

# Enhancing Transport Efficiency by Hybrid Routing Strategy

J.-Q. Dong<sup>1</sup>, Z.-G. Huang<sup>1\*</sup>, Z. Zhou<sup>1</sup>, L. Huang<sup>1</sup>, Z.-X. Wu<sup>1</sup>, Y. Do<sup>2</sup>, and Y.-H. Wang<sup>1</sup>

<sup>1</sup>*Institute of Computational Physics and Complex Systems,  
Lanzhou University, Lanzhou Gansu 730000, China and*

<sup>2</sup>*Department of Mathematics, Kyungpook National University, Daegu, 702-701, Korea*

(Dated: April 17, 2012)

Traffic is essential for many dynamic processes on real networks, such as internet and urban traffic systems. The transport efficiency of the traffic system can be improved by taking full advantage of the resources in the system. In this paper, we propose a dual-strategy routing model for network traffic system, to realize the plenary utility of the whole network. The packets are delivered according to different “efficient routing strategies” [Yan, et al, Phys. Rev. E 73, 046108 (2006)]. We introduce the accumulate rate of packets,  $\eta$  to measure the performance of traffic system in the congested phase, and propose the so-called equivalent generation rate of packet to analyze the jamming processes. From analytical and numerical results, we find that, for suitable selection of strategies, the dual-strategy system performs better than the single-strategy system in a broad region of strategy mixing ratio. The analytical solution to the jamming processes is verified by estimating the number of jammed nodes, which coincides well with the result from simulation.

## I. INTRODUCTION

Recently, the real transportation or communication systems such as the computer networks [1, 2], power grid [3, 4], airport line [5], and so on, have attracted a lot of attention from scientists due to the discovery of the topological features of their self-induced structures. The complex network theory [6–8], as well as the tools inherited from nonequilibrium statistical physics [9] have been successfully applied to study the dynamical properties of these real systems.

The common character for these transportation or communication systems is to perform certain functions by transferring objects among connected elements, which often take the form of large sparse network. Free traffic flow on these networks is key to their normal and efficient functioning. However, they may actually suffer from the overload or traffic jam, which always disable the system partially for a period of time, or even be fatal to the whole system due to the consequential onset of cascades of overload failures [10–15]. Therefore, many recent studies on the traffic networks have analyzed the critical properties of the jamming and congestion transitions [16–26]. And, the schemes to promote the performance of traffic systems are chiefly from *two aspects*, designing efficient routing strategies [27–36] or, optimizing the topology of the underlying network [19, 37–40]. The objectives of these schemes are, on one hand, to avoid the onset of congestion and, on the other hand, to have short delivery times.

The routing algorithm proposed in recent works are relied on the structural properties, as well as the global or local information about the dynamical state of the communication networks [27–36]. For example, the works of biased random walk scheme introduce the probabil-

ity to visit node depending on its degree [28, 29], or the queue length of packets [30]. The works of shortest-path scheme consider the paths with minimized distance from any pair of source and destination [31]. For this scheme, the central nodes (with highest connectivity) are highly overcongested, inducing the bottleneck of the communication capacity. The expanded version of the shortest-path scheme with “effective distance” involving the congestion state (queue length of routers) may bypass the congested nodes locally and thus improve the performance [32]. While, the work of efficient-path schemes [33] propose the routing table of paths with the minimum summary of  $k^\beta$ , with a turnable  $\beta$ . For the value of  $\beta = -1$  this scheme can effectively redistribute the heavy load on central nodes to some of the lower-degree nodes, and the system can reach a more than ten times high capacity of that with shortest-path scheme. We can see that, for certain amount of traffic request, the way to promote the performance of the system is to take full advantage of all kinds of nodes.

These aforesaid researches, have discussed the system with pure routing strategy. While, how the diversity of routing strategy performs is really of curious, and the enhancement of transport capacity by better exertion of all nodes in the system might be expected. In this paper, we put forward a mechanism that the communication system possesses of two different routing strategies. Here we make use of the simple fixed routing scheme, i.e., the efficient-path schemes proposed in Ref. [33], and consider the routing strategies to be denoted by different  $\beta$ . Then, the transport system with this multi-strategy protocol will send packets according to different fixed routing tables of efficient-path schemes. Though the fixed routing algorithm becomes impractical in huge communication systems, it is still widely used in medium-sized or small systems [41, 42], for its obvious advantages in economical and technical costs, compared with the dynamical routing algorithm and information feedback mechanism. In this case, the diversity of the fixed routing strategy is, of

---

\*For correspondence: hangzg@lzu.edu.cn

course, practical if it performs better than pure-strategy system. Actually, through our study, we see that the multi-strategy system may perform better than that of the pure strategy system.

## II. TRAFFIC MODEL

In our traffic model of dual-strategy routing protocol, the packets with given sources and destinations will be sent according to two different fixed routing tables of efficient-path schemes (EPS). For the EPS proposed in Ref.[33], node  $i$  in the graph are weighted by  $w_i = k_i^\beta$ .  $k_i$  is the degree of node  $i$ , and  $\beta$  can be considered as the label of “routing strategy”. A packet with source  $j_1$  and destination  $j_2$  will choose a minimum sum of weight,  $\sum_{i \in \sigma_{j_1 j_2}} k_i^\beta$ , route in the graph.  $\sigma_{j_1 j_2}$  is the path from  $j_1$  to  $j_2$ . Adjusted by the parameter  $\beta$ , the single-strategy system will partial to certain kind of nodes in routing, and may also leave some space to improve the performance further. In our dual-strategy model with two strategies  $\beta_1$ , and  $\beta_2$ , packets are assigned to the two corresponding routing tables, with probability  $1 - p$  and  $p$ , respectively. Here we name  $p$  as the *mixing rate*. Here, for  $p = 0$  (or 1), the system returns to the single-strategy system with  $\beta = \beta_1$  (or  $\beta_2$ ).

Similar to the former work, at each time step,  $R$  packets enter the system with randomly chosen sources and destinations. The delivery capacity of each node is  $C$ , and we set  $C = 1.0$  for simplicity. The maximal queue length of each node is assumed to be unlimited, and the first-in-fist-out discipline is applied at each queue. Once a packet come to its destination, it is removed from the system.

In the previous study, the phase transition of traffic flow is described by the the order parameter [16],

$$H(R) = \lim_{t \rightarrow \infty} \frac{C \langle \Delta W \rangle}{R \Delta t} \quad (1)$$

where  $\Delta W = W(t + \Delta t) - W(t)$ , with  $\langle \cdot \rangle$  indicating average over time windows of width  $\Delta t$ , and  $W(t)$  is the total number of packets in the network at time  $t$ . The critical value  $R_c$  (the packet generation rate) where a phase transition takes place from free flow to congested traffic, can reflect the maximum capability of a system.

The behavior of the critical point  $R_c$  on different networks can be simply explained by their different betweenness centralities (BC) distributions [31, 43, 44]. The BC of a node  $i$  for the single-strategy EPS system [33] is defined as,

$$g_i(\beta) = \sum_{j_1 \neq j_2} \frac{\sigma_{j_1 j_2}(\beta, i)}{\sigma_{j_1 j_2}(\beta)}, \quad (2)$$

where  $\sigma_{j_1 j_2}(\beta)$  is the number of routes going from  $j_1$  to  $j_2$ , according to the EPS routing table with  $\beta$ ; While,  $\sigma_{j_1 j_2}(\beta, i)$  is the number of those also passing through  $i$ .

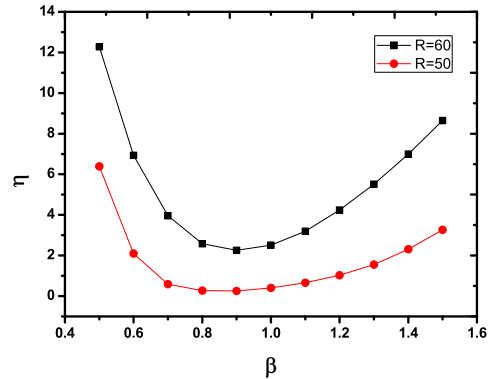


FIG. 1: The accumulate rate  $\eta$  as a function of  $\beta$  in the single-strategy system, for the systems of  $R = 50$  and  $60$ . The results are averaged over 10 realizations for 20 networks, with size  $N = 1225$ .

The critical value  $R_c$  can be estimated by the maximal BC as,

$$R_c = \frac{C \cdot N \cdot (N - 1)}{\text{Max}[g_i(\beta)]}. \quad (3)$$

where  $\text{Max}[g_i(\beta)]$  is the maximal BC of the system with strategy  $\beta$ .

For the dual-strategy system with strategies  $\beta_1$ ,  $\beta_2$ , and probability  $p$ , the efficient BC of one given node  $i$  is,

$$G_i(\beta_1, \beta_2, p) = (1 - p) \cdot g_i(\beta_1) + p \cdot g_i(\beta_2) \quad (4)$$

Then, we have the load of node  $i$ , assigned from the whole transport requirement of the system as,

$$L_i = \frac{G_i(\beta_1, \beta_2, p) \cdot R}{N \cdot (N - 1)} \quad (5)$$

The load of node increases as the  $R$  is increased. Therefore, the critical value  $R_c$  can be estimated as,

$$R_c = \frac{C \cdot N \cdot (N - 1)}{\text{Max}[G_i(\beta_1, \beta_2, p)]}, \quad (6)$$

here,  $\text{Max}[G_i(\beta_1, \beta_2, p)]$  is the maximal efficient BC of the dual-strategy system.

## III. SIMULATION RESULT AND ANALYSIS

The communication networks typically show a scale-free (SF) distribution for the number of links departing from and arriving to a system element. In this paper, we choose Barabási-Albert (BA) network as the communicating network [45]. For this network model, starting from  $m_0 = 3$  fully connected nodes, new node with  $m = 2$  is added in the existing network in turn, until the network size  $N = 1225$ . The network average degree  $\langle k \rangle = 4$ .

For the single-strategy system, the phase transition from free flow to congested traffic has been discussed [33]. When the value of  $R$  increases over  $R_c$ , the number of accumulated packets get to increase with time (i.e., a phase transition takes place from free flow to congested traffic). Similarly, for the multi-strategy system, the phase transition also takes place. The effect of different strategies, in free flow phase, is merely inducing the difference of packet deliver time. While in the congested phase, much more diversified phenomenon appears. We mainly focus on the congested phase as follows.

Firstly, let us revisit the behavior of the single-strategy system in the congested phase. According to the work of Yan [33], the largest  $R_c$  (around 43), i.e. the best performance of the system, is achieved with strategy  $\beta = 1.0$  on BA network of  $N = 1225$  and  $\langle k \rangle = 4$ . From systematic simulation of various  $\beta$  systems in congested phase, we notice that the number of accumulated packets increases linearly with  $t$ . Namely, the accumulate rate  $\eta$  is a constant (with small fluctuation). In Fig. 1, we shows  $\eta$  as a function of strategy  $\beta$ , with  $R$  in the region of congested phase ( $R = 50$  and  $60$ , larger than  $R_c$ ). It is necessary to emphasis that, although the so-called congestion occurs, there still are, on average,  $R - \eta$  packets successfully delivered to their destinations per unit time. This number is actually much larger than  $R - R_c$ . That is to say, while some nodes are jammed as  $R > R_c$ , a noticeable part of transport function still holds in the system. This actually is realized from two aspects, (1) the “free flow” still takes place on the paths which are not entangled with the jammed nodes, and, (2) the packets through the jammed nodes are not stopped but just delayed.

We may say that, the parameter  $R_c$  merely distinguishes the so-called free and congested phases, which actually indicates the free or jammed state of the most “fragile” node [see Eq. (3)].  $R_c$  can not reflect the extent of congestion, and the impact of the jammed nodes to the performance of the system. However, the accumulate rate, defined as,

$$\eta = \lim_{t \rightarrow \infty} \frac{\Delta W}{\Delta t}, \quad (7)$$

is a good parameter to measure the performance of the system in the congested phase. The smaller  $\eta$  denotes better performance of the system.  $\eta$  is the sum of individuals'  $\eta'_i$  over the whole system as,  $\eta = \sum_i \eta'_i H(\eta'_i)$ . Here,  $H(\cdot)$  is the Heaviside function, and  $\eta'_i$  is the individual accumulate rate of node  $i$ , namely, the increase rate of the queue length of packets at node  $i$  per time step.

From Eq. (5), we can get the analytically expression of  $\eta'_i$  as,

$$\eta'_i \equiv L_i - C = \frac{G_i(\beta_1\beta_2, p) \cdot R}{N \cdot (N - 1)} - C \quad (8)$$

with  $L_i$  the load of node  $i$  assigned from the whole transport requirement. We may notice that as  $R$  is increased,

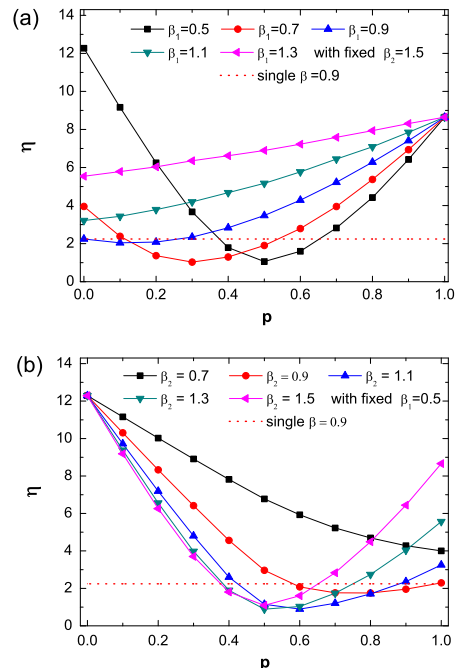


FIG. 2: (Color online.) The accumulate rate  $\eta$  for the dual-strategy system as a function of **mixing ratio**  $p$  of the two strategies  $\beta_1$  and  $\beta_2$ . Here, in (a),  $\beta_2$  is fixed to be 1.5, and in (b),  $\beta_1$  is fixed to be 0.5. The  $\eta$  of the single-strategy system with optimal  $\beta = 0.9$  (the red dot line) is also plotted for comparison. The results shown are averaged over 10 realizations for 20 networks, with size  $N = 1225$ , and  $R = 60$ .

$L_i$  may increases over the capability  $C$  and thus  $\eta'_i$  increases from negative to positive.

In Fig. 1, the non-monotonic behavior of  $\eta$  implies that the medium  $\beta$  system performs better, similar to the results in Ref. [33] from the relationship between  $R_c$  and  $\beta$ .

Then, we will analysis the behavior of the dual-strategy system with  $\beta_1$  and  $\beta_2$  in the congested phase. The packets are assigned to the two strategies with probability  $1 - p$  and  $p$ , respectively. Figure 2 plots  $\eta$  of the system as a function of  $p$ . Here, for  $p = 0$  (or 1),  $\eta$  returns to that of the single-strategy system with  $\beta = \beta_1$  (or  $\beta_2$ ). We can see that, the mix of different strategies is nontrivial and of interest. Take the system with  $\beta_1 = 0.5$  and  $\beta_2 = 1.5$  in Fig. 2(a) as an example, for certain medium value of  $p$ , it performs even better than the optimal state the single-strategy system achieves with  $\beta = 0.9$  (which is also plotted by the red dot line in Fig. 2). Furthermore, as has been shown in Fig. 2, it is also noteworthy that, when  $\beta_1$  and  $\beta_2$  are chosen from each side of 0.9, there always exists an optimal configuration  $p$ , which performs better both than the single-strategy systems of  $\beta_1$  and  $\beta_2$ .

This can be understood as follows. To design routing strategy for the network transportation, there are two factors that should be considered. (1) To bypass the hub nodes which are obviously of heavy burden and prone to jamming. (2) To choose shorter path to reducing deliver time, which is conducive to reduce the occupation (life time) of packets to the resources and thus avoid jam. The system deliver efficiency can be improved from the trade-off of these two factors. However, they are inconsistent in the communicating network with heterogeneous topology. Take the single-strategy system in congested phase as an example (see Fig. 1 the curve with  $R = 60$ ), as  $\beta$  is increased from 0, the traffic through the hub nodes are bypassed to the other smaller degree nodes, while the lengths of the paths adopted are prolonged, which increases the probability of jamming for the other nodes. The system with  $\beta_0$  around 0.9, to certain extent, is compatible of these two factors, and thus achieves the optimal performance. As  $\beta$  is increased further, the utility of the hubs is not sufficient, while the left parts of the system are overworked. Actually, To take a full advantage of each node in the system will return better performance. Therefore, for the dual-strategy system, the strategy inclined to the hubs ( $\beta < \beta_0$ ) and that inclined to the small nodes ( $\beta > \beta_0$ ) may complement each other and perform better than the single strategy one. Thus non-monotonous  $\eta$  can be observed when the  $\beta$  from both side of  $\beta_0$  are mixed.

The effect of multiple strategies in the congested phase can also be understood analytically from the so-called *equivalent generation rate*. In this routing strategy, packets at the head of the queue on node  $i$  will be delivered to the next node  $j$  according to the routing table, no matter node  $j$  is idle or jammed. Current server also has this properties. In this case, congestion in the system will not spread out. Furthermore, counterintuitive, congestion will make the system more “empty”. In each time step,  $\eta$  more packets will queue at the jammed nodes, and as a consequence, the load of the other nodes will be lighten, as if the generation rate for the *subsystem* of these nodes is reduced to a smaller one  $R^*$ , which we name as the *equivalent generation rate*. Here, we have

$$R^* = R - \eta. \quad (9)$$

Different from the case that the servers abandon packets when the queue length is over a threshold, in our model, the queuing packets are not abandoned, and will finally be send to their destination.

We sort nodes by the values of their individual accumulate rates in descending order, as  $\eta'_1 > \eta'_2 > \dots > \eta'_N$ . From Eq. (8), we know that, when  $R$  is increased from 0, all these  $\eta'_i$  increases from  $-C$ . As soon as the maximum one,  $\eta'_1$ , increases from negative to positive, the system transform from free phase to congested phase. Suppose that  $\eta'_2 < 0$ , there are  $\eta'_1$  packets detained at the 1st node per time step. Then, the equivalent generation rate for the subsystem (exclude the 1st node) is  $R^* = R - \eta'_1$ . As  $R$  is increased further, the left nodes will be jammed

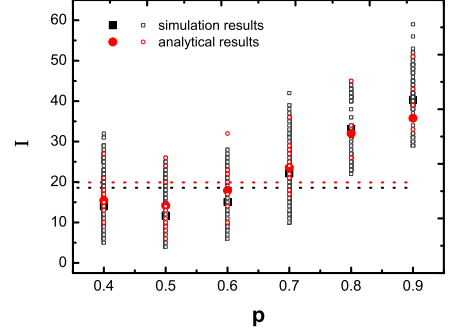


FIG. 3: The number of jammed nodes from analytical and simulation results, for the dual-strategy system with  $\beta_1 = 1.5, \beta_2 = 0.5$ . The sample data of analytical results (red open circle) are from 10 different networks, and that of simulation results (black open square) are from 50 realizations of traffic on these 10 networks. The average number of analytical and simulation results (red solid circle and black solid square) are averaged over the corresponding sample data. The system is of size  $N = 1225$ , and  $R = 60$ . The analytical and simulation results (the red and black dot lines) from single-strategy system with  $\beta = 0.9$  are also plotted for comparison.

one after another (i.e., have positive  $\eta'_i$ ). Accordingly, we may propose the *theory* to predict the number of jammed nodes, and the accumulate rate of the system  $\eta$  from two perspective.

On one hand, from Eqs. (8) and (9), we get,

$$R^* = R - \sum_{i=1}^I \left[ \frac{G_i(\beta_1, \beta_2, p) \cdot R^*}{N \cdot (N-1)} - C \right] \quad (10)$$

with the following constraint applies:

$$L_I = \frac{G_I(\beta_1, \beta_2, p) \cdot R^*}{N \cdot (N-1)} > C, \quad (11)$$

$$L_{I+1} = \frac{G_{I+1}(\beta_1, \beta_2, p) \cdot R^*}{N \cdot (N-1)} < C. \quad (12)$$

By solving this problem, we can get the number of jammed nodes  $I$ , and  $\eta$ , for given values of  $R, \beta_1, \beta_2$  and  $p$ .

On the other hand, we focus on the detailed process of successional jamming which gradually modifies the equivalent generation rate  $R^*$ , as well as the load  $L_i$  of the left nodes. The iterative procedure of  $R^*$  can be written as,

$$R_1^* = R - \frac{G_1(\beta_1, \beta_2, p) \cdot R}{N \cdot (N-1)} + C \quad (13)$$

$$R_2^* = R_1^* - \frac{G_2(\beta_1, \beta_2, p) \cdot R_1^*}{N \cdot (N-1)} + C$$

.....

The iterative formula is,

$$R_i^* = R_{i-1}^* - \frac{G_i(\beta_1, \beta_2, p) \cdot R_{i-1}^*}{N \cdot (N-1)} + C, (i = 1, 2, 3, \dots) \quad (14)$$

$R_i^*$  and  $L_i'$  decrease as the nodes of large load is jammed one after another, until

$$L_I' = \frac{G_I(\beta_1, \beta_2, p) \cdot R_{I-1}^*}{N \cdot (N-1)} > C, \quad (15)$$

$$L_{I+1}' = \frac{G_{I+1}(\beta_1, \beta_2, p) \cdot R_I^*}{N \cdot (N-1)} < C, \quad (16)$$

Different from Eqs. (10) to (12), Eqs. (14) to (16) depicts that the jamming of the first  $I$  nodes steps down  $R^*$  gradually until the value  $R_I^*$ , where the  $(I+1)^{th}$  node, as well as all its following nodes, is capable of treating with its load. Here, from the perspective of successional jamming process described by Eq. (14), one can also get the number of jammed nodes  $I$ , and  $\eta$ , analytically.

In Fig. 3, we plot the analytical and simulation results of the number of jammed nodes  $I$  in the dual-strategy system with  $\beta_1 = 1.5$  and  $\beta_2 = 0.5$ . It can be seen that, the average number of jammed nodes from analysis (red solid circle) coincides well with that from simulation (black solid square). Interestingly, the value of  $I$  also behaves non-monotonically and achieve the minimum around  $p = 0.5$ , which is similar to the accumulate rate  $\eta$  of the same system shown in Fig. 2. Additionally, the analytical results from Eq. (10) and Eq. (14) are very close to each other, thus in Fig. 3 we merely plot the results from Eq. (14).

Here, we can also understand the non-monotonic behavior of  $I$  from the following perspective. The packet generation rate  $R$  can be divided into two parts, the packets using routing table of  $\beta_1$  is  $R^{\beta_1} = (1-p)R$ , and that of  $\beta_2$  is  $R^{\beta_2} = pR$ . From Eq. (5), we can get the corresponding loads of node  $i$  from these two parts of packets, denoted by  $L_i^{\beta_1}$  and  $L_i^{\beta_2}$  (with  $L_i = L_i^{\beta_1} + L_i^{\beta_2}$ ). For the case that the mixing rate  $p = 0$ , we have  $R^{\beta_1} = R$ , and the jamming of nodes are all ascribed to the queue of  $\beta_1$  packets. As  $p$  is increased from 0, the  $R^{\beta_1}$ , as well as the  $L_i^{\beta_1}$  decreases, while that of  $\beta_2$  increases. If the  $\beta_2$  packets prefer to use those *complementary nodes* instead

of the nodes already jammed by  $\beta_2$  packets, the number of jammed nodes  $I$  will decreases with  $p$ . However, as  $p$  is large enough, the increase of load  $L_i^{\beta_2}$  from  $\beta_2$  packets induces new jamming of nodes. Therefore, we can see the non-monotonic behavior of the number of jammed node, when the dual-strategy system is composed of the two strategies from either side of  $\beta_0$ .

#### IV. CONCLUSION

In summary, we propose a hybrid routing strategy for the networked traffic system, which is proved to be a doable and effective way to enhance transport efficiency. Compared with the efficient routing strategy [33], the hybrid routing strategy can make better use of the resources in the traffic system, while there appears no increase in its algorithmic complexity. The performance of the dual-strategy system can be optimized by modulating the mixing rate of the packets, in case that the two strategies share fewer key nodes. Here, we introduce the accumulate rate  $\eta$  to denote the performance of the communication system in congestion phase, which shows richer phenomena than the critical generation rate  $R_c$ . Furthermore, we get analytical descriptions to the jamming processes by the accumulate rate  $\eta$  and the equivalent generation rate  $R^*$ . The number of jammed nodes estimated from analytical formula coincides well with that from simulation.

While our model is based on computer networks, we expect it to be relevant to other practical transport processes in general. Actually, in real system, the hybrid routing is worthy of considering, for the reason that the sources and characters of messages delivering or spreading in complex systems are diversified, which induces the hybrid of various transportation modes. In view of the common features for the networked traffic and spreading, our work may shed some light on the research of packet delivery in technical networks, as well as the rumor and opinion dynamics in social networks.

We gratefully acknowledge T. Zhou and X. Li for helpful discussions.

- 
- [1] R. Albert, H. Jeong, and A.-L. Barabási, Nature (London) **401**, 130 (1999).
  - [2] R. Pastor-Satorras, R. A. Vázquez, and A. Vespignani, Phys. Rev. Lett. **87**, 258701 (2001).
  - [3] D. J. Watts and S. H. Strogatz, Nature (London) **393**, 440 (1998); D. J. Watts, *Small Worlds* (Princeton University Press, Princeton, NJ, 1999).
  - [4] R. Albert, I. Albert, and Gary L. Nakarado, Phys. Rev. E **69**, 025103 (2004).
  - [5] W. Li, and X. Cai Physical Review E **69**, 046106 (2003).
  - [6] R. Albert and A.-L. Barabási, Rev. Mod. Phys. **74**, 47 (2002).
  - [7] M. E. J. Newman, SIAM Rev. **45**, 167 (2003).
  - [8] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D.-U. Hwang, Phys. Rep. **424**, 175 (2006).
  - [9] S. N. Dorogovtsev, A. V. Goltsev, and J. F. F. Mendes, Rev. Mod. Phys. **80**, 1275 (2008).
  - [10] S. H. Strogatz, Nature (London) **410**, 268 (2001).
  - [11] V. Jacobson, Comput. Commun. Rev. **18**, 314 (1988).
  - [12] D. J. Watts, Proc. Natl. Acad. Sci. U.S.A. **99**, 5766 (2002).
  - [13] Y. Moreno, J.B. Gomez, and A.F. Pacheco, Europhys.

- Lett. **58**, 630 (2002).
- [14] A. E. Motter and Y.-C. Lai, Phys. Rev. E **66**, 065102 (2002).
- [15] P. Holme and B. J. Kim, Phys. Rev. E **65**, 066109 (2002); P. Holme, Phys. Rev. E **66**, 036119 (2002).
- [16] A. Arenas, A. Díaz-Guilera, and R. Guimerà, Phys. Rev. Lett. **86**, 3196 (2001).
- [17] R. V. Solé and S. Valverde, Physica A **289**, 595 (2001).
- [18] S. Valverde and R. V. Solé, Physica A **312**, 636 (2002).
- [19] R. Guimerà, A. Díaz-Guilera, F. Vega-Redondo, A. Cabrales, and A. Arenas, Phys. Rev. Lett. **89**, 248701 (2002).
- [20] R. Guimerà, A. Arenas, A. Díaz-Guilera, and F. Giralt, Phys. Rev. E **66**, 026704 (2002).
- [21] B. Tadić and G. J. Rodgers, Adv. Complex Syst. **5**, 445 (2002).
- [22] Z. Toroczkai and K. E. Bassler, Nature (London) **428**, 716 (2004).
- [23] B. Kujawski, J. G. Rodgers, and B. Tadić, Lect. Notes Comput. Sci. **3993**, 1024 (2006).
- [24] B. Tadić, G. J. Rodgers, and S. Thurner, Int. J. Bifurcation Chaos Appl. Sci. Eng. **17**, 2363 (2007).
- [25] S. Sreenivasan, R. Cohen, E. Lopez, Z. Toroczkai, and H. E. Stanley, Phys. Rev. E **75**, 036105 (2007).
- [26] T. Ohira and R. Sawatari, Phys. Rev. E **58**, 193 (1998).
- [27] José J. Ramasco, Marta S. de La Loma, Eduardo López, and Stefan Boettcher, Physical Review E **82**, 036119 (2010).
- [28] W. X. Wang, B. H. Wang, C. Y. Yin, Y. B. Xie, and T. Zhou, Phys. Rev. E **73**, 026111 (2006).
- [29] J. Gómez-Gardeñes and V. Latora, Phys. Rev. E **78**, 065102(R) (2008).
- [30] B. Danila, Y. Yu, S. Earl, J. A. Marsh, Z. Toroczkai, and K. E. Bassler, Phys. Rev. E **74**, 046114 (2006).
- [31] K. I. Goh, B. Kahng, and D. Kim, Phys. Rev. Lett. **87**, 278701 (2001).
- [32] P. Echenique, J. Gómez-Gardeñes, and Y. Moreno, Phys. Rev. E **70**, 056105 (2004); EPL **71**, 325 (2005).
- [33] Gang Yan, Tao Zhou, Bo Hu, Zhong-Qian Fu and Bing-Hong Wang, Phys. Rev. E **73**, 046108 (2006).
- [34] Z. X. Wu, G. Peng, W. M. Wong, and K. H. Yeung, J. Stat. Mech.: Theory Exp. (2008) P11002.
- [35] W.-X. Wang, C.-Y. Yin, G. Yan, and B.-H. Wang, Phys. Rev. E **74**, 016101 (2006).
- [36] X. Ling, M.-B. Hu, R. Jiang, and Q.-S. Wu, Phys. Rev. E **81**, 016113 (2010).
- [37] G.-Q. Zhang, D. Wang, and G.-J. Li, Phys. Rev. E **76**, 017101 (2007).
- [38] Z. Liu, M.-B. Hu, R. Jiang, W.-X. Wang, and Q.-S. Wu, Phys. Rev. E **76**, 037101 (2007).
- [39] L. Zhao, Y.-C. Lai, K. Park, and N. Ye, Phys. Rev. E **71** 026125 (2005); L. Zhao, T.H. Cupertina, K. Park, Y.-C. Lai, and X. Jin, Chaos **17** 043103 (2007).
- [40] V. Cholvi, V. Laderas, L. Lopez, and A. Fernandez, Phys. Rev. E **71** 035103(R) 2005
- [41] A. S. Tanenbaum, Computer Networks (Prentice Hall, Englewood Cliffs, NJ, 1996).
- [42] C. Huitema, Routing in the Internet (Prentice Hall, Upper Saddle River, NJ, 2000).
- [43] M. E. J. Newman, Phys. Rev. E **64**, 016132 (2001).
- [44] M. E. J. Newman and M. Girvan, Phys. Rev. E **69**, 026113 (2004).
- [45] A.-L. Barabási and R. Albert, Science **286**, 509 (1999).