Time reversal symmetry in optics

Gerd Leuchs[1, 2, *] and Markus Sondermann[1, 2, †]

¹Institute of Optics, Information and Photonics,
University of Erlangen-Nuremberg, 91058 Erlangen, Germany
²Max Planck Institute for the Science of Light, 91058 Erlangen, Germany

The utilization of time reversal symmetry in designing and implementing (quantum) optical experiments has become more and more frequent over the past years. We review the basic idea underlying time reversal methods, illustrate it with several examples and discuss a number of implications.
We dedicate this article to Professor Vladilen S. Letokhov, one of the pioneers of laser spectroscopy, laser physics, quantum optics and related fields. His many stimulating ideas influenced the development of these fields and were often the starting point of novel sub fields flourishing ever since. Very remarkably some of his ideas are still waiting to be explored demonstrating that Professor Letokhov was well ahead of his time.

I. INTRODUCTION

Time reversal symmetry is a fundamental concept in physics. Based on every day life observations, time reversal symmetry is far from being obvious. It often seems to be broken in that many evolutions apparently occur only in one direction in time, *i.e.*, they cannot run backwards. In such cases irreversibility comes about, because the reverse direction occurs only with a forbiddingly small probability. One might call such processes thermodynamically irreversible but they are certainly not irreversible in principle. If we could control all degrees of freedom we would retrieve time reversal symmetry. Only in very special cases in particle physics there seems to be a real violation of time reversal symmetry [1]. In all other cases time reversal symmetry is preserved and one may take advantage of it.

Here, we concentrate on the field of optics and quantum optics. Optics is governed by Maxwell's equations which obey time reversal symmetry [2]. As outlined in one problem example below, already simple laboratory tasks may be optimized based on this property. But also in cutting edge problems of modern optics one can benefit from taking guidance from time reversal symmetry. One example is the efficient absorption of a single photon by a single atom. For perfect excitation of the atom the photon should have the shape of the time reversed version of a spontaneously emitted photon. This holds true in *free space* [3–5], in a waveguide [6] as well as for atoms in a resonator [7]. We will discuss the free space absorption problem exemplary in more depth below. Time reversal symmetry arguments play a key role also in the storage and retrieval of photons in atomic ensembles [8, 9].

Methods based on time reversal symmetry have been applied successfully to focusing of electromagnetic radiation onto non-quantum sources, e.g., for microwave emitters embedded in a cavity [10, 11]. Recently, sub-diffraction limited focusing has been demonstrated in the optical domain in the absence of a source by focusing through a scattering medium [12]: The wave front impinging onto a lens has been shaped such that it resembles the phase conjugate, i.e., time reversed version of the wave front that is generated by a hypothetical source emitting a wave through the scattering medium. Another recent example of the application of time reversal techniques is the determination of the optimal wave front for a new variant of 4π microscopy [13].

The paper is outlined as follows: In the next section, we highlight the relation between phase conjugation and time reversal. Then, the application of time reversal techniques in optics is discussed for several different cases. Finally, some issues related to phase conjugation of quantum states of light are reviewed.

II. TIME REVERSAL AND PHASE CONJUGATION

We highlight the relation between time reversal and phase conjugation (see, *e.g.*, Ref. [14]). For this purpose we express an electrical field $\vec{E}(\vec{r}, t)$ by its spectral components $\vec{A}(\vec{r}, \omega)$:

$$\vec{E}(\vec{r}, t) = \int_0^\infty d\omega \left[\vec{A}(\vec{r}, \omega) \mathrm{e}^{i\omega t} + \vec{A}^\star(\vec{r}, \omega) \mathrm{e}^{-i\omega t} \right] \quad . \tag{1}$$

*Electronic address: gerd.leuchs@mpl.mpg.de
†Electronic address: markus.sondermann@physik.uni-erlangen.de
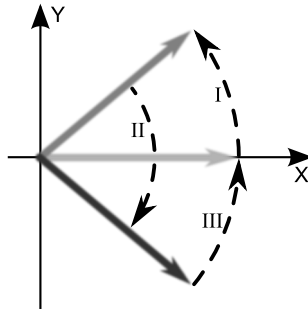
FIG. 1: Most simple example for reversing an evolution in time: Phase space representation of the temporal evolution of a single mode optical field ($E(t) = X(t) + iY(t)$) in its components. (I): evolution starting at $t = 0$ with oscillation $\exp(i\nu t)$. (II): phase conjugation operation. (III): continued evolution according to $\exp(i\nu t)$ back into the initial state. The dynamics resembles the creation of spin echoes.
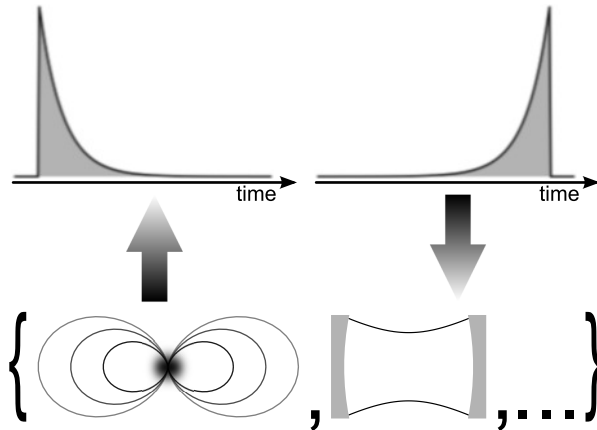

FIG. 2: Paradigmatic systems involving exponentially decaying optical field amplitudes: spontaneous emission from a two-level atom with an electric dipole transition and field amplitude inside an empty optical resonator.

Time reversal is equivalent to replacing t by $-t$. This results in

$$\vec{E}(\vec{r}, -t) = \int_0^\infty d\omega \left[\vec{A}(\vec{r}, \omega) \mathrm{e}^{-i\omega t} + \vec{A}^\star(\vec{r}, \omega) \mathrm{e}^{i\omega t} \right] \quad . \tag{2}$$

Thus, time reversal is equivalent to complex conjugating all spectral amplitudes:

$$t \to -t \quad \Leftrightarrow \quad \vec{A}(\vec{r}, \omega) \to \vec{A}^\star(\vec{r}, \omega) \quad . \tag{3}$$

A graphical representation is given in Fig. 1.

We illustrate the equivalence stated above in a few examples. Consider an electromagnetic wave with exponentially decreasing amplitude, $E(t) \sim \exp(-\beta t + i\nu t) \cdot \theta(t)$, $\theta(t)$ being the Heaviside function. The spectral amplitudes follow a Lorentzian distribution with $A(\omega) \sim [\beta + i(\omega - \nu)]^{-1}$. Complex conjugation gives $A^\star(\omega) \sim [\beta - i(\omega - \nu)]^{-1}$, which is the spectral distribution of a wave with an exponentially increasing amplitude $E(t) \sim \exp(+\beta t + i\nu t) \cdot \theta(-t)$. The latter is clearly the time reversed version of the prior. This example is relevant in the problem of exciting a single atom with a single photon in different geometries [4–7] as well as in the absorption of light pulses by an empty Fabry-Perot resonator [15] (see Fig. 2 for illustration).

As an even simpler example consider a monochromatic (outward moving) spherical wave $E(\vec{r}) \sim \exp(-ikr + i\nu t)/r$. Phase conjugating $A(\vec{r}, \omega) \sim \exp(-ikr)/r \cdot \delta(\omega - \nu)$ gives the amplitude distribution of a wave of same frequency but moving inwards: $E(\vec{r}) \sim \exp(ikr + i\nu t)/r$. This illustrates that time reversal is equivalent to inversion of motion in all degrees of freedom [16].

As a practical application, we mention the temporal inversion of picosecond pulses with distorted temporal profile by phase conjugation in a nonlinear medium [17].

III. EXAMPLES IN CLASSICAL OPTICS

A simple example for the application of time reversal methods is the coupling of light into a standard (single mode) optical fiber. One usually says that one has to mode match the beam which is to be coupled to the optical mode of the fiber in order to achieve 100% coupling efficiency. However, this perfect mode matched light field is nothing else than the time reversed version of the light field that is emitted from the fiber end and collimated by the coupling lens. Of course, upon time reversal the spatial profile remains the same and for a monochromatic beam also the temporal profile is not altered. The only change is in the direction of propagation. Time reversal is equivalent to inversion of motion in all degrees of freedom, i.e., the only difference with the wave originating from the fiber is the opposite sign of the wave vector. If the lens used for coupling exhibits aberrations the spatial profile of the time reversed version must be shaped such that the aberrations introduced by the lens are precompensated: the spatial phase distribution is the complex conjugate of the wave collimated by the lens. This case is included in the spatial dependence of the amplitudes in Eqs. 1 and 2.

Another example is the coupling of light to a Fabry-Perot resonator. What was stated for resonators with atoms inside in Ref. [7] is of course also true for an empty resonator. So consider a resonator consisting of two perfectly reflecting mirrors. Let a wave be traveling back and forth between the two mirrors. If at $t = 0$ the reflectivity of one of the mirrors is suddenly decreased light leaks out of the cavity. The field amplitude measured outside the cavity drops exponentially with the cavity decay constant. Upon time reversal, the light would propagate back into the resonator with an exponentially increasing amplitude. Therefore, the optimum light pulse that is coupled into an empty cavity is an exponentially increasing one (see Ref. [15]).

An example for an apparent lack of time reversal symmetry is an optical isolator (cf. [18, 19]). Such a device is based upon the rotation of the polarization vector of the electric field by the Faraday effect. Typically an isolator consists of an entrance polarizer, the Faraday rotator and an exit polarizer with its transmission axis rotated 45 degree away from the one of the entrance polarizer. The strength of the magnetic field of the Faraday rotator is designed such that it perfectly rotates the input polarization onto the axis of the exit polarizer. If now a beam is back reflected onto the exit polarizer with proper state of polarization (*i.e.*, the time reversed version of a wave leaving the isolator), it passes the exit polarizer and is rotated by 45 degree by the Faraday rotator. However, it is rotated such that it is blocked by the entrance polarizer, because the polarization direction rotation due to the Faraday effect is independent of the direction of light propagation. In the time reversal picture this is to be expected, since it is only the evolution in the degrees of freedom of the wave that have been reversed. Hence the current flow generating the magnetic field of the Faraday rotator (which is of course also a degree of freedom) has also to be reversed for recovering time reversal symmetry.

IV. A QUANTUM OPTICS EXAMPLE

Next, we discuss a quantum optical example: the absorption of a single photon by a single atom in free space. The process of perfect absorption can be understood as the time reversed version of spontaneous emission. Upon spontaneous emission from the excited atomic state, the electromagnetic field (assumed to be in a vacuum state before the emission of the photon) goes into the state of an outward moving single photon dipole wave packet with an exponentially decaying temporal mode profile. The time reversal argument now suggests that perfect absorption will be achieved by an inward moving dipole wave [3] with exponentially increasing temporal profile [4]. The latter requirement has been checked in a theoretical simulation [5], where the atom has been excited with a one-photon Fock state which is a superposition of single frequency mode Fock states with a Lorentzian weight. The first requirement demands illumination from full solid angle, which would require, e.g., an infinitely deep parabolic mirror as the focusing device. Therefore, any finite focusing optics may enable absorption close to but never exactly at 100%. The same holds true for the temporal domain, where in principle infinitely long pulses are required. However, a pulse length of about 4 atomic life times already boosts absorption probabilities to 99.9% [5].

In practice, any focusing device may exhibit aberrations. Since upon spontaneous emission the wave front emitted by the atom is disturbed by the aberrations of the redirecting optics, the time reversed version must exhibit wave front aberrations of opposite sign. In other words, correcting for the aberrations of the focusing optics is nothing else than making use of the equivalence between time reversal and phase conjugation.

The arguments put forward so far are related to the optical degrees of freedom. For perfect absorption also the atomic degrees of freedom have to be reversed. In principle, upon emission of a photon, momentum conservation predicts a recoil of the atom. Before measuring or absorbing the photon, the momenta of the photon and the ground state atom are entangled (see Fig. 3). Using the time reversal argument one would have to generate an incoming single photon dipole wave entangled with an incoming atomic center of mass motion wave. This clearly is a task beyond current technology. Therefore, the idea is taking guidance from the Mössbauer effect: tightly trapping the
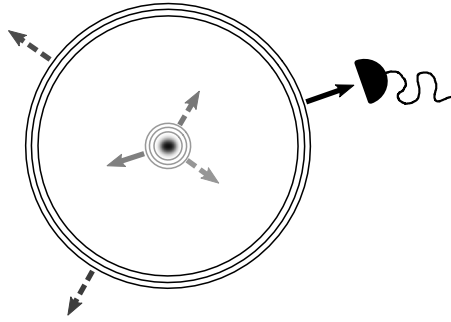
FIG. 3: The momentum of a spontaneously emitted photon and the center of mass motion of the atom left behind are entangled. Therefore, a detector measuring a photon with a certain wave vector will project the atomic center of mass momentum onto a certain direction.

atom at one position in space in a volume well within the Lamb-Dicke regime will effectively enlarge the mass of the atom. In the regime of quantized harmonic oscillation of the atom the photon momentum will be transferred to the whole macroscopic trap, the resulting motion of which is then negligible. For suitable trap parameters see, *e.g.*, Ref. [20]. A detailed theoretical discussion of the recoil problem in spontaneous emission can be found in Ref. [21].

V. PHASE CONJUGATION OF QUANTUM STATES OF LIGHT

In the example of the time reversed version of spontaneous emission one might have the idea of creating the time reversed photon by direct phase conjugation of the spontaneously emitted one. While optical phase conjugation is technically demanding in the classical domain one has to cope with its fundamental noise properties in the quantum domain.

There are several implications when phase conjugating classical light fields. One is related to the spectrum of the light pulse to be phase conjugated. As has been shown by Ou, Bali and Mandel, the pulse shape reflected by a phase conjugating mirror may become independent of the incident pulse, if the incident pulse length is much shorter than the response time of the phase conjugating mirror [22]. This should not pose a problem. The pulse length of a spontaneously emitted photon is practically on the order of a few excited-state life times, which is at least on the order of nanoseconds for most of the atomic dipole transitions. The response time of phase conjugating mirrors is considerably shorter, as evidenced by the successful reversal of picosecond pulses [17].

In contrast to that, the long pulse length does pose a technical problem. Pulse durations of several nanoseconds correspond to spatial pulse lengths on the order of one meter. For successful phase conjugation, the pulse should fit completely into the conjugating material, which is possible in principle but not very practical given the above parameters.

A more serious obstacle has been put forward by Yamamoto and Haus [23] and by Gaeta and Boyd [24] who showed that the phase conjugation process induces excess quantum noise. This excess noise is related to the quantum noise limit of phase insensitive optical amplifiers as explained in the next paragraph. Both, conjugation and amplification are non-unitary operations and can only be implemented imperfectly, i.e. by introducing noise. Unitary operation can be retrieved by embedding the process in a higher dimensional Hilbert space, i.e. by using additional, auxiliary field modes [25].

One might be tempted to reduce the task of phase conjugating a light field to implementing the transformation $\hat{a} \to \hat{c} = a^\dagger$ much like an amplifier would require $\hat{a} \to \hat{c} = \sqrt{G}a^\dagger$, G being the power gain. However, neither of these operators fulfills the commutator relation $[\hat{c}, \hat{c}^\dagger] = 1$. The problem is cured by allowing for an auxiliary field mode $\hat{b}$ with $[\hat{a}, \hat{b}] = 0$. The relations we are looking for are described by the following Bogoliubov transformation:

$$\hat{a} \to \hat{c} = \gamma_1 \hat{a} + \gamma_2 \hat{a}^\dagger + \gamma_3 \hat{b} + \gamma_4 \hat{b}^\dagger \quad . \tag{4}$$

For phase conjugation γ_2 and γ_3 have to be different from zero, $\hat{a} \to \hat{c} = \gamma_2 \hat{a}^\dagger + \gamma_3 \hat{b}$. Using the field commutator for $\hat{c}$ one finds the relation $|\gamma_2|^2 = |\gamma_3|^2 - 1$. Without loss of generality we assume that γ_2 and γ_3 are real, yielding $\gamma_3 = \sqrt{\gamma_2^2 + 1}$.

Some terms in the Hamilton operator for the total field, $\hat{c}^\dagger \hat{c} + \frac{1}{2}$, do not preserve energy, one term *e.g.* leads to the creation of two photons, one each in mode $\hat{a}$ and $\hat{b}$. By adding a (quantized) pump field all terms in the Hamiltonian can be modified to be energy conserving. But this step is not needed when the only purpose is estimating the noise added

by phase conjugation. Here we recall the relation between the variances for the amplitude quadratures $X_i = \frac{1}{2}(\hat{i} + \hat{i}^\dagger)$, $i = a, b, c$ of the input, auxiliary and output field, respectively, the variance being defined as $\langle \Delta X_i^2 \rangle = \langle X_i^2 \rangle - \langle X_i \rangle^2$. If the auxiliary field is in the ground state, the calculation yields

$$\langle \Delta X_c^2 \rangle = \gamma_2^2 \langle \Delta X_a^2 \rangle + (\gamma_2^2 + 1)\langle \Delta X_b^2 \rangle \quad . \tag{5}$$

This quantifies the added noise. If the gain in the phase conjugating process is to be $\gamma_2^2 = 1$ phase conjugation adds two units of vacuum noise:

$$\langle \Delta X_c^2 \rangle = \langle \Delta X_a^2 \rangle + 2\langle \Delta X_b^2 \rangle \quad . \tag{6}$$

This result was reported earlier by Caves [26] and Cerf and Iblisdir [27]. Thus one can say that there is no universal noiseless phase conjugation much like there is no universal 'NOT' operation among the single qubit gates [28, 29].

However, given enough a priori information about a state, the phase conjugated field may nevertheless be generated artificially with 100% fidelity. In conclusion, we underline the usefulness of the time reversal symmetry concept as a powerful tool in designing optical experiments.

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