Safety Science 50 (2012) 861–864

Contents lists available at SciVerse ScienceDirect

Safety Science

journal homepage: www.elsevier.com/locate/ssci

Theoretical equation of gas desorption of particle coal under the non-uniform pressure condition and its analytical solution ☆

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ARTICLE INFO

Article history:
Available online 15 September 2011

Keywords:
Particle coal
The theoretical equation of gas desorption
The non-uniform pressure condition
Analytical solution

ABSTRACT

Based on the second Fick's law, the theoretical equation of gas desorption of particle coal under the non-uniform pressure condition was developed in this paper. The analytical solution of the theoretical equation and the method of gas desorption quantity of particle coal under non-uniform pressure condition were obtained.

0. Introduction

A large quantity of gas internal energy is accumulated in the virgin coal seam when the virgin coal seam is under the condition of the high-pressure gas desorption saturation. When the coal is suddenly exposed by external force, the adsorbed gas in the exposed coal body will be rapidly desorbed to free gas. Owing to the gas pressure gradient, free gas will gush to the entry. As the exposure time increases, the process of desorption will penetrate from the surface of coal seam into the inside. Most of gas desorption occur when the coal seam is exposed to the atmospheric conditions underground. Because of the change of the atmospheric pressure is very slight, basically in a stable constant value, the gas desorption of coal in underground mines can be considered as an isopiestic process. Much research has been done by others on the law of gas desorption in the air medium. The theory equations of gas desorption under the isopiestic condition in the air medium was put forward, and the analytical solution of the equations was withdrawn (Yang, 1986). In the real work process, gas desorption under the non-uniform pressure occurs quite often. During the research, because the gas desorption under the non-uniform pressure condition is complicated, it is difficult to take the influence of the continuously changed external pressure on the gas desorption into consideration. For example, at the time of measuring gas content with surface borehole, the slurry pressure born by the coal is changed continuously during core sampling. Accordingly, the gas desorption of coal in the slurry media is a representative process of desorption under non-uniform pressure condition (Wang, 2001). Owing to the lack of studies on non-uniform pressure desorption, the quantity of coal core lost gas during sampling in the media of drilling mud can only be calculated by the law of isopiestic desorption in the media of air, which leads to the result that the gas content measured in this way is lower than the true value (Gao, 1987). Consequently, it is of great theoretical and practical significance to research the theoretical equation of coal gas desorption under the non-uniform pressure condition and its analytical solution.

1. Theoretical model of particle coal gas desorption under the non-uniform pressure condition

The coal can be seen as a porous material with different particle sizes. Because of its dual pore structure, the gas migrates within the coal seam in three forms: desorption, diffuseness and seepage flow. With the gas pressure in the pore decreasing, the adsorbed gas will change into free gas almost instantaneously. However, the further desorption of gas depends on further decrease of gas pressure in the coal. The diffusion velocity in small physical

☆ The First International Symposium on Mine Safety Science and Engineering (ISMSSE2011) will be held in Beijing on October 26–29, 2011. The symposium is authorized by the State Administration of Work Safety and is sponsored by China Academy of Safety Science Technology (CASST), China University of Mining Technology (Beijing) (CUMTB), Datong Coal Mine Group, McGill University (Canada) and University of Wollongong (Australia) with participation from several other universities from round the world, research institutes, professional associations and large enterprises. The topics will focus on mines safety field: theory on mine safety science and engineering technology, coal mine safety science; engineering technology, metal and nonmetal mines safety science; engineering technology, petroleum and natural gas exploitation safety science; engineering technology, mine safety management and safety standardization science; technology, occupational health and safety in mine, emergent rescue engineering technology in mine, etc.

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doi:10.1016/j.ssci.2011.08.018

dimension like micro-pore is much slower than the seepage velocity in macro-pore and natural fracture. Based on the above, the crucial step of controlling gas emission is the diffusion velocity in micro-pore. It is shown in much research that the movement in natural fracture accords with Darcy's Law, while the migration in small micro-pore complies with Fick's Law (Gao, 1987):

$$V = -D\frac{\partial C}{\partial x} \tag{1}$$

In this formula, V stands for the gas diffusion velocity for getting across one square centimetre's coal, g/(cm^2 s); $\frac{\partial C}{\partial x}$ stands for concentration gradient along the diffusion orientation, g/cm^4; D stands for the diffusion coefficient, cm^2/s.

Suppose that (Yang, 1986; Nie et al., 1999): (1) the coal is sphere particles with homogeneous and isotropy; (2) the coal is ametabolic at the time of gas adsorption and desorption; (3) the migration of gas complies with law of conservation of mass and continuity principle. According to the hypothesis above, the diffusion coefficient has nothing to do with the coordinate. Now, ignore the effect of concentration and time to the diffusion coefficient, and set the coordinate of the sphere (see Fig. 1), and the second Fick's Law can be achieved:

$$\frac{\partial C}{\partial t} = D\left(\frac{\partial^2 C}{\partial r^2} + \frac{2}{r}\frac{\partial C}{\partial r}\right) \tag{2}$$

In this formula, C stands for the gas concentration at the time of the radius of particle coal equals r, cm^3/g; r stands for the radius of particle coal, cm; t stands for the time.

2. Mathematical physics equation of particle coal gas desorption under the non-uniform pressure condition

2.1. Initial condition

For the particle coal, the gas concentration of coal is a fixed value and distributed equably when the adsorption of gas is in equilibrium. Use c_0 as the gas content when adsorption is in equilibrium, and the initial condition is,

$$C|_{t=0} = c_0 = \frac{abP_0}{1+bP_0}(0 < r < r_0) \tag{3}$$

In this formula, c_0 stands for the initial gas concentration of adsorption equilibrium, cm^3/g; P_0 stands for the initial gas pressure of adsorption equilibrium, MPa; a and b are Langmuir constants.

2.2. Boundary condition

2.2.1. The first boundary condition

According to the characteristic of gas diffuseness, the boundary condition of the centre of particle coal is

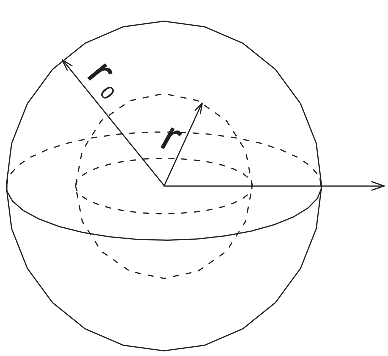

Fig. 1. The coordinate for gas diffusion through coal particle.

$$\left.\frac{\partial C}{\partial r}\right|_{r=0} = 0(t > 0) \tag{4}$$

When the particle coal is exposed, the gas concentration of the surface of the coal decreases. The concentration difference will be formed along the radius orientation of the particle coal, and then the adsorbed gas transforms to the free gas. Meanwhile, the gas diffuses and migrates from the centre to the surface of the particle coal. Under the non-uniform pressure condition, the external pressure of the coal surface is a variety. As it decreases continuously with the change of time, the gas concentration of the coal surface is

$$C = \frac{ab(P_0 - kt)}{1 + b(P_0 - kt)} \tag{5}$$

In this formula, k stands for the velocity according to the change of external pressure.

2.3. Mathematical physics equation of gas desorption under the non-uniform pressure condition

The theory equation of gas desorption under the non-uniform pressure condition can be achieved from the given initial and boundary condition and with the spherical coordinate system:

$$\begin{cases} \frac{\partial C}{\partial t} = D\left(\frac{\partial^2 C}{\partial r^2} + \frac{2}{r}\frac{\partial C}{\partial r}\right) \\ C = \frac{abp_0}{1+bp_0}, \quad 0 < r < r_0, \ t = 0 \\ C = \frac{ab(P_0-kt)}{1+b(P_0-kt)}, \quad r = r_0, \ t > 0 \\ \frac{\partial C}{\partial r} = 0, \quad r = 0, \ t > 0 \end{cases} \tag{6}$$

3. The analytical solution and analysis of the theory equation of particle coal gas desorption under the non-uniform pressure condition

3.1. The analytical solution of the theory equation

Let $U = rC$, and put it into formula (6), then:

$$\begin{cases} \frac{\partial U}{\partial t} = D\frac{\partial^2 U}{\partial r^2} \\ U = \frac{abp_0}{1+bp_0}r, \quad 0 < r < r_0, \ t = 0 \\ U = \frac{ab(P_0-kt)}{1+b(P_0-kt)}r_0, \quad r = r_0, \ t > 0 \\ U = 0, \quad r = 0, \ t > 0 \end{cases} \tag{7}$$

The Eq. (7) is second-order non-homogeneous parabolic partial differential equations, and the analytical solution can be obtained with the separation of variables (2003).

Let $U(r,t) = V(r,t) + W(r,t)$, and put it into formula (7), then formulae (8) and (9) can be achieved:

$$\begin{cases} \frac{\partial^2 W}{\partial r^2} = 0 \\ W(0,t) = 0 \\ W(r_0,t) = \frac{ab(P_0-kt)}{1+b(P_0-kt)}r_0 \\ W(r,0) = \frac{abp_0}{1+bp_0}r \end{cases} \tag{8}$$

$$\begin{cases} \frac{\partial V}{\partial t} = D\frac{\partial^2 V}{\partial r^2} - \frac{\partial W}{\partial t} \\ V(0,t) = 0 \\ V(r_0,t) = 0 \\ V(r,0) = 0 \end{cases} \tag{9}$$

When solving formula (8), let $W(r,t) = A(t)r + B(t)$, and put the boundary condition into the equations, then:

$$W(r,t) = \frac{ab(P_0 - kt)}{1 + b(P_0 - kt)} r \tag{10}$$

So, $\frac{\partial W}{\partial t} = \frac{-abk}{(1+b(P_0-kt))^2} r$, and put it into formula (9), then:

$$\begin{cases} \frac{\partial V}{\partial t} = D \frac{\partial^2 V}{\partial r^2} + \frac{abk}{(1+b(P_0-kt))^2} r \\ V(0,t) = 0 \\ V(r_0,t) = 0 \\ V(r,0) = 0 \end{cases} \tag{11}$$

According to the homogeneous theorem (Gu et al., 2002), the intrinsic system of functions of homogeneous equation corresponding to Eq. (11) is $\left\{\sin\frac{n\pi}{r_0}r\right\}$, let:

$$V(r,t) = \sum_{n=1}^{\infty} V_n(t) \sin\frac{n\pi}{r_0} r \tag{12}$$

Expanding the free term $f(r,t) = \frac{abk}{(1+b(P_0-kt))^2} r$ in accordance with the intrinsic system of functions makes:

$$\frac{abk}{(1+b(P_0-kt))^2} r = \sum_{n=1}^{\infty} f_n(t) \sin\frac{n\pi}{r_0} r \tag{13}$$

where

$$f_n(t) = \frac{2}{r_0}\int_0^{r_0} \frac{abk}{(1+b(P_0-kt))^2} r \sin\frac{n\pi}{r_0} r dr = (-1)^{n+1} \frac{2abkr_0}{n\pi(1+b(P_0-kt))^2} \tag{14}$$

Put Eqs. (12) and (14) into Eq. (11) gives that:

$$\sum_{n=1}^{\infty} [V_n'(t) + \frac{Dn^2\pi^2}{a^2} V_n(t) - f_n(t)] \sin\frac{n\pi}{a} r = 0$$

$$V_n'(t) + \frac{Dn^2\pi^2}{a^2} V_n(t) = f_n(t) = (-1)^{n+1} \frac{2abkr_0}{n\pi(1+b(P_0-kt))^2} \tag{15}$$

From the initial condition of Eq. (11), we can get: $V_n(0) = 0$, and solving Eq. (15) gives:

$$\begin{aligned} V_n(t) &= \int_0^t (-1)^{n+1} \frac{2abkr_0}{n\pi(1+b(P_0-k\tau))^2} e^{-\frac{Dn^2\pi^2}{r_0^2}(t-\tau)} d\tau \\ &= (-1)^{n+1} \frac{2abkr_0}{n\pi} \int_0^t \frac{e^{-\frac{Dn^2\pi^2}{r_0^2}(t-\tau)}}{(1+b(P_0-k\tau))^2} d\tau \\ V(r,t) &= \sum_{n=1}^{\infty} V_n(t) \sin\frac{n\pi}{r_0^2} r \\ &= \frac{2abkr_0}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \left(\int_0^t \frac{e^{-\frac{Dn^2\pi^2}{r_0^2}(t-\tau)}}{(1+b(P_0-k\tau))^2} d\tau \right) \sin\frac{n\pi}{r_0^2} r \\ U(r,t) &= V(r,t) + W(r,t) \\ &= \frac{ab(P_0-kt)}{1+b(P_0-kt)} r + \frac{2abkr_0}{\pi} \sum_{n=1}^{\infty} \\ &\times \frac{(-1)^{n+1}}{n} \left(\int_0^t \frac{e^{-\frac{Dn^2\pi^2}{r_0^2}(t-\tau)}}{(1+b(P_0-k\tau))^2} d\tau \right) \sin\frac{n\pi}{r_0^2} r \\ &\times \quad (n = 1,2,\ldots) \end{aligned} \tag{16}$$

Because of $U = rC$, we get $C = \frac{U}{r}$, then the analytical solution of Eq. (6) is:

$$\begin{aligned} C(r,t) &= \frac{ab(P_0-kt)}{1+b(P_0-kt)} + \frac{2abkr_0}{\pi r} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(-1)^{n+1}}{n} \\ &\times \left(\int_0^t \frac{e^{-\frac{Dn^2\pi^2}{r_0^2}(t-\tau)}}{(1+b(P_0-k\tau))^2} d\tau \right) \sin\frac{n\pi}{r_0^2} \quad (n = 1,2,\ldots) \end{aligned} \tag{17}$$

3.2. The quantity of gas desorption and analysis under the non-uniform pressure condition

$C(r,t)$ is the expression of gas concentration distribution in the coal under the non-uniform pressure condition. The cumulative gas content Q_t desorbed from the particle coal with anytime t can be calculated via Eq. (17). And by the integral of spherical coordinate (Guo et al., 1997), it can be achieved that:

$$\begin{aligned} Q_t &= \int\int_v \int (c_0 - C(r,t)) dv \\ &= \int\int_v \int (c_0 - C(r,t)) r^2 \sin\varphi \, d\theta \, d\varphi \, dr \\ &= \int_0^{2\pi} d\theta \int_0^{\pi} \sin\varphi \, d\varphi \int_0^a (c_0 - C(r,t)) r^2 dr \\ &= \frac{4}{3}\pi r_0^3 \left(c_0 - \frac{ab(P_0-kt)}{1+b(P_0-kt)} \right) - \frac{2abkr_0^3}{\pi^2} \sum_{n=1}^{\infty} \\ &\times \frac{1}{n^2} \left(\int_0^t \frac{e^{-\frac{Dn^2\pi^2}{r_0^2}(t-\tau)}}{(1+b(P_0-k\tau))^2} d\tau \right) \end{aligned} \tag{18}$$

For any variables n and t,

$$0 < e^{-\frac{Dn^2\pi^2}{a^2}(t-\tau)} < 1, 0 < \int_0^t \frac{e^{-\frac{Dn^2\pi^2}{a^2}(t-\tau)}}{(1+b(P_0-k\tau))^2} d\tau < \int_0^t \frac{1}{(1+b(P_0-k\tau))^2} d\tau = \frac{1}{bk}\left(\frac{1}{1+b(P_0-kt)} - \frac{1}{1+bP_0} \right)$$

By the comparison discriminant of infinite series, it can be achieved that infinite series $\sum_{n=1}^{\infty} \frac{1}{n^2} (\int_0^t \frac{e^{-\frac{Dn^2\pi^2}{r_0^2}(t-\tau)}}{(1+b(P_0-k\tau))^2} d\tau)$ is convergent, and the equation is:

$$\lim_{t\to\infty} \sum_{n=1}^{\infty} \frac{1}{n^2} \left(\int_0^t \frac{e^{-\frac{Dn^2\pi^2}{r_0^2}(t-\tau)}}{(1+b(P_0-k\tau))^2} d\tau \right) = 0 \tag{19}$$

When $t \to \infty$, the external pressure equals the atmospheric pressure, namely, $P_0 - kt = P_f = 0.1$ MPa, and at the moment, the gas concentration on the surface of the coal is:

$$C = \frac{ab(P_0-kt)}{1+b(P_0-kt)} = \frac{abP_f}{1+bP_f} = c_f \tag{20}$$

When $t \to \infty$, putting Eqs.(19) and (20) into (18), it can be obtained that the utmost desorbed quantity of gas in the particle coal:

$$Q_\infty = \lim_{t\to\infty} Q_t = \frac{4}{3}\pi a^3 (c_0 - c_f) \tag{21}$$

Referring to the others' research (Yang and Wang, 1986), the utmost desorbed quantity of gas in the particle coal is equal to that in air medium under the isopiestic pressure condition.

It can be seen from Eq. (18) that the laws of gas desorption are different in non-uniform pressure and isopiestic condition. The desorption velocity is not only affiliated with the initial equilibrium concentration and diffusion coefficient, but also related to the structures, size dimensions, and the variational velocity of external pressure.

4. Conclusions

This article established the theoretical equation of particle coal gas desorption under the non-uniform pressure condition, and obtained the analytical solution. Besides, the cumulative gas desorption content Q_t under the non-uniform pressure condition with anytime t was achieved. When $t \to \infty$, the utmost desorbed quantity of gas in the particle coal is equal to that in air medium under the isopiestic pressure condition.

Acknowledgements

This article is supported by the 973 Program (2006CB202200) and the National Natural Science Foundation of China (50974056).

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