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具有操纵面间隙非线性二维翼段的气动弹性分析

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Aeroelastic Analysis of a Two-Dimensional Airfoil with Control Surface Freeplay Nonlinearity

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摘 要: 基于 Theodorsen 理论和 Wagner 函数,提出了不可压缩流作用下三自由度二维翼段任意运动非定常气动力表达式。对操纵面自由度具有的间隙非线性,建立了二维翼段气动弹性系统无量纲分段线性运动方程。数值仿真预示了系统极限环振动的相轨迹、无量纲振动幅值和频率,表明操纵面铰链处存在的间隙非线性将导致整个系统的极限环振动;随着来流速度的增加,系统极限环振动的幅值和频率都存在跳跃现象。

关键词: 气动弹性; 三自由度二维翼段; 非稳态气动力; 间隙非线性; 极限环振动

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Abstract: Based on Theodorsen's theory and Wagner's function, the expressions of unsteady aerodynamic forces for arbitrary motions of a two-dimensional airfoil with three degrees of freedom are presented. It is assumed that the airfoil is subjected to an incompressible flow. The non-dimensional piecewise linear equations of motion for the two-dimensional airfoil with control surface freeplay nonlinearity are established. The phase trajectories, non-dimensional amplitudes and frequencies of limit cycle oscillation are predicted through numerical simulation. Simulation results demonstrate that the freeplay nonlinearity in the control surface hinge moment can result in limit cycle oscillations of the whole system. With the increase of air speed, the jump phenomenon appears in the amplitudes and frequencies of limit cycle oscillations.

Key words: aeroelasticity; two-dimensional airfoil with 3-DOF; unsteady aerodynamic forces; freeplay nonlinearity; limit cycle oscillations

含有结构非线性的翼段结构的非线性气动弹性研究已成为国内外气动弹性力学领域中的一个活跃的方向。其中对间隙非线性的研究大都是针对具有沉浮和俯仰自由度的两自由度二维翼段[1,2],间隙非线性施加在俯仰自由度上。作为两自由度情况的扩展,本文首先利用 Theodorsen 理论和 Wagner 函数得到了三自由度二维翼段任意运动非定常气动力的时域表达式;建立了三自由度二维翼段的无量纲非线性气动弹性动力学方程,然后将动力学方程表达为状态空间中的分段线性形式,进而研究亚音速、不可压缩流作用下带有操纵面间隙非线性的翼段结构的气动弹性响应。

1 三自由度二维翼段的力学模型

带有操纵面的三自由度二维翼段的力学模型如图 1 所示。翼段的 3 个自由度分别为沉浮位移

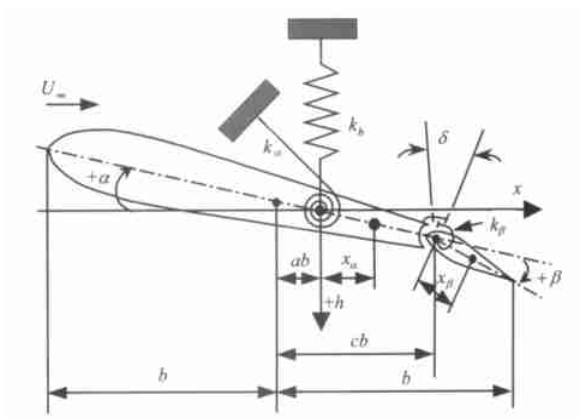

图 1 翼段的力学模型

Fig. 1 The mechanical model of airfoil

h,向下为正;俯仰角 α,顺时针为正;操纵面俯仰角 β,顺时针为正;b 为半弦长;弹性轴与弦中点的距离所占半弦长的比为 a;操纵面铰链与弦中点的距离所占半弦长的比为 c;k_h, k_α 和 k_β 分别表示沉浮刚度、俯仰刚度和操纵面俯仰刚度;U_∞ 为来流速度;x_α 为翼段重心到弹性轴之间的距离;x_β

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为操纵面重心到操纵面铰链中心之间的距离。

操纵面铰链处的间隙非线性回复力矩 $\widetilde{M}_\beta$ 如图 2 所示。δ 为间隙的大小。

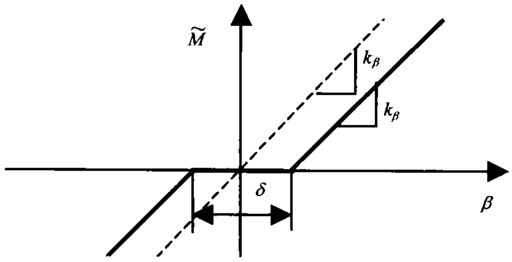

图 2 间隙非线性

Fig. 2 Freeplay nonlinearity

间隙非线性可表示为

$$\widetilde{M}_\beta = \begin{cases} k_\beta(\beta + \delta/2) & \beta < -\delta/2 \\ 0 & -\delta/2 \le \beta \le \delta/2 \\ k_\beta(\beta - \delta/2) & \beta > \delta/2 \end{cases} \quad (1)$$

2 翼段的无量纲动力学方程

考虑间隙非线性在内的三自由度二维翼段的无量纲动力学方程为

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} + \mathbf{H} = \frac{U^2}{\pi\mu}\mathbf{C}_e \quad (2)$$

其中:

$$\mathbf{M} = \begin{bmatrix} 1 & \bar{x}_\alpha & \bar{x}_\beta \\ \bar{x}_\alpha & \bar{r}_\alpha^2 & \bar{r}_\alpha^2 + \bar{x}_\beta(c-a) \\ \bar{x}_\alpha & \bar{r}_\alpha^2 + \bar{x}_\beta(c-a) & \bar{r}_\beta^2 \end{bmatrix}$$

$$\mathbf{C} = \mathrm{diag}\left[2\xi_h\left(\frac{\omega_h}{\omega_\alpha}\right) \quad 2\xi_\alpha\bar{r}_\alpha^2 \quad 2\xi_\beta\bar{r}_\beta^2\left(\frac{\omega_\beta}{\omega_\alpha}\right)\right]$$

$$\mathbf{K} = \begin{cases} \mathbf{K}_m = \mathrm{diag}\left[\left(\frac{\omega_h}{\omega_\alpha}\right)^2 \quad \bar{r}_\alpha^2 \quad \bar{r}_\beta^2\left(\frac{\omega_\beta}{\omega_\alpha}\right)^2\right], & \beta < -\frac{\delta}{2} \\ \mathbf{K}_n = \mathrm{diag}\left[\left(\frac{\omega_h}{\omega_\alpha}\right)^2 \quad \bar{r}_\alpha^2 \quad 0\right], & -\frac{\delta}{2} \le \beta \le \frac{\delta}{2} \\ \mathbf{K}_m = \mathrm{diag}\left[\left(\frac{\omega_h}{\omega_\alpha}\right)^2 \quad \bar{r}_\alpha^2 \quad \bar{r}_\beta^2\left(\frac{\omega_\beta}{\omega_\alpha}\right)^2\right], & \beta > \frac{\delta}{2} \end{cases}$$

$$\mathbf{H} = \begin{cases} \mathbf{F}_1 = \left[0 \quad 0 \quad \bar{r}_\beta^2\left(\frac{\omega_\beta}{\omega_\alpha}\right)^2\right]^{\mathrm{T}}, & \beta < -\frac{\delta}{2} \\ \mathbf{F}_2 = [0 \quad 0 \quad 0]^{\mathrm{T}}, & -\frac{\delta}{2} \le \beta \le \frac{\delta}{2} \\ \mathbf{F}_3 = \left[0 \quad 0 \quad \bar{r}_\beta^2\left(\frac{\omega_\beta}{\omega_\alpha}\right)^2\right]^{\mathrm{T}}, & \beta > \frac{\delta}{2} \end{cases}$$

$$\mathbf{C}_e = [-C_h(\tau) \quad 2C_\alpha(\tau) \quad 2C_\beta(\tau)]^{\mathrm{T}}$$

$$\mathbf{q} = [h/b \quad \alpha \quad \beta]^{\mathrm{T}}$$

其中:$\mathbf{q}$ 为无量纲位移;$\dot{\mathbf{q}} = \mathrm{d}\mathbf{q}/\mathrm{d}\tau$,$\tau = \omega_\alpha t$ 为无量纲时间;$\mu = m/(\pi\rho b^2)$ 为质量比;m 为翼段的质量;ρ 为空气密度;$U = U_\infty/(\omega_\alpha b)$ 为无量纲来流速度;$\omega_h, \omega_\alpha, \omega_\beta$ 分别为无耦合沉浮、俯仰和操纵面俯仰角频率;$\xi_h, \xi_\alpha, \xi_\beta$ 分别为无耦合沉浮、俯仰和操纵面俯仰阻尼;$C_h(\tau)$, $C_\alpha(\tau)$ 和 $C_\beta(\tau)$ 分别为升力系数、俯仰力矩系数和操纵面力矩系数;$\bar{x}_\alpha = x_\alpha/b$ 为翼段重心到弹性轴之间的无量纲距离;$\bar{x}_\beta = x_\beta/b$ 为操纵面重心到操纵面铰链中心之间的无量纲距离;$\bar{r}_\alpha = r_\alpha/b$ 为翼段绕弹性轴的无量纲回转半径;$\bar{r}_\beta = r_\beta/b$ 为操纵面绕操纵面铰链中心的无量纲回转半径。

3 非定常气动力的建立

引入气动力无量纲时间 $\bar{\tau} = U_\infty t/b$,则气动力无量纲时间 $\bar{\tau}$ 与前面定义的 τ 之间的关系为 $\bar{\tau} = U_\infty\tau/(b\omega_\alpha) = U\tau$。根据 Theodorsen 理论并利用 Wagner 函数,则 $C_h(\tau)$, $C_\alpha(\tau)$ 和 $C_\beta(\tau)$ 可表示为时域中的 Duhamel 积分的形式[3]

$$C_h(\tau) = C_h^1(\tau) + 2\pi\cdot\left\{\left[\alpha_{qs}(0)\phi_W(U\tau) + \int_0^\tau \phi_W(U(\tau-\tau_0))\frac{\mathrm{d}\alpha_{qs}(\tau_0)}{\mathrm{d}\tau_0}\mathrm{d}\tau_0\right] + \left[\beta_{qs}(0)\phi_W(U\tau) + \int_0^\tau \phi_W(U(\tau-\tau_0))\frac{\mathrm{d}\beta_{qs}(\tau_0)}{\mathrm{d}\tau_0}\mathrm{d}\tau_0\right]\right\} \quad (3a)$$

$$C_\alpha(\tau) = C_\alpha^1(\tau) + C_\alpha^2(\tau) + \pi\left(a + \frac{1}{2}\right)\cdot\left\{\left[\alpha_{qs}(0)\phi_W(U\tau) + \int_0^\tau \phi_W(U(\tau-\tau_0))\frac{\mathrm{d}\alpha_{qs}(\tau_0)}{\mathrm{d}\tau_0}\mathrm{d}\tau_0\right] + \left[\beta_{qs}(0)\phi_W(U\tau) + \int_0^\tau \phi_W(U(\tau-\tau_0))\frac{\mathrm{d}\beta_{qs}(\tau_0)}{\mathrm{d}\tau_0}\mathrm{d}\tau_0\right]\right\} \quad (3b)$$

$$C_\beta(\tau) = C_\beta^1(\tau) + C_\beta^2(\tau) - \frac{F_{12}}{2}\cdot\left\{\left[\alpha_{qs}(0)\phi_W(U\tau) + \int_0^\tau \phi_W(U(\tau-\tau_0))\frac{\mathrm{d}\alpha_{qs}(\tau_0)}{\mathrm{d}\tau_0}\mathrm{d}\tau_0\right] + \left[\beta_{qs}(0)\phi_W(U\tau) + \int_0^\tau \phi_W(U(\tau-\tau_0))\frac{\mathrm{d}\beta_{qs}(\tau_0)}{\mathrm{d}\tau_0}\mathrm{d}\tau_0\right]\right\} \quad (3c)$$

其中:

$$\alpha_{qs}(\tau) = \frac{1}{U}\left(\frac{\dot{h}}{b}\right) + \alpha + \frac{1}{U}\left(\frac{1}{2} - a\right)\dot{\alpha}$$

$$\beta_{qs}(\tau) = \frac{F_{10}}{\pi}\beta + \frac{F_{11}}{2\pi U}\beta$$

$$C_h^1(\tau) = \frac{\pi}{U}\dot{\alpha} + \frac{\pi}{U^2}\left(\frac{\ddot{h}}{b}\right) - \frac{\pi a}{U^2}\ddot{\alpha} - \frac{F_4}{U}\dot{\beta} - \frac{F_1}{U^2}\ddot{\beta}$$

$$C_\alpha^1(\tau) = -\frac{\pi}{2U^2}\left(\frac{1}{8} + a^2\right)\ddot{\alpha} + \frac{\pi a}{2U^2}\left(\frac{\ddot{h}}{b}\right) + \frac{F_7 + (c-a)F_1}{2U^2}\ddot{\beta}$$

$$C^2_\alpha(\tau) = -\frac{\pi}{2U}\left(\frac{1}{2} - a\right)\dot{\alpha} - \frac{F_4 + F_{10}}{2}\beta - \frac{1}{2U}\left(F_1 - F_8 - (c - a)F_4 + \frac{1}{2}F_{11}\right)\beta$$

$$C^1_\beta(\tau) = -\frac{F_{13}}{U^2}\ddot{\alpha} + \frac{F_1}{2U^2}\left(\frac{\ddot{h}}{b}\right) + \frac{F_3}{2\pi U^2}\ddot{\beta}$$

$$C^2_\beta(\tau) = \frac{1}{2U}\left(2F_9 + F_1 - F_4\left(a - \frac{1}{2}\right)\right)\dot{\alpha} - \frac{1}{2\pi}(F_5 - F_4F_{10})\beta + \frac{1}{4\pi U}F_4F_{11}\beta$$

其中:$\phi_W(\cdot)$ 为 Wagner 函数;$F_1 \sim F_{13}$ 为 Theodorsen 常量。Wagner 函数可近似为

$$\phi_W(U(\tau - \tau_0)) = 1 - \alpha_1 e^{-\beta_1 U(\tau - \tau_0)} - \alpha_2 e^{-\beta_2 U(\tau - \tau_0)} \tag{4}$$

其中:$\alpha_1 = 0.2048$;$\alpha_2 = 0.2952$;$\beta_1 = 0.0557$;$\beta_2 = 0.3333$。

定义气动力状态

$$\eta_1(\tau) = \alpha_1\beta_1 U\int(\alpha_{qs}(\tau_0) + \beta_{qs}(\tau_0))e^{-\beta_1 U(\tau - \tau_0)}d\tau_0 \tag{5a}$$

$$\eta_2(\tau) = \alpha_2\beta_2 U\int(\alpha_{qs}(\tau_0) + \beta_{qs}(\tau_0))e^{-\beta_2 U(\tau - \tau_0)}d\tau_0 \tag{5b}$$

由式(3)、式(4)、式(5)得

$$C_h(\tau) = C^1_h(\tau) + 2\pi((\alpha_{qs}(\tau) + \beta_{qs}(\tau))\cdot\phi_W(0) + \eta_1(\tau) + \eta_2(\tau)) \tag{6a}$$

$$C_\alpha(\tau) = C^1_\alpha(\tau) + C^2_\alpha(\tau) + \pi\left(a + \frac{1}{2}\right)\cdot((\alpha_{qs}(\tau) + \beta_{qs}(\tau))\phi_W(0) + \eta_1(\tau) + \eta_2(\tau)) \tag{6b}$$

$$C_\beta(\tau) = C^1_\beta(\tau) + C^2_\beta(\tau) - \frac{F_{12}}{2}((\alpha_{qs}(\tau) + \beta_{qs}(\tau))\phi_W(0) + \eta_1(\tau) + \eta_2(\tau)) \tag{6c}$$

由式(6)进一步得到

$$\frac{U^2}{\pi\mu}C_e = \frac{1}{\pi\mu}Z_1\ddot{q} + \frac{U}{\pi\mu}Z_2\dot{q} + \frac{U^2}{\pi\mu}Z_3 q + \frac{U^2}{\pi\mu}Z_4\eta \tag{7}$$

其中:

$$Z_1 = \begin{bmatrix} -\pi & \pi a & F_1 \\ \pi a & -\pi(1/8 + a^2) & F_7 + (c - a)F_1 \\ F_1 & -2F_{13} & F_3/\pi \end{bmatrix}$$

$$Z_2 = \begin{bmatrix} -\pi & -\pi\left(\frac{3}{2} - a\right) & F_4 - F_{11}/2 \\ \pi\left(a + \frac{1}{2}\right) & -\pi\left(a - \frac{1}{2}\right)^2 & -F_1 + F_8 + (c - a)F_4 - (1 - 2a)\frac{F_{11}}{4} \\ -\frac{F_{12}}{2} & 2F_9 + F_1 + \left(a - \frac{1}{2}\right)\left(\frac{F_{12}}{2} - F_4\right) & \left(F_4 - \frac{F_{12}}{2}\right)\frac{F_{11}}{2\pi} \end{bmatrix}$$

$$Z_3 = \begin{bmatrix} 0 & -\pi & -F_{10} \\ 0 & \pi\left(a + \frac{1}{2}\right) & \left(a - \frac{1}{2}\right)F_{10} - F_4 \\ 0 & -\frac{F_{12}}{2} & \frac{(-2F_5 + 2F_4F_{10} - F_{12}F_{10})}{2\pi} \end{bmatrix}$$

$$Z_4 = \begin{bmatrix} -2\pi & -2\pi \\ \pi(2a + 1) & \pi(2a + 1) \\ -F_{12} & -F_{12} \end{bmatrix}, \quad \eta = [\eta_1 \quad \eta_2]^T$$

气动力状态满足微分方程

$$\dot{\eta} = U^2 G_1\eta + G_2\dot{q} + UG_3 q \tag{8}$$

其中:

$$G_1 = \begin{bmatrix} -\alpha_1\beta_1^2 & \\ & -\alpha_2\beta_2^2 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} \alpha_1\beta_1 & \alpha_1\beta_1(1/2 - a) & \alpha_1\beta_1 F_{11}/(2\pi) \\ \alpha_2\beta_2 & \alpha_2\beta_2(1/2 - a) & \alpha_2\beta_2 F_{11}/(2\pi) \end{bmatrix}$$

$$G_3 = \begin{bmatrix} 0 & \alpha_1\beta_1 & \alpha_1\beta_1 F_{10}/\pi \\ 0 & \alpha_2\beta_2 & \alpha_2\beta_2 F_{10}/\pi \end{bmatrix}$$

引入状态变量

$$X = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6 \quad x_7 \quad x_8]^T = [h/b \quad \alpha \quad \beta \quad \dot{h}/b \quad \dot{\alpha} \quad \beta \quad \eta_1 \quad \eta_2]^T \tag{9}$$

由式(2)、式(7)、式(8)得到状态空间中翼段的无量纲分段线性动力学方程为

$$\dot{X} = A(U)X - F^1_d, R_1 = \left\{X \in \mathbf{R}^8 \mid x_3 < -\frac{\delta}{2}\right\} \tag{10a}$$

$$\dot{X} = B(U)X, R_2 = \left\{X \in \mathbf{R}^8 \mid -\frac{\delta}{2} \le x_3 \le \frac{\delta}{2}\right\} \tag{10b}$$

$$\dot{X} = A(U)X + F^1_d, R_3 = \left\{X \in \mathbf{R}^8 \mid x_3 > \frac{\delta}{2}\right\} \tag{10c}$$

其中:

$$A(U) = \begin{bmatrix} \mathbf{0}_{3\times3} & I_{3\times3} & \mathbf{0}_{3\times2} \\ -\widetilde{M}^{-1}\overline{K_m} & -\widetilde{M}^{-1}\overline{C} & -\widetilde{M}^{-1}\overline{D} \\ UG_3 & G_2 & U^2G_1 \end{bmatrix}$$

$$B(U) = \begin{bmatrix} \mathbf{0}_{3\times3} & I_{3\times3} & \mathbf{0}_{3\times2} \\ -\widetilde{M}^{-1}\overline{K_n} & -\widetilde{M}^{-1}\overline{C} & -\widetilde{M}^{-1}\overline{D} \\ UG_3 & G_2 & U^2G_1 \end{bmatrix}$$

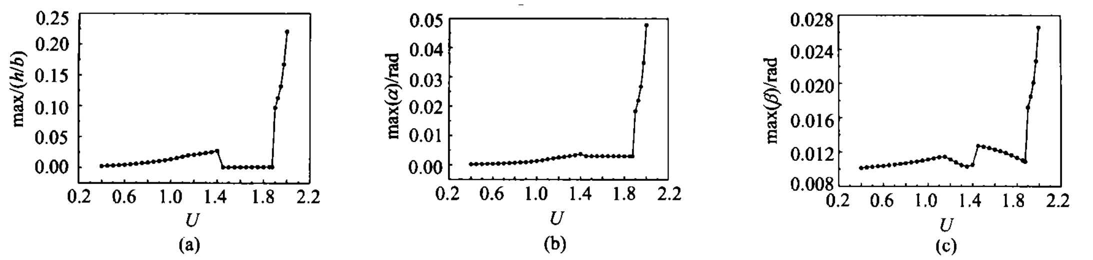

图3 极限环响应的幅值

Fig. 3 The amplitude of limit cycle response

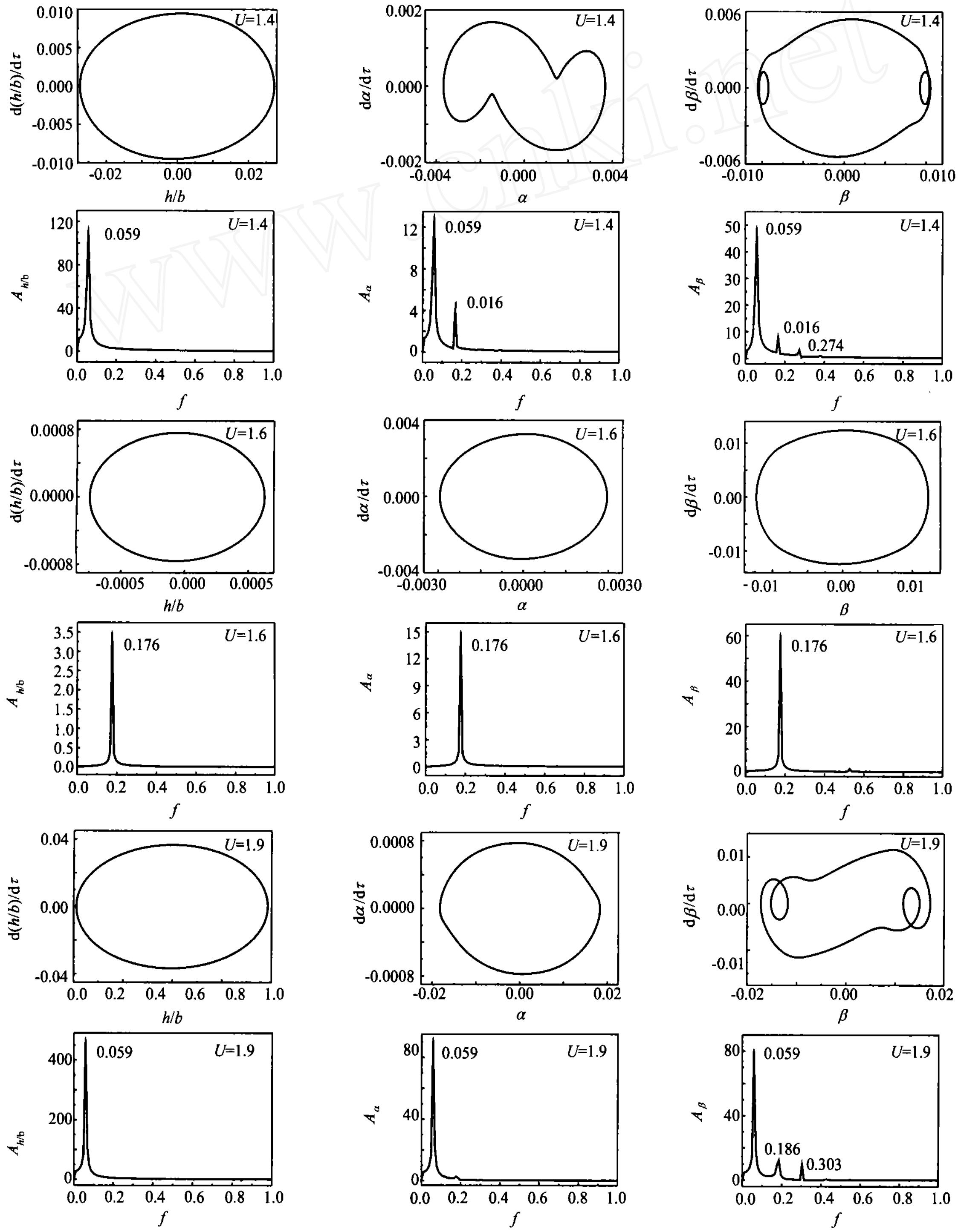

图4 不同 U 下极限环的相图和频谱

Fig. 4 The phase diagrams and spectrum of limit cycle oscillations for various U

$$F_d^1 = \begin{bmatrix} \mathbf{0}_{3\times1} \\ \widetilde{M}^{-1}F^1 \\ \mathbf{0}_{2\times1} \end{bmatrix}, \quad \widetilde{M} = M - \frac{1}{\pi\mu}Z_1$$

$$\overline{K_m} = K_m - \frac{U^2}{\pi\mu}Z_3, \overline{K_n} = K_n - \frac{U^2}{\pi\mu}Z_3$$

$$\overline{C} = C - \frac{U}{\pi\mu}Z_2, \overline{D} = \frac{U^2}{\pi\mu}Z_4$$

4　数值结果

气动弹性响应分析所用到的仿真参数如下: $a = -0.2, c = 0.5, \overline{x_\alpha} = 0.2, \overline{x_\beta} = 0.008, \overline{r_\alpha} = 0.5, \overline{r_\beta} = 0.06, \mu = 30, \omega_h/\omega_\alpha = 0.3, \omega_\beta/\omega_\alpha = 1.5, \zeta_h = 0.016, \zeta_\alpha = 0.006, \zeta_\beta = 0.004, \delta = 1°$。

沉浮运动 h/b,俯仰运动 α 和操纵面俯仰运动 β 的极限环响应幅值和无量纲来流速度的关系如图 3 所示。翼段运动方程(10b) 是导致系统出现极限环振动的原因。通过分析此方程的 Jaccobi 矩阵的特征值,得知系统的 Hopf 分岔点出现在 $U \approx 0.37$ 处,此时在零点附近分化出极限环。结果表明系统的 3 个自由度在 $U \approx 1.4$ 和 $U \approx 1.9$ 处的响应幅值都存在跳跃现象。当 $U > 1.9$ 时,极限环振动的幅值随无量纲来流速度的增加急速增长;当不考虑间隙非线性时,通过根轨迹分析可知系统的线性颤振速度为 $U \approx 2.0$,所以当 $U > 2.0$ 时,系统表现为发散运动。这与文献[3] 指出的间隙非线性不改变系统的颤振速度的结论是一致的。在本文给定的参数下,由间隙非线性导致的极限环振动表现为单谐波极限环和多谐波极限环两种形式,但均表现为周期 - 1 运动(见图 4) 。对不同 U,翼段沉浮运动均表现为单谐波极限环。在极限环振动幅值出现跳跃前($U = 1.4$) ,翼段俯仰运动 α 和和操纵面俯仰运动 β 均表现为多谐波极限环振动;幅值出现跳跃后($U = 1.6$) ,系统 3 个自由度的响应均表现为单谐波极限环振动;图 4 还展示了系统极限环响应的幅值出现第 2 次跳跃之前($U = 1.9$) 的极限环响应的相轨迹和频谱。

图 5 表明系统极限环振动的无量纲基频在 $U \approx 1.4$ 和 $U \approx 1.9$ 处也分别出现一次由低频到高频和由高频到低频的较大的跳跃。

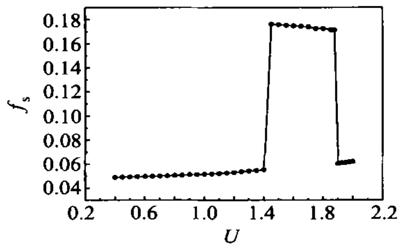

图 5　基频 f_s 与流速的关系

Fig. 5　The relationship between fundamental frequency f_s and air speed U

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