n-j}} + W_j(\Gamma_{p,i}L)^{\frac{p}{n-j}}, \tag{27}$$

with equality for $p=1$ if and only if $\Gamma_{p,i}K$ and $\Gamma_{p,i}L$ are homothetic in (27); for $p>1$ if and only if $\Gamma_{p,i}K$ and $\Gamma_{p,i}L$ dilate each other in (27).

Proof From (25), (26) and polar coordinates representation of volume, we obtain that $\xi = V(K \ddot{+}_p L)$. Hence, from (26), we have

$$\frac{\rho(K \ddot{+}_p L, \cdot)^{n+p-i}}{V(K \ddot{+}_p L)} = \frac{\rho(K, \cdot)^{n+p-i}}{V(K)} + \frac{\rho(L, \cdot)^{n+p-i}}{V(L)}. \tag{28}$$

Using definition (3) and (28), we have

$$h^p_{\Gamma_{p,i}(K \ddot{+}_p L)}(u) = \frac{1}{(n+p)c_{n,p}V(K \ddot{+}_p L)}\int_{S^{n-1}} |u \cdot v|^p \rho^{n+p-i}_{(K \ddot{+}_p L)}(v)\mathrm{d}S(v) = h^p_{\Gamma_{p,i}K}(u) + h^p_{\Gamma_{p,i}L}(u). \tag{29}$$

Together with (29), (10) and (13), for each $Q \in \mathrm{K}_o^n$, we have

$$W_{p,j}(Q, \Gamma_{p,i}(K \ddot{+}_p L)) = W_{p,j}(Q, \Gamma_{p,i}K) + W_{p,j}(Q, \Gamma_{p,i}L) \geqslant$$
$$W_j(Q)^{\frac{n-j-p}{n-j}}(W_j(\Gamma_{p,i}K)^{\frac{p}{n-j}} + W_j(\Gamma_{p,i}L)^{\frac{p}{n-j}}),$$

with equality for $p=1$ if and only if $\Gamma_{p,i}K$ and $\Gamma_{p,i}L$ are homothetic; for $p>1$ if and only if $\Gamma_{p,i}K$ and $\Gamma_{p,i}L$ dilate each other.

Taking $Q = \Gamma_{p,i}(K \ddot{+}_p L)$, and in view of $W_{p,i}(K,K) = W_i(K)$, we obtain the inequality (27). The proof is complete.

Taking $i=0$ in theorem 1, we have the following corollary.

Corollary 1 If $K, L \in S_o^n$, for $p \geqslant 1$ and $j=0,1,\cdots,n-1$,

$$W_j(\Gamma_p(K \mp_p L))^{\frac{p}{n-j}} \geqslant W_j(\Gamma_p K)^{\frac{p}{n-j}} + W_j(\Gamma_p L)^{\frac{p}{n-j}}, \tag{30}$$

with equality for $p=1$ if and only if $\Gamma_p K$ and $\Gamma_p L$ are homothetic; for $p>1$ if and only if $\Gamma_p K$ and $\Gamma_p L$ are dilates each other.

From the case $j=0$ of inequality (30), it follows the corollary.

Corollary 2 If $K, L \in S_o^n$ and $p \geqslant 1$, $V(\Gamma_p(K \mp_p L))^{\frac{p}{n}} \geqslant V(\Gamma_p K)^{\frac{p}{n}} + V(\Gamma_p L)^{\frac{p}{n}}$, with equality for $p=1$ if and only if $\Gamma_p K$ and $\Gamma_p L$ are homothetic, for $p>1$ if and only if $\Gamma_p K$ and $\Gamma_p L$ dilate each other.

Corollary 2 is just a result of ref. [16].

We give pole formal of inequality (27) as follows.

Theorem 2 If $K, L \in S_o^n$, for $p \geqslant 1$ and each real number $i \neq p, n+p$ and real number $j \neq n$, for $j <$

$n+p$,

$$\widetilde{W}_j(\Gamma^*_{p,i}(K \ddot{+}_p L))^{-\frac{p}{n-j}} \geqslant \widetilde{W}_j(\Gamma^*_{p,i}K)^{-\frac{p}{n-j}} + \widetilde{W}_j(\Gamma^*_{p,i}L)^{-\frac{p}{n-j}};$$

for $j \geqslant n+p$,

$$\widetilde{W}_j(\Gamma^*_{p,i}(K \ddot{+}_p L))^{-\frac{p}{n-j}} \leqslant \widetilde{W}_j(\Gamma^*_{p,i}K)^{-\frac{p}{n-j}} + \widetilde{W}_j(\Gamma^*_{p,i}L)^{-\frac{p}{n-j}}.$$

In each inequality, with equality if and only if $\Gamma_{p,i}K$ and $\Gamma_{p,i}L$ dilate each other.

Proof Together with (15), (29) and (5), we obtain that

$$\widetilde{W}_j(\Gamma^*_{p,i}(K \ddot{+}_p L))^{-\frac{p}{n-j}} = \left(\frac{1}{n}\int_{S^{n-1}} h^{-(n-j)}_{\Gamma_{p,i}(K \ddot{+}_p L)} \mathrm{d}S(u)\right)^{-\frac{p}{n-j}} =$$

$$\left(\frac{1}{n}\int_{S^{n-1}} (h^p_{\Gamma_{p,i}K}(u) + h^p_{\Gamma_{p,i}L}(u))^{-\frac{n-j}{p}} \mathrm{d}S(u)\right)^{-\frac{p}{n-j}}.$$

Using the Minkowski integral inequality, we have that for $i < n+p$,

$$\widetilde{W}_j(\Gamma_{p,i}{}^*(K \ddot{+}_p L))^{-\frac{p}{n-j}} \geqslant \left(\frac{1}{n}\int_{S^{n-1}} h^{-(n-j)}_{\Gamma_{p,i}K} \mathrm{d}S(u)\right)^{-\frac{p}{n-j}} + \left(\frac{1}{n}\int_{S^{n-1}} h^{-(n-j)}_{\Gamma_{p,i}L} \mathrm{d}S(u)\right)^{-\frac{p}{n-j}} =$$

$$\widetilde{W}_j(\Gamma^*_{p,i}K)^{-\frac{p}{n-j}} + \widetilde{W}_j(\Gamma^*_{p,i}L)^{-\frac{p}{n-j}};$$

for $j \geqslant n+p$,

$$\widetilde{W}_j(\Gamma^*_{p,i}(K \ddot{+}_p L))^{-\frac{p}{n-j}} \leqslant \left(\frac{1}{n}\int_{S^{n-1}} h^{-(n-j)}_{\Gamma_{p,i}K} \mathrm{d}S(u)\right)^{-\frac{p}{n-j}} + \left(\frac{1}{n}\int_{S^{n-1}} h^{-(n-j)}_{\Gamma_{p,i}L} \mathrm{d}S(u)\right)^{-\frac{p}{n-j}} =$$

$$\widetilde{W}_j(\Gamma^*_{p,i}K)^{-\frac{p}{n-j}} + \widetilde{W}_j(\Gamma^*_{p,i}L)^{-\frac{p}{n-j}}.$$

By using the condition of equality of Minkowski integral inequality, we obtain that the equality is true if and only if $\Gamma_{p,i}K$ and $\Gamma_{p,i}L$ dilate each other. The proof is complete.

For $i=0$ in theorem 2, we have the following corollary.

Corollary 3 If $K, L \in S^n_o$, for $p \geqslant 1$ and each real number $j \neq n$, for $i < n+p$,

$$\widetilde{W}_j(\Gamma^*_p(k \mp_p L))^{-\frac{p}{n-j}} \geqslant \widetilde{W}_j(\Gamma^*_p K)^{-\frac{p}{n-j}} + \widetilde{W}_j(\Gamma^*_p L)^{-\frac{p}{n-j}}; \tag{31}$$

for $j \geqslant n+p$, $\widetilde{W}_j(\Gamma^*_p(k \mp_p L))^{-\frac{p}{n-j}} \leqslant \widetilde{W}_j(\Gamma^*_p K)^{-\frac{p}{n-j}} + \widetilde{W}_j(\Gamma^*_p L)^{-\frac{p}{n-j}}$. In each inequality, with equality if and only if $\Gamma_p K$ and $\Gamma_p L$ dilate each other.

From the case $j=0$ of inequality (31), it follows the corollary.

Corollary 4 If $K, L \in S^n_o$ and $p \geqslant 1$, $V(\Gamma^*_p(k \mp_p L))^{-\frac{p}{n}} \geqslant V(\Gamma^*_p K)^{-\frac{p}{n}} + V(\Gamma^*_p L)^{-\frac{p}{n}}$; with equality if and only if $\Gamma_p K$ and $\Gamma_p L$ dilate each other.

Corollary 4 is just a result of ref. [16].

4 Monotonicity Inequality of L_p-Mixed Centroid Body $\Gamma_{p,i}K$ and New Geometric Body

Let $p \geqslant 1$ and $i=0,1,\cdots,n-1$, and let $Z_{-p,i}$ denote the subset of K^n containing the origin-symmetric convex bodies affected by operator $\Gamma_{-p,i}$, namely, $Z_{-p,i} = \{\Gamma_{-p,i}K, K \in \mathrm{K}^n_s\}$. Let $Z^*_{-p,i}$ denote the subset of K^n containing the origin-symmetric convex bodies affected by operator $\Gamma^*_{-p,i}$, namely, $Z^*_{-p,i} = \{\Gamma^*_{-p,i}K, K \in \mathrm{K}^n_s\}$. In this section, we establish some monotonicity inequalities for operator $\Gamma_{p,i}$ ($p \geqslant 1, i \in \mathbf{R}$) and operator $\Gamma_{-p,i}$ ($p \geqslant 1, i=0,1,\cdots,n-1$).

Lemma 1 If $K, L \in \mathrm{K}^n_s$, $p \geqslant 1$, and $i=0,1,\cdots,n-1$,

$$\frac{W_{p,i}(K, \Gamma^*_{-p,i}L)}{V(K)} = \frac{W_{p,i}(L, \Gamma^*_{-p,i}K)}{V(L)}. \tag{32}$$

Proof According to (10), (5) and (24), we have

$$W_{p,i}(K, \Gamma^*_{-p,i}L) = \frac{1}{n}\int_{S^{n-1}} h^p_{\Gamma^*_{-p,i}L}(v) \mathrm{d}S_{p,i}(K,v) = \frac{1}{n}\int_{S^{n-1}} \rho^{-p}_{\Gamma_{-p,i}L}(v) \mathrm{d}S_{p,i}(K,v) =$$

$$\frac{1}{nV(L)}\int_{S^{n-1}}\int_{S^{n-1}}|u\cdot v|^p dS_{p,i}(L,u)dS_{p,i}(K,v)=$$
$$\frac{V(K)}{nV(L)}\int_{S^{n-1}}\rho_{\Gamma_{-p,i}K}^{-p}(u)dS_{p,i}(L,u)=$$
$$\frac{V(K)}{nV(L)}\int_{S^{n-1}}h_{\Gamma_{-p,i}^{*}K}^{p}(u)dS_{p,i}(L,u)=$$
$$\frac{V(K)}{V(L)}W_{p,i}(L,\Gamma_{-p,i}^{*}K).$$

So we obtain (32). The proof is complete.

Theorem 3　If $K,L\in \mathcal{K}_s^n$, $\Gamma_{-p,i}K\subseteq\Gamma_{-p,i}L$, $p\geqslant 1$ and $i=0,1,\cdots,n-1$, then

$$\frac{W_{p,i}(K,Q)}{V(K)}\geqslant\frac{W_{p,i}(L,Q)}{V(L)}\tag{33}$$

for each $Q\in Z_{-p,i}^{*}$.

Proof　Since $Q\in Z_{-p,i}^{*}$, there exists a positive $M\in\mathcal{K}_s^n$, such that $Q=\Gamma_{-p,i}^{*}M$. Hence, from lemma 1, we obtain

$$\frac{W_{p,i}(K,Q)}{V(K)}=\frac{W_{p,i}(K,\Gamma_{-p,i}^{*}M)}{V(K)}=\frac{W_{p,i}(M,\Gamma_{-p,i}^{*}K)}{V(M)},\tag{34}$$

$$\frac{W_{p,i}(L,Q)}{V(L)}=\frac{W_{p,i}(M,\Gamma_{-p,i}^{*}L)}{V(M)},\tag{35}$$

since $\Gamma_{-p,i}K\subseteq\Gamma_{-p,i}L$, we have $\Gamma_{-p,i}^{*}K\supseteq\Gamma_{-p,i}^{*}L$, namely, $h_{\Gamma_{-p,i}^{*}K}(u)\geqslant h_{\Gamma_{-p,i}^{*}L}(u)$ is true for each $u\in S^{n-1}$.

From this result and (10), we have $W_{p,i}(M,\Gamma_{-p,i}^{*}K)\geqslant W_{p,i}(M,\Gamma_{-p,i}^{*}L)$. By using (34) and (35) on above inequality, the inequality (33) is true. The proof is complete.

For $i=0$, from (9), we know that theorem 3 is the generalization of the corresponding result in ref. [16].

Lemma 2　If $K\in S_o^n$, $L\in\mathcal{K}_s^n$, $p\geqslant 1$ and $i=0,1,\cdots,n-1$,

$$\widetilde{W}_{-p,i}(K,\Gamma_{-p,i}L)=\frac{(n+p)c_{n,p}V(K)}{V(L)}W_{p,i}(L,\Gamma_{p,i}K).\tag{36}$$

Proof　From (18), (10), (24), and the Fubini theorem, we have

$$\widetilde{W}_{-p,i}(K,\Gamma_{-p,i}L)=\frac{1}{n}\int_{S^{n-1}}\rho_K^{n+p-i}(u)\rho_{\Gamma_{-p,i}L}^{-p}(u)dS(u)=\frac{1}{nV(K)}\int_{S^{n-1}}\rho_K^{n+p-i}(u)\int_{S^{n-1}}|u\cdot v|^p dS_{p,i}(L,v)dS(v)=$$
$$\frac{(n+p)c_{n,p}V(K)}{nV(L)}\int_{S^{n-1}}h_{\Gamma_{p,i}K}^{p}(v)dS_{p,i}(L,v)=\frac{(n+p)c_{n,p}V(K)}{V(L)}W_{p,i}(L,\Gamma_{p,i}K).$$

So we obtain (36). The proof is complete.

For $i=0$ and $p=2$, the equality (36) is proved in ref. [17]. For $i=0$ and $p\geqslant 1$, the equality (36) is proved in ref. [16].

The following theorem is the dual form of theorem 2.

Theorem 4　If $K,L\in S_o^n$, $\Gamma_{p,i}K\subseteq\Gamma_{p,i}L$, $p\geqslant 1$, and $i=0,1,\cdots,n-1$,

$$\frac{\widetilde{W}_{-p,i}(K,Q)}{V(K)}\leqslant\frac{\widetilde{W}_{-p,i}(L,Q)}{V(L)},\tag{37}$$

for each $Q\in Z_{-p,i}$.

Proof　From the condition $\Gamma_{p,i}K\subseteq\Gamma_{p,i}L$ and the definition (10), for each $M\in\mathcal{K}_o^n$, we have

$$W_{p,i}(M,\Gamma_{p,i}K)\leqslant W_{p,i}(M,\Gamma_{p,i}L).\tag{38}$$

Since $Q\in Z_{-p,i}$, there exists a positive $M\in\mathcal{K}_s^n$, such that $Q=\Gamma_{-p,i}M$. Hence, from lemma 2, we have

$$\frac{\widetilde{W}_{-p,i}(K,Q)}{V(K)}=\frac{\widetilde{W}_{-p,i}(K,\Gamma_{-p,i}M)}{V(K)}=\frac{(n+p)c_{n,p}W_{p,i}(M,\Gamma_{p,i}K)}{V(M)},$$

and

$$\frac{\widetilde{W}_{-p,i}(L,Q)}{V(L)}=\frac{\widetilde{W}_{-p,i}(L,\Gamma_{-p,i}M)}{V(L)}=\frac{(n+p)c_{n,p}W_{p,i}(M,\Gamma_{p,i}L)}{V(M)}.$$

By using (38), we obtain (37). The proof is complete.

For $i=0$, from (17) we know that theorem 3 is the generalization of the corresponding result in ref. [18].

Lemma 3 If $K, L \in S_o^n, p \geqslant 1, i \neq n, i \neq n+p$,

$$\widetilde{W}_{-p,i}(K,Q) = \widetilde{W}_{-p,i}(L,Q), \tag{39}$$

if and only if $K=L$ for any $Q \in S_o^n$.

Proof For $K=L$, we easily know (39) is true. Conversely, for $i<n$ (or $i>n+p$), taking $Q=K$ in (39), and using (19) and inequality (20), we have $\widetilde{W}_i(K)^{n-i} = \widetilde{W}_{-p,i}(L,K)^{n-i} \geqslant \widetilde{W}_i(L)^{n+P-i}\widetilde{W}_i(K)^{-p}$, namely, $\widetilde{W}_i(L)^{n+p-i} \geqslant \widetilde{W}_i(K)^{n+p-i}$. That yields $\widetilde{W}_i(L) \geqslant \widetilde{W}_i(K)$ (or $\widetilde{W}_i(L) \leqslant \widetilde{W}_i(K)$), with equality if and only if K and L dilate.

Again let $Q=L$ in (39), and we get $\widetilde{W}_i(K)^{n+p-i} \geqslant \widetilde{W}_i(L)^{n+p-i}$, namely, $\widetilde{W}_i(K) \geqslant \widetilde{W}_i(L)$ (or $\widetilde{W}_i(K) \leqslant \widetilde{W}_i(L)$), with equality if and only if L and K dilate.

Therefore, $\widetilde{W}_i(K) = \widetilde{W}_i(L)$, and K and L must dilate. Thus $K=L$.

Similar to the above proof, for $n<i<n+p$, we may prove lemma 3.

To sum up, the proof of lemma 3 is completed.

Lemma 4 If $K \in \mathcal{K}_o^n, L \in S_o^n, i=0,1,\cdots,n-1$, and $p \geqslant 1$,

$$W_{p,i}(K, \Gamma_{p,i}L) = \frac{\omega_n}{V(L)}\widetilde{W}_{-p,i}(L, \Pi_{p,i}^*K). \tag{40}$$

Proof From (10) and (18), and the definitions (3), (21) and (5), we have

$$W_{p,i}(K, \Gamma_{p,i}L) = \frac{1}{n}\int_{S^{n-1}} h^p_{\Gamma_{p,i}L}(u)\mathrm{d}S_{p,i}(K,u) = \frac{1}{n(n+p)c_{n,p}V(L)} \cdot$$
$$\int_{S^{n-1}}\int_{S^{n-1}} |u\cdot v|^p \rho_L^{n+p-i}(v)\mathrm{d}S(v)\mathrm{d}S_{p,i}(K,u) = \frac{\omega_n}{nV(L)} \cdot$$
$$\int_{S^{n-1}} \rho_L^{n+p-i}(v) h^p_{\Pi_{p,i}K}\mathrm{d}S(v) = \frac{\omega_n}{nV(L)} \cdot$$
$$\int_{S^{n-1}} \rho_L^{n+p-i}(v)\rho^{-p}_{\Pi^*_{p,i}K}(v)\mathrm{d}S(v) =$$
$$\frac{\omega_n}{V(L)}\widetilde{W}_{-p,i}(L, \Pi^*_{p,i}K).$$

So we get (40). The proof is complete.

Lemma 5 If $K, L \in S_o^n$ and $p \geqslant 1$, for any real number $i \neq n$ and $i \neq n+p$,

$$\frac{\widetilde{W}_{-p,i}(K, \Gamma^*_{p,i}L)}{V(K)} = \frac{\widetilde{W}_{-p,i}(L, \Gamma^*_{p,i}K)}{V(L)}. \tag{41}$$

Proof From (12), (5) and (3), we have

$$\widetilde{W}_{-p,i}(L, \Gamma^*_{p,i}K) = \frac{1}{n}\int_{S^{n-1}} \rho_L^{n+p-i}(u)\rho^{-p}_{\Gamma^*_{p,i}K}(u)\mathrm{d}S(u) = \frac{1}{n(n+p)c_{n,p}V(K)}$$
$$\int_{S^{n-1}}\int_{S^{n-1}} \rho_L^{n+p-i}(u)\, \rho_K^{n+p-i}(v)\, |u\cdot v|^p \mathrm{d}S(v)\mathrm{d}S(u) = \frac{V(L)}{nV(K)} \cdot$$
$$\int_{S^{n-1}} \rho_K^{n+p-i}(v)\rho^{-p}_{\Gamma^*_{p,i}L}(v)\mathrm{d}S(v) = \frac{V(L)}{V(K)}\widetilde{W}_{-p,i}(K, \Gamma^*_{p,i}L).$$

So we obtain (41). The proof is complete.

Theorem 5 If $K, L \in \mathcal{K}_o^n, p \geqslant 1$ and $i=0,1,\cdots,n-1$, and for any $Q \in S_o^n$, we have $\widetilde{W}_{-p,i}(K,Q) \leqslant \widetilde{W}_{-p,i}(L,Q)$, then

$$\frac{W_i(\Gamma_{p,i}K)^{-\frac{p}{n-i}}}{V(K)} \geqslant \frac{W_i(\Gamma_{p,i}L)^{-\frac{p}{n-i}}}{V(L)}, \tag{42}$$

$$\frac{\widetilde{W}_i(\Gamma^*_{p,i}K)^{\frac{p}{n-i}}}{V(K)} \geqslant \frac{\widetilde{W}_i(\Gamma^*_{p,i}L)^{\frac{p}{n-i}}}{V(L)}, \tag{43}$$

with equality if and only if $K=L$.

Proof Since for $p\geqslant 1$ and $i=0,1,\cdots,n-1$, and for any $Q\in S_o^n$, we have $\widetilde{W}_{-p,i}(K,Q)\leqslant\widetilde{W}_{-p,i}(L,Q)$. For any $M\in \mathcal{K}_o^n$, taking $Q=\Pi_{p,i}^*M$, we have

$$\widetilde{W}_{-p,i}(K,\Pi_{p,i}^*M)\leqslant\widetilde{W}_{-p,i}(L,\Pi_{p,i}^*M). \tag{44}$$

From lemma 3, the equality is true if and only if $K=L$.

From lemma 3, we have

$$\widetilde{W}_{-p,i}(K,\Pi_{p,i}^*M)=\frac{V(K)}{\omega_n}W_{p,i}(M,\Gamma_{p,i}K),$$

and

$$\widetilde{W}_{-p,i}(L,\Pi_{p,i}^*M)=\frac{V(L)}{\omega_n}W_{p,i}(M,\Gamma_{p,i}L).$$

Thus, together with (44), we get $V(K)W_{p,i}(M,\Gamma_{p,i}K)\leqslant V(L)W_{p,i}(M,\Gamma_{p,i}L)$. Let $M=\Gamma_{p,i}L$, since $p\geqslant 1$ and $i=0,1,\cdots,n-1$, by using (12), we obtain

$$V(L)W_i(\Gamma_{p,i}L)\geqslant V(K)W_{p,i}(\Gamma_{p,i}L,\Gamma_{p,i}K)\geqslant V(K)W_i(\Gamma_{p,i}L)^{\frac{n-i-p}{n-i}}W_i(\Gamma_{p,i}K)^{\frac{p}{n-i}}.$$

Thus, we easily get (42). And from (44) and (13), we know that the equality holds if and only if $K=L$.

Since $p\geqslant 1$, $i=0,1,\cdots,n-1$, and for any $Q\in S_o^n$, we have $\widetilde{W}_{-p,i}(K,Q)\leqslant\widetilde{W}_{-p,i}(L,Q)$, for any $M\in \mathcal{K}_o^n$. Taking $Q=\Gamma_{p,i}^*M$, we have

$$\widetilde{W}_{-p,i}(K,\Gamma_{p,i}^*M)\leqslant\widetilde{W}_{-p,i}(L,\Gamma_{p,i}^*M), \tag{45}$$

from lemma 3, the equality is ture if and only if $K=L$.

From lemma 5, we have

$$\widetilde{W}_{-p,i}(K,\Gamma_{p,i}^*M)=\frac{\widetilde{W}_{-p,i}(M,\Gamma_{p,i}^*K)V(K)}{V(M)},\ \widetilde{W}_{-p,i}(L,\Gamma_{p,i}^*M)=\frac{\widetilde{W}_{-p,i}(M,\Gamma_{p,i}^*L)V(L)}{V(M)}.$$

Thus, together with (45), we get

$$\frac{V(K)\widetilde{W}_{-p,i}(M,\Gamma_{p,i}^*K)}{V(M)}\leqslant\frac{V(L)\widetilde{W}_{-p,i}(M,\Gamma_{p,i}^*L)}{V(M)},$$

namely, $V(L)\widetilde{W}_{-p,i}(M,\Gamma_{p,i}^*L)\geqslant V(K)\widetilde{W}_{-p,i}(M,\Gamma_{p,i}^*K)$, taking $M=\Gamma_{p,i}^*L$, we have $V(L)\widetilde{W}_i(\Gamma_{p,i}^*L)\geqslant V(K)\widetilde{W}_{-p,i}(\Gamma_{p,i}^*L,\Gamma_{p,i}^*K)$.

Since $i<n$, by using inequality (20), we have

$$V(L)\widetilde{W}_i(\Gamma_{p,i}^*L)\geqslant V(K)\widetilde{W}_{-p,i}(\Gamma_{p,i}^*L,\Gamma_{p,i}^*K)\geqslant V(K)\widetilde{W}_i(\Gamma_{p,i}^*L)^{\frac{n+p-i}{n-i}}\widetilde{W}_i(\Gamma_{p,i}^*K)^{-\frac{p}{n-i}}.$$

From this, we immediately obtain (43); and from (45) and (19), we know that the equality holds if and only if $K=L$. The proof is complete.

For $i=0$, from (7) and (15), we know that theorem 5 is the generalization of the corresponding result in ref. [12].

Lemma 6[14] If $K,L\in \mathcal{K}_o^n$, $0\leqslant i<n$, $p\geqslant 1$, $n-i\neq p$, for any $Q\in \mathcal{K}_o^n$, $W_{p,i}(K,Q)=W_{p,i}(L,Q)$ if and only if $K=L$.

Theorem 6 If $K,L\in \mathcal{K}_o^n$, $p\geqslant 1$, $i=0,1,\cdots,n-1$, $n-i\neq p$, and for any $Q\in \mathcal{K}_o^n$, we have $W_{p,i}(K,Q)\leqslant W_{p,i}(L,Q)$. Then

$$\frac{\widetilde{W}_i(\Gamma_{-p,i}K)^{\frac{p}{n-i}}}{V(K)}\geqslant\frac{\widetilde{W}_i(\Gamma_{-p,i}L)^{\frac{p}{n-i}}}{V(L)}, \tag{46}$$

and

$$\frac{W_i(\Gamma_{-p,i}^*K)^{-\frac{p}{n-i}}}{V(K)}\geqslant\frac{W_i(\Gamma_{-p,i}^*L)^{-\frac{p}{n-i}}}{V(L)}, \tag{47}$$

with equality if and only if $K=L$.

Proof From $p\geqslant 1$, $i=0,1,\cdots,n-1$, and for any $Q\in \mathcal{K}_o^n$, $W_{p,i}(K,Q)\leqslant W_{p,i}(L,Q)$. So for any $M\in \mathcal{K}_o^n$, taking $Q=\Gamma_{p,i}M$, we have

$$W_{p,i}(K,\Gamma_{p,i}M)\leqslant W_{p,i}(L,\Gamma_{p,i}M). \qquad (48)$$

From lemma 6, the equality holds if and only if $K=L$.

From lemma 2, we have

$$W_{p,i}(K,\Gamma_{p,i}M)=\frac{V(K)}{(n+p)c_{n,p}V(M)}\widetilde{W}_{-p,i}(M,\Gamma_{-p,i}K),$$

and

$$W_{p,i}(L,\Gamma_{p,i}M)=\frac{V(L)}{(n+p)c_{n,p}V(M)}\widetilde{W}_{-p,i}(M,\Gamma_{-p,i}L).$$

together with (48), we obtain $V(L)\widetilde{W}_{-p,i}(M,\Gamma_{-p,i}L)\geqslant V(K)\widetilde{W}_{-p,i}(M,\Gamma_{-p,i}K)$.

Taking $M=\Gamma_{-p,i}L$, we have $V(L)\widetilde{W}_{i}(\Gamma_{-p,i}L)\geqslant V(K)\widetilde{W}_{-p,i}(\Gamma_{-p,i}L,\Gamma_{-p,i}K)$.

Since $i<n$, by using (20), we have

$$V(L)\widetilde{W}_{i}(\Gamma_{-p,i}L)\geqslant V(K)\widetilde{W}_{-p,i}(\Gamma_{-p,i}L,\Gamma_{-p,i}K)\geqslant V(K)\widetilde{W}_{i}(\Gamma_{-p,i}L)^{\frac{n+p-i}{n-i}}\widetilde{W}_{i}(\Gamma_{-p,i}K)^{-\frac{p}{n-i}}.$$

So we easily obtain (46). And from (48) and (19), we know that the equality holds if and only if $K=L$.

Besides, since $p\geqslant 1, i=0,1,\cdots,n-1$, and for any $Q\in \mathcal{K}_o^n$, we have $W_{p,i}(K,Q)\leqslant W_{p,i}(L,Q)$. From lemma 6, the equality holds if and only if $K=L$.

For any $M\in \mathcal{K}_o^n$, we take $Q=\Gamma^*_{-p,i}M$. Then we have

$$W_{p,i}(K,\Gamma^*_{-p,i}M)\leqslant W_{p,i}(L,\Gamma^*_{-p,i}M), \qquad (49)$$

with equality if and only if $K=L$.

From lemma 5, we have

$$W_{p,i}(K,\Gamma^*_{-p,i}M)=\frac{V(K)W_{p,i}(M,\Gamma^*_{-p,i}K)}{V(M)},$$

and

$$W_{p,i}(L,\Gamma^*_{-p,i}M)=\frac{V(L)W_{p,i}(M,\Gamma^*_{-p,i}L)}{V(M)}.$$

According to (49), we easily have

$$V(L)W_{p,i}(M,\Gamma^*_{-p,i}L)\geqslant V(K)W_{p,i}(M,\Gamma^*_{-p,i}K).$$

Taking $M=\Gamma^*_{-p,i}L$ in above inequality, and using (13), we immediately obtain

$$V(L)W_{i}(\Gamma^*_{-p,i}L)\geqslant V(K)W_{p,i}(\Gamma^*_{-p,i}L,\Gamma^*_{-p,i}K)\geqslant V(K)W_{i}(\Gamma^*_{-p,i}L)^{\frac{n-i-p}{n-i}}W_{i}(\Gamma^*_{-p,i}K)^{\frac{p}{n-i}}.$$

Thus, we immediately obtain (47). And from (49) and (13), we know that the equality holds if and only if $K=L$. The proof is complete.

If $i=0$, from (7), we know that theorem 6 is the generalization of the corresponding result in ref. [16].

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L_p -混合质心体均质积分和对偶均质积分 Brunn-Minkowski 不等式

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摘　要:定义了新几何体 $\Gamma_{-p,i}K$ 和 L_p -混合调和 Blaschke 加 $K \ddot{+}_p L$ 的概念,建立了 L_p -混合质心体 $\Gamma_{p,i}K$ 的均质积分和对偶均质积分的 Brunn-Minkowski 不等式,并研究了算子 $\Gamma_{p,i}$ 和 $\Gamma_{-p,i}$ 的单调性.

关键词:L_p -质心体;L_p -混合质心体;L_p -混合投影体;均质积分;对偶均质积分;L_p -混合调和 Blaschke 加
中图分类号:O184　　**文献标识码**:A

(责任编辑　向阳洁)