A CHARACTERIZATION OF ORDINARY MODULAR EIGENFORMS WITH CM

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ABSTRACT. We show that a p-ordinary modular eigenform f of weight $k \geq 2$, with p-adic Galois representation ρ_f and mod p^m reductions $\rho_{f,m}$, and with complex multiplication(CM) is characterized by the existence of p-ordinary CM companion forms h_m modulo p^m for all integers $m \geq 1$ (in the sense that $\rho_{f,m} \sim \rho_{h_m,m} \otimes \chi^{k-1}$). As an application we give an alternative proof of the well-known result that if f has CM then the restriction of ρ_f to a decomposition group at p splits.

1. INTRODUCTION

For a rational prime $p \geq 3$ let f be a primitive, modular eigenform with q-expansion $\sum_n a_n(f)q^n$ and associated p-adic Galois representation ρ_f. In this paper, we prove some interesting arithmetic properties of such a form which, in addition, has complex multiplication (CM). One of the reasons that CM forms have historically been an important subclass of modular forms is that the simplicity with which they can be expressed – they arise from algebraic Hecke characters of imaginary quadratic fields – makes them ideal initial candidates to check deep conjectures in the theory of modular forms on. For instance, Hecke showed that the L-function of a CM form is precisely the L-function of the corresponding Grössencharacter. This can be viewed as a precursor to the work of Eichler, Shimura, Deligne and Serre, almost half a century later, on attaching Galois representations to eigenforms. Similarly, Shimura established the modularity of CM elliptic curves over $\mathbb{Q}$ two decades before the modularity theorem over $\mathbb{Q}$ was proven in full generality.

The specific arithmetic property we establish involves higher congruence companion forms which were introduced in [AM]. The precise definition and properties of these forms are given in Section 2, but for now we will only remark that companion forms mod p^n are defined as natural analogues of the classical (mod p) companion forms of Serre and Gross [Gro90].

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The main theorem of this work (Theorem 4.1, stated and proved in Section 4) establishes that given a p-ordinary CM form f, one can always find a CM companion form mod p^m for any integer $m \geq 1$. The proof explicitly finds the desired companion forms in a Hida family of CM forms in which f specializes and thus circumvents the deformation theory and modularity lifting approach of the companion form theorem in [AM]. As an application of the main theorem, we show in Section 5 that the converse is true as well: any p-ordinary form f which has CM companions mod p^m for each $m \geq 1$ must necessarily have CM. We therefore have a complete arithmetical characteriztion of p-ordinary CM forms and this allows us insight into another deep and conjectural characterization of such forms – Greenberg's local nonsemisimplicity conjecture. This conjecture states that if the the restriction splits (*i.e.* is diagonal *w.r.t* some basis) then f has CM. The converse of this conjecture is well-known to be true and we apply the main theorem to give an alternative proof.

2. Higher companion forms

Let $p \geq 3$ be a rational prime and let f be a primitive cusp form of weight $k \geq 2$, level N prime to p and q-expansion $\sum_n a_n(f)q^n$. Here, f is primitive in the sense that it is a normalised newform that is a common eigenform of all the Hecke operators. For a place $v|p$ in K_f, the number field generated by the $a_n(f)$'s, let $\rho_f : G_{\mathbb{Q}} \longrightarrow GL_2(K_{f,v})$ be a continuous, odd, irreducible Galois representation that can be attached to f. We may, after conjugation, assume that ρ_f takes values in the ring of integers of some finite extension of $\mathbb{Q}_p$. This allows us to consider reductions of ρ_f modulo p^m (for integers $m \geq 1$) which, with the exception of the mod p reduction $\overline{\rho}$, we will denote by $\rho_{f,m}$. In fact, if $\overline{\rho}$ is absolutely irreducible then there is no ambiguity in defining these reductions. We can, and will, define congruences and reduction mod p^m even when the elements don't lie in Z_p by using the notion of congruence due to Wiese and Ventosa [TiVW10]. With f as above and $2 \leq k \leq p^{m-1}(p-1)+1$ we define a companion form of f.

Definition 2.1. A **companion form** g of f, modulo p^m, is a normalised eigenform of level prime to p and weight k' where $k' \geq 2$ is the smallest integer such that $\rho_{f,m} \simeq \rho_{g,m} \otimes \chi^{k-1} \mod p^m$. An equivalent formulation of the above criterion in terms of the Fourier expansions is: $a_n(f) \equiv n^{k-1} a_n(g) \mod p^m$ for $(n,p) = 1$.

We make the following remarks on companion forms.

- The equivalence between the Galois side and the coefficient side in the above definition perhaps needs further justification. One direction is immediate if we take the traces of Frobenii of the Galois representations. The other direction follows from the absolute irreducibility of $\overline{\rho}$, Chebotarev and [Car94,

Théorème 1] which is essentially a generalization of the Brauer-Nesbitt theorem to arbitrary local rings.

- The weight k' can be easily deduced from the definition and the determinant condition on ρ_f to be the smallest integer such that $k' \geq 2$ and $k' + k - 2 \equiv 0 \mod \phi(p^m)$.
- The companionship between f and g is symmetric in the sense that the congruence between their associated representations may be written as $\rho_{g,m} \simeq \rho_{f,m} \otimes \chi^{k'-1} \mod p^m$.

Example. For $p = 7$, let f be a newform of weight 3, level 12 and character of order 2. It has the following Fourier expansion,

$$q - 3q^3 + 2q^7 + 9q^9 - 22q^{13} + 26q^{19} - 6q^{21} + 25q^{25} - \cdots$$

We find a companion form g modulo 49 of weight 41, level 12 and character of order 2 with the Fourier expansion

$$q + 3486784401q^3 - 153603710655044926q^7 + 12157665459056928801q^9 +$$
$$9249847954500085824674q^{13} - 6041129147325477931519326q^{19} -$$
$$5355830224772813993099326q^{21} + 909494701772928237915039062q^{25} \cdots$$

Note that we need only to check the congruence for coefficients up to the Sturm bound

Following Wiles [Wil88], we say that f is **ordinary** at p (or simply p-**ordinary**) if $a_p \not\equiv 0 \mod v$ for each prime $v|p$ (in K_f). Then, by Wiles [Wil88], and Mazur-Wiles [MW86], for every prime $\mathfrak{p}|p$ we have

$$\rho_f|_{G_p} \sim \begin{pmatrix} \chi^{k-1}\psi^{-1} & * \\ 0 & \psi \end{pmatrix}$$

where G_p is a decomposition group at $\mathfrak{p}$ and ψ is an unramified character.

A natural question is to ask when the restriction(s) $\rho_f|_{G_p}$ actually split. If $\rho_{f,\wp} \mod \wp$ is absolutely irreducible then we note that the splitting behaviour of $\rho_{f,m}|_{G_p} = \rho_f|_{G_p} \mod \wp^m$ is independent of the choice of a lattice used to define ρ_f.

The following proposition gives us a sufficient condition for $\rho_{f,m}$ to split at p.

Proposition 2.2. *If f is p-ordinary and has a p-ordinary companion form modulo p^m then $\rho_{f,m}$ splits at p.*

Proof. Let g be the p-ordinary companion form of f with weight k'. We know from the preceding discussion that with respect to some basis $\rho_{f,m}|_{G_p}$ is 'upper-triangular' as is $\rho_{g,m}|_{G_p}$. In fact,

$$\rho_{f,m}|_{G_p} \sim \begin{pmatrix} \chi^{k-1}\psi^{-1}(a_p(f)) & * \\ & \\ 0 & \psi(a_p(f)) \end{pmatrix}, \rho_{g,m}|_{G_p} \sim \begin{pmatrix} \chi^{k'-1}\psi^{-1}(a_p(g)) & * \\ & \\ 0 & \psi(a_p(g)) \end{pmatrix}.$$

Since f and g are companion forms, $\rho_{f,m} \simeq \rho_{g,m} \otimes \chi^{k-1} \mod p^m$. So,

$$\rho_{g,m}|_{G_p} \otimes \chi^{k-1} = \begin{pmatrix} \chi^{k'+k-2}\psi^{-1}(a_p(g)) & * \\ & \\ 0 & \chi^{k-1}\psi(a_p(g)) \end{pmatrix}$$

$$\equiv \begin{pmatrix} \psi^{-1}(a_p(g)) & * \\ & \\ 0 & \chi^{k-1}\psi(a_p(g)) \end{pmatrix} \mod p^m$$

since $\chi^{k'+k-2} \equiv \chi^{\phi(p^m)} \equiv 1 \mod p^m$. After applying an appropriate change of basis so that the unramified character ψ appears as the lower right entry we get,

$$\begin{pmatrix} \chi^{k-1}\psi^{-1}(a_p(f)) & * \\ & \\ 0 & \psi(a_p(f)) \end{pmatrix} \equiv \begin{pmatrix} \chi^{k-1}\psi(a_p(g)) & 0 \\ & \\ * & \psi^{-1}(a_p(g)) \end{pmatrix} \mod p^m$$

and conclude that $* \equiv 0$. (Note that when $k = \phi(p^m) + 1$, we need the additional assumption that $a_p(g)^2 \neq \psi(p)$ so that we can distinguish between $\psi^{-1}(a_p(g))$ and $\psi(a_p(g))$.) $\qquad\square$

That the splitting at p of $\rho_{f,m}$ implies the existence of a companion form mod p^m is considerably more difficult to prove. This was shown in [AM] but only under the hypothesis that $\overline{\rho}_f$ has large image which was necessary to make the deformation theory work. (Specifically, it was required that the $\text{Im}(\overline{\rho}_f)$ contains $SL_2(\boldsymbol{k})$, where $\boldsymbol{k}$ is a finite field of characteristic p.) However, when f has CM, $\text{Im}(\overline{\rho}_f)$ is necessarily projectively dihedral. In the sequel we avoid the use of lifting theorems altogether and exhibit the companion forms by working directly with the Hecke character associated to the CM form and visualising them as part of a Hida family.

3. Hecke characters and CM forms

In this section we describe the connection between forms with complex multiplication (CM forms) and Hecke characters of imaginary quadratic fields. The material

in this section is well known. The interested reader may consult [Rib77, §3] and [Neu99, Chapter VII, §3].

Let K be a number field, $\mathcal{O} = \mathcal{O}_K$ its ring of integers and $\mathfrak{m}$ an ideal of K. We denote by $J^{\mathfrak{m}}$ the group of fractional ideals of $\mathcal{O}_K$ that are coprime to $\mathfrak{m}$.

Definition 3.1. A Hecke character ψ of K, of modulus $\mathfrak{m}$, is then a group homomorphism $\psi : J^{\mathfrak{m}} \to \mathbb{C}^*$ such that there exists a character $\psi_f : (\mathcal{O}_K/\mathfrak{m})^* \to \mathbb{C}^*$ and a group homomorphism $\psi_\infty : K_\mathbb{R}^* \to \mathbb{C}^*$, where $K_\mathbb{R} := K \otimes_\mathbb{Q} \mathbb{R}$, such that:

$$(3.1) \qquad\qquad \psi((\alpha)) = \psi_f(\alpha)\psi_\infty(\alpha) \qquad \text{for all } \alpha \in K.$$

We refer to ψ_f and ψ_∞ as the finite and the infinite type of ψ respectively. The conductor of ψ is the conductor of ψ_f and we will call ψ primitive if the conductor is equal to its modulus.

We now specialize to the imaginary quadratic field case. Let $K = \mathbb{Q}(\sqrt{-d})$, where d is a square free positive integer. We will denote by D its discriminant and by $(D|.)$ the Kronecker symbol associated to it. The only possible group homomorphisms ψ_∞ are then of the form σ^u, where σ is one of the two conjugate complex embeddings of K and u is a non negative integer. The following well known theorem due to Hecke and Shimura then associates a modular form to the Hecke character ψ of K ([Miy06, Theorem 4.8.2]):

Theorem 3.2. *Given a Hecke character ψ of infinity type σ^u and finite type ψ_f with modulus $\mathfrak{m}$, assume $u > 0$ and let $\mathbf{N}\mathfrak{m}$ be the norm of $\mathfrak{m}$. We then have that*

$$f = \sum_n (\sum_{\mathfrak{a}=n} \psi(\mathfrak{a}))q^n$$

is a cuspidal eigenform in $S_{u+1}(N, \epsilon)$, where $N = |D|\mathbf{N}\mathfrak{m}$ and $\epsilon(m) = (D|m)\psi_f(m)$ for all integers m.

The eigenform associated with the Hecke character is new if and only if ψ is primitive. (See Remark 3.5 in [Rib77].)

An important feature of forms which arise in this way is that coincide with CM forms whose definition we recall.

Definition 3.3. A newform f is said to have **complex multiplication**, or just CM, by a quadratic character $\phi : G_\mathbb{Q} \longrightarrow \{\pm 1\}$ if $a(q, f) = \phi(q)a(q, f)$ for almost all primes q. We will also refer to CM by the corresponding quadratic extension.

It is clear that the cusp form f in the above Theorem has CM by $(D|.)$. Indeed, the coefficient $a_q(f)$ in the Fourier expansion of f is 0 if no ideal of K has norm equal to q. Since $(D|q) = -1$ exactly when this holds for q, $a_q(f) = (D|q)a_q(f)$ and f has CM. On the other hand, Ribet [Rib77] shows that if f has CM by an imaginary quadratic field K then it is induced from a Hecke character on K.

4. THE MAIN THEOREM

Theorem 4.1. *Let $p \geq 3$ be a rational prime, and $f = \sum_n a_n(f)q^n$ a p-ordinary CM eigenform of weight $k \geq 2$ and level N prime to p. Then for every integer $m \geq 1$, there exists a p-ordinary CM eigenform $h_m = \sum_n a_n(h_m)q^n$ of weight k_m that is a companion form for $f \mod p^m$, where k_m is the smallest integer ≥ 2 such that $k + k_m \equiv 2 \mod \phi(p^m)$.*

Proof. Let $K = \mathbb{Q}(\sqrt{-d})$ be the imaginary quadratic field by which f has CM, D be the discriminant of K and $\sigma : K \hookrightarrow \mathbb{C}$ one of its two complex embbedings which we fix for the rest of this section. Let g be the (p-old) eigenform of level Np and weight k, whose p-th coefficient has p-adic valuation equal to $k - 1$ and whose q-expansion agrees with that of f at all primes n coprime to p. Clearly g has CM by K as well. Let ψ be the Hecke character over K, of conductor $\mathfrak{m}$ and infinity type σ^{k-1} associated with the CM form g. The p-distinguishedness of f implies that $p = \mathfrak{p}\bar{\mathfrak{p}}$ is split in K. If r is a rational prime coprime to $\mathfrak{m}$ that splits in K, say $r = \mathfrak{r}\bar{\mathfrak{r}}$, then we have:

$$a_r(g) = \psi(\mathfrak{r}) + \psi(\bar{\mathfrak{r}})$$

In the proof of Proposition 7.1 in [Col96] Coleman shows how to obtain a p-ordinary (CM) p-adic eigenform h_{2-k} of weight $2 - k$ such that

$$\theta^{k-1}(h_{2-k}) = g,$$

where θ is the operator $q\dfrac{d}{dq}$ on q-expansions. It can be easily seen that θ^{k-1} has the following effect on q-expansions:

$$\theta^{k-1}\left(\sum c_n q^n\right) = \sum n^{k-1} c_n q^n.$$

Therefore,

$$(4.1) \qquad\qquad a_n(f) = n^{k-1} a_n(h_{2-k})$$

Clearly, then, it is enough to find classical forms h_{k_m}, of weight k_m, that are congruent to $h_{2-k} \mod \mathfrak{P}^m$, where $\mathfrak{P}$ is the ideal above p in an appropriate finite extension of $\mathbb{Q}_p$.

By Proposition 7.1 in [Col96]

$$h_{2-k} = \sum_{\mathfrak{a}} \bar{\psi}^{-1}(\mathfrak{a})q^{N\mathfrak{a}}$$

where the sum runs over all the integral ideals of K away from $\mathfrak{m}$. Ghate in [Gha05, pp 234-236], following Hida, shows how to construct a p-adic CM family admitting a

specific CM form as a specialization. We outline the construction. Let λ be a Hecke character of conductor $\mathfrak{p}$ and infinity type σ. We have that $\mathcal{O}_E^\times \cong \mu_E \times W_E$, where W_E is the pro-p part of $\mathcal{O}_E^\times$. Let $\langle\rangle$ denote the projection from $\mathcal{O}_E^\times$ to W_E. One then one gets (part of) the family mentioned above by

$$G(w) := \sum_{\mathfrak{a}} \bar\psi^{-1}(\mathfrak{a})\langle\lambda(\mathfrak{a})\rangle^{w-(2-k)} q^{\mathbf{N}\mathfrak{a}}.$$

For any integer $w \geq 2$, $\psi_w(\mathfrak{a}) = \bar\psi^{-1}(\mathfrak{a})\langle\lambda(\mathfrak{a})\rangle^{w-(2-k)}$ defines a Hecke character of infinity type $w - 1$, so that by Theorem 3.1, $G(w)$ a p-adic CM eigenform of weight w. Moreover all of them are p-ordinary ([Gha05, pp 236]) and therefore classical for weight $w \geq 2$ ([Hid86, Theorem I]). Clearly $G(2 - k) = h_{2-k}$. Notice also that all the ψ_w have coefficients in the field generated by the field of coefficients of ψ and λ. We will denote by L the extension of $\mathbb{Q}_p$ generated by the coefficients of ψ and all the ψ_w and by $\mathfrak{P}$ its prime ideal above p. We will also denote by E the extension of $\mathbb{Q}_p$ in which λ takes its values. In this setting $K \subset L$.

Let k_E, k_L be the residue fields of E and L respectively and consider the composition $\mathcal{O}_E^\times \to k_E^\times \to k_L^\times$, where the first map is the obvious surjection and the second one is the obvious injection. The image has prime-to-p order so the kernel contains W_E. In particular $\langle\lambda\rangle \equiv 1 \mod \mathfrak{P}$. This implies:

$$\langle\lambda(\mathfrak{a})\rangle^{w-w'} \equiv 1 \mod \mathfrak{P} \qquad \text{for all } w, w' \in \mathbb{Z}_p.$$

It then follows easily that if $w \equiv w' \mod \phi(p^m)$ then:

$$\langle\lambda(\mathfrak{a})\rangle^{w-w'} \equiv 1 \mod \mathfrak{P}^m.$$

Consider the members of the family with weight k_m where k_m is the smallest integer greater than 2 such that $k + k_m \equiv 2 \mod \phi(p^m)$. The previous identity then gives:

$$(4.2) \qquad\qquad\qquad \langle\lambda(\mathfrak{a})\rangle^{k_m-(2-k)} \equiv 1 \mod \mathfrak{P}^m.$$

Since $G(w) = \sum_n \left(\sum_{\mathfrak{a}=n} \bar\psi^{-1}(\mathfrak{a})\langle\lambda(\mathfrak{a})\rangle^{w-(2-k)} \right) q^n$, the r-th coefficient of $G(w)$ (for r a rational prime is):

$$a_r(G(w)) = \bar\psi^{-1}(\mathfrak{r})\langle\lambda(\mathfrak{r})\rangle^{w-(2-k)} + \bar\psi^{-1}(\bar{\mathfrak{r}})\langle\lambda(\bar{\mathfrak{r}})\rangle^{w-(2-k)}.$$

The identity $\mathfrak{r}\bar{\mathfrak{r}} = r$ along with (4.2) then shows that $a_q(G(k_m)) \equiv a_q(h_{2-k}) \mod \mathfrak{P}^m$.

For the primes r that are inert in K the above equivalence is trivially true since in this case $a_r(G(k_m)) = 0 = a_r(h_{2-k})$. We thus get that for all primes r away from Np the following holds:

$$a_r(G(k_m)) \equiv a_r(h_{2-k}) \mod \mathfrak{P}^m.$$

As we mentioned before, all the members of G with weight $w \geq 2$ are classical forms so every $h_m := G(k_m)$ is classical. Finally the last identity implies that h_m is congruent to h_{2-k} modulo $\mathfrak{P}^m$ almost everywhere, as required.

$\square$

Note that the Fourier coefficient version of Definition 2.1 was used to show companionship in the above proof. As noted in the remarks following the definition, this formulation can be reconciled with the Galois representation formulation if one knows that $\overline{\rho}_f$ is absolutely irreducible. For f as in the theorem above, $\overline{\rho}_f$ is reducible because it has projectively dihedral image. Absolute irreducibility then follows because $\overline{\rho}_f$ is odd and $p \geq 3$.

5. APPLICATIONS

As an immediate application, we use Theorem 4.1 to in fact prove its converse; thereby giving a complete arithmetic characterization of p-ordinary CM forms.

Theorem 5.1. *Let f be a p-ordinary cuspidal eigenform such that for every $m \geq 1$ there exists a CM cuspidal eigenform h_m which is a companion of f modulo p^m. Then f has CM.*

Proof. Assume that f has CM companions h_m for all m 1 and assume that h_m is CM with respect to a non-trivial quadratic character $\epsilon_m : (\mathbb{Z}/D_m\mathbb{Z})^\times \longrightarrow \{\pm 1\} \subset \mathbb{C}^\times$. The companionship property between f and each of the h_m's enforces the following compatibility congruences:

$$h_{m_1} \equiv' h_{m_2} \mod p^{m_1} \qquad \text{for all } m_2 \geq m_1$$

$$\epsilon_{m_1} \equiv' \epsilon_{m_2} \mod p^{m_1} \qquad \text{for all } m_2 \geq m_1$$

where $\equiv'$ means "away from p". The second compatibility congruence, combined with the fact that the characters ϵ_m are valued in ± 1 and $p \geq 3$, implies that $\epsilon_m = \epsilon$ for all $m \geq 1$. In particular ϵ is also a non-trivial quadratic character.

Furthermore by Theorem 4.1 each of the h_m's has companions everywhere and, in particular, there exist CM forms f_m, such that:

$$f_m \equiv' f \mod p^m \qquad \text{for all } m.$$

Each of the f_m has CM *w.r.t* the same character ϵ for all $m \geq 1$. This, combined with the previous congruence implies that:

$$a_\ell, (f) = a_\ell, (f)\epsilon(\ell) \qquad \text{for all } (\ell, cp) = 1.$$

If $p|c$ then it is clear that f has complex multiplication by ϵ as well. If $(c, p) = 1$ then let $\epsilon' : (\mathbb{Z}/cp\mathbb{Z})^\times \longrightarrow \mathbb{C}^\times$ be the quadratic character that is trivial on $(\mathbb{Z}/p\mathbb{Z})^\times$ and ϵ on $(\mathbb{Z}/c\mathbb{Z})^\times$. Then it is also clear that f has complex multiplication with respect to ϵ'.

$\square$

A second application provides insight into a deep conjectural equivalence between f having CM and ρ_f being split at p. In one direction we have Greenberg's local non-semisimplicity conjecture.

Conjecture 5.2. *If f is ordinary at p and $\rho_f|_{G_p}$ splits then f has complex multiplication.*

This is known for classical p-ordinary eigenforms of weight 2. The converse to this conjecture is well-known to be true. For instance, the reader may refer to Ghate's paper [Gha04] for a proof as well as a survey of results for the weight 2 case. We apply our main theorem to give an alternative proof of the converse.

Theorem 5.3. *Let f be a p-ordinary cuspidal eigenform of weight $k \geq 2$ and level N prime to p. If f has CM then $\rho_f|_{G_p}$ splits.*

Proof. As stated in Section 2, since f is p-ordinary, $\rho_f|_{G_p}$ is "upper-triangular". Specifically, we fix a basis under which it has the following shape.

$$\rho_f|_{G_p} \sim \begin{pmatrix} \chi^{k-1}\psi^{-1} & * \\ 0 & \psi \end{pmatrix}$$

By Theorem 4.1, since f has CM, it has companions h_m for each m. In particular, $\rho_{f,m} \sim \rho_{h_m,m} \otimes \chi^{k-1}$. By Proposition 2.2 this implies that $\rho_{f,m}|_{G_p}$ splits for each m. Finally the observation that $\rho_f|_{G_p} \equiv \rho_{f,m}|_{G_p} \mod p^m$ finishes off the proof. $\square$

We conclude with the following picture which summarizes the relationship, known and conjectural, between CM forms and higher companions forms on the one hand and local splitting behaviour of their associated Galois representations on the other.
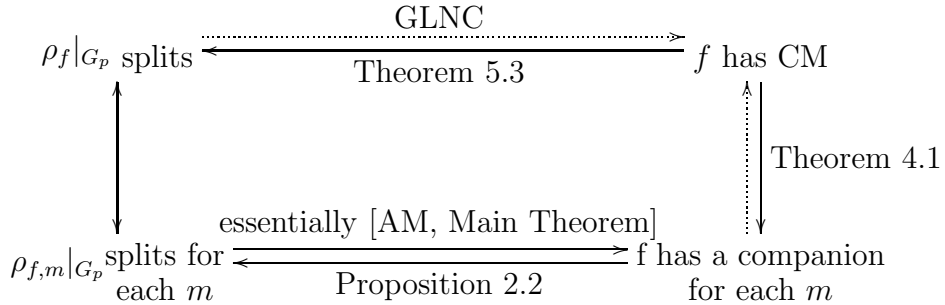
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