Principles for a Unified Picture of Fermions

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The principles and conceptual foundations required for a unified picture of fermions are clarified, which in turn suggest that the standard theory may be reducible in a far simpler form. The resultant three generation model describes quarks and leptons as quasi excitations of a single chiral doublet, while electromagnetic and strong interactions as secondary interactions mediated by Nambu-Goldstone bosons originated from spontaneous violations of global SU(2) and Lorentz symmetries. The model also provides an alternative scenario for baryon and lepton asymmetries of the Universe.

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I. INTRODUCTION

There are several facts left unanswered or assumed from the beginning in the standard theory:

1. The number of generations for leptons and quarks.
2. Parity violation: gauge symmetries of the standard model consist of SU(3)×SU(2)$_L$×U(1), of which only SU(2) is chiral.
3. Large mass disparities observed in a weak doublet.
4. The origins of quark mixings and neutrino mixings.
5. Many unspecified parameters in the Yukawa couplings with Higgs bosons.
6. The reason for the existence of the fundamental scalar bosons, if ever.
7. Baryon and lepton asymmetries in the Universe.

Answers for all the questions seem to require the unified picture of fermions.

Historically, a unified picture of elementary particles was once proposed by Nambu [1] based on an analogy to the superconductivity theory [2], though the model concerned only hadrons and the principles for model building was left untouched.

However, the quasi fermion picture, in which fundamental fermions are viewed as collective excitations of primary fermions is in favor of the requirement. Then the problem is to clarify the principles for model construction.

The unified picture of fermions is not irrelevant to the problems of Higgs bosons [3]. The Higgs bosons play several roles in the standard model. They generate masses of weak bosons and fermions, and possibly the quark flavor mixings and the neutrino mixings[4]. If masses of fermions are not due to the vacuum expectation values of scalar fields, we may have to seek also for an alternative mechanism of generating weak boson masses. Accordingly, we are led to begin with considering an alternative origin of the weak boson masses.

We show in Sec.II that a non-abelian gauge theory has an ability to violate its own symmetry by self interactions. The requirements that the vacuum polarization generates a positive mass squared for SU(2) as well as zero mass for SU(3) gauge bosons determine essentially the number of multiplets coupling to each gauge force. When there is no fundamental scalar boson, the required numbers of multiplets for SU(2) and SU(3) interactions can be viewed as consistent with real observations, provided that both multiplets are chiral. A further scrutiny on the fundamental multiplets leads us to suppose that the minimal system of the primary fermions will be a pair of a left-handed and a charge conjugate of a right-handed Weyl spinor. As a result, the SU(2) gauge interaction will be the most fundamental for the primary fermions.

Whereas the fundamental chiral multiplet offers a natural origin of parity violation, there arises in turn a serious problem for the mechanism of fermion mass generation, since Lorentz invariant quantum field theories require both left- and right-handed spinors for constructing a mass term. Then the vacuum expectation values of scalar bosons do not serve for making a chiral fermion massive.

As shown in Sec.III, the vacuum expectation values of vector bosons offer the desired effect, since a left-handed spinor field can turn into a right-handed one when coupled with a vector field. Consequently, the mechanism of dynamical mass generation suitable for the quasi fermion theory will be the spontaneous Lorentz violation (SLV). Sec.IV shows that this is really the case and the vacuum expectation values of SU(2) gauge potentials generate quasi fermions interpretable as leptons and quarks. This is quite a distinct type of mechanism for the fermion mass generation, compared to those hitherto considered [5–10].

If the SU(2) gauge interaction is the most fundamental, a question naturally arises where electromagnetic and strong interactions come from. As argued in Sec.V, the quasi fermion theory suggests strongly that they are induced interactions by Nambu-Goldstone bosons. We will

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see that when SLV occurs in a chiral model, the Nambu-Goldstone bosons become vector bosons. Examinations on the constitution of NG bosons as well as the mechanism of dynamical generation of right-handed fermions evidence that part of NG-bosons mediate electromagnetic interactions.

An evidence that Nambu-Goldstone bosons also mediate strong interactions relates to the origin of color degrees of freedom of quarks. As shown in Sec.III, the vacuum expectation values of SU(2) gauge potentials constitute "the mass crystal" or "the core of mass", which determines masses of quasi fermions, and a quark mass generated by the core shows spatial anisotropy. One of the consequences from the anisotropic mass is the emergence of extra degeneracies originated from the revolution of the core. The color triplicity corresponds to the core angular momentum $l = 1$, since then a state of a quark type quasi fermion will split up in three degenerate states, which naturally represent the color degrees of freedom. Though the quasi fermions are generated by SLV, the emergent theory remains Lorentz invariant. Therefore, as discussed in detail in Sec.V and Sec.VI, the mass anisotropy should be "concealed". In this respect, we can expect the emergence of SU(3) gauge bosons to conceal the anisotropy of a quark mass. Then what is really observed will be not the anisotropy of quark mass, but the quark confinement.

Sec.VII shows that our construction of a unified model of fermions also satisfies the requirements for baryon and lepton asymmetries in Cosmology. The baryon number violating interaction constitutes the first of the Sakhalov's criteria [11] which are thought to be inevitable for baryon asymmetry[12, 13]. In our quasi fermion theory, the primary SU(2) interactions conserve, by construction, the left-handed minus right-handed primary fermion number: $L - R$, which is analogous to the baryon minus lepton number: $B - L$ in the SU(5) GUT model[14]. According to the Sakhalov's criteria, the fermion number violating interaction alone is not adequate to generate fermion asymmetry, but CP violations are also necessary. The spontaneous Lorentz violation serves also in this respect. As seen in Sec.III, the vacuum expectation values of SU(2) gauge potentials provide not only masses for chiral fermions, but also CP violating chemical potentials. The Sakhalov's last criterion, "the allow of time" is also naturally provided by the phase transition from primary to quasi fermions.

Finally, as a simple demonstration that the quasi fermion theory can explain flavor mixing phenomena, we calculate an amplitude of $\nu_e \leftrightarrow \nu_\mu$ mixing in Sec.VIII.

II. RECONSIDERATION ON THE VACUUM POLARIZATION

As mentioned in Introduction, the reconsideration on the vacuum polarization of non-abelian gauge theories offers the first principle required for constructing a unified model of fermions. This suggests the dynamical mass generation of Yang-Mills gauge bosons without recourse to any ad hoc scalar boson. The mass of gauge boson depends on the matter contents and the consistency with real observations in turn offers insights into the relations between the primary fermions and the observed fermions.

We first remark that though the current conservation requires the transversality of vacuum polarization, an explicit one-loop calculation generally gives a non zero value. For example, if the Ward-Takahashi identity is applied to the Schwinger-Dyson equation for the vacuum polarization in QED, we find [15]

$$q_\nu \Pi^{\mu\nu}(q) = e^2 \int \frac{d^4p}{(2\pi)^4} i\mathrm{Tr}\gamma^\mu \left(S(p) - S(p+q)\right). \quad (1)$$

Though translations of integral variables would make this to be zero, this operation is illegitimate for divergent integrals. The evaluation of Eq.(1) conventionally assumes the regularization scheme by which the shift of integration variables becomes legitimate. However, if the integrand is expanded with respect to q^μ by assuming q^μ small, we find for a massless electron

$$q_\nu \Pi^{\mu\nu}(q) = -2e^2 q^\mu k_1 + O(q^2), \quad (2)$$

where

$$k_1 = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 + i\epsilon} = \frac{\Lambda^2}{8\pi^2} > 0, \quad (3)$$

in terms of a 3-momentum cut off Λ. Then the exact form of the vacuum polarization will be of the form

$$\Pi^{\mu\nu}(q) = (g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2})\Pi(q), \quad (4)$$

with

$$\Pi(0) = -2e^2 k_1 + O(e^4). \quad (5)$$

This result can be generalized as

$$\Pi(0) = e^2 k_1 (B_1 - F_1) + O(e^4), \quad (6)$$

when the photon couples to F_1 chiral fermions and B_1 massless scalar bosons. Since $\Pi(0)$ corresponds to the mass squared of the photon, the relation (6) implies the dynamical generation of the photon mass. The massless pole appearing in (4) exhibits the nature of the spontaneous breaking of gauge symmetry, in accordance with the Goldstone theorem[16]. The massless Nambu-Goldstone boson will not be observed since it emerges only in a pure longitudinal part.

The fact clearly revealing the possibility of dynamical generation of gauge boson mass seems to have long been unnoticed. In a finite order perturbation theory, the gauge invariance $q_\nu \Pi^{\mu\nu}(q) = 0$ has been interpreted as indicating $\Pi^{\mu\nu}(0) = 0$. However, as evident from the appearance of a massless pole in (4), $\Pi^{\mu\nu}(0) \neq 0$ in the one-loop calculation exhibits that this is essentially a non perturbative effect. Moreover, the evaluation of divergent integrals with some regulators prepared so as to satisfy the gauge invariance conceals the possibility of dynamical mass generation by the vacuum polarization. The dimensional regularization evaluates k_1 as zero, which is also apt to bury the simple fact (6) obtained without any regulator. According to (6), photons remain massless only when $B_1 = F_1$. If $\Pi(0) > 0$, photons will acquire mass dynamically, and if $\Pi(0) < 0$, QED will become inconsistent or, at least, unstable.

Similar arguments hold for non abelian gauge theories. In the case of SU(N) Yang-Mills theory, the one-loop approximation gives

$$\begin{aligned}\Pi_N(0) &= \frac{g^2}{2}k_1[-\frac{9}{2}N + \frac{1}{2}N + 6N - F_N + B_N] + O(g^4)\\ &= \frac{g^2}{2}k_1[2N - F_N + B_N] + O(g^4),\end{aligned} \tag{7}$$

for $N \geq 2$, where $\Pi_N{}^{\mu\nu}_{ab}(q) = \delta_{ab}(g^{\mu\nu} - q^\mu q^\nu/q^2)\Pi_N(q)$, and F_N and B_N are the numbers of N-plets of chiral fermions and massless scalar bosons, respectively. The first term $-9N/2$ in the upper parentheses is a contribution from the gauge boson loop made up with two 3-point vertices, $N/2$ from the ghost loop and $6N$ from the loop made by the 4-point vertex. These values are obtained in the Feynman gauge. The Landau gauge gives for each $-3N$, $N/2$ and $9N/2$, respectively, though the total is the same as in the Feynman gauge. The consequence that the gauge bosons can become dynamically massive by self interactions is intrinsic for Yang-Mills theories, which abelian gauge theories do not have.

The case $N = 2$ corresponds to weak bosons. Massive weak bosons require $F_2 \leq 3$ for $B_2 = 0$. As presently shown in Sec.IV, we can actually conclude $F_2 = 3$.

On the other hand, gluons correspond to $N = 3$. Massless gluons require $F_3 = 6$ for $B_3 = 0$, which seems to imply that QCD is consistent only for 6 chiral flavors. In particular, these results strongly suggest that the fundamental multiplet which may generate leptons and quarks will be chiral.

Evidently, three SU(2) doublets and six SU(3) triplets can not be regarded as fundamental separately, since six color triplets also constitute nine weak doublets. The fermions existent in nature have electromagnetic, weak and strong interactions, of which the electromagnetic interaction excludes neutrinos and the strong interaction excludes leptons, while all fermions have the weak interaction. Therefore it is reasonable to suppose that a chiral doublet is the most fundamental as the primary fermion multiplet. Accordingly, we are led to suppose that a single chiral doublet generates one leptonic doublet as well as three quark doublets with colors simultaneously.

We further proceed to think that three chiral doublets are still not fundamental, but different excitation modes of the primary spinor fields. The ultimate simplicity is achieved when the primary spinor fields are assumed to be a pair of a left-handed Weyl spinor φ_L and a charge conjugate of a right-handed Weyl spinor $i\sigma_2\varphi_R^*$, as suggested by the group structure of four dimensional Lorentz symmetry.

In our analysis we have not used any specific regulators. The quadratically divergent integral k_1 determines the gauge boson mass, which might be a meaningless result were it not for the fundamental scale of energy in Nature. However, the fundamental scale of energy or length is inevitable for the completion of quantum field theory. This is a missing concept in the standard description of present day physics. Then it will be reasonable to consider that if the ultimate law of physics yet to be discovered includes the third fundamental constant with mass dimension, all the ultraviolet divergences in the present theory would have finite counterparts. Then in the ffollowing we treat k_1 and therefore Λ in (3) as finite.

III. LEPTONS AND QUARKS

In the previous section, we were led to construct the primary chiral doublet by a left-handed and a charge conjugate of a right-handed Weyl spinor. Under this assumption, however, there arises a problem. Ordinarily in relativistic quantum field theory, a mass term is not constructible only by left-handed fermions. The only solution to avoid this dilemma seems to be the spontaneous violation of Lorentz symmetry, since then the vacuum can provide requisite spins for making right handed fermions. We will return to this respect in Sec.V and Sec.VI. This section shows that the vacuum expectation values of SU(2) gauge potentials actually generate masses of chiral fermions.

We consider the mass matrix of the form

$$\bar{M} = \bar{\sigma}^\mu \frac{\rho_a}{2} m_{a\mu}, \tag{8}$$

where $\sigma^\mu = (1, \boldsymbol{\sigma})$, $\bar{\sigma}^\mu = (1, -\boldsymbol{\sigma})$ and $\rho_a/2$ are the generators of SU(2) transformations. Three constant 4-vectors ${m_a}^\mu$ constitute the core of the mass matrix. We further assume that all the time-components of ${m_a}^\mu$ are equal to zero: ${m_a}^\mu = (0, \boldsymbol{m_a})$. Then the core forms a parallelepiped in three dimensional space.

The dynamics of free quasi fermions is determined by the Lagrangian

$$\mathcal{L}_\psi = \psi^\dagger(\bar{\sigma}^\mu i\partial_\mu - \bar{M})\psi, \tag{9}$$

where $\psi = {}^T(\varphi_L, i\sigma_2\varphi_R^*)$. The equation of motion dictates the four-momentum p^μ to satisfy

$$\begin{aligned}0 &= |\bar{\sigma}\cdot p - \bar{M}|\\ &= \left(p_0^2 - \boldsymbol{p}^2 - \tfrac{3}{4}l_m^2\right)^2 - 4\mu^2\boldsymbol{p}^2 + p_0 V_m - \tfrac{3}{4}S_m^2,\end{aligned} \tag{10}$$

where

$$\begin{aligned}&\mu^2 = \textstyle\sum_a(\boldsymbol{e}_p\cdot\boldsymbol{m}_a)^2/4, \qquad l_m^2 = \sum_a \boldsymbol{m}_a^2/3,\\ &S_m^2 = \textstyle\sum_{ab}(\boldsymbol{m}_a\times\boldsymbol{m}_b)^2/6,\ \ V_m = \boldsymbol{m}_1\cdot(\boldsymbol{m}_2\times\boldsymbol{m}_3),\end{aligned} \tag{11}$$

and a unit vector $\boldsymbol{e}_p$ is parallel to the momentum $\boldsymbol{p}$: $\boldsymbol{e}_p = \boldsymbol{p}/|\boldsymbol{p}|$. The geometrical meanings of l_m, S_m and $|V_m|$ are the root mean squares of edges, faces and the volume of the core, respectively. We also notice that the dispersion relation (10) is invariant under SU(2) transformations of the mass matrix:

$$|\bar{\sigma}\cdot p - \bar{M}'| = |U(\bar{\sigma}\cdot p - \bar{M})U^{-1}| = |\bar{\sigma}\cdot p - \bar{M}| = 0. \quad (12)$$

Four solutions of Eq.(10) have the asymptotic forms

$$p_0 = \begin{cases} \pm\sqrt{\boldsymbol{p}^2 + m_+^2} - \mu \\ \pm\sqrt{\boldsymbol{p}^2 + m_-^2} + \mu \end{cases} + O(1/\boldsymbol{p}^2), \quad (13)$$

where

$$m_\pm^2 = \frac{1}{4}\left(3l_m^2 - 4\mu^2 \pm \frac{V_m}{\mu}\right). \quad (14)$$

The positivity of $m_\pm^2$ is immediately verified if the rotational invariance of the expression (14) is taken into account. What is noticeable in (13) is the appearance of an extra energy μ. We may call it the fermi potential, since it suggests a correspondence to the fermi energy or the chemical potential.

Concerning the negative energy solutions, it is natural to extend the hole interpretation to be applied, not to solutions with negative energies, but to those with negative roots. Only by this convention a particle and an anti-particle have one to one correspondence. Then a quasi fermion and a quasi anti-fermion with the same mass have opposite fermi potentials. This is consistent with the interpretation of μ as the chemical potential. In addition, two fermions composing a doublet have opposite fermi potentials.

The fermi potential and the quasi fermion masses are generally not constant but depend on the direction of motion. The averaged fermi potential $\bar{\mu}$ and the averaged masses $\bar{m}_\pm$ satisfy the relations

$$|\bar{\mu}| = \frac{l_m}{2}, \;\; \bar{m}_\pm = 2\bar{\mu}\sqrt{\frac{1\pm\gamma^3}{2}}, \quad (15)$$

where we have introduced the quantity $\gamma = \sqrt[3]{V_m}/l_m$, which characterizes how close the core approaches to a cubic form.

The fermi potential and masses become constant only when three mass vectors are orthogonal and have the same length: $\boldsymbol{m}_a = m'\boldsymbol{e}'_a$ with $\boldsymbol{e}'_a\cdot\boldsymbol{e}'_b = \delta_{ab}$, what is the same thing, when the core becomes a cube. In this case the mass matrix can be brought to the form

$$\bar{M}_L = \frac{m}{2}\boldsymbol{\rho}\cdot\boldsymbol{\sigma} \quad (16)$$

by an appropriate SU(2) transformation. The mass parameters m and m' differ only by the sign: $m'/m = \boldsymbol{e}'_1\cdot(\boldsymbol{e}'_2\times\boldsymbol{e}'_3)$. Then Eq.(10) has exact solutions

$$p_0 = \pm\omega - \frac{m}{2}, \;\; \pm|\boldsymbol{p}| + \frac{m}{2}, \quad (17)$$

where $\omega = \sqrt{\boldsymbol{p}^2 + m^2}$. The mass matrix with a cubic core is particular due to the generation of a massless quasi fermion and dispersion relations in exactly covariant forms, except for additive constants. We may regard ψ with the mass matrix $\bar{M}_L$ or its SU(2) equivalent as a leptonic doublet, while that with a non-cubic type mass matrix as a quark doublet.

The reversal of the signs of three mass vectors makes the sign of V_m reversed. This operation is equivalent to reverse the sign of μ in (13). For a cubic core, this implies to change the sign of m. Quasi fermions with the same mass but the opposite fermi potential may be called reciprocal fermions, which are on another vacuum distinct from the original.

We are to interpret the observed leptons and quarks as the members of quasi fermion doublets of various types. In this respect the quantity γ serves to characterize weak doublets. Three generations of leptonic doublets have $\gamma = 1$, while the quark doublets have $\gamma =$0.8434, 0.9958 and 0.9996 for the first, second and third generations, where we have assumed $m_u : m_d = 1 : 2$ for the first generation. It is clear that γ is very close to 1 for all the quark and leptonic doublets, except for the quark doublet of the first generation. In particular, a quark doublet has smaller the value γ for higher the generation. Taking into account the asymptotic freeness, this tendency will be consistent with the view that the absence of a massless component in a quark doublet may be due to the effects of strong interactions.

IV. A MODEL OF SPONTANEOUS LORENTZ VIOLATION

In the previous section, it was shown that if the mass term of the primary fermion doublet is of the same form as that obtained from the vacuum expectation values of SU(2) potentials, both quark and leptonic type quasi fermions are generated. This section shows that SLV occurs if massive SU(2) vector bosons couple to a chiral doublet. Then the currents and gauge potentials develop vacuum expectation values to reproduce the mass term of a leptonic doublet discussed in Sec.III.

We consider the Lagrangian

$$\mathcal{L} = \psi^\dagger\bar{\sigma}^\mu i\partial_\mu\psi - \boldsymbol{j}^\mu\cdot\boldsymbol{A}_\mu - \frac{1}{4}\boldsymbol{F}^{\mu\nu}\cdot\boldsymbol{F}_{\mu\nu} + \frac{1}{2}m_A^2\boldsymbol{A}^\mu\cdot\boldsymbol{A}_\mu, \quad (18)$$

where $\boldsymbol{F}^{\mu\nu}$ is the field strength of SU(2) gauge potentials and

$$\boldsymbol{j}^\mu = g\psi^\dagger\bar{\sigma}^\mu\frac{\boldsymbol{\rho}}{2}\psi, \quad (19)$$

is the SU(2) current. For the time being, the mechanism generating the mass of vector bosons is left unspecified. It may be either by an ordinary Higgs mechanism or by the vacuum polarization described in Sec.II. The analysis presented in this section is not influenced by the details of the origin of gauge boson mass.

Ordinarily, perturbation theory divides $\mathcal{L}$ into a free part $\mathcal{L}_0$ and an interaction part $\mathcal{L}_1$, where

$$\begin{aligned}\mathcal{L}_0 =\ & \psi^\dagger \bar{\sigma}^\mu i\partial_\mu \psi - \tfrac{1}{4}(\partial^\mu \boldsymbol{A}^\nu - \partial^\nu \boldsymbol{A}^\mu)\cdot(\partial_\mu \boldsymbol{A}_\nu - \partial_\nu \boldsymbol{A}_\mu) \\ & + \tfrac{1}{2} m_A^2 \boldsymbol{A}^\mu \cdot \boldsymbol{A}_\mu, \\ \mathcal{L}_1 =\ & -\boldsymbol{j}^\mu \cdot \boldsymbol{A}_\mu + g\partial^\mu \boldsymbol{A}^\nu \cdot (\boldsymbol{A}_\mu \times \boldsymbol{A}_\nu) \\ & -\frac{g^2}{4}(\boldsymbol{A}^\mu \times \boldsymbol{A}^\nu)\cdot(\boldsymbol{A}_\mu \times \boldsymbol{A}_\nu).\end{aligned} \tag{20}$$

This division is conventional and we may divide $\mathcal{L}$ into $\mathcal{L}_0'$ and $\mathcal{L}_1'$, where

$$\mathcal{L}_0' = \mathcal{L}_0 - \psi^\dagger \bar{M}_L \psi, \ \ \mathcal{L}_1' = \mathcal{L}_1 + \psi^\dagger \bar{M}_L \psi, \tag{21}$$

analogously to the renormalization method by counter terms. We show that there is a non-trivial solution for $\bar{M}_L$ with $m \neq 0$, where m is determined by the condition that the fermion self energy should vanish.

If the dynamics of quasi fermions is governed by the free Lagrangian $\mathcal{L}_0'$, the current $\boldsymbol{j}^\mu$ develops a vacuum expectation value

$$\langle j_a^\mu \rangle = -\int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left(g\bar{\sigma}^\mu \frac{\rho_a}{2} \frac{i}{\bar{\sigma}\cdot p - \bar{M}_L}\right) \simeq \frac{mgk_1}{2}\delta_a^\mu, \tag{22}$$

where we have estimated it by expanding with respect to $\bar{M}_L$ to see that the same constant k_1 in Sec.II appears in (22).

The self energy is calculated as follows. The method of Feynman diagram represents $\langle j_a^\mu \rangle$ as a tadpole graph, which would be quadratically divergent. Loop diagrams other than tadpoles contribute to the self energy at most logarithmically divergent terms, which are comparatively negligible. Contributions from self interactions will effectively survive when the four point vertex couples to three tadpoles. Since a tadpole carries no momentum, the contribution from the three point vertex coupled to two tadpoles will vanish. Taking into account the above considerations we have the self energy Σ:

$$g\bar{\sigma}^\mu \frac{\boldsymbol{\rho}}{2}\cdot\left[\frac{\langle \boldsymbol{j}_\mu\rangle}{m_A^2} + \frac{g^2}{m_A^2}\frac{\langle \boldsymbol{j}^\nu\rangle}{m_A^2}\times\left(\frac{\langle \boldsymbol{j}_\mu\rangle}{m_A^2}\times\frac{\langle \boldsymbol{j}_\nu\rangle}{m_A^2}\right)\right] - \bar{M}_L = 0. \tag{23}$$

The inversion of (23) under the assumption $g\langle \boldsymbol{j}_\mu\rangle/m_A^3 \ll 1$ gives

$$g\langle j_a^\mu\rangle = m(m_A^2 + 2m^2)\delta_a^\mu. \tag{24}$$

Combining (24) with (22) we have

$$\frac{g^2}{2}k_1 = m_A^2 + 2m^2 \ \text{ or } \ m = m_A\sqrt{\frac{(\Lambda/\Lambda_c)^2 - 1}{2}}, \tag{25}$$

where $\Lambda_c = 4\pi m_A/g$. If m_A is identified with the averaged weak boson mass: $\sqrt{(2m_W^2 + m_Z^2)/3} \simeq 84$ GeV and g with the weak coupling constant $g \simeq 0.6315$, then $\Lambda_c \simeq 1.67$ TeV, which will be the fundamental scale of energy in our model. Generally, the value Λ/Λ_c is extremely close to 1: $\Lambda/\Lambda_c - 1 = 10^{-11} \sim 10^{-4}$ for leptonic doublets. Only the quark doublet of third generation is exceptional, for which $m_{\text{top}}/m_A \sim 2.05$ and $\Lambda_{\text{top}} \sim 3.06\Lambda_c$.

If the symmetry breaking is due to the vacuum polarization, we have $m_A^2 = \Pi_2(0) = g^2 k_1(4 - F_2)/2$ from (7), while $m_A^2 \simeq g^2 k_1/2$ for $m \simeq 0$ from (25). Then we conclude $F_2 = 3$, which implies that the consistency of massive weak bosons by the vacuum polarization with the fermion mass generation by SLV requires three generations for quarks and leptons.

The gauge potential $\boldsymbol{A}^\mu$ also develops a vacuum expectation value through the first and higher order perturbation corrections. The same approximation gives

$$\langle \boldsymbol{A}^\mu\rangle = \frac{\langle \boldsymbol{j}^\mu\rangle}{m_A^2} + \frac{g^2}{m_A^2}\cdot\frac{\langle \boldsymbol{j}_\nu\rangle}{m_A^2}\times\left(\frac{\langle \boldsymbol{j}^\mu\rangle}{m_A^2}\times\frac{\langle \boldsymbol{j}^\nu\rangle}{m_A^2}\right). \tag{26}$$

Comparing (23) with (26), we see that

$$\bar{M}_L = g\bar{\sigma}^\mu \frac{\boldsymbol{\rho}}{2}\cdot\langle \boldsymbol{A}_\mu\rangle, \tag{27}$$

which proves the conjecture that the mass matrix may be generated by the vacuum expectation values of the SU(2) gauge potentials.

Incidentally, we may invert (26) approximately in the following form

$$\langle \boldsymbol{j}^\mu\rangle = m_A^2\langle \boldsymbol{A}^\mu\rangle - g^2\langle \boldsymbol{A}^\nu\rangle \times (\langle \boldsymbol{A}^\mu\rangle \times \langle \boldsymbol{A}_\nu\rangle), \tag{28}$$

which is rather exact and nothing but the vacuum expectation value of the equations of motion:

$$\langle \nabla_\nu \boldsymbol{F}^{\nu\mu} + m_A^2 \boldsymbol{A}^\mu - \boldsymbol{j}^\mu\rangle = 0. \tag{29}$$

We can further show that the vacuum $|\Omega\rangle$ with $\langle j_a^\mu\rangle \neq 0$ and $\langle A_a^\mu\rangle \neq 0$ is energetically favorable than the normal one $|0\rangle$ with $\langle j_a^\mu\rangle = \langle A_a^\mu\rangle = 0$. The vacuum expectation value of the energy density $\langle T^{00}\rangle$ of the system (18) is reducible in the form

$$\begin{aligned}&\langle \psi^\dagger(i\boldsymbol{\sigma}\cdot\nabla + \bar{M}_L/2)\psi\rangle - \tfrac{g^2}{4}\langle(\boldsymbol{A}^\mu \times \boldsymbol{A}^\nu)\cdot(\boldsymbol{A}_\mu \times \boldsymbol{A}_\nu)\rangle \\ &= -\int \frac{d^4p}{(2\pi)^4} i\mathrm{Tr}\left[(-\boldsymbol{\sigma}\cdot\boldsymbol{p} + \bar{M}_L/2)(\bar{\sigma}\cdot p - \bar{M}_L)^{-1}\right] \\ &\qquad -\frac{3m^4}{2g^2},\end{aligned} \tag{30}$$

with the help of the equations of motion, from which we find

$$\langle : T^{00} : \rangle \simeq -m^4\left[\frac{\ln(2\Lambda/m) - 3/4}{16\pi^2} + \frac{3}{2g^2}\right] < 0, \tag{31}$$

where

$$\langle : T^{\mu\nu} : \rangle = \langle \Omega | T^{\mu\nu} | \Omega\rangle - \langle 0 | T^{\mu\nu} | 0\rangle. \tag{32}$$

Thus the vacuum $|\Omega\rangle$ is the true ground state of the system.

It may be worth mentioning that the VEV of the energy-momentum tensor $\langle : T^{\mu\nu} : \rangle$ will not be a source of classical gravitation to an observer on the vacuum $|\Omega\rangle$. Since the VEV $\langle :T^{\mu\nu}:\rangle$ expresses the value measured from the normal vacuum $|0\rangle$, gravitational effects from $\langle :T^{\mu\nu}:\rangle$ will be detectable only by an observer on the normal vacuum. Since the vacuum is the fiducial point of every quantum measurement, the VEV of any observable is intrinsically zero to an observer on the true vacuum. This does not imply, of course, that VEVs have no physical effect. It should be recalled that the formation of the true ground state $|\Omega\rangle$ already takes into account the physical effects of VEVs. Thus the observer already on the true vacuum has no reason to expect additional gravitational effects from $\langle :T^{\mu\nu}:\rangle$.

It seems that the possibility of SLV attracted attention first in string theories as a signal of the compactification of extra dimensions[17]. Accordingly, some physicists consider SLV in the contexts of classical and quantum gravity, expecting an experimental or an observational evidence for Lorentz violation as an indication of the Planck scale physics [18]. However, as examined in this paper, the presented model does not indicate any connection of SLV with the space-time compactification nor with the Planck scale physics, since SLV can occur even in the electro-weak scale.

One more comment will be in order. One may wonder why the clear evidence of SLV has not been noticed for so long time, though the model belongs to a type with which we are well acquainted. As has been mentioned in Sec.II, this possibility requires to accept the cut off Λ as the fundamental scale of energy. In numerical simulations, for example, Λ corresponds to the inverse of the lattice spacing a. However, in renormalizable theories, the reliable results are expected to be obtained by an extrapolation: $a \to 0$. Then the results depending sensitively on a variation of a would be regarded as unphysical. Therefore, it will not be surprising even if this type of solution should drop out of researcher's attention.

V. PHOTONS AND GLUONS

The model examined in the previous section consists of a chiral doublet interacting only with SU(2) gauge bosons. Compared with the standard theory, this model seems to lack electromagnetic and strong interactions. Moreover, due to anomaly, it appears difficult to incorporate U(1) and SU(3) gauge interactions with a chiral model. However, there are circumstantial evidences revealing that photons and gluons can emerge as Nambu-Goldstone bosons.

The model (18) breaks global SU(2) and Lorentz symmetries spontaneously. Then nine NG mesons are expected to appear[16]. As presently seen, NG bosons in our model become vector bosons. These may be partly degenerate and partly unobservable as expected from the Higgs mechanism.

A simple fact suggesting the dynamical generation of photons and gluons is that the dimension of the group SU(2)×SO(1,3) is equal to that of the group U(1)×SU(3).

We begin with a formal proof of the emergence of massless bosons by SSB in a form suitable even for the case of SLV.

We suppose that the symmetries of a system are generated by hermitian operators G_α ($\alpha = 1, \cdots, n$), which include transformation parameters linearly, and broken by the vacuum: $G_\alpha|\Omega_\eta\rangle \neq 0$. The vacuum $|\Omega_\eta\rangle$ is assumed to be an eigenstate of the 4-momentum operator P^μ: $P^\mu|\Omega_\eta\rangle = \eta^\mu|\Omega_\eta\rangle$, where η^μ is a constant 4-vector. Due to Lorentz symmetry, G_α and P^μ will have the following commutation relations

$$[G_\alpha, P^\mu] = -i\omega_\alpha{}^\mu{}_\nu P^\nu, \tag{33}$$

where $\omega_{\alpha\mu\nu}$ are constant anti-symmetric tensors: $\omega_{\alpha\mu\nu} + \omega_{\alpha\nu\mu} = 0$. When some $G_{\alpha'}$ is a pure gauge generator, then $\omega_{\alpha'\mu\nu} = 0$.

Taking a matrix element of the relation (33) between two distinct vacua, we find

$$(\eta^\mu - \eta'^\mu)\langle\Omega_{\eta'}|G_\alpha|\Omega_\eta\rangle = -i\omega_\alpha{}^\mu{}_\nu \eta^\nu \delta^4(\eta - \eta'), \tag{34}$$

from which

$$\langle\Omega_{\eta'}|G_\alpha|\Omega_\eta\rangle = -i\omega_\alpha{}^\mu{}_\nu \eta^\mu \frac{\partial}{\partial\eta^\nu}\delta^4(\eta - \eta'), \tag{35}$$

results. In particular, we have for $\eta' = \eta$

$$\langle\Omega_\eta|G_\alpha|\Omega_\eta\rangle = 0, \tag{36}$$

which shows that

$$|g_\alpha\rangle = \frac{G_\alpha|\Omega_\eta\rangle}{\sqrt{\langle\Omega_\eta|G_\alpha^2|\Omega_\eta\rangle}} \neq 0, \tag{37}$$

is interpretable as a particle state orthonormal to the vacuum. The calculation of the 4-momentum of a g_α-boson gives

$$\begin{aligned} p^\mu &= \langle g_\alpha|P^\mu|g_\alpha\rangle - \eta^\mu \\ &= \langle([G_\alpha, P^\mu] + P^\mu G_\alpha)\, G_\alpha\rangle/\langle G_\alpha^2\rangle - \eta^\mu \\ &= (-i\omega_\alpha{}^\mu{}_\nu \eta^\nu\langle G_\alpha\rangle + \eta^\mu\langle G_\alpha^2\rangle)/\langle G_\alpha^2\rangle - \eta^\mu \\ &= 0, \end{aligned} \tag{38}$$

which implies that g_α is a massless boson with momentum zero. When the number of independent $|g_\alpha\rangle$ is $r \leq n$, there will be r massless bosons emergent.

In the case of our model, global SU(2) and Lorentz transformations deform the mass vectors $\boldsymbol{m}_a$ and the mass matrix $\bar{M}$ as

$$\begin{aligned} &SU(2): \ \delta_\omega \boldsymbol{m}_a = \epsilon_{abc}\omega_b \boldsymbol{m}_c, \ \ \delta_\omega \bar{M} = \epsilon_{abc}\tfrac{\rho_a}{2}\omega_b \boldsymbol{\sigma}\cdot\boldsymbol{m}_c, \\ &SO(3): \ \delta_\theta \boldsymbol{m}_a = \boldsymbol{\theta}\times\boldsymbol{m}_a, \ \ \ \delta_\theta \bar{M} = \tfrac{\rho_a}{2}\boldsymbol{\sigma}\cdot(\boldsymbol{\theta}\times\boldsymbol{m}_a), \\ &SB(3): \ \delta_v m_a^\mu = g^{\mu 0}\boldsymbol{v}\cdot\boldsymbol{m}_a, \ \ \delta_v \bar{M} = \tfrac{\rho_a}{2}\boldsymbol{v}\cdot\boldsymbol{m}_a, \end{aligned} \tag{39}$$

where SB(3) represents a group of proper Lorentz boost transformations. Under a variation $\delta\bar{M}$, the vacuum suffers a change according to

$$\delta q_i|\Omega\rangle + q_i\delta|\Omega\rangle = 0, \tag{40}$$

where q_i are annihilation operators of quasi fermions. More explicitly, we find that $\delta|\Omega\rangle$ has the form

$$\begin{aligned}\delta|\Omega\rangle &= iG|\Omega\rangle, \qquad (41)\\ G &= \sum_{\boldsymbol{p}}\big[ic^*_{13\boldsymbol{p}}q_{1\boldsymbol{p}}\bar{q}_{1-\boldsymbol{p}} + ic^*_{14\boldsymbol{p}}q_{1\boldsymbol{p}}\bar{q}_{2-\boldsymbol{p}}\\ &\quad + ic^*_{23\boldsymbol{p}}q_{2\boldsymbol{p}}\bar{q}_{1-\boldsymbol{p}} + ic^*_{24\boldsymbol{p}}q_{2\boldsymbol{p}}\bar{q}_{2-\boldsymbol{p}}\big] + \text{h.c.}, \qquad (42)\end{aligned}$$

where the coefficients $c_{ij\boldsymbol{p}}$ are given by

$$c_{ij\boldsymbol{p}} = \frac{1}{\omega_{i\boldsymbol{p}} - \omega_{j\boldsymbol{p}}}\psi^\dagger_{i\boldsymbol{p}}\delta\bar{M}\psi_{j\boldsymbol{p}} \quad (i \neq j). \qquad (43)$$

Wave functions $\psi_{i\boldsymbol{p}}$ are defined in the expansion of the operator ψ:

$$\psi(x) = \sum_{i\boldsymbol{p}} f_{i\boldsymbol{p}}\psi_{i\boldsymbol{p}}\frac{1}{\sqrt{V}}e^{i\boldsymbol{p}\cdot\boldsymbol{x} - i\omega_{i\boldsymbol{p}}t}, \qquad (44)$$

where $f_{i\boldsymbol{p}} = (q_{1\boldsymbol{p}}, q_{2\boldsymbol{p}}, \bar{q}^\dagger_{1-\boldsymbol{p}}, \bar{q}^\dagger_{2-\boldsymbol{p}})$. From the constitution of G in (42), we see that g_α are vector bosons, since q_1 and q_2 are left-handed, while $\bar{q}_1$ and $\bar{q}_2$ are right-handed. The number of independent NG bosons does not reflect the freedom of helicity states, since the proof concerns only NG bosons with momentum zero.

In the case of a leptonic doublet $\boldsymbol{m}_a = m\boldsymbol{e}_a$, SU(2) transformations degenerate into spatial rotations;

$$\begin{aligned}&SU(2): \ \delta_\omega\bar{M} = -\tfrac{m}{2}\boldsymbol{\omega}\cdot\boldsymbol{\rho}\times\boldsymbol{\sigma},\\ &SO(3): \ \delta_\theta\bar{M} = \tfrac{m}{2}\boldsymbol{\theta}\cdot\boldsymbol{\rho}\times\boldsymbol{\sigma},\\ &SB(3): \ \delta_v\bar{M} = \tfrac{m}{2}\boldsymbol{v}\cdot\boldsymbol{\rho}.\end{aligned} \qquad (45)$$

This reduction of independent modes of NG-bosons corresponds to the decoupling of the strong interactions from leptons. If we denote $q_1 = e$ and $q_2 = \nu$, then explicit calculations using free wave functions given in Appendix show that $c_{24\boldsymbol{p}} = 0$ for both global SU(2) and Lorentz transformations, which demonstrates that neutral components of NG bosons are decoupled from neutrinos. This is one of the indirect evidences showing that part of NG bosons mediate electromagnetic interactions.

Another evidence for the emergence of photons concerns the dynamical generation of right-handed fermions. We have seen in Sec.IV that left-handed fermions can have mass by SLV. This is interpretable as, in light of the ordinary relativistic field theory, the dynamical generation of the right-handed fermions.

A right-handed spinor is constructible from a left-handed one by multiplying a global vector field, which is easily understandable by observing that $\bar{A}\varphi_L$ transforms as a right-handed spinor, where $\bar{A} = \bar{\sigma}^\mu A_\mu$. However, in order that $\bar{A}\varphi_L$ acquires dynamical degrees of freedom independent of φ_L, A_μ should be a zero mode of some dynamical vector field. On the other hand, if all the right-handed fermions are generated by the coupling to some zero momentum vector boson, this boson should not couple to neutrinos. Considering that all the massive fermions have electric charges, whereas massless neutrinos have no electric charge, the property of the required vector boson coincides with that of a photon. Then the dynamical generation of the electromagnetic interaction is expected.

We next turn to the strong interactions. As has been observed in Sec.III, quark type quasi fermions reveal mass anisotropy. If this is really the character of quarks, an expected observable effect will be the emergence of the rotational spectrum associated with the spatial quantization of the core. If the core has angular momentum l, there will appear $2l + 1$ degeneracies. The color degrees of freedom observed for quarks seems to originate from the rotation of the core with $l = 1$.

However, relativistic quantum theories do not allow an anisotropic mass for elementary particles. So the states with an extra angular momentum sould be "concealed", as argued again in Sec.VI. The concealment of an extra angular momentum will be accomplished by confining quarks in bound states to form a closed shell; quarks with $l = 0$ will be realized only in meson states and those with $l = 1$ only in three body bound states. Thus we come to an alternative understanding of color confinement. The SU(3) interactions and therefore gluons are emergent in order to conceal the anisotropic nature of the quark mass, which necessarily cause the quark confinement.

According to the new understanding of color degrees of freedom, quarks can form also a bound state made up of five quarks with $l = 2$. A penta-quark baryon[22–25] will be composed of either five quarks, or four quarks and one anti-quark. Though the latter composition seems easier to realize, the intrinsic five quark baryons may be also produced in a higher scale of energy by a reaction

$$5(qqq) \to 3(qqqqq). \qquad (46)$$

The existence of penta-quark baryons will not imply that quarks can have five colors, since the the strong interactions are mediated not by SU(5), but by the same NG-bosons equivalent to SU(3) interactions.

VI. LORENTZ INVARIANCE OF THE EMERGENT THEORY

The Lorentz invariance of the theory which is emergent from spontaneous Lorentz violation is at a glance paradoxical, but certainly legitimate, since SLV implies simply that the vacuum is not invariant under Lorentz transformations, while the equations of motions and the conservation laws are not influenced by the property of the vacuum.

Still, it may be not inutile to see how the Lorentz violating properties of quasi fermions can reconcile with the Lorentz invariance of the emergent theory.

First of all, we show that the dispersion law (10) is the same in any Lorentz frame. Since the Lagrangian (18) is Lorentz invariant, the Lagrangian in another Lorentz frame $\mathcal{L}'$ is obtained by simply substituting ψ' and $\boldsymbol{A}'_\mu$ for ψ and $\boldsymbol{A}_\mu$ in $\mathcal{L}$. If the quasi fermions with the mass matrix $\bar{M}$ are the solutions in one Lorentz frame, then the division of $\mathcal{L}'$ into a free part $\mathcal{L}_0''$ and an interaction part $\mathcal{L}_1''$ by

$$\mathcal{L}_0'' = \mathcal{L}_0' - \psi'^\dagger \bar{M}\psi', \ \ \mathcal{L}_1'' = \mathcal{L}_1' + \psi'^\dagger \bar{M}\psi', \tag{47}$$

will generate the same quasi fermions also in another Lorentz frame, which demonstrates the statement.

The dispersion relations (13) and (17) deviate from the ordinary ones in two respects; the additional potential energies and the anisotropy of mass for quark type quasi fermions. The existence of extra energies seems to violate conservation of energy in some reactions. For example, we consider the decay of a W^- boson at rest to an electron and an anti-neutrino.

$$W^- \to e^- + \bar{\nu}_e. \tag{48}$$

According to the dispersion relations (17), the energy of the right-hand side is $(\omega_e - m_e/2) + (|\boldsymbol{p}_\nu| - m_e/2)$, which is smaller than the ordinary energy assignment $\omega_e + |\boldsymbol{p}_\nu|$ by one electron mass. A larger deviation from energy conservation would be expected when we consider the mu-on decay,

$$\mu^- \to \nu_\mu + e^- + \bar{\nu}_e. \tag{49}$$

Then, the energy conservation seems to demand that

$$m_\mu/2 = (|\boldsymbol{p}_1| + m_\mu/2) + (\omega_e - m_e/2) + (|\boldsymbol{p}_2| - m_e/2), \tag{50}$$

where $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ are momenta of ν_μ and $\bar{\nu}_e$, respectively. The energy of mu-on at rest is $m_\mu/2$ in our theory. Since the right-hand side is greater than $m_\mu/2$, the mu-on decay appears impossible, otherwise the energy conservation would be violated.

However, the above reasoning neglects non perturbative effects of NG-bosons emergent from SLV. Our theory maintains the conservations of charge and energy-momnentum by virtue of NG-bosons. Actually, we can see explicitly that massless bosons contribute to the conservations of charge and energy-momentum as follows. If we write matrix elements of the SU(2) Noether current $\boldsymbol{J}^\mu = \boldsymbol{j}^\mu + g\boldsymbol{F}^{\mu\nu} \times \boldsymbol{A}_\nu$ and the energy-momentum tensor $T^{\mu\nu}$ in the forms

$$\langle p'|J_a^\mu|p\rangle = g\psi_{p'}^\dagger X_a^\mu \psi_p e^{iqx}, \tag{51}$$

$$\langle p'|T^{\mu\nu}|p\rangle = \psi_{p'}^\dagger X^{\mu\nu} \psi_p e^{iqx}, \tag{52}$$

where

$$X_a^\mu = \bar{\sigma}^\mu \frac{\rho_a}{2} + \Lambda_a^\mu, \tag{53}$$

$$X^{\mu\nu} = \frac{1}{4}[\bar{\sigma}^\mu (p'+p)^\nu + \bar{\sigma}^\nu (p'+p)^\mu] + \Lambda^{\mu\nu}, \tag{54}$$

then conservations of J_a^μ and $T^{\mu\nu}$ require

$$q_\mu \Lambda_a^\mu = [\frac{\rho_a}{2}, \bar{M}], \tag{55}$$

$$q_\mu \Lambda^{\mu\nu} = \frac{1}{4}[\bar{\sigma}\cdot q\sigma^\nu \bar{M} - \bar{M}\sigma\cdot q\bar{\sigma}^\nu], \tag{56}$$

which imply that the longitudinal parts of non perturbative corrections Λ_a^μ and $\Lambda^{\mu\nu}$ have the following forms:

$$\Lambda_{la}^\mu = \frac{q^\mu}{q^2}[\frac{\rho_a}{2}, \bar{M}], \tag{57}$$

$$\begin{aligned}\Lambda_l^{\mu\nu} = \frac{1}{4q^2}[&\bar{M}(q^\mu\sigma^\nu + q^\nu\sigma^\mu)\bar{\sigma}\cdot q \\ &-(q^\mu\bar{\sigma}^\nu + q^\nu\bar{\sigma}^\mu)\sigma\cdot q\bar{M}].\end{aligned} \tag{58}$$

The massless poles appearing in (57) and (58) clearly show the contributions from NG-bosons to the conservation laws of charge and energy-momentum. As an intuitive expression, we may say that NG-bosons "conceal" the violations of symmetries caused by the quasi fermions. Needless to say, this does not mean that NG-bosons nullify the consequences from broken symmetries.

According to the observations above, reactions (48) and (49) should be understood instead as

$$W^- \to e^- + \bar{\nu}_e + g_\alpha, \tag{59}$$

$$\mu^- + g_\alpha \to \nu_\mu + e^- + \bar{\nu}_e. \tag{60}$$

The symbol g_α in each reaction does not necessarily imply a single quantum. NG-bosons g_α have an energy m_e in the first reaction, while $m_\mu - m_e$ in the second reaction. The detection of g_α in each reaction will be difficult, unless experiments are devised to measure fermi potentials directly. One should not expect to detect g_α-bosons as Lorentz-violating phenomena, since Lorentz invariance of the effective theory is maintained by virtue of g_α-bosons.

Finally, as a summary of the Lorentz invariance of the emergent theory, we recall the anisotropy of a quark mass. As has been mentioned, a free quark mass depends on the direction of motion through the anisotropy of the fermi potential μ. However, since the effective theory is Lorentz invariant, the anisotropy of mass should be concealed. This argument gives an alternative reason why quarks are permanently confined in three body bound states or in meson states, as discussed in Sec.V.

We next recall the generation of right-handed fermions. The mass of a chiral fermion implies the supply of spins from the vacuum, which would be impossible in ordinary relativistic quantum field theories. It was argued in Sec.V that the photon mode of NG bosons with zero momentum can provide an expected effect.

The violation of Lorentz invariance in the presented model is caused by the vacuum expectation values $\langle A_a^\mu\rangle$. The number of generators broken by $\langle A_a^\mu\rangle \neq 0$ is equal to that of NG-bosons, which take over the lost degrees of symmetries to make the vacuum Lorentz covariant, and conceal those properties of quasi fermions, which would otherwise manifestly contradict to Lorentz invariance.

VII. BARYON AND LEPTON ASYMMETRIES

As has been mentioned in Introduction, our unified model of fermions satisfies all the criteria for generating baryon and lepton asymmetries of the Universe.

The model presented in Sec.IV breaks the primary fermion number $F_p = L + R$, but conserves the quasi fermion number $F_q = L - R$ corresponding to the quark plus lepton number. Then even if the initial condition of the Universe demands the equality of the numbers of particles and antiparticles: $F_p = 0$, there can remain a non-zero quark plus lepton number: $F_q = 2L$. As a result, the baryon number violation is not necessary in our scheme for generating baryons, which is in favor of the stability of baryons. Moreover, the fermi potentials generated by SLV provide large CP-violations, and the phase transition from primary to quasi fermions also provides irreversible processes.

We consider as an examination on the subject a primordial quasi fermion doublet created immediately after the phase transition, which would transmute into a quark doublet and a leptonic doublet with branching ratios r and $1 - r$, respectively. The number density n_q of the quasi fermion with chemical potential μ and mass m_q at finite temperature β^{-1} is approximately given by

$$\begin{aligned} n_q &= 2\int \frac{d^3p}{(2\pi)^3}\left(\frac{1}{e^{\beta(\omega-\mu)}+1} - \frac{1}{e^{\beta(\omega+\mu)}+1}\right) \\ &\simeq \frac{\mu\beta}{3\beta^3}\left[1 + (\frac{\beta}{\pi})^2(\mu^2 - \frac{3}{2}m_q^2)\right], \end{aligned} \tag{61}$$

for $\mu\beta \ll 1$ and $m_q\beta \ll 1$, where we have taken into account the state of right-handed quasi fermion generated by NG-bosons as an independent degree of freedom. The quasi fermion asymmetry η_q is defined by n_q/n_γ, where $n_\gamma = 2\zeta(3)/(\pi^2\beta^3)$ is the number density of photons. Then the primordial "up" and "down" quark asymmetries $\eta_\pm$, the charged lepton asymmetry η'_l and the neutrino asymmetry η'_ν generated at the critical temperature β_c^{-1} are given by

$$\begin{aligned} \eta_+ &= -\frac{\pi^2}{6\zeta(3)}\mu\beta_c r\left[1 - (\frac{\mu\beta_c}{\pi})^2(2+3\gamma^3)\right], \\ \eta_- &= \frac{\pi^2}{6\zeta(3)}\mu\beta_c r\left[1 - (\frac{\mu\beta_c}{\pi})^2(2-3\gamma^3)\right], \\ \eta'_l &= -\frac{\pi^2}{6\zeta(3)}\mu\beta_c(1-r)\left[1 - 5(\frac{\mu\beta_c}{\pi})^2\right], \\ \eta'_\nu &= \frac{\pi^2}{6\zeta(3)}\mu\beta_c(1-r)\left[1 + (\frac{\mu\beta_c}{\pi})^2\right], \end{aligned} \tag{62}$$

where we have used the second relation in (15) and assumed that the fermi potentials are common for primordial quark and leptonic doublets: $\bar{\mu}_q = \mu_l = \mu$. The sign of the fermi potential has been determined so that the sign of resultant baryon asymmetry becomes positive.

The branching ratio r may be calculated by requiring the charge neutrality of emergent fermions:

$$\frac{2}{3}\eta_+ - \frac{1}{3}\eta_- - \eta'_l = 0, \tag{63}$$

which is approximately guaranteed by

$$r \simeq \frac{1}{2} - (\frac{\mu\beta_c}{2\pi})^2(3 - \gamma^3). \tag{64}$$

Then we have the baryon asymmetry

$$\eta_b = (\eta_+ + \eta_-)/3 = \frac{(\mu\beta_c\gamma)^3}{6\zeta(3)}. \tag{65}$$

As the Universe cools down to $kT \sim 100$MeV, all the anti-quarks and anti-leptons would disappear and remaining fermions would be only protons, neutrons, electrons and neutrinos. The electron asymmetry η_e and the neutrino asymmetry η_ν, which includes all types of neutrinos, are determined at this epoch by conservations of the electric charge and the lepton number:

$$\eta_e = \frac{\eta_b}{1+\xi}, \;\; \eta_\nu = (\frac{3}{\gamma^3} - \frac{1}{1+\xi})\eta_b, \tag{66}$$

where ξ is the n/p ratio.

The quasi fermion picture of quarks and leptons holds irrespective of the origin of weak boson mass. As a rough estimation, we take as the critical temperature T_c that of the electro-weak phase transition expected from the standard model where the masses of weak bosons are assumed to originate from the VEV of a Higgs doublet Φ described by the Lagrangian

$$\mathcal{L}_\Phi = D^\mu\Phi^\dagger D_\mu\Phi - \frac{\lambda}{4}(\Phi^\dagger\Phi - \eta^2)^2. \tag{67}$$

By the simplest evaluation in which only the first order contribution from the term $(\Phi^\dagger\Phi)^2$ is taken into account, we obtain the finite temperature correction: $\eta^2 \to \eta^2 - (kT)^2/4$. Then the weak boson mass at a temperature T is given by

$$m_A(T) = m_A\sqrt{1 - \left(\frac{T}{T_c}\right)^2}, \tag{68}$$

where

$$kT_c = \sqrt{8}m_A/g = 376 \text{ GeV}. \tag{69}$$

If we further assume $\gamma = 1$ for the primordial quark doublet, equate the quark mass with that of a τ lepton, and identify the critical temperature with that given by (69), then $\mu = 888$MeV and $\beta_c^{-1} = 376$GeV, from which we have

$$\eta_b = 1.826 \times 10^{-9}. \tag{70}$$

At temperatures below $kT \sim 1$ MeV, $e^\pm$ annihilation is expected to have increased n_γ by 11/4 times [19, 20]. Then the value of the present baryon asymmetry will be 4/11 times smaller than the value given by (70), which equals 6.64×10^{-10}. This value is comparable with that obtained from observations $\eta_b = 6.11 \times 10^{-10}$ [21].

On the other hand, if the mass of weak boson is originated from the vacuum polarization, the thermal history of the Universe will drastically change. The weak boson mass $m_A^2 = \Pi_2(0)$ is determined by the number of chiral doublets which are supposed to be generated by the primary spinor fields. If there is no fundamental scalar boson, m_A^2 will vanish suddenly when the primary spinor fields generate the fourth chiral doublet. Then the vacuum would recover Lorentz invariance, quarks and leptons would lose their masses and identities, and electromagnetic and strong interactions would dissapear. Though the estimation of the critical temperature of this phase transition is not possible at present, it will not considerably differ from the value given by (69) owing to the coincidence of the observed value of η_b with that calculated by the Higgs model.

VIII. FLAVOR MIXINGS

The quasi fermion picture offers also a rather natural basis for flavor mixings, since all the fundamental fermions are merely various collective excitation modes of common primary fermions.

We here consider leptons as an example and define the column of operators for the g-th generation of leptons with momentum $\boldsymbol{p}$ by

$$l_{g\boldsymbol{p}} = {}^T(e_{g\boldsymbol{p}}, \nu_{g\boldsymbol{p}}, \bar{e}^\dagger_{g-\boldsymbol{p}}, \bar{\nu}^\dagger_{g-\boldsymbol{p}}), \tag{71}$$

where

$$\begin{cases} e_{1\boldsymbol{p}} = e_{\boldsymbol{p}}, \\ e_{2\boldsymbol{p}} = \mu_{\boldsymbol{p}}, \\ e_{3\boldsymbol{p}} = \tau_{\boldsymbol{p}}, \end{cases} \begin{cases} \nu_{1\boldsymbol{p}} = \nu_{e\boldsymbol{p}}, \\ \nu_{2\boldsymbol{p}} = \nu_{\mu\boldsymbol{p}}, \\ \nu_{3\boldsymbol{p}} = \nu_{\tau\boldsymbol{p}}. \end{cases} \tag{72}$$

On the other hand, we denote the annihilation operators of the primary fermions and anti-fermions with momentum $\boldsymbol{p}$ by $a_{\boldsymbol{p}L}$ and $b_{\boldsymbol{p}R}$ for the upper component of ψ, and $b_{\boldsymbol{p}L}$ and $a_{\boldsymbol{p}R}$ for the lower component. Then the column of operators of the primary fermions

$$f_{\boldsymbol{p}} = {}^T(a_{L\boldsymbol{p}}, b_{L\boldsymbol{p}}, b^\dagger_{R-\boldsymbol{p}}, a^\dagger_{R-\boldsymbol{p}}), \tag{73}$$

relate to (71) by $l_{g\boldsymbol{p}} = U_g f_{\boldsymbol{p}}$, where U_g is a unitary matrix. Leptons in different generations are connected by

$$l_{g'\boldsymbol{p}} = U_{g'} U_g^{-1} l_{g\boldsymbol{p}}. \tag{74}$$

We can find the matrix elements of U_g and the vacuum of the g-th generation $|\Omega_g\rangle$ by expanding ψ in both primary and quasi fermion basis as follows

$$\begin{cases} e_{g\boldsymbol{p}} = \lambda_{g+\boldsymbol{p}} h_{\boldsymbol{p}} + \lambda_{g-\boldsymbol{p}} \bar{h}^\dagger_{-\boldsymbol{p}}, \\ \nu_{g\boldsymbol{p}} = \sin\frac{\theta}{2} a_{\boldsymbol{p}L} + \cos\frac{\theta}{2} e^{-i\phi} b_{\boldsymbol{p}L}, \end{cases}$$
$$\begin{cases} \bar{e}^\dagger_{g-\boldsymbol{p}} = \lambda_{g-\boldsymbol{p}} h_{\boldsymbol{p}} - \lambda_{g+\boldsymbol{p}} \bar{h}^\dagger_{-\boldsymbol{p}}, \\ \bar{\nu}^\dagger_{g-\boldsymbol{p}} = \cos\frac{\theta}{2} e^{i\phi} b^\dagger_{-\boldsymbol{p}R} + \sin\frac{\theta}{2} a^\dagger_{-\boldsymbol{p}R}, \end{cases} \tag{75}$$

where

$$\lambda_{g\pm\boldsymbol{p}} = \frac{1}{2}\left(\sqrt{1+\frac{m_g}{\omega}} \pm \sqrt{1-\frac{m_g}{\omega}}\right), \tag{76}$$

and $m_g = (m_e, m_\mu, m_\tau)$. The subsidiary operators $h_{\boldsymbol{p}}$ and $\bar{h}^\dagger_{-\boldsymbol{p}}$ are defined by

$$\begin{cases} h_{\boldsymbol{p}} &= \cos\frac{\theta}{2} a_{\boldsymbol{p}L} + \sin\frac{\theta}{2} e^{-i\phi} b_{\boldsymbol{p}L}, \\ \bar{h}^\dagger_{-\boldsymbol{p}} &= -\sin\frac{\theta}{2} e^{i\phi} b^\dagger_{-\boldsymbol{p}R} + \cos\frac{\theta}{2} a^\dagger_{-\boldsymbol{p}R}, \end{cases} \tag{77}$$

where (θ, ϕ) are the polar coordinates of momentum $\boldsymbol{p}$. Then $|\Omega_g\rangle$ is expressible in the form

$$|\Omega_g\rangle = \prod_{\boldsymbol{p}} \left[\lambda_{g+\boldsymbol{p}} + \lambda_{g-\boldsymbol{p}} (h_{\boldsymbol{p}} \bar{h}_{-\boldsymbol{p}})^\dagger\right] |0\rangle, \tag{78}$$

which satisfies $(e_g, \bar{e}_g, \nu_g, \bar{\nu}_g)|\Omega_g\rangle = 0$. Incidentally, we notice from the expression (78) that the vacuum is composed of Cooper pairs with spin 1, which contrasts to NJL [1] and BCS [2] theories constructed on scalar Cooper pairs.

We consider the mixing $\nu_e \leftrightarrow \nu_\mu$. Then

$$\langle \nu_\mu | \nu_e \rangle = \langle \Omega_2 | \nu_\mu \nu_e^\dagger | \Omega_1 \rangle = \langle \Omega_2 | \Omega_1 \rangle. \tag{79}$$

From the asymptotic behaviors

$$\lambda_{+g\boldsymbol{p}} \sim 1 - \frac{m_g^2}{8\boldsymbol{p}^2}, \;\; \lambda_{-g\boldsymbol{p}} \sim 1 - \frac{m_g}{2|\boldsymbol{p}|}, \tag{80}$$

we find

$$\langle \Omega_2 | \Omega_1 \rangle \simeq \prod_{\boldsymbol{p}} \left[1 - \frac{(\Delta m)^2}{8\boldsymbol{p}^2}\right] \simeq \exp\left[-\frac{V(\Delta m)^2 \Lambda}{16\pi^3}\right], \tag{81}$$

where $\Delta m = m_\mu - m_e$. If the quantization volume V is replaced by E^{-3} where E is the typical energy scale of the phenomenon and the three-momentum cut off Λ by Λ_c, then the energy scale where $\langle \nu_\mu | \nu_e \rangle \sim 1$ is estimated as $E \sim 0.473$GeV.

IX. SUMMARY AND CONCLUSIONS

The principles and conceptual foundations are laid down, which enable to construct a unified model of fermions.

The suggestions for the principles come partly from the fact that non-abelian gauge bosons can develop mass dynamically due to the vacuum polarization by self interactions without any external cause. Then the interrelation of the number of quark flavors with the mass of gluons, and the number of generations with the mass of weak bosons become manifest.

The consistency with real observations requires that the primary spinor field which can generate quarks and leptons dynamically is not a Dirac spinor, but a doublet composed of a left-handed and a charge-conjugate of a right-handed Weyl spinor.

The SU(2) interaction introduced to this doublet breaks the primary fermion number, which satisfies one of the requisite properties for baryon asymmetry.

The chiral structure of the primary spinor fields requires the spontaneous violation of Lorentz symmetry for generating massive fermions. Massive SU(2) gauge bosons by the vacuum polarization are consistent with massive fermions by SLV only for three generations of quarks and leptons.

The spontaneous Lorentz violation also adds to the energy of a quasi fermion a potential term μ inducing a large CP-violating effect, which satisfies another requirement for baryon asymmetry.

The Nambu-Goldstone mesons generated associated with the spontaneous violations of global SU(2) and Lorentz symmetries can mediate electromagnetic and strong interactions.

The color degrees of freedom inherent in quarks find a natural origin as the degeneracy of rotational states arising from the anisotropy of quark mass.

We now return to the questions enumerated in Introduction. The new insights obtained by the unified picture of fermions are

1. Six quark flavors and three leptonic generations suggest that they are quasi excitations of the primary chiral doublet and the origin of weak boson masses are due not to scalar bosons but to the vacuum polarization.

2. Among $SU(3)\times SU(2)_L\times U(1)$ interactions in the standard model, only SU(2) is genuine and the others are secondary interactions induced by spontaneous Lorentz and global SU(2) violations. Parity violation reflects that the primary fermions are chiral.

3. Large mass disparities in a weak doublet reflects that their masses originate not by the VEV of scalar bosons, but those of SU(2) vector gauge potentials.

4. Flavor mixings and neutrino mixings reflect that all the quarks and leptons are generated by the same primary spinor fields.

5. Since the roles of Higgs bosons are replaced by the vacuum polarization of non-abelian gauge theory and the VEVs of gauge potentials, there is no need for extra Yukawa terms for the quasi fermion theory.

6. Accordingly, there is no need for fundamental scalar bosons.

7. The primary interactions are fermion number violating and spontaneous Lorentz violation provides CP-violating fermi potentials. The resultant baryon asymmetry well reproduces the value obtained from observations.

As a result, it seems that we can construct a realistic unified model of fermions rather in the absence of the fundamental scalars. Though the simple model presented in Sec.IV is not yet immediately replaceable with the standard theory, the model suggests a possibility of the standard model intension, namely that the standard theory will be reducible in a far simpler form.

Finally, we should mention on the remaining problem. The consequences from the presented model are based on the existence of the fundamental scale of energy, which is absent in present day physics. Therefore the quasi fermion picture of leptons and quarks may offer in turn a clue to the last foundation of theoretical physics.

Appendix: The wave functions of the leptonic doublet

The wave functions $\psi_{i\boldsymbol{p}}$ appeared in Sec.V are explicitly given by

$$\begin{array}{ll} p_0=\omega-m/2 & :\ \psi_{1\boldsymbol{p}}=\lambda_+\chi_R\varphi_L+\lambda_-\chi_L\varphi_R, \\ p_0=|\boldsymbol{p}|+m/2 & :\ \psi_{2\boldsymbol{p}}=\chi_L\varphi_L, \\ p_0=-\omega-m/2 & :\ \psi_{3\boldsymbol{p}}=\lambda_+\chi_L\varphi_R-\lambda_-\chi_R\varphi_L, \\ p_0=-|\boldsymbol{p}|+m/2 & :\ \psi_{4\boldsymbol{p}}=\chi_R\varphi_R, \end{array} \tag{A.1}$$

where $\lambda_\pm=\lambda_{g\pm\boldsymbol{p}}$ defined in (76), and

$$\begin{array}{l} \varphi_R=\begin{pmatrix}\cos(\theta/2)\\ \sin(\theta/2)e^{i\phi}\end{pmatrix},\quad \varphi_L=\begin{pmatrix}-\sin(\theta/2)e^{-i\phi}\\ \cos(\theta/2)\end{pmatrix}, \\ \chi_R=\begin{bmatrix}\mathbf{1}\cos(\theta/2)\\ \mathbf{1}\sin(\theta/2)e^{i\phi}\end{bmatrix},\quad \chi_L=\begin{bmatrix}-\mathbf{1}\sin(\theta/2)e^{-i\phi}\\ \mathbf{1}\cos(\theta/2)\end{bmatrix}. \end{array} \tag{A.2}$$

φ_R and φ_L are 2-spinors, while χ_R and χ_L are SU(2) doublets. We immediately see from (A.1) that $c_{24\boldsymbol{p}}=0$ in (42).

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