

# Bilateral Credit Valuation Adjustment of an Optional Early Termination Clause

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## Abstract

*Is an option to early terminate a swap at its market value worth zero? At first sight it is, but in presence of counterparty risk it depends on the criteria used to determine such market value. In case of a single uncollateralised swap transaction under ISDA between two defaultable counterparties, the additional unilateral option to early terminate the swap at predefined dates requires a Bermudan credit valuation adjustment. We give a general pricing formula assuming a default-free close-out amount, and apply it in a simplified setting with deterministic intensity and one single date of optional early termination, showing that the impact on the fair value of the transaction at inception might be non negligible.*

**JEL** Classification codes: G12, G13

**AMS** Classification codes: 62H20, 62L15, 91B25, 91B70

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## 1 Summary

The impact of close-out conventions in the ISDA setting has been highlighted in [1], [2] with particular regard to the consequences on bilateral counterparty risk adjustments. The authors show how crucial is the contractual definition of the close out amount (“risk-free” vs “substitution”) in case of default, even on the fair price at inception of the transaction; moreover, the ISDA protocol extends such conventions to any Additional Early Termination event (ATE), see 6(e)(ii)(1) and pag 17 (par 5) in [11]. A wide debate [5], [6] has taken place on the implication of the close-out definition and has focused in particular on the rating-triggered ATEs [4].

Here, we focus instead on how to value a single uncollateralised transaction where a party has the option to early terminate the transaction at predefined dates by exchanging the default-free amount with the other party. This kind of ATE is commonly introduced to shorten the horizon of the credit line between the parties; in the following we will refer to it as a *break clause* (BC). Therefore this

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case is different, and to a certain extent simpler, from those examined in previous literature where the exercise date of the early termination is a priori unknown. We will get a general but remarkably simple pricing formula for the case of default-free close out amount, a particularly interesting choice, being the only counterparty-independent one, therefore a good candidate to a less arbitrary close-out convention [8] even if subject to criticism [3]. Another reason of interest of this close-out convention is that, as we will show, in case of a bilateral BC it is effective in reducing the counterparty risk of the transaction by canceling its contribution after the first exercise date, thus supporting capital relief discussed in [9]. Quasi-analytical results are available, at least assuming deterministic intensities for the default processes, using the well known Geske-Johnson approach (see e.g. [10]) to bermudan derivatives pricing, which is an effective technique when the exercise dates are few - as is often the case (e.g. for medium dated swaps i.e. with maturity between 7 and 10 years). For illustrative purposes we apply the pricing formula in a simplified setting, but more general results are straightforward; the results show a considerable impact on the par strike.

Even if our approach is at a transaction level and cannot easily include the netting effects with possible other transaction referenced to the same ISDA agreement, we point out that our work has practical implications, due to the wide presence in the market of uncollateralized swap referenced to a single-transaction agreement, or of portfolios composed by a few large transactions with the same side (e.g. a portfolio of payer interest rate swaps). For instance, consider the case of a derivative between a bank and a corporate, a bank and a sovereign (see the discussion in [8] for a notable example), and the so-called back-swap linked to securitization or covered bond. The last two cases often concern long-dated swaps and typically involve break clause.

## 2 Notation

In the following we refer to the notation and the results of [2]. The value  $V_B^{AB}(t_0)$  of a derivative contract between two defaultable counterparty  $A$  and  $B$  as seen from  $B$  in  $t_0$  is

$$V_B^{AB}(t_0) = V_B^0(t_0) - \mathbb{E} \left\{ L_A \mathbb{I}_A(t_0, T) D(t_0, \tau_A) [V_B^0(\tau_A)]^+ \middle| t_0 \right\} + \mathbb{E} \left\{ L_B \mathbb{I}_B(t_0, T) D(t_0, \tau_B) [-V_B^0(\tau_B)]^+ \middle| t_0 \right\}, \quad (1)$$

where  $D(t_1, t_2)$  is the stochastic discount factor between two dates,  $\tau_X$  is the default time of counterparty  $X$ ,  $\mathbb{I}_A(t_1, t_2) = \mathbb{I}_{t_1 < \tau_A < \min(\tau_B, t_2)}$ ,  $\mathbb{I}_B(t_1, t_2) = \mathbb{I}_{t_1 < \tau_B < \min(\tau_A, t_2)}$ ,  $T$  is the last payment date of the derivative,  $L_X$  the loss given default of counterparty  $X$ , and  $V_B^0(t)$  is the value of the equivalent default-free derivative as seen from  $B$ ,

$$V_B^0(t) = \mathbb{E} \{ \Pi_B(t, T) | t \} = \mathbb{E} \left\{ \sum_{i=1}^N D(t, T_i) C_i(T_i) \middle| t \right\}, \quad (2)$$

with  $T_1 > t$ ,  $T_N = T$ , and  $C_i$  the cashflow paid in  $T_i$  that depends on the values of the risk factors (e.g. interest rates, stock prices) in  $T_i$ . We also define  $\tau = \min(\tau_A, \tau_B)$ , and we will use the notation  $\mathbb{P}_A(t_1, t_2) = \mathbb{E} \{ \mathbb{I}_A(t_1, t_2) \}$ , and analogous for  $B$ , for the unconditioned probabilities. We will omit conditioning on  $t_0$  henceforth.

The last two terms in eq. (1) define the Bilateral Credit Valuation Adjustment (BCVA) and Debt Valuation Adjustment (BDVA) respectively as seen from  $B$ ,

$$BCVA_B(t, T) = \mathbb{E} \left\{ L_A \mathbb{I}_A(t, T) D(t, \tau_A) [V_B^0(\tau_A)]^+ \middle| t \right\}, \quad (3)$$

$$BDVA_B(t, T) = \mathbb{E} \left\{ L_B \mathbb{I}_B(t, T) D(t, \tau_B) [-V_B^0(\tau_B)]^+ \middle| t \right\}, \quad (4)$$

so that we can write the well known formula

$$V_B^{AB}(t_0) = V_B^0(t_0) - BCVA_B(t_0, T) + BDVA_B(t_0, T). \quad (5)$$

### 3 General Results

#### 3.1 Unilateral Break Clause

Let us assume that party  $B$  has a BC at time  $\hat{t} < T$ , i.e. she has the right to terminate the derivative at time  $\hat{t}$  by liquidating its default-free fair value  $V_B^0(\hat{t})$ . It is easy to show that the equivalent of eq. (1) becomes

$$\begin{aligned} \hat{V}_B^{AB}(t_0) &= V_B^0(t_0) - \\ &\mathbb{E} \left\{ L_A \mathbb{I}_A(t_0, \hat{t}) D(t_0, \tau_A) [V_B^0(\tau_A)]^+ \right\} + \mathbb{E} \left\{ L_B \mathbb{I}_B(t_0, \hat{t}) D(t_0, \tau_B) [-V_B^0(\tau_B)]^+ \right\} - \\ &\mathbb{E} \left\{ \mathbb{I}_{V_B^0(\hat{t}) \leq V_B^{AB}(\hat{t})} \left( L_A \mathbb{I}_A(\hat{t}, T) D(t_0, \tau_A) [V_B^0(\tau_A)]^+ - L_B \mathbb{I}_B(\hat{t}, T) D(t_0, \tau_B) [-V_B^0(\tau_B)]^+ \right) \right\}, \end{aligned} \quad (6)$$

the exercise condition of the BC being

$$V_B^0(\hat{t}) \geq V_B^{AB}(\hat{t}). \quad (7)$$

By conditioning on  $\hat{t}$  the inner part of the last term in (6), we obtain our main result:

$$\begin{aligned} \hat{V}_B^{AB}(t_0) &= V_B^0(t_0) - BCVA_B(t_0, \hat{t}) + BDVA_B(t_0, \hat{t}) + \\ &\mathbb{E} \left\{ \mathbb{I}_{\tau > \hat{t}} D(t_0, \hat{t}) [BDVA_B(\hat{t}, T) - BCVA_B(\hat{t}, T)]^+ \right\} \end{aligned} \quad (8)$$

The last term is equivalent to

$$\mathbb{E} \left\{ \mathbb{I}_{\tau > \hat{t}} D(t_0, \hat{t}) [V_B^{AB}(\hat{t}) - V_B^0(\hat{t})]^+ \right\},$$

where one easily recognizes the payoff linked to the continuation value of the option to early liquidate the derivative in  $\hat{t}$ .

#### 3.2 Multiple Unilateral Break Clause

Define the last three terms of eq. (8) as  $UBC(t_0, \hat{t})$  so that we can write  $\hat{V}_B^{AB}(t_0) = V_B^0(t_0) + UBC(t_0, \hat{t})$ . If we add an additional BC in  $\hat{t}_2 > \hat{t}$  we can repeat the same arguments of the previous section to see that eq. (8) becomes

$$\hat{V}_{B,multiple}^{AB}(t_0) = V_B^0(t_0) - BCVA_B(t_0, \hat{t}) + BDVA_B(t_0, \hat{t}) + \mathbb{E} \left\{ \mathbb{I}_{\tau > \hat{t}} D(t_0, \hat{t}) [UBC(\hat{t}, \hat{t}_2)]^+ \right\}, \quad (9)$$

and analogous results can be obtained for an arbitrary number of BC.

#### 3.3 Mutual Break Clause

In case of mutual BC (i.e. both parties have the right to liquidate the position in  $\hat{t}$ ), it is easy to see that, if both parties apply the same mark-to-market rules, the BC is automatically exercised. In fact, the exercise condition for party  $A$  is

$$V_A^0(\hat{t}) \geq V_A^{AB}(\hat{t}), \quad (10)$$

and since  $V_A^0 = -V_B^0$  and (see again [2])  $V_A^{AB} = -V_B^{AB}$ , it becomes

$$V_B^0(\hat{t}) \leq V_B^{AB}(\hat{t}). \quad (11)$$

Hence, either  $A$  or  $B$  will exercise, and we can write

$$\hat{V}_{B,mutual}^{AB}(t_0) = V_B^0(t_0) - BCVA_B(t_0, \hat{t}) + BDVA_B(t_0, \hat{t}). \quad (12)$$

## 4 Application and results in a simplified setting

We apply the pricing formula (8) to the case of an equity swap on a non dividend paying stock  $S_t$  following Black and Scholes dynamics, with strike  $K$ , maturity  $T$  and unilateral BC at a single date  $\hat{t} < T$ , and we will adopt simplifying assumptions in order to obtain analytical results. Like in [2] we assume that the default times have a bivariate exponential distribution obtained combining exponential marginal distributions  $\mathbb{P}(\tau_X > t) = e^{-\lambda_X t}$  with a Gumbel copula, disallowing simultaneous default. Furthermore we assume independence between default events and the market risk factors. For the payer case we get from (3) and (4) respectively

$$BCVA_B(t, T) = L_A \int_{t_0}^T d\tau_A \int_{\tau_A}^{+\infty} d\tau_B p(\tau_A, \tau_B) Call(t, K, \tau_A), \quad (13)$$

$$BDVA_B(t, T) = L_B \int_{t_0}^T d\tau_B \int_{\tau_B}^{+\infty} d\tau_A p(\tau_A, \tau_B) Put(t, K, \tau_B). \quad (14)$$

where  $Call(t, K, \tau)$  stands for the value in  $t$  of a European call option with maturity  $\tau$  on the contingent claim  $D(\tau, T)(S_T - K)$ . To simplify the subsequent formulas let us assume that default can happen up to  $\hat{t}^-$  or  $T^-$ , and in case of default the parties exchange the amount as evaluated at  $\hat{t}$  or  $T$  respectively instead of at the time to default. Therefore we obtain the pricing formula without BC from (5)

$$\begin{aligned} V_{B,payer}^{AB}(t_0) &= S_0 - KP(t_0, T) - \\ &L_A \mathbb{P}_A(t_0, \hat{t}) Call(t_0; KP(\hat{t}, T), \hat{t}) + L_B \mathbb{P}_B(t_0, \hat{t}) Put(t_0; KP(\hat{t}, T), \hat{t}) - \\ &L_A \mathbb{P}_A(\hat{t}, T) Call(t_0; K, T) + L_B \mathbb{P}_B(\hat{t}, T) Put(t_0; K, T). \end{aligned} \quad (15)$$

We point out here that the inclusion of the counterparty risk terms BCVA and BDVA changes the par strike i.e. the strike that brings the value of the swap to zero at inception. Only in case of a contract between two identical counterparties ( $L_A = L_B = L$  and  $\lambda_A = \lambda_B = \lambda$ ), the par strike remains unchanged, even if the value of the swap is changed.

Now, introducing the break clause at  $\hat{t}$  in favor of  $B$  we have to calculate the last expectation term of (6) or (8), which under our hypothesis becomes

$$\mathbb{E} \left\{ D(t_0, \hat{t}) [L_B \mathbb{P}_B(\hat{t}, T) Put(\hat{t}; K, T) - L_A \mathbb{P}_A(\hat{t}, T) Call(\hat{t}; K, T)]^+ \right\},$$

which is positive when the continuation condition

$$L_B \mathbb{P}_B(\hat{t}, T) Put(\hat{t}; K, T) \geq L_A \mathbb{P}_A(\hat{t}, T) Call(\hat{t}; K, T) \quad (16)$$

is satisfied. In order to apply the Geske-Johnson technique we determine the boundary of such region, that is we look for a  $S_{\hat{t}} = K_e$  such that

$$L_B \mathbb{P}_B(\hat{t}, T) Put(\hat{t}; K, T) = L_A \mathbb{P}_A(\hat{t}, T) Call(\hat{t}; K, T); \quad (17)$$

this allows us to write the call and put payoffs in terms of indicator functions, arriving at the final formula

$$\begin{aligned} \hat{V}_{B,payer}^{AB}(t_0) &= S_0 - KP(t_0, T) - \\ &L_A \mathbb{P}_A(t_0, T) Call(t_0; KP(\hat{t}, T), \hat{t}) + L_B \mathbb{P}_B(t_0, T) Put(t_0; KP(\hat{t}, T), \hat{t}) + \\ &L_B \mathbb{P}_B(\hat{t}, T) \mathbb{E} \left\{ D(T_0, T) (K - S_T) \mathbb{I}_{S_{\hat{t}} < K_e} \mathbb{I}_{S_T < K} \right\} - \\ &L_A \mathbb{P}_A(\hat{t}, T) \mathbb{E} \left\{ D(T_0, T) (S_T - K) \mathbb{I}_{S_{\hat{t}} < K_e} \mathbb{I}_{S_T > K} \right\}. \end{aligned} \quad (18)$$

It is straightforward to price analytically these barrier options within a Black and Scholes approximation, computing a two-dimensional Gaussian integral<sup>1</sup>. Analogous results are obtained for a receiver equity swap.

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<sup>1</sup>a multiple break clause with  $n$  exercise dates would require a  $n + 1$ -dimensional Gaussian integral.

## 5 Numerical Results

In this section we evaluate the impact of the BC by comparing the at-the-money strike implied by eq. (18) with that coming from eq. (15). All the results are obtained for  $S_0 = 1$ ,  $\sigma_t = 0.3$ ,  $L_A = L_B = 100\%$  and zero interest rates. The Gumbel copula for the default times gives the following survivalship probability

$$\mathbb{P}(\tau_A > t_A, \tau_B > t_B) = e^{-[(\lambda_A t_A)^\theta + (\lambda_B t_B)^\theta]^{1/\theta}}, \quad (19)$$

with  $\theta \in [1, +\infty)$ . Observe that Kendall's Tau  $\tau_K$  is worth

$$\tau_K = 1 - \frac{1}{\theta}, \quad (20)$$

so that the independent case corresponds to  $\theta = 1$ , and the comonotonic to  $\theta = \infty$ . We will report the results also for the receiver swap, that are almost perfectly antithetic to those of the payer swap (this effect is enhanced by the choice of zero interest rate and dividend yield, and of equal loss given default). We point out that this instrument is particularly sensitive to counterparty risk, since all cash flows are concentrated at the end of its life, and is therefore an ideal case to illustrate the effects of the BC.

We examine first the effect of the distance between  $\hat{t}$  and  $T$  for  $\tau_K = 0$  and  $\tau_K = 0.75$ . In figure (1), where  $\lambda_A = 0.1$ ,  $\lambda_B = \lambda_A/2$ , we see that as  $\hat{t}$  approaches 0 the counterparty risk is completely removed (the par strike is equal to 1, the at-the-money forward when counterparty risk is not included), since it is convenient to exercise the BC as soon as possible, being A the riskier counterparty. As  $\hat{t}$  increases, the effect of the BC is reduced, and the par strike approaches that implied by the full BCVA and BDVA contributions. When  $\hat{t}$  is halfway through the life of the swap, it reduces by almost a factor of two the counterparty risk. In figure (2), with the same set of parameters but  $\lambda_A = 0.05$ ,  $\lambda_B = 2\lambda_A$ , we do not notice any efficient removal of counterparty risk, an effect that is common to most examples below when the counterparty that has the BC is more risky than the other, i.e. when  $\lambda_B > \lambda_A$ .

In figure (3) we show for  $\tau_K$  ranging from 0 to 0.75 that the correction due to the BC decreases as  $\lambda_B$  grows larger than  $\lambda_A$  and goes to zero faster for large  $\theta$ . This effect can be explained by noticing that the more probable is that  $\tau_B < \tau_A$ , the less useful is the BC to B.

Figure (4) shows, again with  $\lambda_A = 0.1$ ,  $\lambda_B = \lambda_A/2$ , how effective the BC is in reducing the expected length of the transaction across all values of  $\theta$ : the par strike of a 4 years and a 2 year swap with a BC after 1 year is almost identical to that of the 1 year swap. As expected, the case with  $\lambda_B > \lambda_A$  (not shown) displays almost no dependence on the BC.

## 6 Conclusions

The financial effect of the BC (both unilateral and bilateral) is quite strong for a large range of the parameter space. This result must be contrasted with the historically low propensity of counterparties to exercise the BC in order not to compromise their relationship with clients.

Our approach can be generalized to more complex break clauses, for instance conditioned to rating downgrade, that we leave for future work. Another development will be to investigate the effect of stochastic default rates that will add volatility to the credit adjustment and also alter the order of default of the counterparties by weakening the dependence of the result on the ratio between  $\lambda_B$  and  $\lambda_A$  that is kept constant in our examples.

## References

- [1] D. BRIGO, M. MORINI, *Dangers of Bilateral Counterparty Risk: the fundamental impact of close out conventions*, November 16, 2010, available at [HTTP://WWW.DEFAULTRISK.COM](http://www.defaultrisk.com)
- [2] D. BRIGO, C. BUESCU, M. MORINI, *Impact on the first to default time on a Bilateral CVA*, June 20, 2011, available at [HTTP://WWW.DEFAULTRISK.COM](http://www.defaultrisk.com)

- [3] D. BRIGO, M. MORINI, *Close-out convention tensions*, *Risk* December 2011
- [4] F. MERCURIO, R. CACCIA AND M. CUTULI, *Downgrade termination costs*, *Risk* April 2012
- [5] L. CARVER, *ISDA to review close-out value definitions*, *Risk online* September 2011
- [6] L. CARVER, *CVA's cousin: Dealers try to value early termination clauses*, *Risk* September 2011
- [7] M. CAMERON, *Italy could face more swap terminations*, *Risk online* March 2012
- [8] M. CAMERON, *Dealers call for revised language on break clause close-outs*, *Risk online* April 2012
- [9] M. CAMERON, *Breaking with tradition*, *Risk* March 2012
- [10] O. BROCKHAUS, A. FERRARIS, C. GALLUS, D. LONG, R. MARTIN, M. OVERHAUS, *Modelling and hedging equity derivatives*, 1999, RISK BOOKS
- [11] INTERNATIONAL SWAP AND DERIVATIVES ASSOCIATION, *ISDA Close-out Amount Protocol*, February 27, 2009, available at [HTTP://WWW.DEFAULTRISK.COM](http://www.defaultrisk.com)

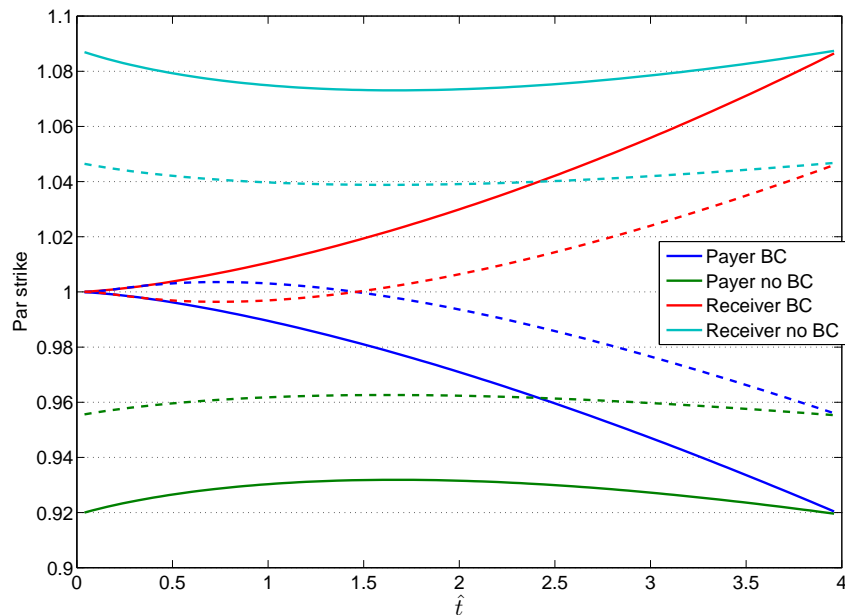


Figure 1: Par strike as a function of  $\hat{t}$  for a 4y equity swap with  $\lambda_A = 0.1$ ,  $\lambda_B = 0.05$ ,  $\theta = 4$  (full lines) and  $\theta = 1$  (dashed lines). As  $\hat{t}$  approaches 0 the counterparty risk is completely removed, since it is convenient to exercise the BC as soon as possible, being A the riskier counterparty.

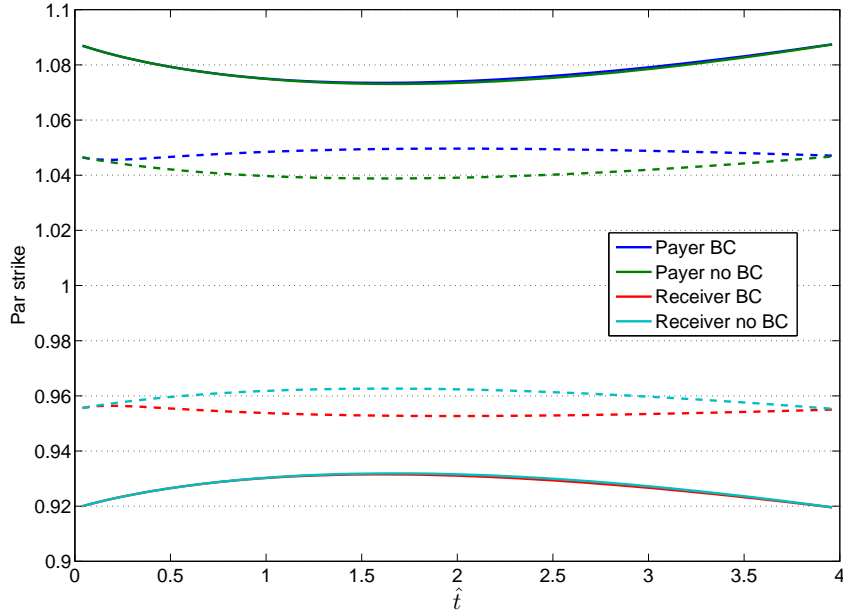


Figure 2: Par strike as a function of  $\hat{t}$  for a 4y equity swap with  $\lambda_A = 0.05$ ,  $\lambda_B = 0.1$ ,  $\theta = 4$  (full lines) and  $\theta = 1$  (dashed lines). Contrary to the  $\lambda_B < \lambda_A$  case, now the results depend very weakly on the BC.

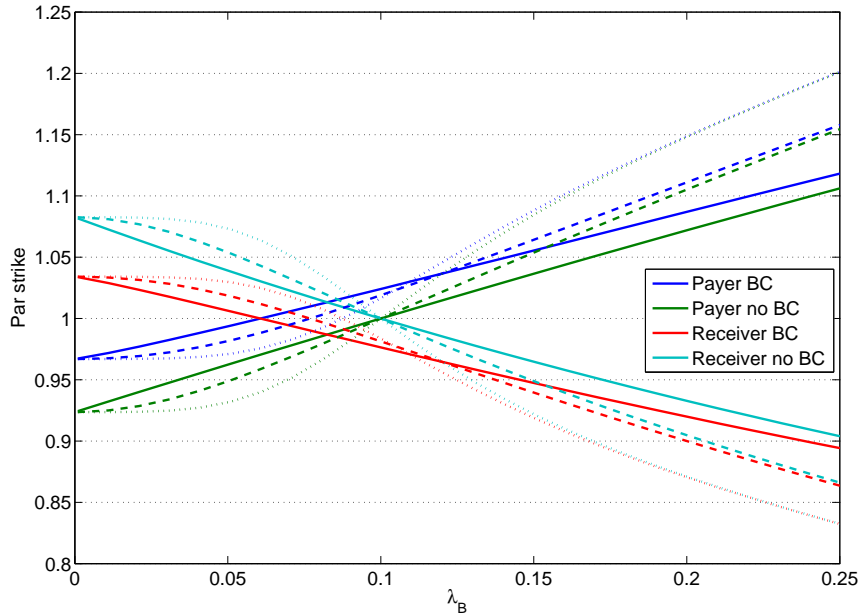


Figure 3: Par strike as a function of  $\lambda_B$  for an equity swap with  $\lambda_A = 0.1$ ,  $T = 4$ ,  $\hat{t} = 2$ ,  $\theta = 1$  (full lines),  $\theta = 2$  (dashed lines) and  $\theta = 4$  (dotted lines). The correction due to the BC decreases as  $\lambda_B$  grows larger than  $\lambda_A$  and goes to zero faster for large  $\theta$ .

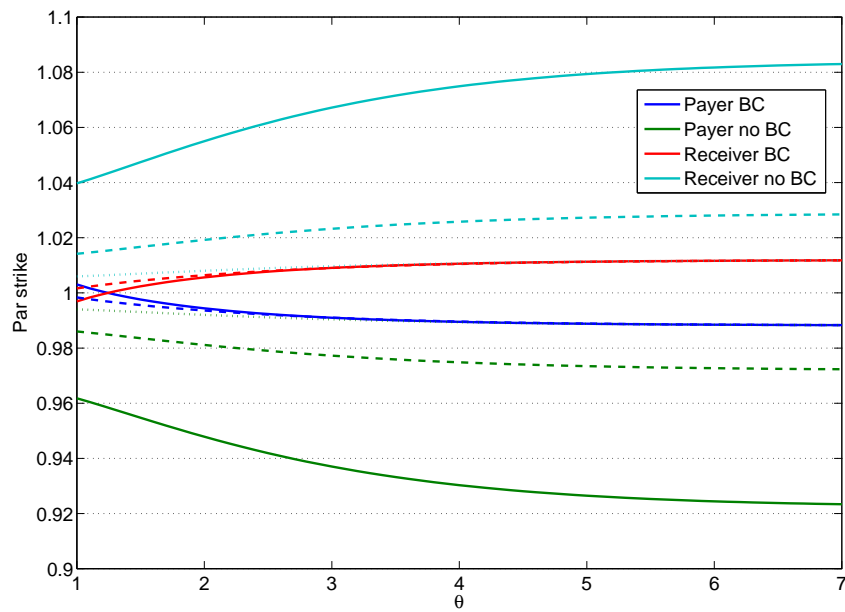


Figure 4: Par strike as a function of  $\theta$  for an equity swap with  $\lambda_A = 0.1$ ,  $\lambda_B = 0.05$ ,  $\hat{t} = 1$ ,  $T = 4$  (full lines) and  $T = 2$  (dashed lines). Notice how the break clause reduces the 4 years and the 2 years cases to the 1y with no BC (dotted line).