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RESEARCH PAPER

# A plastic indentation model for sandwich beams with metallic foam cores

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**Abstract** Light weight high performance sandwich composite structures have been used extensively in various load bearing applications. Experiments have shown that the indentation significantly reduces the load bearing capacity of sandwiched beams. In this paper, the indentation behavior of foam core sandwich beams without considering the globally axial and flexural deformation was analyzed using the principle of virtual velocities. A concisely theoretical solution of loading capacity and denting profile was presented. The denting load was found to be proportional to the square root of the denting depth. A finite element model was established to verify the prediction of the model. The load-indentation curves and the profiles of the dented zone predicted by theoretical model and numerical simulation are in good agreement.

**Keywords** Indentation · Sandwich beam · Metallic foam

## 1 Introduction

Metallic foam has attracted much interest due to its characteristics, such as low density, high specific strength and excellent performance of energy absorption [1,2]. Recently, the study of various structures with foam cores has attracted much attention [3–5]. The use of foam cores can increase the flexural bearing capacity of beams, but the improvement strongly depends on the indentation involved in the deformation. Hence the study on the indentation behavior of foam core sandwich beams is of great significance.

Much previous work has contributed to the knowledge of indentation behavior of foam core sandwich structures [6-10]. The indentation of sandwich beams with glass-fiber-reinforced plastic skins and poly foam cores was investigated by Shuaeib and Soden [11], where an analytical model with a linear elastic upper skin on the elastic-plastic core foundation was used to predict the loads at which the core yields and the upper skin fails. Foam core sandwich beams subjected to static indentation and subsequent unloading were analyzed using the Winkler foundation model [12], in which the deflection of the face sheet with bending stiffness was assumed linear elastic, and compressive response of the foam core was assumed elastic-perfectly plastic. The inelastic quasi-static response of sandwich beams and panels with a foam core to localized loads was obtained using the Kirchhoff–Love static theory [13]. The indentation behavior of foam core sandwich composite panels loaded by a spherical indenter was examined by Rizov et al. [14] through experimental and computational methods. In order to further understand the failure modes caused by local static indentation, numerical simulation was also conducted [15].

Although many researchers have investigated the indentation behavior of sandwich structures, their analysis was generally based on the assumptions that the face sheet material was elastic and the foam core was elastic-perfectly plastic. However, the elasticity of foam cores, which may lead to a complicated analysis and cover the understanding of the underlying mechanisms, is not significant. In many engineering applications, a deep indentation will significantly reduce the bending resistance of sandwich structures and result

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in large deflection. Hence the plastic behavior of face sheets should be considered. In this paper, we propose an analytical indentation model for sandwich structures with rigid-perfectly plastic face sheets and metallic foam cores, and the prediction is compared with the results of finite element simulations.

## 2 Analytical model

A sandwich beam with a foam core under denting loading usually deforms with a localized zone, as represented schematically in Fig. 1. According to practical applications, a face sheet is much thinner than a foam core, and the interface between them is fully bonded. The indentation induced by strongly localized interactions is significant in practical applications. Here, we present an analytical model based on the principle of virtual velocities to determine the deformation region.

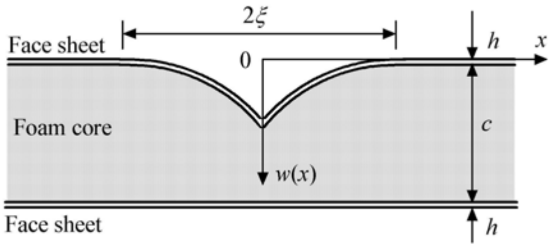


**Fig. 1** Schematic representation of the plastically deforming zone

In the present analysis, our model is based on the following assumptions:

(1) The deforming zone in denting process is much smaller than the engineering structures, and no globally longitudinal and flexural deformation is considered.

(2) The sheet of the sandwiched structure is rigid-perfectly plastic. The foam core has a constant plateau stress.

(3) The radius of the indenter is neglected, and the denting loading is in a line.

(4) The velocity field of the dented zone is assumed to vary linearly with $x$, according to the equation

$$\dot{w} = \dot{\delta}(1 - x/\xi), \tag{1}$$

where the dot signifies the time derivative, $\dot{\delta}$ is the rate of denting depth, and $\xi$ is the half width of the dented region. Thus, the change in the longitudinal curvature becomes zero, $\dot{\kappa}_{xx} = 0$, and the rate of internal work from flexural deformation can be neglected. The strain rate is defined according to the theory of moderately large deflections of beams

$$\dot{\varepsilon} = \frac{\mathrm{d}w}{\mathrm{d}x}\frac{\mathrm{d}\dot{w}}{\mathrm{d}x}. \tag{2}$$

(5) The thickness of the face sheet remains constant during deformation.

The contribution of the rate of energy dissipation by membrane forces in the dented region is calculated by

$$2\int_0^{\xi} N_0\dot{\varepsilon}b\mathrm{d}x = -(2N_0b\dot{\delta}/\xi)\int_{\delta}^{0}\mathrm{d}w = 2N_0b\delta\dot{\delta}/\xi, \tag{3}$$

where $N_0 = \sigma_0 h$ and $\sigma_0$ is flow stress of the face material, $b$ is the width of the sandwich beam, and $h$ is the face sheet thickness. It is noteworthy that the form of the skin deflection $w(x)$ is still unknown and we only use the conditions $w(0) = \delta$ and $w(\xi) = 0$ to determine Eq. (3).

The contribution of the rate of energy dissipation due to compressive deformation of foam cores is calculated by

$$\int_V \sigma_{\mathrm{p}}\mathrm{d}\dot{V} = 2\int_0^{\xi}\sigma_{\mathrm{p}}\dot{\delta}(1 - x/\xi)b\mathrm{d}x = \sigma_{\mathrm{p}}b\xi\dot{\delta}, \tag{4}$$

where $\sigma_{\mathrm{p}}$ is the plateau stress of foam materials.

The total rate of energy dissipation is summed up to

$$\dot{D} = 2N_0b\delta\dot{\delta}/\xi + \sigma_{\mathrm{p}}b\xi\dot{\delta}. \tag{5}$$

The statement of equilibrium between the rate of external energy $\dot{E}$ and the rate of internal work $\dot{D}$ is expressed via the principle of virtual velocities [16]

$$\dot{E} = \dot{D}, \tag{6}$$

where $\dot{E}$ and $\dot{D}$ represent the rate of work of external force and the internal energy, respectively. The rate of external work is given by

$$\dot{E} = P\dot{\delta}, \tag{7}$$

where $P$ is the denting load. The rate of internal work of the sandwich beam is mainly given by the sum of contributions due to extensional deformation of the face sheet and the compressive deformation of the foam core.

Based on Eqs. (5)–(7), an expression for load $P$ is obtained

$$P = 2N_0b\delta/\xi + \sigma_{\mathrm{p}}b\xi, \tag{8}$$

which depends on $\xi$ and the actual load corresponds to the minimum $P$. Using the minimal condition of the load [17], $\partial P/\partial\xi = 0$, we have a relation between $\xi$ and $\delta$, i.e.

$$\xi = (2N_0\delta/\sigma_{\mathrm{p}})^{1/2}. \tag{9}$$

Substituting this relation back into Eq. (8), we can determine the desired load-indentation relation of the indentation process

$$P/b = 2(2N_0\sigma_{\mathrm{p}}\delta)^{1/2}. \tag{10}$$

This expression shows that the denting load is proportional to the square root of the denting depth.

The deflection profile of the dented zone $w(x, t)$ is unknown as indicated previously, but it can be determined by the following derivations due to the dependence of $\xi$ on $\delta$. The extent of the deformed zone $\xi$ varies with $\delta$ and so changes in time $t$. For a given position $x$, the skin remains undeformed until $\xi > x$. Therefore, the resulting permanent deflection profile is calculated by a time-integral of velocity as

$$w(x,t) = \int_{T(x)}^{t} \dot{w}(x,t)\mathrm{d}t = \int_{T(x)}^{t} \dot{\delta}[1 - x/\xi(t)]\mathrm{d}t, \tag{11}$$

where $T(x)$ is the time at which $\xi = x$. By changing the integration variable from $t$ to $\delta$, Eq. (11) is rewritten as

$$w(x,\delta) = \int_{\delta(x)}^{\delta(\xi)} [1 - x/(2N_0\delta/\sigma_{\mathrm{p}})^{1/2}]\mathrm{d}\delta, \tag{12}$$

where $\delta(x) = (\sigma_{\mathrm{p}}/2N_0)x^2$ at time $T(x)$. To simplify the integrating process, Eq. (12) is rewritten in the following form by further changing the integration variable from $\delta$ to $\xi$,

$$w(x,\delta) = \frac{\sigma_{\mathrm{p}}}{N_0}\int_{x}^{\xi} (\xi - x)\mathrm{d}\xi. \tag{13}$$

Integrating this expression, we determine the deflection profile as

$$w(x,\delta) = \delta(1 - x/\xi)^2, \tag{14}$$

which varies non-linearly with the position $x$.

## 3 Modeling the indentation behavior of sandwich structures

In order to verify the validity of the above theoretical results and evaluate the error caused by the assumptions introduced in the model, a 2-D finite element analysis was conducted using the ABAQUS code. The numerical model was established for half of the structure due to symmetry, see Fig. 2. The half-length of the sandwich beam is 100 mm while the thicknesses of the face and core are 2 mm and 20 mm, respectively.

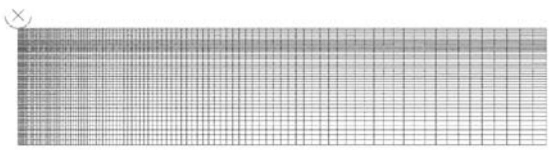

**Fig. 2** Finite element model of a sandwich beam indented by a cylindrical indenter

The indenter was modeled as a rigid cylinder, and a relatively small indenter diameter of 5 mm was chosen to simulate a line load. The face sheet and foam core were modeled using 4-node bilinear plane strain quadrilateral finite elements (CPE4R). In total, 4 500 elements were used. The finite element mesh was condensed towards the contact area between the indenter and the face sheet. The contact between the face sheet and foam core was set as TIE of surface to surface. All rigid body motions of the indenter were restricted except the displacement in the vertical direction. The lower and end boundaries of the sandwich beam were fully clamped.

The face sheet material was assumed as elastic-perfectly plastic. The mechanical properties of the face and core materials are listed in Table 1. The plastic part of the mechanical behavior of the core was modeled using the *CRUSHABLE FOAM and the *CRUSHABLE FOAM HARDENING options in the ABAQUS package as shown in Fig. 3.

**Table 1** Mechanical properties of structural constituents used in the FE model

| | $\rho/(\mathrm{kg{\cdot}m^{-3}})$ | $E$/GPa | $\nu$ | $\sigma_0$/MPa | $\sigma_{\mathrm{p}}$/MPa |
|---|---|---|---|---|---|
| Face | 2700 | 50 | 0.2 | 150 | – |
| Core | 400 | 0.25 | 0 | – | 1.0 |

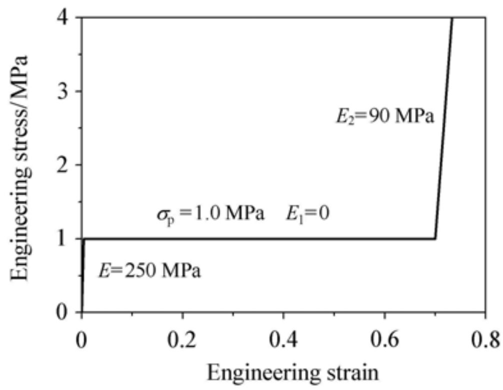


**Fig. 3** Engineering stress–strain relation of foam core used in the FE model

## 4 Results and discussion

A comparison between the load-indentation curves predicted by the theoretical model and calculated by the numerical simulation is presented in Fig. 4. In the first initial phase, the indenter diameter leads to more deformation region than that caused by a line load, so the numerical result is a little higher than the analytical one. As the deformation region expands, the effect of indenter diameter would gradually decrease. In addition, elongation of the face sheet would gradually reduce its thickness, so finally the simulation load is a little lower than that from the analysis. On the whole, results from the two different methods are in good agreement.

Numerical results show that the dented zone is continuously expanded with the increasing of the denting depth as predicted by Eq. (9). The profile in the dented zone obtained from numerical simulation is consistent with that by theoretical prediction, except that a little difference was found around the contact area, as shown in Fig. 5. The difference is due to the assumption equation (3) in the theoretical analysis. If finite size of the indenter is taken in the theoretical modeling, the linear assumption of the velocity distribution in the dented zone needs to be changed, which will lead to a complex derivation.

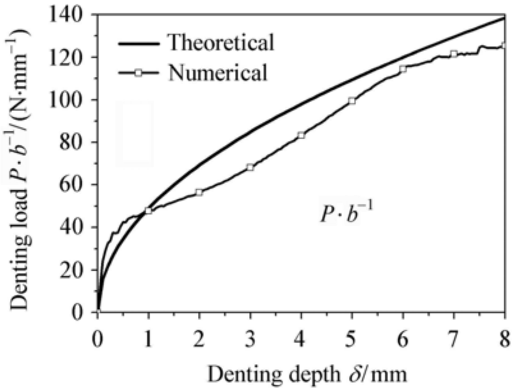


**Fig. 4** Load-indentation curves of sandwich beam

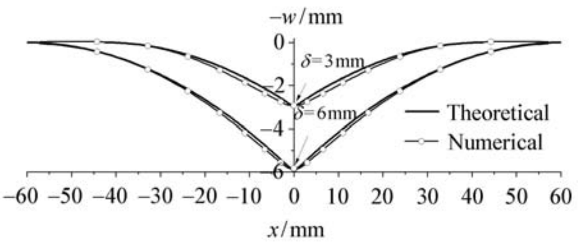


**Fig. 5** Theoretical and simulated profiles during denting

## 5 Conclusions

The indentation behavior of foam core sandwich beams without considering globally axial and flexural deformation was analyzed using the principle of virtual velocities. A concisely theoretical solution of loading capacity and denting profile was presented. The denting load was found to be proportional to the square root of the denting depth. A 2D finite element model was established to verify the validity of the theoretical result. It was found that the two results are in good agreement. The influence of the diameter of the indenter on the load-indentation curves and the profiles of the dented zone were also discussed. The analytical method may be extended to study the coupled deformation of indentation and flexure.

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