

Role of photonic bandgap in transverse localization of light in a disordered waveguide lattice

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The role of a prominent photonic bandgap (PBG) on the phenomenon of transverse localization of light in an evanescently coupled disordered one-dimensional semi-infinite lossless waveguide lattice has been investigated numerically. The interplay between the underlying photonic bandgap due to inherent periodicity and various levels of deliberately induced transverse disorder in it in the form of refractive index perturbation has been studied. We show that the PBG indeed plays an important role and it could help in achieving localized light even in a partially disordered lattice. An important outcome of this study revealed that PBG could be gainfully exploited to tailor the spectral window of light localization for specific applications.

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Raedt et al.¹ re-visited the idea of light localization² and introduced the concept of transverse light localization in a semi-infinite disordered geometry in which light confinement occurs only in a plane perpendicular to the direction of light propagation. This interesting phenomenon of light localization in 1D/2D disordered dielectric structures analogous to Anderson localization, which proved to be experimentally realizable, has emerged as a research field of intense contemporary interest.^{3,4} It is now known that in temporarily or permanently realized lattices with deliberately introduced disorder, light confinement occurs due to the sole effect of disorder.^{5,7} The backbone periodic structures in these disordered geometries play a key role to show the features of localization as a combined effect of order and disorder. Ongoing intensive research on photonic bandgap structures, deal with confinement of light within a localized defect region in an otherwise periodic structure (the periodicity accounting for the photonic bandgap) and with the advent of discrete photonic systems, this new avenue of light localization has emerged as a contemporary field of research in the context of disordered optical structures.^{5,9}

However, with the state-of-the art fabrication process for developing such photonic structures, having features of micrometer/sub-micrometer scale, it is not an easy task to realize a perfect periodic structure. In this context, transverse localization (TL) of light has added a new dimension to light guidance in PBG structures. Hence, due to this unwanted deviation, the migration of new accessible states from the band edge towards the center of the inherent bandgap of a targeted periodic structure comes into picture. This effect eventually smears out the desired effect of decreased density of states (DOS) because of filling the PBG by band-tail localized states.¹⁰ Hence, investigation on the stability of the photonic bandgap in the presence of disorder is an important aspect even from the point of view of estimating the quality of an intended PBG in an optical structure.^{11,12} This particular aspect (simultaneous presence of PBG and disorder) which has eventually synthesized two distinctly different phenomena (bandgap guidance and localization of light) is of contemporary interest from the application as well as pure theoretical stand-point, but not yet fully explored.

In these above mentioned studies^{5,8} on the localization effect, though there exist an underlying periodicity, the significance of a prominent bandgap in the context of TL of light seems to have been not discussed, in particular, in the literature. In a bandgap geometry, the phenomenon of light localization due to the inherent Bragg scattering by the ordered structure is fundamentally different from the effect of TL in the presence of a disorder. Moreover, the interplay between the backbone bandgap-forming periodic structures with the deliberate controlled disorder may play a key role in disordered photonic structures. This motivated us to investigate the interesting aspect of TL of light in the presence of a prominent bandgap.

In this paper, we study the influence of the existence of a prominent photonic bandgap on the controlled disorder-assisted TL of light in a 1D waveguide lattice. The sample lattice geometry is chosen to be such that at the operating wavelength there exists a prominent PBG and it is assumed that disorder is introduced into it in the form of a perturbed refractive index in transverse direction. Our results show that even only a small disorder (of much lower strength than the threshold disorder required for the case of solely disorder-assisted TL in absence of PBG) is sufficient to achieve localization in the simultaneous presence of a bandgap. Also, one may perhaps exploit this by appropriately choosing the lattice parameters as an additional tool to tune the PBG along with the embedded disorder to attain spectral selectivity of the localization phenomenon in specific applications like random/disordered¹³ lasing.

We consider an evanescently coupled waveguide lattice consisting of a large number (N) of unit cells, and in which all the waveguides spaced equally apart are buried inside a medium of constant refractive index n_0 .^{6,8} The overall structure is homogeneous in the longitudinal (z) direction along which the optical beam is assumed to propagate. The change in refractive index $\Delta n(x)$ (over the uniform background of n_0) due to disorder in this 1D waveguide lattice is assumed to be of the form

$$\Delta n(x) = \Delta n_p (H(x) + C\delta(x)) \quad (1)$$

here C is a dimensionless constant, whose value governs the level/strength of disorder; the periodic function $H(x)$ takes the value 1 inside the higher-index regions and is zero

elsewhere; $\Delta n(x)$ consists of a deterministic periodic part Δn_p of spatial period Λ and a spatially periodic random component δ (uniformly distributed over a specified range varying from 0 to 1). This particular choice of randomly perturbed refractive index the high index as well as low index layers enables us to model the diagonal and off-diagonal disorders to study the localization of light.⁶ Wave propagation through the lattice is governed by the standard scalar Helmholtz equation, which under paraxial approximation could be written as

$$i \frac{\partial A}{\partial z} + \frac{1}{2k} \left(\frac{\partial^2 A}{\partial x^2} \right) + \frac{k}{n_0} \Delta n(x) A = 0 \quad (2)$$

where $A(x, z)$ is amplitude of an input CW optical beam having its electric-field as $E(x, z, t) = R e[A(x, z) e^{i(kz - \omega t)}]$; $k = n_0 \omega / c$. To study the effect of a PBG on the phenomena of transverse localization of light, one could write, for the lattice, the following equations for the two configurations:

$$\left. \begin{aligned} \Delta n(x) &= \Delta n_p (H(x) + C \delta(x)); C \neq 0 \\ d_1 + d_2 &= \Lambda \end{aligned} \right\} \quad (3a)$$

$$\left. \begin{aligned} \Delta n(x) &= \Delta n_p (H(x) + C \delta(x)); C = 0 \\ d_1(\pm \delta) + d_2(\pm \delta) &= \Lambda (\text{randomly varying}) \end{aligned} \right\} \quad (3b)$$

where the Eq. (3a) and Eq. (3b) represents the index and spatial disorder, respectively. However, the results presented in this paper are for the particular case of refractive index disorder. We solve Eq. (2) numerically through the scalar beam propagation method, which we implemented in Matlab. In our optimized 10 mm long 1D lattice geometry of 150 identical evanescently coupled waveguides, we consider the high index regions of width 3 μm , which are separated by equal distances of 6 μm . The value of Δn_p was chosen to be 0.016 over the background material of refractive index $n_0 = 1.454$. We have chosen this particular distribution of the transverse refractive index of the perfectly ordered lattice such that it spawns a prominent bandgap. In Fig. 1(a) we have depicted the schematic of the designed lattice geometry without any disorder introduced, whereas Fig. 1(b) shows the numerically estimated DOS plot in the presence of the PBG

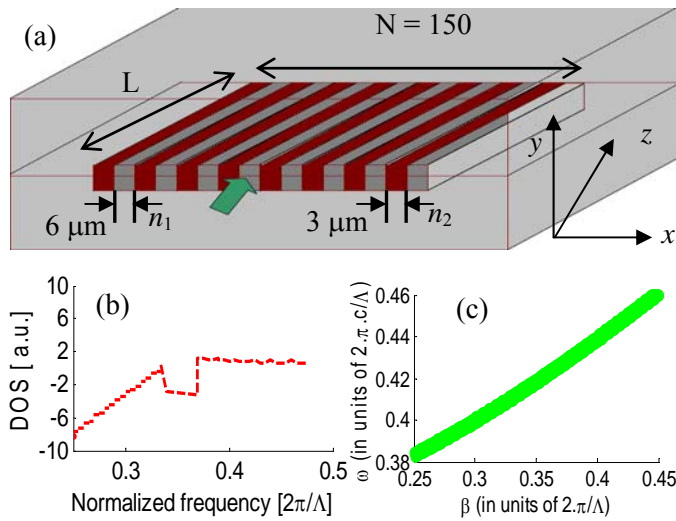


FIG. 1. (Color online) a) Schematics of the chosen 1D coupled waveguide lattice. b) Calculated density of states (DOS) and c) lowest order stopband of the lattice.

due to a perfectly periodic structure. The extent of the corresponding fundamental bandgap in the operating frequency scale of the lattice has also been shown in Fig. 1(c). Once the band structure is obtained, before introducing the different levels of refractive index perturbations in the lattice we have identified the wavelength window of the stopband and accordingly three important regimes of operation have been chosen: one well inside the bandgap, one just outside its stop-band (i.e. near the band edge) and one well outside the bandgap, respectively. With these particular choices of the operating wavelengths, it should be possible to investigate the interplay between transverse disorder and photonic bandgap from the point of view of localization.

In order to investigate quantitatively the effect of having a photonic bandgap on its localization in a disordered medium, we have studied the beam dynamics (with the input Gaussian beam of FWHM 8 μm) for different lengths of the lattice at different operating wavelength regimes. A measure of the localization is assumed to be quantifiable through decrease in the average effective width (ω_{eff}) (as defined in⁸)

$$P \equiv \left[\int I(x, L)^2 dx \right] / \left[\int I(x, L) dx \right]^2$$

$$\omega_{eff} = \langle P \rangle^{-1} \quad (4)$$

of the propagating beam after including the statistical nature of the localization phenomenon in a finite system; where $\langle \dots \rangle$ represents a statistical average over several realizations of the same level of disorder. In Fig. 2(a), we have shown the spectral dependence of ω_{eff} (a key parameter which is directly proportional to the localization length of the state) around the center of the fundamental bandgap for a chosen lattice length of 10 mm when the values of C are set at 0 (absence of disorder), 0.05, and 0.40, respectively. This particular plot clearly shows the signature of the presence of a PBG (when $C = 0$); whereas even in the presence of a little disorder of 5%, the bandgap loses its efficiency when more number of accessible states appear inside the bandgap and the corresponding ω_{eff} variation near the edge becomes relatively flat. As we increase the C to 0.40, the disorder destroys the bandgap effect and the wavelength selectivity of ω_{eff} inside the lattice almost disappears as could be seen from Fig. 2(a). The variations in spectral derivatives of the ω_{eff} corresponding to five different strengths of disorder are depicted in Fig. 2(b). The trend in these variations clearly manifests the existence of a boundary (near $\lambda = 1020$ nm) at the band center between two distinct categories of modes inside a PBG. As we increase the magnitude of C , in the structure the two prominent peaks around the band center diminishes and the overall behavior becomes almost flat. Physically, the ordered lattice loses its standing wave-like feature inside the bandgap as we increase the C to a value > 0.25 and it forms a band consisted of localized states ($C = 0.40$). Therefore, there exist a critical level of disorder for a given lattice above which signature of the underlying photonic bandgap is completely destroyed. The disorder takes over to control the nature of the states (standing-wave like state due to interference effect inside the bandgap to a localized state) to form band.

To appreciate the after-effect of the deliberate disorder in a lattice in the presence of a PBG, we have chosen certain wavelengths from Fig. 2(b) in three different regimes in and around the bandgap. We first choose a wavelength near the center of the PBG ($\lambda = 1020$ nm), then one well outside the PBG ($\lambda = 1400$ nm), and finally one near the band edge ($\lambda =$

1060 nm) respectively. To quantify the effect of disorder in the waveguide lattice, we introduce C from 0 to 0.50 upwards in steps of 0.10 and investigate the beam dynamics inside the lattice. In the simultaneous presence of disorder and PBG, the effect of bandgap is gradually reduced as we increase the level of disorder and at the same time the sole effect of disorder in terms of localization effect comes into picture. The

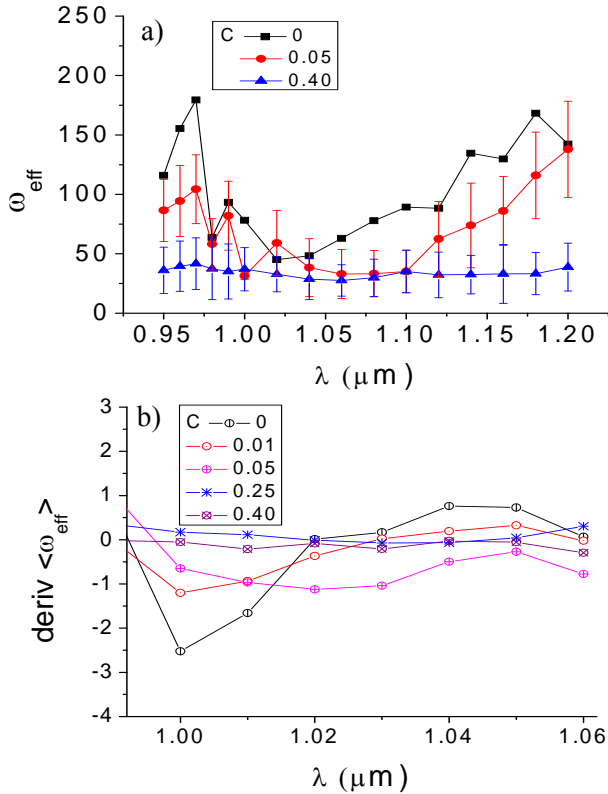


FIG. 2. Variation in the a) ensemble averaged effective width of the output beam (ω_{eff}) for three different C values, $C = 0$ (ordered lattice), 0.05, and 0.4 respectively; b) spectral derivative of (ω_{eff}) with the operating wavelength for an input Gaussian beam (FWHM 8 μm) for four different strengths of disorder ($C = 0.01, 0.05, 0.25$ and 0.4), other than the case of perfectly ordered lattice after propagation through 10 mm sample lengths. The range of wavelength considered (x -axis) is dictated by the PBG of the lattice. The error bars indicate the statistical standard deviations (ssd) of beam widths for 100 realizations.

dynamics of this subtle interplay between PBG effect and localization effect is different at different operating wavelength regimes with respect to the bandgap. Results in terms of spectral dependence of ω_{eff} which contains crucial information regarding the spectral properties of the natural states of the lattices are shown in Fig. 3. It can be seen from Fig. 3(a) that when we operate well inside the bandgap (i.e. $\lambda = 1020$ nm), the propagating beam get localized due to the bandgap effect (at $C = 0$) and evolves to a state with relatively smaller ω_{eff} . Inside the PBG, a Bloch state with its exponential envelope covers relatively less number of lattice units. However, as we introduce disorder, following the trend as shown in Fig. 2 the influence of bandgap becomes less which results in a relatively poor confinement of the output beam (upto $C = 0.10$). Thus for C upto 0.10, the beam width increases (unlike the signature of localization).⁸ However as C is increased beyond this value, disorder-assisted localization

effect takes over the bandgap-disorder interplay and the light beam get localized for a $C \geq 0.25$. Beyond that point the variation of ω_{eff} follow the universal behavior of TL i.e. ω_{eff} decreases with C . It could be concluded from these results that disorder overtakes bandgap effect at C around 0.10. Hence it reveals that the transition from a ballistic mode of propagation to a localized mode in a disordered lattice without having a PBG⁸ is smoother than the transition from a bandgap guided state to a localized state. In other words, in the presence of a prominent bandgap, localization of light would occur even in the presence of relatively low level of disorder contrary to its PBG-less counterpart. With this unique feature these lattices could be one of the best candidate/ platform to achieve localized light. Even from the point of view of spectral control of the TL effect, these lattices are more suitable. Whereas when we operate the lattice with a beam at an operating wavelengths far away from the bandgap ($\lambda = 1400$ nm and 950 nm on either side of the bandgap); the trend for variation in ω_{eff} follows the characteristic universal feature for TL of light. Accordingly, as we increase C , the beam width decreases beyond the threshold level of disorder for the chosen lattice.⁸ An interesting behavior is observed when the operating condition is chosen near the band edge

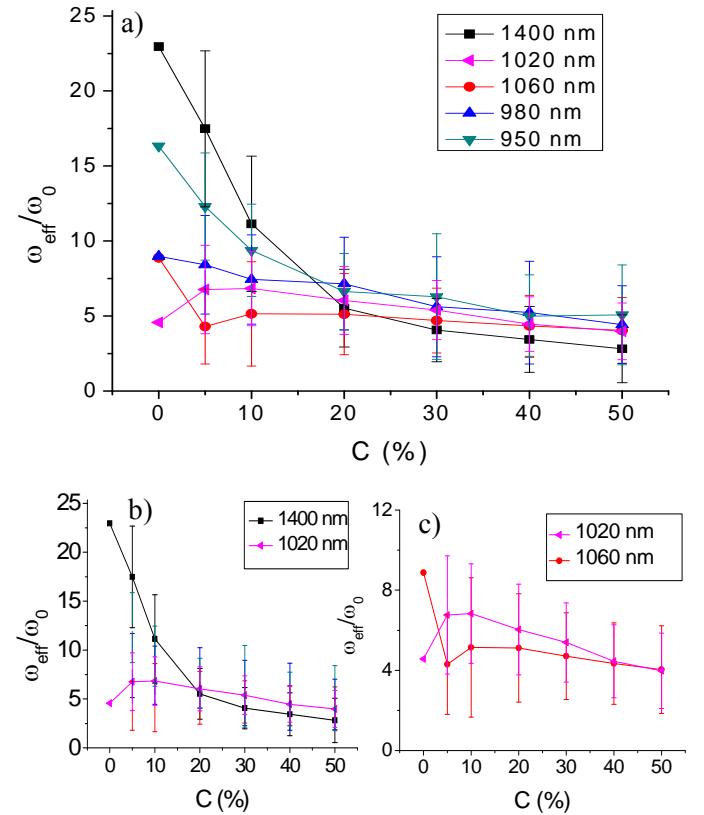


FIG. 3. (Color online) a) Variation in the ensemble averaged effective width (along with the ssd) of the output beam for an input Gaussian beam (FWHM 8 μm) for various levels of disorder after propagation through 10 mm. The figure shows the interplay between the refractive index disorder and PBG; b) outside the bandgap window (i.e. $\lambda = 1400$ nm) and around the band-centre (i.e. $\lambda = 1020$ nm); c) near the band-edge (i.e. $\lambda = 1060$ nm) and around the band-centre (i.e. $\lambda = 1020$ nm).

(i.e. $\lambda = 1060$ nm). In a perfectly ordered lattice the edge states are always localized. They carry signatures of both bandgap

guidance (similar to exponentially localized states within the bandgap) and ballistic mode of propagation (similar to extended states outside the bandgap in the absence of disorder) as they correspond to intermediate states. Hence this particular class of localized modes exhibits the characteristic decaying tail at a rate slower than exponential. When disorder is introduced, the spectral window of bandgap is slightly enhanced but with reduced efficiency (as shown in Fig. 2), subsequently the combined effect of interplay between disorder and PBG^{11,14}; and sole effect of transverse disorder together play the role to define the nature of these states. When the level of disorder is not sufficiently strong (i.e. relatively small C); the influence of underlying bandgap is reduced though still exists while TL effect is visible as the ballistic feature of propagation disappears. In this regime as we increase C (~ 0.05), the beam width gets reduced rapidly. However, when a stronger ($C \geq 0.2$) level of disorder is chosen, the bandgap effect is destroyed and the modes are more dominated by the signature of the TL. In this regime as we increase C , the beam width also decreases. In between for moderate values of C , a state of transition could be seen, which is marked by the peak (near $C = 0.10$) in the variation

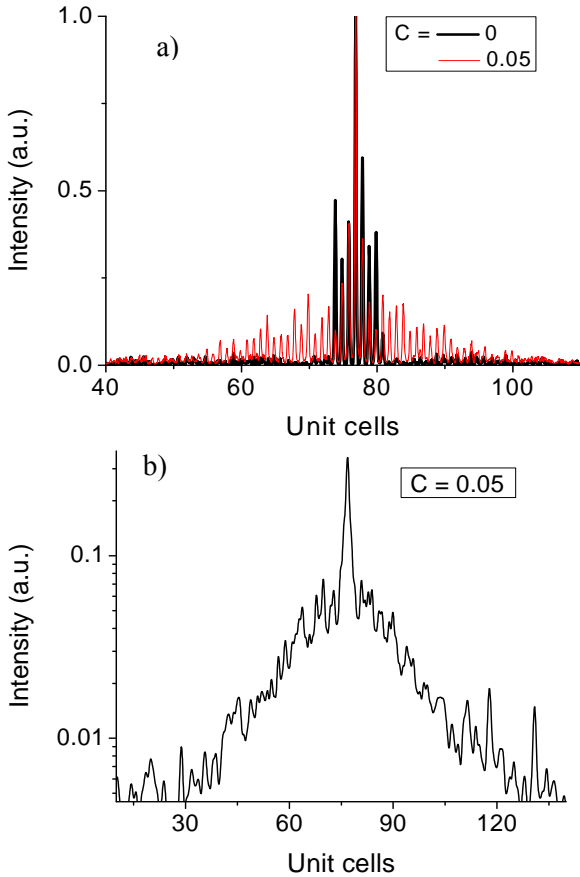


FIG. 4. (Color online) a) Ensemble averaged output intensity profiles for the case of absence of disorder (in black) and a deliberate disorder of 5% (in red) from a 10 mm long lattice, respectively, when a Gaussian beam (FWHM 8 μm) was injected at the input of wavelength 1020 nm. b) Output intensity profile on a semi-log scale shows a nearly linear variation in the tails, thereby manifesting the signature of transverse localization in a partially disorder lattice in the presence of a PBG.

of ω_{eff} with λ . To better understand the above mentioned beam dynamics, we have separately plotted the behavior well inside

the PBG and far away from the bandgap window in Fig. 3(b) whereas in Fig. 3(c) corresponding behavior well inside the bandgap and near the band edge are shown. It may be noted that a very similar behavior has been observed when the transverse disorder was introduced in spatial separation between the waveguides as well as in individual waveguide widths in the lattice (cf. Eq. (3b)).

Thus the above results of our study clearly show that there are two quantitatively different regimes of localization if PBG exists simultaneously in 1D waveguide array forming a disordered lattice. States outside the PBG and near the band edge is categorized as regime-1, which follow the normal behavior of TL, whereas the regime-2 consists of the states well inside the PBG. This particular regime shows the anomalous behavior unlike TL below a certain critical value of disorder. However, above that critical disorder level both the regimes follow normal behavior according to the well known signature of TL. It can be seen that ω_{eff} decreases with increase of disorder at high enough C in both the regimes. As the over all behavior of Fig. 2 is directly related to the bandgap feature of the parent periodic structure, the transition of ω_{eff} variation with C (as shown in Fig. 3) from one shape to another occurs within a narrow spectral band.

For a deeper appreciation of this interplay between PBG and disorder on the degree of localization, we have plotted in Fig. 4(a) the ensemble averaged intensity profiles (averaged over 100 output intensity profiles) at the output end of the 10 mm long lattice when we choose the operating wavelength near the center of the PBG and chosen C values are set at 0 and 0.05, respectively. The output profile from the periodic lattice carries the signature of localization due to PBG. In this case light is exponentially localized due to the presence of imaginary part of the Bloch wave-vector and it covers relatively lesser number of lattice units. With C set at 0.05, the transition from a bandgap influenced state to a localized state under the combined influence of disorder and PBG is evident. Hence the ω_{eff} slightly increases and profile eventually acquires two linearly decaying tails when plotted in semi-log scale in Fig. 4(b); which is indeed the hallmark of transverse localization. Also this state covers a relatively large number of lattice units compared to a PBG-assisted state.

To appreciate the interesting behavior (as discussed in Fig. 2) of competition between disorder and PBG near the band edge (i.e. at $\lambda = 1060$ nm) we have plotted in Fig. 5 the ensemble-averaged (over 100 realizations) output intensity profiles from the 10 mm long waveguide lattice when the level of disorder is set at $C = 0, 0.05$ and 0.10 respectively. In Fig. 5(a) we have shown a localized edge state in the periodic structure. These are states intermediate between the ballistic and bandgap guided (Bloch) states as they carry the characteristic feature of both the states (ballistic side lobes with slowly decaying tails). These localized edge states are very sensitive to disorder, which affects PBG effect and at the same time favors localization. In Fig. 5(b) we have plotted such a state (when C is set to be 0.05), in which the ballistic nature of the tails are now less prominent and simultaneously the exponentially decaying nature (due to combined effect of PBG with slightly extended spectral window and TL) dominate. But as we further increase C beyond 0.10, the bandgap effect dies off and dominance of transverse disorder becomes evident (as shown in Fig. 5(c) with $C = 0.10$).

Until now, we have elaborated the interplay between a PBG and the presence of a deliberate and controlled transverse

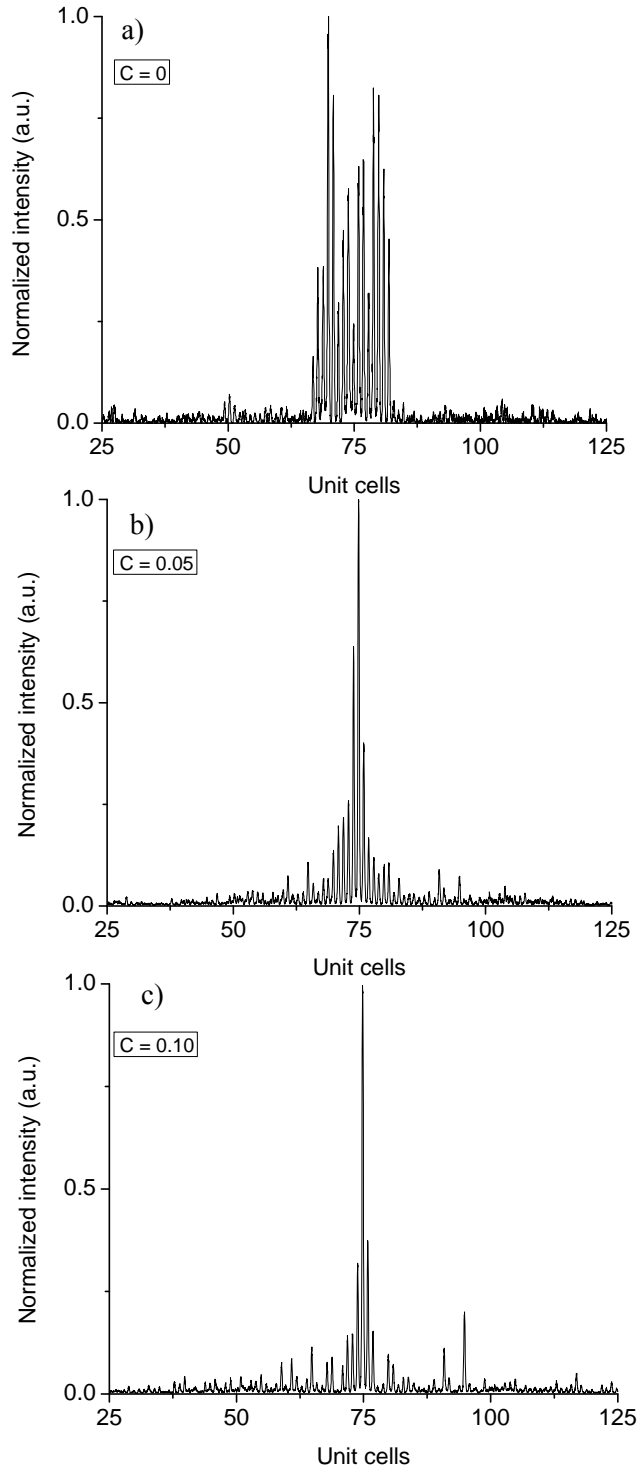


FIG. 5. Ensemble averaged output intensity profiles from a 10 mm long sample lattice with C as 0, 0.05 and 0.1, respectively when an input Gaussian beam (FWHM 8 μm) was assumed at a wavelength, $\lambda = 1060$ nm near the band edge. a) The output for $C = 0$ clearly shows the ballistic feature in the tails along with the exponentially decaying envelope. b) Output intensity profile for $C = 0.05$; it evidently indicates the signature of localization due to relatively less-prominent PBG but with enhanced bandwidth and a transition towards TL (with reduced ballistic feature) is also evident. c) Output intensity profile for an intermediate state having $C = 0.1$, where the effect of PBG is almost absent and the influence of TL dominates.

disorder; and subsequently established the fact that the phenomena of TL is more interesting in the presence of a bandgap rather than the case in the absence of bandgap. In the absence of PBG, propagation dynamics in the lattice is solely dictated by long-range disorder in lower dimensions. However, interesting additional features are revealed when we

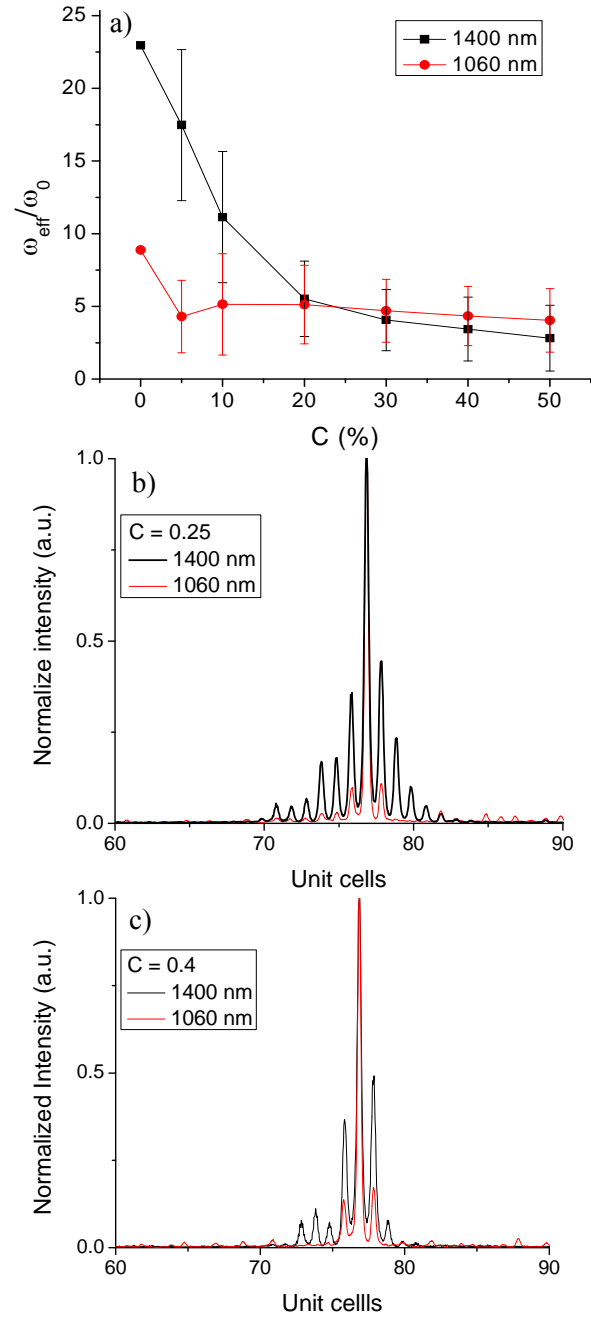


FIG. 6. (Color online) a) Variation in the ensemble averaged effective width (along with the ssd) of the output beam intensity versus levels of disorder for an input Gaussian beam (FWHM 8 μm) after propagation through 10 mm for two wavelengths: one at a wavelength ($\lambda = 1400$ nm) outside the bandgap window and the other, which is near the band-edge ($\lambda = 1060$ nm). b) Defect mode profiles by considering a defect waveguide at 77th unit cell of width 3.7 μm (different from the uniformly chosen width of 3 μm for all other waveguides) for these wavelengths for a $C = 0.25$ and c) 0.4, respectively.

compare the behavior of the states in different localized regimes (well inside the bandgap, near the band edge and far away from the bandgap). In Fig. 6(a) we have re-plotted the beam dynamics with various levels of disorder for two particular cases near band edge ($\lambda = 1060$ nm) and far away from the gap ($\lambda = 1400$ nm). From this figure, it is evident that the localization effect at the operating wavelength of 1400 nm takes over the localization behavior at 1060 nm for C beyond 0.25. With any further increase in C , ω_{eff} assumes smaller values at the operating wavelength of 1400 nm as compared at $\lambda = 1060$ nm. To visualize the contribution of the underlying bandgap of the chosen lattice to the localized states as a result of combined effect of PBG and disorder, we introduce an isolated defect waveguide in the lattice near the central region (77th high index region). As the transverse disorder is in the form of perturbed refractive index, the defect waveguide was chosen to be of a spatial width of 3.7 μm , different from 3 μm chosen for all other waveguides. This particular choice of the defect introduces a defect state inside the bandgap of the lattice. Accordingly, we assume launch of two different wavelengths ($\lambda = 1060$ and 1400 nm respectively) into the defect waveguide. In the presence of this defect inside the lattice, we plot the ensemble averaged output profiles for C as 0.25 (as shown in Fig. 6(b)) and 0.40 (as shown in Fig. 6(c)) respectively in the respective localized regimes (as shown in Fig. 6(a)). In both the cases, while operating at 1060 nm, it excites a localized defect state, which is more confined compared to its counterpart defect state. This additional confinement factor of the localized defect state is attributed to the bandgap effect (at $\lambda = 1060$ nm) in addition to the sole effect of disorder (which is present both at $\lambda = 1060$ nm and 1400 nm) to realize these localized states. A direct comparison

between the states plotted in Fig. 4(a) and those in Figs. 6(b) & (c), confirms that the localized defect states at 1060 nm is more strongly localized than the pure disorder-induced transverse localized state or a pure bandgap state. It may be noted that a very similar behavior has been observed while the operating wavelength was chosen very close to the center of the PBG (i.e. $\lambda = 1020$ nm).

To conclude, we have studied the significance of a prominent underlying photonic bandgap spawned by the waveguide lattice at the operating wavelength in the context of transverse localization of light. The results of our extensive numerical simulations reveal that in the presence of a prominent bandgap, localization of light would occur even in the presence of relatively low level of disorder compared to its counterpart, in which PBG is absent. Our study also establishes the fact that disorder induced localization is achieved both inside as well as outside of the underlying PBG. However, it is seen that existence of a backbone PBG for the waveguide lattice introduces a new tool to enhance the wavelength selectivity of the phenomenon in such 1D waveguide lattice; one could easily control the spectral window of light localization for a specific application. Hence, we envision that these results should be of interest in designing disordered lasers and other applications involving localization in imperfect lattice structures that spawn PBG.

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