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Adaptive generalized synchronization of simple piecewise linear chaotic system and SETMOS chaotic system

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Abstract: The generalized synchronization of different structure chaotic systems based on SETMOS with unknown parameters, double-scroll-like chaotic system and simple piecewise linear chaotic system is investigated with respect to an assumed function. By analyzing characteristics of the chaotic systems and definition of the generalized synchronization, based on Lyapnuov stability theorem, novel and simple adaptive controllers and corresponding parameter update law are proposed for generalized synchronization of different chaotic systems with unknown parameters. Further, if the function is changed, the theory can also be applied for other synchronization for different structure chaotic systems, such as adaptive generalized anti-synchronization. Numerical simulation results are provided to show the effectiveness and feasibility of the proposed theory.

Key words: nonlinear optics; adaptive generalized synchronization; numerical simulation; SETMOS; unknown parameter

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简单分段线性混沌系统与 SETMOS 混沌系统的自适应广义同步

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摘 要： 研究了基于 SETMOS 构成的、参数未知的类双涡卷混沌系统与结构不同的简单分段线性混沌系统的广义自适应同步方法。通过分析混沌系统的特点和广义同步的定义，基于李雅普诺夫稳定性理论，提出了一种新颖的、结构简单的自适应控制器和参数更新律，来实现不同结构、驱动系统参数未知的混沌系统的广义同步。这种方法还可以应用于不同结构或相同结构的其他同步问题，如自适应广义反同步等，应用范围较广。仿真结果进一步证实了该方法的有效性和可行性。

关键词： 非线性光学；自适应广义同步；数值仿真； SETMOS; 参数未知

1 Introduction

Due to its many potential applications, such as in secure communication, biological systems and information science, chaos synchronization has attracted a great deal of attentions from various fields since 1990

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when Pecora and Carroll introduced the pioneer work of synchronizing identical chaotic systems with different initials[1]. In recent years, a variety of types of synchronization of identical or different chaotic systems have been studied[1~9], such as complete synchronization[2~4], generalized synchronization[5,6], phase synchronization, lag synchronization[7], projective synchronization[8] and anti-synchronization[9] and so on.

The hybrid device of single electron transistor and metal oxide semiconductor (SETMOS) is a novel hybrid device based on single electron[10]. It has been taken more attentions due to its potential advantages[10], such as small structure size, low power consummation, high integration and negative differential resistance, which make it possible to design chaotic system based on SETMOS[10]. Recently, many chaotic systems based on SETMOS have been designed or realized[10]. For example, by using SETMOS, Chua chaotic circuit in structure of cellular neural networks has been realized. From these designs of chaotic systems based on SETMOS, it can be concluded that the chaotic systems have many advantages, such as simple structures and fast response rates[10].

In this paper, the double-scroll-like chaotic system based on SETMOS is analyzed. Then the generalized synchronization of the double-scroll-like chaotic system based on SETMOS and the simple piece-wise linear chaotic system[11] with unknown parameters is studied. Based on the Lyapunov stability theorem[3,4,12], the adaptive controller and the corresponding parameters update laws are designed to synchronize the two different chaotic systems. Because SETMOS is based on the nanoscale, the study on the synchronization of chaotic systems based on SETMOS is very important for the future applications for the secure communication in nano-technology[8,10].

2 Double-scroll-like chaotic system based on SETMOS

The double-scroll-like chaotic system can be implemented by transconductance amplifiers which are realized by SETMOS[10]. The circuit of chaotic system based on SETMOS is just as the Ref.[10]. We can obtain the equations

$$\frac{\mathrm{d}v_x}{\mathrm{d}t}=\frac{g_2}{C_3}v_y,\quad \frac{\mathrm{d}v_y}{\mathrm{d}t}=\frac{g_1}{C_2}v_z,\quad \frac{\mathrm{d}v_z}{\mathrm{d}t}=\frac{1}{C_1}[-g_3v_x-g_2v_y-g_1v_z+f(v_x)],\tag{1}$$

where

$$f(v_x)=\begin{cases} I_{\mathrm{b}}, v_x\geq 0\\ -I_{\mathrm{b}}, v_x<0\end{cases},$$

where I_{b} is constant. Here it assumes that $g_1=g_2=g_3=g, C_1=C/a, C_2=C_3=C, \tau=\dfrac{g}{C}t$ and $x=\dfrac{gv_x}{I_{\mathrm{b}}}, y=\dfrac{gv_y}{I_{\mathrm{b}}}, z=\dfrac{gv_z}{I_{\mathrm{b}}}$. Then the equation (1) can be expressed as

$$\frac{\mathrm{d}x}{\mathrm{d}\tau}=y,\quad \frac{\mathrm{d}y}{\mathrm{d}\tau}=z,\quad \frac{\mathrm{d}z}{\mathrm{d}\tau}=a[-x-y-z+f(x)],\tag{2}$$

where

$$f(x)=\begin{cases} 1, & x\geq 0\\ -1, & x<0\end{cases}.$$

Just setting proper values of parameters of the circuit, the chaos will be observed. Here setting $a=0.5$, and the initial states are -0.1, 0.3, 0.5, respectively. Fig.1 shows the state trajectories in x-y, and y-z planes.

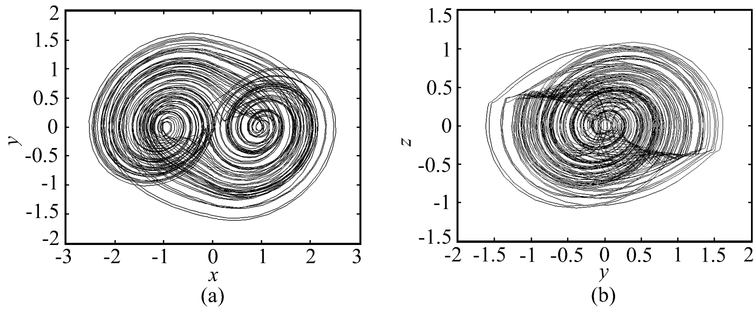

Fig.1 The state trajectories in (a) x-y plane and (b) y-z plane

In this section, the double-scroll-like chaotic system realized by SETMOS is analyzed. And the adaptive generalized synchronization of the mentioned chaotic system and the simple piecewise linear chaotic system with unknown parameter will be investigated in next section.

3 Adaptive generalized synchronization of different chaotic systems

The drive system is Sprott chaotic system which is the sample piecewise linear chaotic system as in the form of[11]

$$\dot{x}_1 = y_1, \quad \dot{y}_1 = z_1, \quad \dot{z}_1 = -\widetilde{A}z_1 - y_1 + |x_1| - 2, \tag{3}$$

where $\widetilde{A}$ is the evaluation of the unknown parameter A, and $X = [x_1, y_1, z_1]^{\mathrm{T}}$ is the state variable of the drive system.

The response system is a forementioned double-scroll-like chaotic system based on SETMOS. And the controlled response system can be written as

$$\dot{x}_2 = y_2 + u_1, \quad \dot{y}_2 = z_2 + u_2, \quad \dot{z}_2 = a[-x_2 - y_2 - z_2 + f(x_2)] + u_3, \tag{4}$$

where u_1, u_2, u_3 are controllers, and $Y = [x_2, y_2, z_2]^{\mathrm{T}}$ is the state variable of the response system.

A differentiable function is defined by the form of $G(x)$. The goal of adaptive generalized synchronization is to find out the proper controllers to synchronize the two different chaotic systems with regard to the differentiable function. Here give the definition of generalized synchronization.

Definition For two different chaotic systems described by (3) and (4), it's said that they are generalized synchronous with respect to the differentiable function $G(x)$ if there exists controllers $U = [u_1, u_2, u_3]^{\mathrm{T}}$, such that all trajectories $(X(t), Y(t))$ in (3) and (4) with any initial conditions $(X(0), Y(0))$ approach the manifold $M = \{X(t), Y(t) : Y(t) = G[X(t)]\}$ as time t goes to infinity, that is to say,

$$\lim_{t\to\infty} \|Y(t) - G[X(t)]\| = 0. \tag{5}$$

Let $e_1 = x_2 - G(x_1), e_2 = y_2 - G(y_1)$, and $e_3 = z_2 - G(z_1)$ be the errors between the states of systems (4) and (3). Because the function $G(x)$ is differentiable, the error dynamical system can be obtained.

$$\begin{aligned}
\dot{e}_1 &= \dot{x}_2 - \frac{\mathrm{d}G(x_1)}{\mathrm{d}t} = \dot{x}_2 - \frac{\mathrm{d}G(x_1)}{\mathrm{d}x_1}\dot{x}_1 = y_2 - \dot{G}y_1 + u_1, \\
\dot{e}_2 &= \dot{y}_2 - \frac{\mathrm{d}G(y_1)}{\mathrm{d}t} = \dot{y}_2 - \frac{\mathrm{d}G(y_1)}{\mathrm{d}y_1}\dot{y}_1 = z_2 - \dot{G}z_1 + u_2, \\
\dot{e}_3 &= \dot{z}_2 - \frac{\mathrm{d}G(z_1)}{\mathrm{d}t} = \dot{z}_2 - \frac{\mathrm{d}G(z_1)}{\mathrm{d}z_1}\dot{z}_1 = \\
&\quad a[-x_2 - y_2 - z_2 + f(x_2)] - \dot{G}[-\widetilde{A}z_1 - y_1 + |x_1| - 2] + u_3.
\end{aligned} \tag{6}$$

From the definition of generalized synchronization, it can be known that it is sufficient for generalized synchronizing the system (3) and (4) that the trivial solution of the error system (6) is asymptotically stable. In the following, the theorem is proposed for adaptive generalized synchronizing systems (3) and (4).

Theorem Let the adaptive controllers be

$$\begin{aligned} u_1 &= k_1e_1 - G(y_1) + \dot{G}(x_1)y_1, \\ u_2 &= k_2e_2 - G(z_1) + \dot{G}(y_1)z_1, \\ u_3 &= k_3e_3 + a(G(x_1) + G(y_1) + G(z_1) - f) + \dot{G}(z_1)(-y_1 + |x_1| - 2 - z_1A), \end{aligned} \tag{7}$$

and the corresponding parameter update law is

$$\dot{\widetilde{A}} = -\dot{G}(z_1)z_1e_3, \tag{8}$$

where k_1, k_2, k_3 subject to

$$k_1 < 0, \quad k_1k_2 - \frac{1}{4} > 0, \quad (1 - k_1 - k_2)a^2 + (2k_1 - 4k_1k_2)a + 4k_1k_2k_3 - k_1 - k_3 < 0. \tag{9}$$

Then, the generalized synchronization with respect to the differentiable function $G(x)$ of systems (3) and (4) can be achieved.

Proof Construct a Lyapunov function candidate in the form of

$$V = \frac{1}{2}[e_1^2 + e_2^2 + e_3^2 + (\widetilde{A} - A)^2]. \tag{10}$$

The time derivative of V along the trajectories in the error system (6) is

$$\begin{aligned} \dot{V} &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + (\widetilde{A} - A)\dot{\widetilde{A}} = (y_2 - \dot{G}y_1 + u_1)e_1 + (z_2 - \dot{G}z_1 + u_2)e_2 + \\ &\{a(-x_2 - y_2 - z_2 + f(x_2)) - \dot{G}[-\widetilde{A}z_1 - y_1 + |x_1| - 2] + u_3\}e_3 + (\widetilde{A} - A)\dot{\widetilde{A}}. \end{aligned} \tag{11}$$

Substitute Equs. (7) and (8) into (11), we can obtain

$$\begin{aligned} \dot{V} &= k_1e_1^2 + k_2e_2^2 + (k_3 - a)e_3^2 + e_1e_2 - ae_1e_3 + (1 - a)e_2e_3 = \\ &(e_1, e_2, e_3)\begin{bmatrix} k_1, & \frac{1}{2}, & -\frac{a}{2} \\ \frac{1}{2}, & k_2, & \frac{1-a}{2} \\ -\frac{a}{2}, & \frac{1-a}{2}, & k_3 - a \end{bmatrix}(e_1, e_2, e_3)^{\mathrm{T}}. \end{aligned} \tag{12}$$

And from the Equ (9), it can be obtained that

$$\begin{aligned} &(-1)^1|k_1| = -k_1 > 0, \\ &(-1)^2\begin{vmatrix} k_1 & \frac{1}{2} \\ \frac{1}{2} & k_2 \end{vmatrix} = k_1k_2 - \frac{1}{4} > 0, \\ &(-1)^3\begin{vmatrix} k_1 & \frac{1}{2} & -\frac{a}{2} \\ \frac{1}{2} & k_2 & \frac{1-a}{2} \\ -\frac{a}{2} & \frac{1-a}{2} & k_3 - a \end{vmatrix} = \\ &\quad -[(1 - k_1 - k_2)a^2 + (2k_1 - 4k_1k_2)a + 4k_1k_2k_3 - k_1 - k_3] > 0. \end{aligned} \tag{13}$$

Then by the Hurwitz theorem, we achieve that

$\dot{V} < 0.$

From the Lyapunov stability theorem, the trivial solution of the error system (6) is asymptotically stable which implies the generalized synchronization with respect to the function $G(x)$ of the systems (3) and (4) is achieved.

4 Numerical simulations

In this section, by using MATLAB, we simulate the generalized synchronization of the double-scroll-like chaotic system based on SETMOS and the simple piecewise linear chaotic system, just described as equations (3) and (4). The fourth-order Runge-Kutta method is used. The unknown parameter of the driving system is assumed as $A = 0.6$, and the initial evaluation of A is chose as $\widetilde{A} = 0$. The parameter of the response system is set as $a = 0.5$. And the initial states of the driving and response system are set as [0.2, 0.3, 0.1], [0, 0, 0], respectively. Assume the differentiable function is, $G(x) = -x$, and $k_1 = -2.5, k_2 = -2, k_3 = 0$. The simulation results are shown in Fig.2∼4.

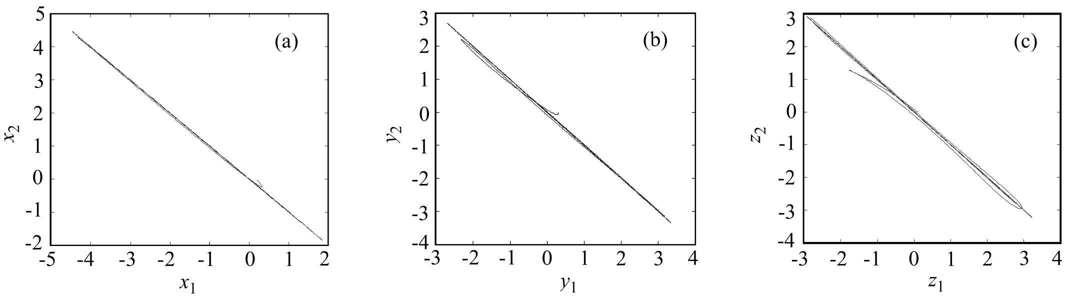

Fig.2 Generalized synchronization of the state variables of the drive (3) and response system (4)
(a) x_1-x_2 plane, (b) y_1-y_2 plane, (c) z_1-z_2 plane

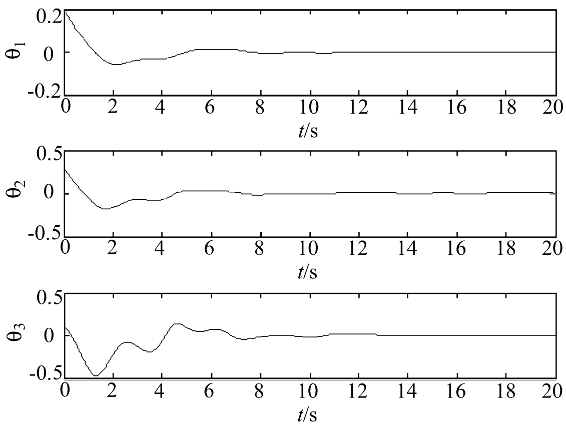

Fig.3 Generalized synchronization errors e_1, e_2, e_3 between systems (3) and (4)

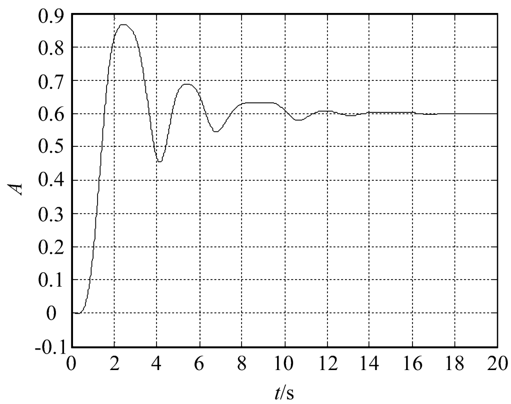

Fig.4 Estimated parameter A of the drive system

From Fig.2 and Fig.3, it can be concluded that the response system and the driving system realize synchronization with respect to the function $G(x)$ as time increases. Although the initial errors between the driving and the response systems are very large, they converge to zero when time is less than 10 s. And the adjustable parameters $k_i (i = 1, 2, 3)$ control the synchronization speed. From Fig.4, the unknown parameter A of the drive system can be identified very well. And the identified value of the unknown parameter is just

0.6 which is the same as the parameter A of the driving system. All of the simulation results are provided to show the effectiveness and feasibility of the method.

5 Conclusions

This paper is concerned with generalized synchronization of different chaotic system with unknown parameters, especially the response system is a double-scroll-like chaotic system realized by SETMOS[10]. Based on the Lyapunov stability theorem, the control schemes and the corresponding parameter update law has been proposed for generalized synchronization of the double-scroll-like and the simple piecewise linear chaotic systems with unknown parameters. Further, this method can generalize synchronize not only between different chaotic systems but also identical. And adaptive generalized anti-synchronization of different chaotic systems will be achieved if the function $G(x)$ were changed to $-G(x)$. Numerical simulations are provided to show the effectiveness and feasibility of the proposed method.

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