Fast and Secure Root-Finding for Code-based Cryptosystems

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Abstract. In this work we analyze four previously published respectively trivial approaches to the task of finding the roots of the error locator polynomial during the decryption operation of code-based encryption schemes. We compare the performance of these algorithms and show that optimizations concerning finite field element representations play a key role for the speed of software implementations. Furthermore, we point out a number of timing attack vulnerabilities that can arise in root-finding algorithms, some aimed at recovering the message, others at the secret permutation. We give experimental results showing that manifestations of these vulnerabilities are present in straightforward implementations of most of the root-finding variants presented in this work. As a result, we find that one of the variants provides security with respect to all vulnerabilities as well as highly competitive results in terms of performance for code parameters that minimize the public key size.

Keywords: side channel attack, timing attack, implementation, code-based cryptography

1 Introduction

Implementations of code-based cryptosystems like the McEliece[1] and Niederreiter[2] schemes have received growing interested from researchers in the past years and been analyzed with respect to efficiency on various platforms [3–7]. Furthermore, a growing number of works has investigated the side-channel security of code-based cryptosystems [8–12].

In this work, we will turn to an algorithmic task that arises in the decryption operation of both Niederreiter and McEliece cryptosystems. This is the root-finding algorithm. It deserves attention for two reasons: first of all, as addressed already in previous work, it is in general the most time-consuming part of the decoding algorithm [3, 5]. The second aspect is that of side-channel security of the root-finding and has so far, to the best of our knowledge, only been considered in [13]. We point out basically two types of timing side-channel vulnerabilities

** To the most part, this work was done in the author's private capacity, a part of the work was done at[2]

that can arise in the root-finding procedure. One is aimed at recovering the message to a given ciphertext, the other at finding the permutation that is part of code-based private keys.

Based on these considerations, we chose four different root-finding algorithms, which we describe in Sec. 3 after providing some elementary preliminaries about code-based cryptosystems in Sec. 2. In Sec. 4, we perform a timing side-channel analysis of these algorithms including theoretical discussions and experimental results. Afterwards, in Sec. 5, we give the results of a performance evaluation of the chosen algorithms on modern x86 architectures.

As the main result from our work, we find that the root-finding variant according to [14], that to the best of our knowledge has not been taken into consideration for code-based cryptosystems so far, achieves both competitive results in terms of running time and security with respect to all potential timing side-channel vulnerabilities in the root-finding. But we wish to stress that it is not the aim of this work to give a definitive answer to the question which root-finding variant is the best to use in a code-based cryptosystem. We will come back to this in the Conclusion in Sec. 6.

As the starting point for the implementation that the experimental results of this work are based on, we used the HyMES open source implementation [15] of the McEliece scheme presented in [3]. We added the countermeasure previously proposed in [11], and removed some fault attack vulnerabilities, the latter is addressed in App. C. Furthermore, we completely adopted the root-finding algorithm variant given by the Berlekamp Trace Algorithm [16], as it is found in HyMES. We added the remaining root-finding variants that are presented in Sec. 3. In this context, we want to emphasize that we do not take any credit for the the choice of the Berlekamp Trace Algorithm for the purposes of root-finding in code-based cryptosystems, the implementation of this algorithm used for the results in this work, and its description as given in this work, since all of this is adopted from [3] resp. [15].

2 Preliminaries

In the following, we describe those aspects of the encryption and decryption of code-based cryptosystems as McEliece[1] and Niederreiter[2] schemes that are relevant for the topics of this work. As these only depend on properties common to both types of cryptosystems, it is possible for us to basically omit any distinction between them.

Code-based cryptosystems build on error correcting codes. Specifically, the only codes known to be secure for this use are Goppa Codes [17]. The fundamental properties of such a code are its length n, which is the length of the code words, where we restrict the following discussion to the case $n = 2^m$; the dimension k (with $k < n$), which is the length of the message words; and the error correcting capability t (all lengths refer to lengths of the binary representation). For such a code, if a message word is encoded into a code word, up to t bit flip errors in the code word may be corrected by a corresponding error correction

algorithm, thus allowing to recover the message word. In the following, we will also make use of the expression of "adding errors" to the code word, by which we mean carrying out the "exclusive or" (XOR) operation with the code word $\boldsymbol{c}$ and the error vector $\boldsymbol{e}$, i.e. $\boldsymbol{c} \oplus \boldsymbol{e}$.

In both McEliece and Niederreiter schemes, the encryption involves the creation of an error vector $\boldsymbol{e}$, whose Hamming weight $\mathrm{wt}(\boldsymbol{e})$ is equal to the error correcting capability t of the employed code. The concrete realization of the encryption is different in both schemes, but in either case, it is vital for the privacy of the encrypted message that $\boldsymbol{e}$ remains secret.

In the McEliece and Niederreiter cryptosystems, syndrome decoding through Patterson's Algorithm [18] plays a key role. As the details of this algorithm are irrelevant for purposes of understanding the topics of this work, we give only a brief outline of this algorithm. In the McEliece cryptosystem, the syndrome is computed from the ciphertext by multiplying the ciphertext with the so called parity check matrix, in the Niederreiter scheme, the syndrome is the ciphertext itself. The syndrome decoding algorithm takes as input the so called syndrome vector $\boldsymbol{s}$ and outputs the error vector $\boldsymbol{e}$ that was added to the code word.

As already mentioned, this work deals with one part of the syndrome decoding, this is the finding of the roots, i.e. zeros of the so called error locator polynomial $\sigma(Y) \in \mathbb{F}_{2^m}[Y]$ which is computed in the course of syndrome decoding. In case of $w = \mathrm{wt}(\boldsymbol{e}) \leqslant t$ it holds that

$$\sigma(Y) = \Pi_{i=1}^{w}(\gamma_{E'_i} - Y), \tag{1}$$

where $\{\gamma_i\} \in \mathbb{F}_{2^m}$ are in lexicographical ordering and $\{E'_i\}$ are the positions of the bits having value 1 in the error vector $\boldsymbol{e}'$, whose relation to $\boldsymbol{e}$ will be explained in the following. From this equation we see that the indices of the roots in $\mathbb{F}_{2^m}$ correspond to the error positions in $\boldsymbol{e}'$. If $w > t$, then $\sigma(Y)$ will have degree less or equal than t, where the probability for the latter is very high, but it will not be of the form given in Equ. 1.

When error correcting codes are employed for the task they were originally designed for, the syndrome decoding outputs the error vector that was added to the code word, i.e. $\boldsymbol{e} = \boldsymbol{e}'$. In code-based cryptosystems, however, first a secret permutation P has to be applied to $\boldsymbol{e}'$ to recover the error vector $\boldsymbol{e}$ that was added during encryption:

$$E_i = P_{E'_i}, \tag{2}$$

where $\{E_i\}$ are the (usually t) indices of those positions in $\boldsymbol{e}$ that have value one, and P_j is the the value for which the value j has to be substituted according to the permutation P. The employment of the secret permutation, that is part of the private key, is fundamental for code-based encryption schemes: in this way the encrypting party is able to encode code words in a public code, which is connected to the private code via P.

In Sec. 3, we will present four different concrete algorithms for the task of finding the roots of the error locator polynomial $\sigma(Y)$.

3 Variants of Root Finding

Before we start with the descriptions of the root-finding algorithms we compare in this work, we want to point out some details concerning the costs of the basic $\mathbb{F}_{2^m}$ operations that are involved, i.e. addition and multiplication.

While $C_{\text{gf\_add}}$, the cost of an addition in $\mathbb{F}_{2^m}$, is given by a simple XOR operation, the multiplication in $\mathbb{F}_{2^m}$ is much more complex and has a number of variants. An efficient software implementation of finite field arithmetics with characteristic 2 and small extension degrees is realized by the use of one lookup table for the logarithm of each non-zero element to the base of some primitive element, and the corresponding anti-logarithm table.

The standard multiplication, as it is for instance implemented by the "C" macro `gf_mul()` in HyMES [15], which is used throughout their code, takes arguments in the normal representation and outputs the result in normal representation. This type of multiplication, we refer to as *mul_nnn* . Its cost is two conditional branches to check whether the arguments are zero, three table lookups, one arithmetic ADD, and reduction of the result modulo the fields multiplicative order, which in turn consists of several instructions. In the general case, this multiplication is needed, as in most places in the algorithms involved in the syndrome decoding, multiplication and addition in $\mathbb{F}_{2^m}$ are intermixed, and moreover, operands having value zero cannot be excluded.

However, when operands are known to be non-zero, and multiplications are carried out subsequently, other forms of the multiplication, which have results (a in the algorithm description *mul_abc*) or operands (b and c) in the logarithmic representation, are more efficient:

- *mul_lll* consists only of one arithmetic ADD (a certain number of these multiplications can be carried out before a reduction modulo the multiplicative order becomes necessary to avoid overflowing the register)
- *mul_nln* saves one conditional branch and one table lookup compared with *mul_nnn*

This rough review of the finite field arithmetic implementations in software makes it obvious that speaking of the cost of multiplication in $\mathbb{F}_{2^m}$ is not sufficient to describe actual computation costs, there exist drastic differences in the cost with respect to how the multiplication is embedded into the algorithm.

3.1 Exhaustive Evaluation with and without Division

The most straightforward implementation of the root finding is to simply evaluate the polynomial $\sigma(Y)$ for each element of the code.

The complexity of this algorithm is given as

$$C_{\text{eval}-\text{rf}} = nt(C_{\text{gf\_add}} + C_{\text{mul\_nln}})$$

Taking a look at the Horner Scheme evaluation used here, we see that when evaluating $\sigma(x)$, we can transform $x \neq 0$ to the logarithmic representation, avoiding some unnecessary table lookups, i.e. make use of *mul_nln* .

The algorithm can be sped up by dividing the polynomial $\sigma(Y)$ by each root found. Such a division has basically the same complexity as the evaluation of the polynomial for one single element of $\mathbb{F}_{2^m}$. In the following, we will call these two variants *eval-rf* and *eval-div-rf*.

3.2 Berlekamp Trace Algorithm

As stated in the introduction, our implementation is based on the HyMES implementation [3, 15]. There, the root finding is achieved by the so called Berlekamp Trace Algorithm [16]. For completeness, we provide the description of this algorithm as originally given in [3] in Alg. 1. The initial call to this recursive algorithm is given as $\mathrm{BTA}(\sigma(Y), 1)$, which we will refer to as *bta-rf* for the remainder of this work. The trace function is defined as $\mathrm{Tr}(Y) = Y + Y^2 + Y^{2^2} + \ldots + Y^{2^{m-1}}$, and $\{\beta_1, \beta_2, \ldots, \beta_m\}$ is a standard basis of $\mathbb{F}_{2^m}$.

Algorithm 1 The recursive Berlekamp Trace Algorithm $\mathrm{BTA}(\sigma(Y), i)$.

Require: the error locator polynomial $\sigma(Y)$
Ensure: the roots $\mathcal{E}'$ of $\sigma(Y)$
1: **if** $\deg(\sigma(Y) \leqslant 1)$ **then**
2: **return** root of $\sigma(Y)$
3: **end if**
4: $\sigma_0(Y) \leftarrow \gcd(\sigma(Y), \mathrm{Tr}(\beta_i, z))$
5: $\sigma_1(Y) \leftarrow \gcd(\sigma(Y), 1 + \mathrm{Tr}(\beta_i, z))$
6: **return** $\mathrm{BTA}(\sigma_0(Y), i+1) \cup \mathrm{BTA}(\sigma_1(Y), i+1)$

In [19], the complexity of the BTA is given as $\mathcal{O}(mt^2)$. The concrete cost in terms of $\mathbb{F}_{2^m}$ operations is not given there and cannot be easily derived.

3.3 Root-finding with linearized Polynomials

In this section, we explain a root finding method based on decomposing a polynomial in $\mathbb{F}_{2^m}[Y]$ into linearized polynomials [14]. The idea of this approach is based on the fact that the exhaustive evaluation of a linearized polynomial can be done with much less computational complexity than for general polynomials.

Definition 1. *A polynomial $L(Y)$ over $\mathbb{F}_{2^m}$ is called a linearized polynomial if $L(Y) = \sum_i L_i Y^{2^i}$, where $L_i \in \mathbb{F}_{2^m}$.*

As shown in [14], an affine polynomial of the form $A(Y) = L(Y) + \beta$ with $\beta \in \mathbb{F}_{2^m}$ can be evaluated for the value $Y = x_i$ as

$$A(x_i) = A(x_{i-1}) + L(\Delta_i), \Delta_i = x_i - x_{i-1} = \alpha^{\delta(x_i, x_{i-1})}, \tag{3}$$

where $\{\alpha^0, \alpha^1, \ldots, \alpha^{m-1}\}$ is a standard basis of $\mathbb{F}_{2^m}$ and $\mathrm{wt}\,(x_i \oplus x_{i-1}) = 1$, i.e. their Hamming distance is 1. A generic decomposition of a polynomial $f(Y) = \sum_{i=0}^{t} f_i Y^i$, also given in [14], is

$$f(Y) = f_3 Y^3 + \sum_{i=0}^{\lceil (t-4)/5 \rceil} Y^{5i} A_i(Y), \tag{4}$$

where

$$A_i(Y) = f_{5i} + \sum_{j=0}^{3} f_{5i+2^j} Y^{2^j}. \tag{5}$$

The evaluation of each $A_i(x_i)$ is done efficiently according to Equ. 3. To this end, the exhaustive evaluation of Equ. 4 is done with the x_i being in Gray Code ordering, i.e. for all i we have that x_i and x_{i+1} differ only in one single bit. Specifically, we use the Gray Code generated by $x_i = (\mathrm{i} >> 1) \oplus i$, where ">>" denotes logical right shift. The actual computation cost is given by the sum of the precomputations, i.e. the computation of the $A_i(Y)$. This cost is given in [14], it is however negligible for secure code parameters. The dominating cost is that of computing $f(Y)$ for all n code elements:

$$C_{\mathrm{dcmp-rf}} = (n-1)\,(2C_{\mathrm{look}} + C_{\mathrm{mul\_nll}} + \lceil (t+1)/5 \rceil\,(2C_{\mathrm{gf\_add}} + C_{\mathrm{mul\_lll}} + C_{\mathrm{mul\_nln}}))$$

where C_{look} refers to the cost of looking up $\log(x^3)$ resp. $\log(x^5)$ from precomputed tables. This optimization we use to avoid the computation of $\log(x^3)$ and $\log(x^5)$ has only a small benefit in speed, as the computation of these values through subsequent *mul_lll* operations is also fast.

4 Security Aspects of the Root Finding in Code-based Cryptosystems

In this section, we show that a dependency of the root finding algorithm's running time on the number of the roots of the Error Locator Polynomial $\sigma(Y)$ introduces vulnerabilities to timing attacks against the cleartext, and that other effects threaten the secrecy of the secret permutation P.

In order to be able to judge the relevance of the timing results provided in this section, it is important to know that the syndrome decoding, which we consider to include the root-finding, is at least for the McEliece cryptosystem the only source of variable running time, under some assumptions[3].

[3] The assumptions are: multiplication of the ciphertext with the parity check matrix is either constant time of linear in the ciphertext's Hamming weight (these are the straightforward implementation choices); the CCA2 conversion is constant time (see for instance [20]); the computation according to Equ. 2 is constant time (cache effects could subvert this).

4.1 Security against Message-aimed Attacks

The first important fact to know is that $\sigma(Y)$ output by Patterson's Algorithm has $\mathrm{wt}(\boldsymbol{e})$ roots in case $\mathrm{wt}(\boldsymbol{e}) \leqslant t$, and only a fraction of t roots if $\mathrm{wt}(\boldsymbol{e}) > t$ (refer to Equ. 1). For instance, for $n = 2048$ and $t = 50$, $\sigma(Y)$ typically has about five roots in the latter case.

A dependence of the running time on the number of roots thus potentially creates the following problem: if the case $w > t$ can be inferred from the running time, an attack similar to that described in [9] is possible. In such an attack, the attacker flips a bit in ciphertext he wishes to decrypt, observes the decryption, and from the running time tries to guess whether $t + 1$ or $t - 1$ errors resulted from his bit flip. This clearly gives him information about the error positions piece by piece.

Note that the case of $w < t$ is covered by the countermeasures proposed in [11]. In the presence of these countermeasures, the decryption of a ciphertext with $\mathrm{wt}(e) < t$ also results in $\sigma(Y)$ with degree t and very few roots. But it is important to be aware that even if $w < t$ and $w > t$ are not distinguishable based on the timings, but $w = t$ can be distinguished from $w \neq t$, an attack is still possible: by flipping two bits in a ciphertext and trying to find those cases where $w = t$, the attacker will learn whenever he flipped one non-error and one error position.

In Figures 1(b), 1(a), 1(c) and 1(d), we give plots of the running time of the root-finding for the four different algorithms. The timings were taken on an Atmel ATMega1284P Microcontroller. We chose this platform, as it provides far more deterministic cycle counts than a modern x86 CPU, and thus is more suited to identify possible timing vulnerabilities. We used a Goppa Code with parameters $n = 512$ and $t = 33$ for the syndrome decoding performed on the microcontroller and created 30 different syndromes for each value of w between 20 and 40. The cycle counts apply to the running time of the respective root-finding algorithm. For each value of w the center mark indicates the mean of the set of the 30 different syndromes, and the bar shows the minimal and maximal values from this set.

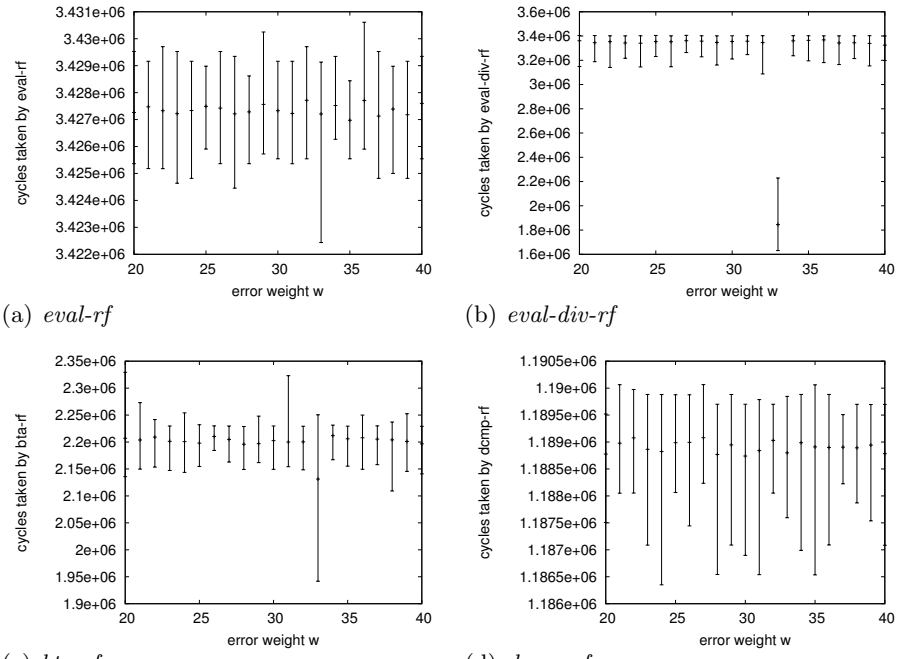
In the syndrome decoding of our implementation, the countermeasure proposed in [11] is included, so that for $w < t$ we still have $\deg(\sigma(Y)) = t$, but the number of roots of $\sigma(Y)$ is much less than t, as already mentioned. This happens automatically for $w > t$ in Patterson's Algorithm, so that we can expect to find major differences in the running time of the root-finding algorithm only for the cases $w \neq t$ and $w = t$.

The *eval-rf* algorithm's running time, depicted in Fig. 1(a), shows the mean running time of $w = t$ in line with the those of $w \neq t$. However, there seem to be cases of $w = t$ with considerably lower running time than for $w \neq t$, as can be seen by the depicted minimal value. Neither did we find the reason for this, nor did we analyze whether this effect can be used for actual attacks. We justify these omissions by the fact that, as already apparent from the results given in this section, *eval-rf* is not a competitive candidate for root-finding in code-based cryptosystems.

Fig. 1(b) shows clearly the speedup by a factor of two by *eval-div-rf* compared to *eval-rf*. However, also the inherent timing vulnerability [13] of this algorithm cannot be overlooked: the case $w = t$, where the benefit of the divisions has its real impact, is almost twice as fast as for $w \neq t$. This renders it an insecure choice.

The timing results of *bta-rf*, as implemented in HyMES, are shown in Fig. 1(c). Here, we can realize that already the mean of the running times for $w = t$ is below most of the minimal values of sets for $w \neq t$, clearly indicating a vulnerability. We did however not find the reason for this effect.

Finally, Fig. 1(d) shows the results for *dcmp-rf*. There is no apparent difference between the cases $w = t$ and $w \neq t$.



(a) *eval-rf* (b) *eval-div-rf*

(c) *bta-rf* (d) *dcmp-rf*

Fig. 1: Cycle counts taken on an ATMega1284P for the various root-finding algorithm variants with parameters $n = 512$ and $t = 33$.

4.2 Security with respect to Attacks aiming at the secret Support

We now show that other vulnerabilities can arise in the root-finding algorithm, which allow attacks against the secret permutation of the code-based scheme. This is for instance the case, when the running time of the root-finding algorithm depends on the values of the roots found. To understand that this is a

vulnerability one has to consider that an attacker can create a ciphertexts with $\boldsymbol{e}$ known to him. Then, according to Equ. 2 any information about the roots is information about P.

Fig. 2(a) and 2(b) shows running times on the AVR platform of the syndrome decoding with *eval-div-rf* for $n-(t-1)$ error vectors created in the following way: a random error pattern of weight $t-1$ was fixed, and the position of the last error, E'_t was varied over the remaining free positions, resulting in an error vector with Hamming weight t. On the x-axis, $P^{-1}_{E_t} = E'_t$ is shown. We will refer to this type of plot as "support scan"[4] henceforth. The result is a relatively clear linear ascend, which is not surprising when considering the *eval-div-rf* algorithm: Starting evaluation at $Y = 0$, the earlier a root is found, the more beneficial is the reduction of the degree of $\sigma(Y)$ by one through the subsequent division.

Thus, the task for an attacker amounts to bringing the measured timings into an ascending ordering, giving him P. Obviously, there is some distortion of this ordering in Fig. 2(a), which stems from other operations of variable duration in the syndrome decoding. We leave it open whether in this manner the permutation P becomes known to the attacker in its entirety, it is however clearly obvious, that a large amount of information about P becomes available. We discuss possible countermeasures for *eval-div-rf* in App. B; it is not in the focus of this work to advise a timing side-channel secure implementation of this rather uncompetitive root-finding algorithm.

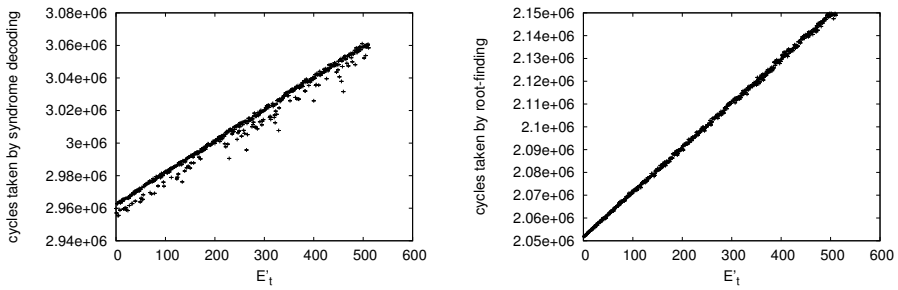
We also wish to point out that an attack exploiting this vulnerability, in contrast to other previously published timing attacks [11, 10] , cannot be detected: The ciphertext carries t errors and will pass the CCA2 integrity test[5]. This is important, because the other attacks, which cannot be carried out in a clandestine manner in this sense, can be thwarted by countermeasures which detect the irregularity of the ciphertext, and for instance add an enormous delay or enforce constant running time if possible on the respective platform[6].

We also analyzed the *dcmp-rf* and *bta-rf* algorithms with respect to these vulnerabilities. As one should expect from an algorithm that performs an exhaustive search, *dcmp-rf* does not exhibit any dependency of the running time on the root positions. The results are given in Fig. 3 in App. A. On the other hand, for *bta-rf*, we see some "clouding" effect in the running times, which is also apparent for timings of the whole syndrome decoding, as shown in Fig. 2(c). Thus it remains unclear whether there is a vulnerability in the *bta-rf* allowing attacks against the secret permutation, we discuss this a little further in App. A.

[4] From another point of view, P^{-1} can be seen as the so called support of the employed code.

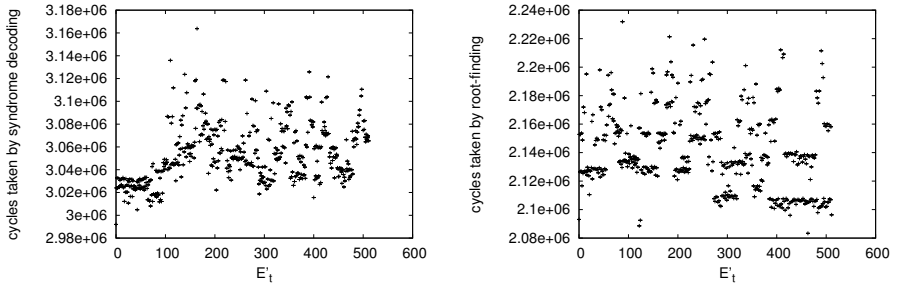
[5] A CCA2 conversion is necessary for any Niederreiter or McEliece like code-based encryption scheme, see for instance [21].

[6] In the presence of the threat of power analysis attacks, however, such countermeasures would in most cases not suffice as adding delays after the actual computation will most likely be detectable in the power trace.



(a) Timing for the syndrome decoding with *eval-div-rf*.

(b) Timing for the root-finding with *eval-div-rf*.



(c) Timings for syndrome decoding with *bta-rf*.

(d) Timings for root-finding with *bta-rf*.

Fig. 2: Running times of *eval-div-rf* and *bta-rf* for $n-(t-1)$ ciphertexts, where $t-1$ error positions are fixed and the $t-th$ position varies, with code parameters $n = 512$ and $t = 33$.

But there is one caveat concerning the implementation of *dcmp-rf*: this is the multiplication by σ_3 in Equ. 4. In our implementation, we precompute the logarithmic representation of σ_3 to subsequently use *mul_nll* for the computation $\sigma_3 Y^3$. In the unprotected variant of our implementation we cover the case $\sigma_3 = 0$ by a conditional branch that bypasses this multiplication. However, in this case, the timings clearly allow identification of a syndrome decoding where $\sigma_3 = 0$. This is shown in Fig. 3(a) and 3(b) in App. A. The information gained by such an observation is, according to Equ. 1:

$$0 = \sigma_3 = \epsilon_1\epsilon_2 \dots \epsilon_{w-3} \oplus \epsilon_1\epsilon_2 \dots \epsilon_{w-4}\epsilon_w \oplus \dots,$$

i.e. the sum of products of all possible combinations of $w-3$ different $\epsilon_i \in \mathbb{F}_{2^m}$, $i = 1, \dots, w$, where w is the error vector's Hamming Weight, usually $w = t$, and $\epsilon_i = \gamma_{E_i'} = \gamma_{P^{-1}_{E_i}}$. It is certainly not trivial to exploit this information, however, in combination with other vulnerabilities it might be useful to provide a means of verifying guesses for P. The countermeasure to protect against this vulnerability is trivial and comes at a low computational cost, it is explained in App. A.

5 Performance of the Root-finding Variants

In this section, we give a comparison of the performance of *eval-div-rf*, *bta-rf*, and *dcmp-rf*. We omit *eval-rf* as it is a hopeless candidate in terms of running time as we have already seen in Sec. 4. The code was compiled with GCC version 4.5.2 with the optimization options `-finline-functions -O3 -fomit-frame-pointer`.

Tab. 1 summarizes the results for two parameter sets based on the propositions given in [22], which are based on code parameter choices aiming at the minimization of the public key size, which is known to be the most problematic feature of code-based cryptosystems. However, since our implementation only supports $n = 2^m$ and the addition of no more than t errors (both of which restrictions do not apply to the assumptions made by the authors in that paper), the security levels given in the table are not exact. For these parameters, we can see that *bta-rf* and *dcmp-rf* are close competitors.

However, it must be pointed out, that for parameter choices using large values of m while trying to minimize t, *bta-rf* clearly wins against *dcmp-rf*. In Tab. 2, we give the root-finding running times for these two algorithms for some parameter sets taken from the results given in [3]. The drawback of such a parameter choice is a large public key size.

6 Conclusion and Outlook

In this work we have evaluated four different root-finding algorithms with respect to their performance and timing side-channel security in code-based cryptosystems. We have shown that timing vulnerabilities can be present in all of these variants. The variant *eval-rf* and *eval-div-rf* can be ruled out as they are both

parameters	**security level**	**root-finding algorithm**	**running time / 10^5 cycles**
$m = 12$, $t = 57$	≈ 128 bit	*bta-rf*	8.31
		dcmp-rf	8.67
		eval-div-rf	15.98
$m = 13$, $t = 115$	≈ 256 bit	*bta-rf*	34.54
		dcmp-rf	29.29
		eval-div-rf	74.10

Table 1: Comparison of the root-finding algorithm performance on an x86 Intel Intel(R) Core(TM)2 Duo CPU U7600 for code parameters as suggested in [22].

parameters	**root-finding algorithm**	**running time / 10^5 cycles**
$m = 11$, $t = 32$	*bta-rf*	3.16
	dcmp-rf	3.21
$m = 12$, $t = 21$	*bta-rf*	1.47
	dcmp-rf	5.34
$m = 13$, $t = 18$	*bta-rf*	1.28
	dcmp-rf	10.10

Table 2: Comparison of the root-finding algorithm performance on an x86 Intel Intel(R) Core(TM)2 Duo CPU U7600 for code parameters with large m and small t.

not competitive in terms of computation speed (which is a well known fact). The latter, which has at least considerable performance advantages over the former, exhibits a timing side-channel vulnerability with respect to message-aimed attacks, which is beyond repair.

Considering the remaining two candidates, we find that for code parameters that minimize the public key size, *dcmp-rf* and *bta-rf* are close competitors. Since timing side-channel security of *bta-rf* is problematic at least with respect to message-aimed attacks, *dcmp-rf* can be seen as the winner of this evaluation.

But as we stated in the introduction, we do not want to postulate this as the definitive answer concerning the choice of root-finding algorithms in code-based cryptosystems. If one would achieve a timing side-channel secure variant of the *bta-rf*, running time advantages could be achieved at the expense of public key size. Furthermore, the most important task certainly is the implementation of code-based cryptosystems on smart cards and related platforms. To achieve competitive performance on such resource constrained platforms, hardware support certainly would have to be present, as it is the case for RSA and elliptic curve based algorithms today. Thus, the real question is that of an optimal choice of algorithms and hardware support, achieving both good performance and side-channel security on these platforms. In this context, among other aspects, it will become relevant how easily an algorithm can be parallelized[7]. Thus we encourage future research investigating implementations with efficient hardware support on resource constrained platforms.

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[7] Note that *dcmp-rf* can easily be parallelized by starting independent evaluations at 2^x elements in the Gray Code ordered $\mathbb{F}_{2^m}$.

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A Further Results to the running Time Dependencies on the Root Values

Fig. 3 shows the support scan results for the whole syndrome decoding operation resp. the root-finding with *dcmp-rf* alone on the AVR platform, both with and without countermeasures to protect against the vulnerability that reveals $\sigma_3 = 0$. The threat of detection of $\sigma_3 = 0$ through the syndrome decoding timing is obvious from Fig. 3(a) and 3(b) . The countermeasure is realized by assigning the precomputed value of the logarithm of σ_3 a dummy value during the initialization phase of *dcmp-rf* if $\sigma_3 = 0$, and carrying out the multiplication $Y^3\sigma_3$ with both operands in logarithmic representation regardless of the value of σ_3. Afterwards, a logical AND operation is performed on the result with a mask having all bits set in case of $\sigma_3 \neq 0$ and having value 0 otherwise. The timings for support scans in these cases are given in Fig. 3(c) and 3(d). The latter, showing the timings of only the root-finding operation, shows a multi-level structure that we have not analyzed further, but must be assumed to result from the precomputation phase of *dcmp-rf*. Whether additional countermeasures are necessary to remove these timing differences remains an open question.

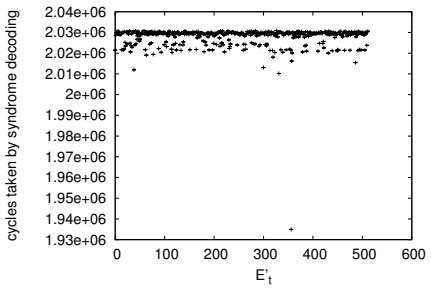
Turning back to the security discussion of *bta-rf*, we look at the support scan results shown in Fig. 2(c) and 2(d). It is obvious, that these running times are neither constant nor random. There seems to be a tendency to build "clouds"; by that we mean that it seems that an attacker should be able to build hypotheses of the type $|E'_i - E'_j| < x$ which have a higher probability of being true than possible without any side-channel information. However, how exactly these hypotheses are to be build, and what actual attack potential arises from them, is an open question.

Note for instance the values of E'_t below 100 in 2(c), which have consequently lower timings than $3.04 \cdot 10^6$. Though such a dramatic effect was not obvious in all support scans we conducted, it corroborates the notion of "clouding" effects in the timings for *bta-rf*. Thus we strongly suggest that the running time properties of the *bta-rf* be subject to thorough analysis before considering its use in real world implementations of code-based schemes.

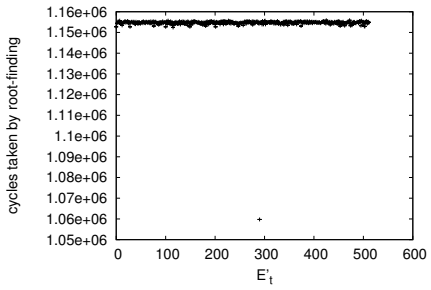
B Consideration of Countermeasures against the key-aimed vulnerability of *eval-div-rf*

A straightforward countermeasure would be to randomize the starting point of the evaluation of $\sigma(Y)$ in $\mathbb{F}_{2^m}$. But this is not enough, as one can see from the following considerations: in this case, a long sequence of roots following each other with only a few non-root elements in between them [8] may be detected, since if they are evaluated soon after the starting position, they cause shorter timings than a shorter such "sequence" of nearby roots. However, evaluating $\sigma(Y)$ in a completely permuted order would solve the problem, the creation of a

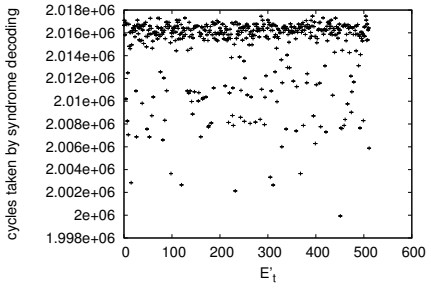
[8] Concerning the evaluation order, i.e. usually lexicographical ordering of $\mathbb{F}_{2^m}$



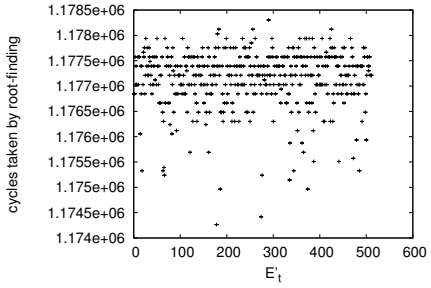
(a) Running times of the syndrome decoding without countermeasures to hide $\sigma_3 = 0$ in *dcmp-rf*. The outlier with smallest running time shows such a case.



(b) Running times of the root-finding with *dcmp-rf* without countermeasures to hide $\sigma_3 = 0$. The outlier with smallest running time shows such a case.



(c) Running times of the syndrome decoding with countermeasures to hide $\sigma_3 = 0$ in *dcmp-rf*.



(d) Running times of the root-finding with *dcmp-rf* with countermeasures to hide $\sigma_3 = 0$.

Fig. 3: Running times of the syndrome decoding with *dcmp-rf* for $n - (t-1)$ ciphertexts, where $t-1$ error positions are fixed and the t-th position E'_t varies.

pseudorandom permutation however incurs a considerable computational effort. Furthermore, a countermeasure removing the vulnerability with respect to the message-aimed timing attacks from Sec. 4.1 of *eval-div-rf* cannot be implemented in a simple manner.

C Vulnerabilities in HyMES Syndrome Decoding Implementation

While working with the HyMES implementation [15], we encountered a number of vulnerabilities, potentially enabling both timing and fault attacks. We list them here, because this is good example showing what problems can arise when the syndrome decoding is implemented without implementation security in mind (which was not in the scope of that work).

All code relevant to the syndrome decoding in HyMES is found in the file `decrypt.c`, all line numbers given in the following refer to this file. In line 270, when $\deg(\sigma(Y)) \neq t$, decryption is aborted with an error. This is only a problem if the countermeasures proposed in [11] are not implemented, in this case it allows message-aimed fault-attacks of the type explained in Sec. 4.1 (highly likely that $w > t$ leads to $\deg(\sigma(Y)) = t$, and always that $w < t$ leads to $\deg(\sigma(Y)) = w$).

In line 276, if the root-finding did not return t roots decryption is also aborted with an error. This is clearly allowing message-aimed fault attacks with the two-bit-flip attack described earlier in Sec. 4.1, such a check must not be present in a secure implementation.

In line 285, a Quick Sort algorithm is applied to the set of roots to sort the array. Though we did not seek for attacks against this algorithm, it is certainly clear that the running time of Quick Sort depends on the number of roots, and in general also on their positions. As far as understood by the the author, the sorting is needed in HyMES for the constant weight word encoding[9]. We want to point out that a real world implementation must ensure that such sorting is done without giving information about the roots positions. The number of roots, however, can be, as a countermeasure, artificially increased to t before the algorithm is applied, as a deviating number of roots indicates an irregular ciphertext anyway.

[9] The constant weight word encoding (CWE) is needed in the implementation [15] to encode additional information in the error vector; in this aspect their scheme deviates from the original McEliece scheme [1]. Note, however, that in most CCA2 conversions proposed for the McEliece scheme, CWE is used [21]. Furthermore, CWE is needed in the Niederreiter scheme [2].