# Improved Integral Attacks on Reduced Round Camellia 

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#### Abstract

In this paper a method is presented to extend the length of integral distinguisher of Feistel-SP structure, based on which a new 8 -round distinguisher of Camellia is proposed. Moreover, we improve integral attacks on reduced round Camellia without $F L / F L^{-1}$. We attack 11-round Camellia-128 with the data complexity of $2^{120}$ and the time complexity of $2^{125.5}$, and 12 -round Camellia- 256 with the data complexity of $2^{120}$ and the time complexity of $2^{214.3}$. The result is the best one of integral attacks on reduced round Camellia so far.


Key words. Block cipher, Distinguisher, Integral attack, Camellia, Partial sum technique

## 1 Introduction

The block cipher Camellia was proposed by NTT and Mitsubishi in $2000^{[1]}$. It is based on Feistel structure with SP-type F function and $F L / F L^{-1}$ functions layers, and it supports the block length of 128 bits and a variable key length of 128/192/256 bits. Camellia has been accepted by ISO/IEC as an international standard. It is also a winner of NESSIE, CRYPTREC project and IETF. The security of Camellia was initially analyzed by the algorithm designers. Efficient attacks on Camellia include linear cryptanalysis ${ }^{[14]}$, differential cryptanalysis ${ }^{[13,14]}$, impossible differential cryptanalysis ${ }^{[17]}$, truncated differential cryptanalysis ${ }^{[7,9]}$, higher order differential cryptanalysis ${ }^{[5]}$, collision attack ${ }^{[16]}$ and Square attack ${ }^{[19,3]}$. The best attacks on Camellia without $F L / F L^{-1}$ function layer were impossible differential cryptanalysis ${ }^{[18]}$, which can attack 12-round Camellia-128 and 16 - round Camellia-256 without $F L / F L^{-1}$.

Integral attack was extended from Square attack, which is one of the best attacks on AES ${ }^{[2]}$. Ferguson et al. in [4] improved this attack to 8 rounds version of Rijndael-128 with the partial sum technique and the herd technique. Knudsen and Wagner first proposed the definition of integral and analyzed it as a dual to differential attacks particularly applicable to block ciphers with bijective components ${ }^{[8]}$. Several years later, Reza Z'aba et al. presented bit-pattern based integral attack ${ }^{[12]}$. The integral attack applied to many kinds of block ciphers so far, such as Rijndael ${ }^{[11]}$, ARIA ${ }^{[10]}$, and Serpent ${ }^{[12]}$. Higher order differential attack and Square attack are different from integral attack, however, their distinguisher length can be extended by using the integral property. In this paper a method is presented to extend the length of distinguisher of Camellia, based on which the effect of integral attack will be improved. Moreover, this method also can be used even to any Feistel-SP structure. Then a new 8-round distinguisher of Camellia without $F L / F L^{-1}$ is proposed. Finally, we attack 11 -round Camellia-128 with the data complexity of $2^{120}$ and the time complexity of $2^{125.5}$, and 12 -round Camellia- 256 with the data complexity of $2^{120}$ and the time complexity of $2^{214.3}$. The result is the best one of integral attacks on reduced round Camellia so far.

This paper is organized as follows: Section 2 provides a brief description of preliminaries. Section 3 describes a method to extend the length of distinguisher. Section 4 describes the attacks on 11/12-round Camellia. Finally, Section 5 concludes this paper.

Camellia is a Feistel-SP style block cipher with $F L / F L^{-1}$ layers, and the number of rounds are $18 / 24 / 24$ corresponding to key length of $128 / 192 / 256$ bits. Additionally, $F L / F L^{-1}$ function is inserted every 6 rounds (Fig.1). The round keys are derived from the master key by means of key scheduling (Fig.2), and the key schedule constants are listed in Table.1. In this paper the input and output of round function are treated as two 8-byte vectors over $F_{2^{8}}^{8}$.

Table 1. The key schedule constants

| $\sum_{1}$ | 0xA09E667F3BCC908B | $\sum_{4}$ | 0x54FF53A5F1D36F1C |
| :--- | :--- | :--- | :--- |
| $\sum_{2}$ | 0xB67AE8584CAA73B2 | $\sum_{5}$ | 0x10E527FADE682D1D |
| $\sum_{3}$ | 0xC6EF372FE94F82BE | $\sum_{6}$ | 0xB05688C2B3E6C1FD |



Fig. 1. The Structure of Camellia-128
The round function of Camellia includes three basic operations: Round Key Addition, Substitution Layer and Diffusion Layer (Fig.3). These three basic operations are defined as follows:

Round Key Addition (RKA): The 64-bit round key is Xored to the state.
Substitution Layer (SL): A non-linear byte substitution operation is applied to each byte of the state independently. In Camellia this is implemented by 4 S -boxes with the relationship as follows.

$$
s_{2}(a)=s_{1}(a) \lll 1 ; s_{3}(a)=s_{1}(a) \ggg 1 ; s_{4}(a)=s_{1}(a \lll 1)
$$

Diffusion Layer (DL): The diffusion layer is a function $P: F_{2^{8}}^{8} \rightarrow F_{2^{8}}^{8}$, which is given by

$$
\begin{array}{ll}
m_{1}=x_{1} \oplus x_{3} \oplus x_{4} \oplus x_{6} \oplus x_{7} \oplus x_{8} & m_{5}=x_{1} \oplus x_{2} \oplus x_{6} \oplus x_{7} \oplus x_{8} \\
m_{2}=x_{1} \oplus x_{2} \oplus x_{4} \oplus x_{5} \oplus x_{7} \oplus x_{8} & m_{6}=x_{2} \oplus x_{3} \oplus x_{5} \oplus x_{7} \oplus x_{8} \\
m_{3}=x_{1} \oplus x_{2} \oplus x_{3} \oplus x_{5} \oplus x_{6} \oplus x_{8} & m_{7}=x_{3} \oplus x_{4} \oplus x_{5} \oplus x_{6} \oplus x_{8} \\
m_{4}=x_{2} \oplus x_{3} \oplus x_{4} \oplus x_{5} \oplus x_{6} \oplus x_{7} & m_{8}=x_{1} \oplus x_{4} \oplus x_{5} \oplus x_{6} \oplus x_{7}
\end{array}
$$



Fig. 2. The Round Function of Camellia


Fig. 3. The Key Schedule of Camellia

### 2.2 Notations

In the following, we introduce some notations used throughout this paper. The plaintext are denoted as Plantext $=\left(X_{1}, X_{0}\right)$, where $X_{i}=\left(x_{i, 1}, x_{i, 2}, \cdots, x_{i, 8}\right), i=0, \ldots, r-1$. Other notations that will be used in this paper are described as follows:
$B_{r}$ : the output of RKA in $r$-th round.
$O_{r}$ : the output of SL in $r$-th round.
$M_{r}$ : the output of DL in $r$-th round.
$K_{r}$ : the subkey of the $r$-th round.
$b_{r, i}$ : the $i+1$-th byte of $B_{r}$.
$o_{r, i}$ : the $i+1$-th byte of $O_{r}$.
$m_{r, i}$ : the $i+1-t h$ byte of $M_{r}$.
$k_{r, i}$ : the $i+1$-th byte of $K_{r}$.
$\lambda_{i}$ : the active bytes.

### 2.3 Integral Attack and the Partial Sum Technique

Integral Attack. The integral attack has many interesting features. It can saturate S-Box Layer, and Round Key Addition Layer will not affect this property of saturation. However, the linear transformation influences the length of the integral distinguisher. Integral attack considers a particular collection of $m$ bytes in the plaintexts and ciphertexts. The aim of this attack is to predict the values in the sums (i.e. the integral) of the chosen bytes after a certain number of rounds of encryption. In [8], Knudsen and Wagner also generalized this approach to higher order integrals: the original set to consider becomes a set of $m^{d}$ vectors which differ in $d$ components and where the sum of this set is predictable after a certain number of rounds. The sum of this set is called a $d^{\text {th }}$-order integral. In this paper we not only pay attention to the sum, but also to the appearing times of the sum value.

The Partial Sum Technique. In our attack we will use the partial sum technique. For a value $c_{0}, c_{1}, c_{2}, \ldots, c_{l}$, we define

$$
x_{u}:=\sum_{j=0}^{u} S_{j}\left[c_{j} \oplus k_{j}\right] .
$$

Guessing the values of $k_{0}$ and $k_{1}$, we will complete the transformation $\left(c_{0}, c_{1}, c_{2}, \ldots, c_{l}\right) \rightarrow$ $\left(x_{1}, c_{2}, \ldots, c_{l}\right)$. Guessing the values of $k_{i}$, we will complete the transformation $\left(x_{i-1}, c_{i}, c_{i+1}, \ldots, c_{l}\right)$ $\rightarrow\left(x_{i}, c_{i+1}, \ldots, c_{l}\right)$. In order to obtain the value of $x_{l}$, there should be processed $l-1$ steps. If $c_{i} s$ are in byte pattern, the time complexity of the count of $x_{l}$ is $2^{8 l} \times 2^{16} \times(l-1)$ times S-box lookups. For the details of the complexities of each step, the readers can refer to [4].

## 3 Integral Distinguishers Based on Feistel-SP Structure

In this section we first explain how to construct a 2 nd-order 5-round integral distinguishers (Sec. 3.1), then introduce a method to extend the length of integral distinguishers and the proof also given in detail (Sec. 3.2).

### 3.1 The 2nd-Order 5-Round Integral Distinguisher

The idea of constructing a 2 nd-order 5 -round integral distinguisher is like that of constructing 5 -round higher order differential distinguishers in Fig. $4^{[5]}$. Now we will give different explain as follows.


Fig. 4. The 5-Round Higher Order differential Distinguisher

$$
\begin{gathered}
X_{1} \oplus M_{2} \oplus M_{4} \oplus M_{6}=X_{7} \Leftrightarrow X_{1} \oplus X_{7}=M_{2} \oplus M_{4} \oplus M_{6} \\
\Leftrightarrow P^{-1}\left(X_{1} \oplus X_{7}\right)=O_{2} \oplus O_{4} \oplus O_{6}
\end{gathered}
$$

For the 6-th byte, the above equation is equate to

$$
\begin{equation*}
\left[P^{-1}\left(X_{1}\right)\right]_{6} \oplus\left[P^{-1}\left(X_{7}\right)\right]_{6}=o_{2,6} \oplus o_{4,6} \oplus o_{6,6} \tag{1}
\end{equation*}
$$

where $\left[P^{-1}\left(X_{1}\right)\right]_{6}$ is the 6th byte of $P^{-1}\left(X_{1}\right)$.
According to the above description, we will obtain the following result.
Lemma 1. Let the bytes of $x_{0,1}, x_{0,2}$ are active, and other bytes of $X_{0}, X_{1}$ are constants, each value of $t$ will appear even times.

$$
t=s_{3}\left(x_{6,6} \oplus k_{6,6}\right) \oplus\left[P^{-1}\left(X_{7}\right)\right]_{6} .
$$

Proof. According to Equ.(1), $s_{3}\left(X_{6,6} \oplus k_{6,6}\right) \oplus\left[P^{-1}\left(X_{7}\right)\right]_{6}=\left[P^{-1}\left(X_{1}\right)\right]_{6} \oplus o_{2,6} \oplus o_{4,6}$, where $\quad\left[P^{-1}\left(X_{1}\right)\right]_{6}$ $\oplus s_{2,6}$ is constant, so the value of $o_{4,6}$ is important. We find

$$
\begin{aligned}
o_{4,6}= & s_{3}\left(s_{1}\left(o_{2,1} \oplus c_{1}\right) \oplus s_{2}\left(o_{2,1} \oplus o_{2,2} \oplus c_{2}\right)\right. \\
& \left.\oplus s_{2}\left(o_{2,1} \oplus o_{2,2} \oplus c_{3}\right) \oplus s_{3}\left(o_{2,1} \oplus o_{2,2} \oplus c_{4}\right) \oplus c_{5}\right) .
\end{aligned}
$$

Let $o_{2,1} \oplus o_{2,2}=y$, for each value of $y, o_{2,1}$ is active, so each value of $o_{4,6}$ will appear 256 times. Then each value of $t$ will appear 256 times.

### 3.2 A Method to Extend the Length of Integral Distinguisher

In the structure of Feistel-SP the Xor operation and the permutation $P$ are linear transformations (Fig.5), which can influence the general integral property and also can be used to extend the integral distinguisher. Let some bytes of $X_{0}$ be active, and the bytes of $X_{1}$ be constant, and then the input of a known $t$-round integral distinguisher is $\left(X_{1}, X_{0}\right)$. Now we extend it forward one round by the following formula:

$$
X_{1}=F\left(X_{0}\right) \oplus X_{-1}=P \circ S\left(K\left(X_{0}\right)\right) \oplus X_{-1}=P\left[S\left(K\left(X_{0}\right)\right) \oplus P^{-1}\left(X_{-1}\right)\right] .
$$

Choose $X_{-1}$, which satisfying that the bytes of $P^{-1}\left(X_{-1}\right)$ corresponding to the no-constant bytes of $S\left(K\left(X_{0}\right)\right)$ are active and other bytes are constants. Such $\left(X_{0}, X_{-1}\right)$ will lead to several sets of ( $X_{1}, X_{0}$ ) after one round encryption, and then we will obtain a $t+1$-round integral distinguisher. The related proof will be presented in Lemma 2.


Fig. 5. The Feistel-SP Structure


Fig. 6. Example 1

Example 1. In Fig. $6 P$ is an identical transformation. The input of $\left(X_{0}, X_{-1}\right)$ will lead to a $t+1$-round integral distinguisher. The $\lambda_{0}$ and $\lambda_{1}$ are two active bytes, so there are $2^{16}$ values of $\left(X_{0}, X_{-1}\right)$, which will lead to $2^{8}$ sets. In each set $x_{0,1}$ denoted as $\lambda_{0}$ is active and other bytes of $\left(X_{1}, X_{0}\right)$ are constants denoted as $c$.

Usually the $P$ isn't identical transformation, so the choice of $X_{-1}$ will be more complex than described in Example 1. Take Camellia for example, the input of a $2^{\text {nd }}$ order 5-round distinguisher is $\left(X_{1}, X_{0}\right)$, where the bytes of $x_{0,1}, x_{0,2}$ are active and other bytes are constants. Extending three more rounds forward we will obtain

$$
\begin{aligned}
& X_{-1}=P\left(\lambda_{2}, \lambda_{3}, c, c, c, c, c, c\right) \\
& X_{-2}=P\left(\lambda_{4}, \lambda_{5}, \lambda_{6}, \lambda_{7}, \lambda_{8}, \lambda_{9}, c, \lambda_{10}\right), \\
& X_{-3}=P\left(\lambda_{11}, \lambda_{12}, \lambda_{13}, \lambda_{14}, \lambda_{15}, \lambda_{16}, \lambda_{17}, \lambda_{18}\right) .
\end{aligned}
$$

The $\lambda_{i} s, 0 \leq i \leq 18$ are active bytes. Because all bytes of $X_{-3}$ are active, we can't improve it one more round. Then the following lemma is obtained.

Lemma 2. Let the 7th byte of $P^{-1}\left(X_{-2}\right)$ be constant and other 15 bytes of $\left(P^{-1}\left(X_{-2}\right), X_{3}\right)$ be active. After 3 rounds iterative encryption we will obtain $2^{104}$ sets and in each set the bytes of $x_{0,1}, x_{0,2}$ are active and other bytes are constants.

Proof. We will proof this lemma in three steps as follows:

1) The $2^{32}$ values of $\left(X_{1}, X_{-1}\right)$ will lead to $2^{16}$ sets of $\left(X_{1}, X_{0}\right)$ after one round encryption, in each set the first two bytes of $X_{1}$ are active and other bytes are constant.
The proof of this step is just like that of example 1 . We will not repeat it here.
2) The $2^{72}$ values of $\left(X_{-1}, X_{-2}\right)$ will lead to $2^{40}$ sets of $\left(X_{1}, X_{-1}\right)$ after one round encryption. In each set $x_{1,0}, x_{1,1}$ and the first two bytes of $P^{-1}\left(X_{-1}\right)$ are active, and other bytes are constants.
Let all active bytes of $\left(P^{-1}\left(X_{-1}\right), P^{-1}\left(X_{-2}\right)\right)$ be denoted as $x_{1}, x_{2}, x_{3}, x_{4}, x_{5}, x_{6}, x_{7}, x_{8}, x_{9}$, and all
constants be denoted as 0 . Let the active bytes of $\left(X_{1}, P^{-1}\left(X_{-1}\right)\right)$ be denoted as $y_{3}, y_{4}, y_{1}, y_{2}$. One round encryption will be decrypted as the following equations.

$$
\left\{\begin{array}{l}
x_{1}=y_{1} \\
x_{2}=y_{2} \\
s_{1}\left(x_{1} \oplus k_{1}\right) \oplus x_{3}=y_{4} \\
s_{2}\left(x_{1} \oplus x_{2} \oplus k_{2}\right) \oplus x_{4}=y_{3} \\
s_{3}\left(x_{1} \oplus x_{2} \oplus k_{3}\right) \oplus x_{5}=y_{3} \oplus y_{4} \\
s_{4}\left(x_{2} \oplus k_{4}\right) \oplus x_{6}=y_{3} \oplus y_{4} \\
s_{2}\left(x_{1} \oplus x_{2} \oplus k_{5}\right) \oplus x_{7}=y_{3} \oplus y_{4} \\
s_{3}\left(x_{2} \oplus k_{6}\right) \oplus x_{8}=y_{4} \\
s_{1}\left(x_{1} \oplus k_{8}\right) \oplus x_{9}=y_{3}
\end{array}\right.
$$

Simplified the above equations, we will obtain the equivalent equations as follows.

$$
\left\{\begin{array}{l}
x_{1}=y_{1} \\
x_{2}=y_{2} \\
s_{1}\left(x_{1} \oplus k_{1}\right) \oplus x_{3}=y_{4} \\
s_{2}\left(x_{1} \oplus x_{2} \oplus k_{2}\right) \oplus x_{4}=y_{3} \\
s_{1}\left(x_{1} \oplus k_{1}\right) \oplus x_{3} \oplus s_{2}\left(x_{1} \oplus x_{2} \oplus k_{2}\right) \oplus x_{4} \oplus s_{3}\left(x_{1} \oplus x_{2} \oplus k_{3}\right) \oplus x_{5}=0 \\
s_{1}\left(x_{1} \oplus k_{1}\right) \oplus x_{3} \oplus s_{2}\left(x_{1} \oplus x_{2} \oplus k_{2}\right) \oplus x_{4} \oplus s_{4}\left(x_{2} \oplus k_{4}\right) \oplus x_{6}=0 \\
s_{1}\left(x_{1} \oplus k_{1}\right) \oplus x_{3} \oplus s_{2}\left(x_{1} \oplus x_{2} \oplus k_{2}\right) \oplus x_{4} \oplus s_{2}\left(x_{1} \oplus x_{2} \oplus k_{5}\right) \oplus x_{7}=0 \\
s_{1}\left(x_{1} \oplus k_{1}\right) \oplus x_{3} \oplus s_{3}\left(x_{2} \oplus k_{6}\right) \oplus x_{8}=0 \\
s_{2}\left(x_{1} \oplus x_{2} \oplus k_{2}\right) \oplus x_{4} \oplus s_{1}\left(x_{1} \oplus k_{8}\right) \oplus x_{9}=0
\end{array}\right.
$$

According to the simplified equations above, we find that there is only one solution. For $2^{32}$ values of $y_{3}, y_{4}, y_{1}, y_{2}$, we get a set of $2^{32}$ solutions, and each value of $x_{5}, x_{6}, x_{7}, x_{8}, x_{9}$ is decided by $x_{1}, x_{2}, x_{3}, x_{4}$. After taking over all $2^{40}$ values of $x_{5}, x_{6}, x_{7}, x_{8}, x_{9}, 2^{40}$ sets will be obtained, i.e. $2^{72}$ values of $x_{1}, x_{2}, x_{3}, x_{4}, x_{5}, x_{6}, x_{7}, x_{8}, x_{9}$ will lead to $2^{40}$ sets after one round encryption, in each set the bytes of $y_{1}, y_{2}, y_{3}, y_{4}$ are active.
3) The $2^{120}$ values of $\left(X_{-2}, X_{-3}\right)$ will lead to $2^{48}$ sets of $\left(X_{-1}, X_{-2}\right)$ after one round encryption, and in each set the nine bytes of $\left(P^{-1}\left(X_{-2}\right), P^{-1}\left(X_{-1}\right)\right)$ are active, and other bytes are constants.
The proof of this step is just like that of the above step.
According to the above three steps, we will obtain $2^{104}$ sets of ( $X_{1}, X_{0}$ ) from $2^{120}$ values of $\left(X_{-2}, X_{-3}\right)$ described in the lemma 2. In each set the bytes of $x_{0,1}, X_{0,2}$ are active and other bytes are constant.

In line with Lemma 1 and Lemma 2, we can construct a new $15^{\text {th }}$ order 8-round distinguisher of Camellia as depicted in Theorem 1(Fig.7).

Theorem 1. Let $\left(X_{1}, X_{0}\right)$ are input of Camellia without $F L / F L^{-1}$. If the bytes of $P^{-1}\left(X_{1}\right)_{7}$ are constants and other bytes of $\left(X_{1}, X_{0}\right)$ take all values of $F_{2^{8}}^{15}$, then each value of $t=s_{3}\left(X_{9,6} \oplus\right.$ $\left.k_{9,6}\right) \oplus P^{-1}\left(X_{10}\right)_{6} \quad$ will appear even times.


Fig. 7. The $15^{\text {th }}$-Order 8 -Round Distinguisher

## 4 Attacks on Reduced Round Camellia

### 4.1 Integral Attack on 10/11-Round Camellia-128

Based on the $15^{\text {th }}$-order 8 -round distinguisher described above, we will attack 10 rounds of Camellia-128 without $F L / F L^{-1}$ functions now, which is illustrated in Fig.8.


Fig. 8. Integral Attacks on Reduced Round Camellia

1. Choose a structure of plaintexts $\left(X_{1}, X_{0}\right)$. It satisfies

$$
X_{1}=P\left(\lambda_{4}, \lambda_{5}, \lambda_{6}, \lambda_{7}, \lambda_{8}, \lambda_{9}, c, \lambda_{10}\right), X_{0}=P\left(\lambda_{11}, \lambda_{12}, \lambda_{13}, \lambda_{14}, \lambda_{15}, \lambda_{16}, \lambda_{17}, \lambda_{18}\right),
$$

where $\lambda_{i} \mathrm{~s}(4 \leq i \leq 18)$ are active bytes, and $c$ is a constant. $\left(X_{1}, X_{0}\right)$ take all values of $F_{2^{8}}^{15}$. Encrypt all this plaintexts and set $2^{56}$ counters for the seven bytes of $x_{11,6}, x_{10,2}, x_{10,3}, x_{10,5}, x_{10,6}$, $x_{10,7}, x_{10,8}$ and the corresponding counters plus one.
2. For the $2^{56}$ values of ciphertexts, there are $2^{56}$ values at most in bytes of $x_{11,6}, x_{10,2}, x_{10,3}, x_{10,5}$, $x_{10,6}, x_{10,7}, x_{10,8}$. We choose those values that the corresponding counters are odd times (the Xor value of the same value is zero).

Guessing the key bytes of $k_{10,2}, k_{10,3}, k_{10,5}, k_{10,7}, k_{10,8}$ and $k_{9,6}$, we do a partial decrypt to the single
value of $t$. In this phase we need the partial sum technique in order to reduce the workfactor of computing the value of $s_{3}\left(x_{9,6} \oplus k_{9,6}\right)$.

$$
\begin{align*}
t= & {\left[P^{-1}\left(X_{10}\right)\right]_{6} \oplus s_{3}\left(x_{9,6} \oplus k_{9,6}\right) } \\
= & {\left[P^{-1}\left(x_{10,2}, x_{10,3}, x_{10,5}, x_{10,6}, x_{10,8}\right)\right]_{6} }  \tag{2}\\
& \oplus s_{3}\left[s_{2}\left(x_{10,2} \oplus k_{10,2}\right) \oplus s_{3}\left(x_{10,3} \oplus k_{10,3}\right) \oplus s_{2}\left(x_{10,5} \oplus k_{10,5}\right)\right. \\
& \left.\oplus s_{4}\left(x_{10,7} \oplus k_{10,7}\right) \oplus s_{1}\left(x_{10,8} \oplus k_{10,8}\right) \oplus x_{11,6} \oplus k_{9,6}\right] .
\end{align*}
$$

For the values of $x_{11,6}, x_{10,2}, x_{10,3}, x_{10,5}, x_{10,7}, x_{10,8}$, the number of which appear odd times, we do the following steps:
(a) Guess the two bytes of $k_{10,2}$ and $k_{10,3}$, and we obtain the corresponding 5 bytes value

$$
\left(x_{1}, x_{10,5}, x_{10,7}, x_{10,8}, x_{11,6}\right) .
$$

(b) Guess the value of $k_{10,5}$, and compute the partial sum, then we obtain 4 bytes value

$$
\left(x_{2}, x_{10,7}, x_{10,8}, x_{11,6}\right) .
$$

(c) Guess the value of $k_{10,7}$, and compute the partial sum, then we obtain 3 bytes value

$$
\left(x_{3}, x_{10,8}, x_{11,6}\right) .
$$

(d) Guess the value of $k_{10,8}$, and compute the partial sum, then we obtain 2 bytes value

$$
\left(x_{4}, x_{11,6}\right)
$$

(e) Guess the value of $k_{9,6}$, and compute the partial sum, then we obtain 1 byte value

$$
x_{5}=s_{3}\left(x_{9,6} \oplus k_{9,6}\right) .
$$

We sum the values of $x_{5}$ and $\left[P^{-1}\left(X_{10}\right)\right]_{6}$ over all the encryptions, and check the value of $t$ if it is even times. If each value of $t$ appears even times, the guessed key bytes are right, otherwise they are wrong guesses.

In Step 1, we choose $2^{120}$ plaintexts and need encrypt $2^{120}$ times. In Step 2-(a), we guess 16 bits key, and process $2^{48}$ ciphertexts, which cost $2^{64}$ S-box applications. Step 2-(b) costs workfactor as $2^{16} \times 2^{8} \times 2^{40}=2^{64}$ at most. This is same to the other phases of Step 2, so the workfactor of Step 2 is $2^{72} \times 7$ S-box applications. There are also additional works to compute $\left[P^{-1}\left(X_{10}\right)\right]_{6}$ in each phase of Step 2, however, using rough equivalence of 8 S -box applications to a round encryption with a new key, the work to compute $\left[P^{-1}\left(X_{10}\right)\right]_{6}$ can be ignored. For a wrong key, the probability that it satisfies each value of $t$ appearing even times is less than $2^{-169}$ (Appendix A). After analyzing a structure of plaintexts, there is $2^{48} \times 2^{-169}=2^{-121}$ wrong key that will pass Step 2. So the data complexity of $2^{120}$ is enough and the time complexity is $2^{48} \times 2^{16} \times 5 /\left(2^{3} \times 10\right)=2^{60}$ encryptions.

We also can improve this attack by adding one more round. For 11-round Camellia-128, the key schedule could be used (Appendix B). Guessing the key bytes of $k_{1,3}, k_{1,4}, \ldots, k_{1,8}$, and looking up S-boxes for $2^{56}$ values of $X_{1}$, the values of six bytes of $P^{-1}\left(X_{0} \oplus X_{2}\right)$ will be obtained. Then choosing the six bytes of $P^{-1}\left(X_{2}\right)$ as constant, we can get the corresponding six bytes of $P^{-1}\left(X_{0}\right)$. Let the first two bytes of $P^{-1}\left(X_{2}\right)$ be active, and we will obtain $2^{16}$ values of $X_{0}$ in the chosen plaintexts. For each key of $k_{1,3}, k_{1,4}, \ldots, k_{1,8}$, encrypt all $2^{72}$ plaintexts. According to the key schedule, $K_{11}=K_{1} \lll 60$, so the key bytes of $K_{11}$ are known except for $k_{11,2}, 4$ bits of $k_{11,1}$ and 4 bits of $k_{11,3}$. With the partial sum technique we will decrypt three rounds to obtain the value of $t$. The $x_{10, i}, i=2,3,5,6,7,8$ used in Equ.(2) can be represented as follows.

$$
\begin{aligned}
& x_{10,2}=s_{1}\left(x_{11,1} \oplus k_{11,1}\right) \oplus s_{2}\left(x_{11,2} \oplus k_{11,2}\right) \oplus c_{2} \\
& x_{10,3}=s_{1}\left(x_{11,1} \oplus k_{11,1}\right) \oplus s_{2}\left(x_{11,2} \oplus k_{11,2}\right) \oplus s_{3}\left(x_{11,3} \oplus k_{11,3}\right) \oplus c_{3}
\end{aligned}
$$

$$
\begin{aligned}
& x_{10,5}=s_{1}\left(x_{11,1} \oplus k_{11,1}\right) \oplus s_{2}\left(x_{11,2} \oplus k_{11,2}\right) \oplus c_{5} \\
& x_{10,6}=s_{2}\left(x_{11,2} \oplus k_{11,2}\right) \oplus s_{3}\left(x_{11,3} \oplus k_{11,3}\right) \oplus c_{6} \\
& x_{10,7}=s_{3}\left(x_{11,3} \oplus k_{11,3}\right) \oplus c_{7} \\
& x_{10,8}=s_{1}\left(x_{11,1} \oplus k_{11,1}\right) \oplus c_{8}
\end{aligned}
$$

where $c_{i}, i=2,3,5,6,7,8$ should be pre-computed in order to reduce the time complexity. For example, $\quad c_{2}=s_{4}\left(x_{11,4} \oplus k_{11,4}\right) \oplus s_{2}\left(x_{11,5} \oplus k_{11,5}\right) \oplus s_{4}\left(x_{11,7} \oplus k_{11,7}\right) \oplus s_{1}\left(x_{11,8} \oplus k_{11,8}\right) \oplus x_{12,2} \quad$. The time complexity of pre-computation is about $2^{48} \times 2^{72} \times \frac{2}{3} \approx 2^{119.4}$ rounds encryption, which can be ignored compared with the consuming later. For the $2^{72}$ ciphertexts, we do the following steps.

1. Guess $k_{11,2}$ and 4 bits of $k_{11,3}$, and we obtain the corresponding 8 bytes value

$$
\left(x_{11,1}, x_{11,6}, c_{2}^{\prime}, c_{3}^{\prime}, c_{5}^{\prime}, x_{10,6}, x_{10,7}, c_{8}\right)
$$

where $\quad c_{2}^{\prime}=s_{2}\left(x_{11,2} \oplus k_{11,2}\right) \oplus c_{2}$, and $c_{3}^{\prime}, c_{5}^{\prime}$ are similar.
2. Guess 4 bits of $k_{11,1}$, and we obtain the corresponding 7 bytes value

$$
\left(x_{10,2}, x_{10,3}, x_{10,5}, x_{10,6}, x_{10,7}, x_{10,8}, x_{11,6}\right)
$$

The remainder two rounds decryptions are just as the above attack on 10-round Camellia. The computation required in Step 1 is the main workfactor, which is equivalent to $2^{48} \times 2^{72} \times 2^{12}$ $=2^{132} \mathrm{~S}$-box lookups. The total time complexity is $2^{132} /\left(2^{3} \times 3\right) \approx 2^{125.5}$ encryptions.

### 4.2 Integral Attack on Camellia-192/256

In this subsection, we describe improved integral attacks on 11-round Camellia-192 and 12-round Camellia-256. The key schedule can't be used here. We add 1 round after 10 -round Camellia, and guess all bytes of $K_{11}$, and decrypt the last round, where the workfactor is the equivalent of $2^{64} \times 2^{64}=2^{128}$ F function operations and $2^{64} \times 2^{48} \times 2^{48}=2^{160}$ Xor operations. The rest workfactor can be ignored, so the main time complexity of the attack on 11 rounds Camellia-192/256 is $2^{160} /\left(2^{6} \times 11\right) \approx 2^{150.5}$. In a similar way the time complexity of attack on 12 rounds Camellia- 256 is $2^{214.3}$. There no relation of subkeys can be used.

## 5 Conclusion

The improved integral attacks on reduced round Camellia were described in this paper. We gave a method of forward-extending the length of integral distinguisher, by which a new 8 -round integral distinguisher for Camellia was proposed. And then we attacked 11-round Camellia-128 and 11/12-round Camellia-192/256 with the partial sum technique. Table 2 summarizes our integral attacks together with the previously known integral-like attacks on Camellia.

Table 2. Results of Integral-like Attacks on Camellia

| Camellia-b | Rounds | FL/FL | Method | D-Rounds | Data | Time | Notes |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Camellia-128 | 8 | Yes | SA | 4 | $2^{48}$ | $2^{116}$ | $[19]$ |
|  | 9 | No | SLA | 5 | $2^{66}$ | $2^{84.8}$ | $[3]$ |
|  | 10 | No | IntA | 8 | $2^{120}$ | $2^{120}$ | Sec4.1 |
|  | 11 | No | IntA | 8 | $2^{120}$ | $2^{123.9}$ | Sec4.1 |
| Camellia-192 | 9 | No | HODC | 5 | $2^{21}$ | $2^{188}$ | $[5]$ |
|  | 9 | Yes | SA | 4 | $2^{60.5}$ | $2^{202}$ | $[19]$ |
|  | 10 | No | SLA | 5 | $2^{66}$ | $2^{167.3}$ | $[3]$ |
|  | 11 | No | SLA | 5 | $2^{66}$ | $2^{211.6}$ | $[3]$ |
|  | 11 | N/Y | HODC | 5 | $2^{66}$ | $2^{255.6}$ | $[5]$ |
|  | 11 | No | IntA | 8 | $2^{120}$ | $2^{150.5}$ | Sec4.2 |
|  | 12 | No | SLA | 5 | $2^{66}$ | $2^{249.6}$ | $[3]$ |
|  | 12 | No | IntA | 8 | $2^{114.3}$ | Sec4.2 |  |

Note 1. D-Rounds: Distinguisher Rounds; SA: Square Attack; IntA: Integral Attack; HODC: Higher Order Differential Attack; SLA: Square Like Attack.
Note 2. Time complexity is measured in encryption units.
According to Table 2, the integral attacks presented in this paper make significant improvements on both data and time complexities. However, the full round Camellia provides a sufficient safety margin. Our new method also can be used for any Feistel-SP structure. How to evaluate the security of Feistel-SP structure against integral attack is more important, which will be our future work.

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## A The probability of $2^{-169}$

The probability is obtained from the following analysis. For $2^{56}$ times appearing of $t$, we need study the number of each different value appearing even times according to the following situations. First, if each value appears 2 times, the number of arrangements is

$$
\binom{256}{128} \times\left(\frac{256!}{2!2!\cdots 2!}\right) \times\left(\frac{128!}{128!}\right)
$$

Second, if one value appears 4 times and each other value appears 2 times, the number of arrangements is

$$
\binom{256}{127} \times\left(\frac{256!}{4!2!\cdots 2!}\right) \times\left(\frac{127!}{1!126!}\right)
$$

Likewise, if there are $i$ different values appearing, the number of each different value appearing even times is

$$
\binom{256}{i} \times\left(\frac{256!}{\left(2 j_{1}\right)!\left(2 j_{2}\right)!\cdots\left(2 j_{i}\right)!}\right) \times\left(\frac{i!}{n_{1}!n_{2}!\cdots n_{l}!}\right)
$$

where $\left(j_{1}, j_{2}, \cdots, j_{i}\right)$ is the partition of $i$ parts of 128 , and $n_{1}$ denotes the appearing times of $j_{1}$ in the set of $\left(j_{1}, j_{2}, \cdots, j_{i}\right)$. The vector $\left(n_{0}, n_{1}, \ldots, n_{l}\right)$ satisfies the equation $n_{1}+n_{2}+\ldots+n_{l}=i$, i.e. there are $l$ different $j_{m} \mathrm{~S}(m=1,2, \ldots, i)$ in $\left(j_{1}, j_{2}, \cdots, j_{i}\right)$. When $i$ takes all values from 128 to 1 , we will obtain all the number of arrangements, the sum of which is defined as

$$
f(128)=\bigoplus_{i=1}^{128}\binom{256}{i} \times\left(\frac{256!}{\left(2 j_{1}\right)!\left(2 j_{2}\right)!\cdots\left(2 j_{i}\right)!}\right) \times\left(\frac{i!}{n_{1}!n_{2}!\cdots n_{1}!}\right)
$$

By searching with computer, we only can obtain

$$
f(64)=\oplus_{i=1}^{64}\binom{256}{i} \times\left(\frac{128!}{\left(2 j_{1}\right)!\left(2 j_{2}\right)!\cdots\left(2 j_{i}\right)!}\right) \times\left(\frac{i!}{n_{1}!n_{2}!\cdots n_{l}!}\right) \approx 2^{855}
$$

However, it is easy to get the inequation $f(128)<f(64) \times 256^{128}$. Therefore, for 256 times appearing of $t$, the probability of each different value appearing even times is less than $2^{855} \times 256^{128} / 256^{256}=2^{-169}$.

## B The Relation of Subkeys

The relationship of subkeys for Camellia-128 is listed in Table 3.
Table 3. Subkeys for 128-bit keys

| 128-bit keys | subkey | value |
| :--- | :--- | :--- |
| Prewhitening | $k w_{1}$ <br> $k w_{2}$ | $\left(K_{L} \lll 0\right)_{L}$ <br> $\left(K_{L} \lll 0\right)_{R}$ |
| F(Round 1) | $k_{1}$ | $\left(K_{A} \lll 0\right)_{L}$ |
| F(Round 2) | $k_{2}$ | $\left(K_{A} \lll 0\right)_{R}$ |
| F(Round 3) | $k_{3}$ | $\left(K_{L} \lll 15\right)_{L}$ |
| F(Round 4) | $k_{4}$ | $\left(K_{L} \lll 15\right)_{R}$ |
| F(Round 5) | $k_{5}$ | $\left(K_{A} \lll 15\right)_{L}$ |
| F(Round 6) | $k_{6}$ | $\left(K_{A} \lll 15\right)_{R}$ |
| FL | $k l_{1}$ | $\left(K_{A} \lll 30\right)_{L}$ |
| FL $^{-1}$ | $k l_{2}$ | $\left(K_{A} \lll 30\right)_{R}$ |
| F(Round 7) | $k_{7}$ | $\left(K_{L} \lll 45\right)_{L}$ |
| F(Round 8) | $k_{8}$ | $\left(K_{L} \lll 45\right)_{R}$ |
| F(Round 9) | $k_{9}$ | $\left(K_{A} \lll 45\right)_{L}$ |
| F(Round 10) | $k_{10}$ | $\left(K_{L} \lll 60\right)_{R}$ |
| F(Round 11) | $k_{11}$ | $\left(K_{A} \lll 60\right)_{L}$ |
| F(Round 12) | $k_{12}$ | $\left(K_{A} \lll 60\right)_{R}$ |
| FL | $k l_{3}$ | $\left(K_{L} \lll 77\right)_{L}$ |
| FL | $k l_{4}$ | $\left(K_{L} \lll 77\right)_{R}$ |
| F(Round 13) | $k_{13}$ | $\left(K_{L} \lll 94\right)_{L}$ |
| F(Round 14) | $k_{14}$ | $\left(K_{L} \lll 94\right)_{R}$ |
| F(Round 15) | $k_{15}$ | $\left(K_{A} \lll 94\right)_{L}$ |
| F(Round 16) | $k_{16}$ | $\left(K_{A} \lll 94\right)_{R}$ |
| F(Round 17) | $k_{17}$ | $\left(K_{L} \lll 111\right)_{L}$ |
| F(Round 18) | $k_{18}$ | $\left(K_{L} \lll 111\right)_{R}$ |
| Postwhitening | $k w_{3}$ | $\left(K_{A} \lll 11\right)_{L}$ |
| $k w_{4}$ | $\left(K_{A} \lll 111\right)_{R}$ |  |

