

# 3D hybrid depth migration and four-way splitting scheme

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## Abstract

The alternately directional implicit (ADI) scheme is usually used in 3D depth migration. It splits the 3D square-root operator along crossline and inline directions alternately. In this paper, based on the ideal of data line, the four-way splitting schemes for the finite-difference (FD) method and the hybrid or Fourier finite-difference (FFD) method are investigated. All schemes can be used in 3D post-stack or prestack depth migration. Numerical results of 3D post-stack depth migration show that the ADI FD migration has visible numerical anisotropic errors, and that the hybrid or FFD method has much less splitting errors than that of FD method. Moreover, the ADI hybrid or FFD method can image complex structures with large velocity variations. Numerical calculations with the ADI hybrid scheme for SEG/EAGE benchmark model are completed and very good imaging results are yielded. The MPI parallel algorithm which based on shot parameter are adopted and improve the computational efficiency further. The results in this paper show that 3D ADI hybrid shot profile migration has large potential practical values.

**keywords:** 3D depth migration, prestack, poststack, multiway splitting, finite-difference, hybrid method, SEG/EAGE model, MPI parallel.

## 1 Introduction

3D prestack depth migration is an important tool for complex structure imaging. There are two kinds of 3D prestack depth migration methods. One is the Kirchhoff integral method which based on ray tracing. The other is the non-Kirchhoff integral method which based on wavefield extrapolation. Kirchhoff integral method is a high-frequency approximation method, which has difficulties in imaging complex structures. However, it can adapt sources and receivers configuration easily and has the advantage of less computational cost. So it is still the dominant method for 3D prestack migration in oil industry. Non-Kirchhoff integral method, such as the finite-difference method, the phase-shift method (Gazdag, 1978), the split-step Fourier (SSF) method (Stoffa et al., 1990) and the Fourier finite-difference (FFD) method (Ristow and Ruhil, 1995), do wavefield extrapolation with one-way wave equation.

For 3D one-way wave equation, a direct solution with stable implicit finite-difference scheme may lead to a non tri-diagonal system, which is computationally expensive. In order to decrease computational cost, the alternatively directional implicit (ADI) scheme is usually used. It splits the finite-difference equation along two directions which are perpendicular to each other, i.e. the  $0^\circ$  and  $90^\circ$  directions, and then implements wavefield extrapolation by solving two tri-diagonal equations successively. By doing so, it saves large computational cost. However, the ADI scheme will lead to azimuthal errors with maximum at  $45^\circ$  and  $135^\circ$ . In order to eliminate these errors, Li (1991) derived an error-correction equation to correct the azimuthal anisotropy. Wang (2001) proposed a so-called ADI plus interpolation method, which uses a simple, efficient interpolation step to accomplish the azimuthal-error correction and the evanescent-wave suppression two jobs. In 1994, Ristow and Ruhl (1994) proposed the ideal of multiway splitting method which splits the migration operator or the square-root operator along three, four and six ways, in order to reduce splitting errors. The commonly used splitting method is a four-way splitting method which does splitting along  $45^\circ$  and  $135^\circ$  two directions in addition to the original  $0^\circ$  and  $90^\circ$  two directions. Claebout (1998, 1999) proposed the ideal of helix and migration in helix can be found in the works of Rickett (Rickett et al. 1999) and Zhang (Zhang et al., 1999, 2000, 2001). In helix, helical boundary conditions are considered, which makes the absorbing boundary more simple.

In this paper, we will discuss another type of error-correction method, namely the multi-way splitting method on a data line. We implement the computations of wavefield extrapolation on a data line, which makes four-way computations more easily and has a better generality and adaptability as well. The implicit scheme is used in wavefield extrapolation and so it is stable unconditionally. After deriving the relevant formulae, numerical analysis for a impulse response with constant velocity and a field data with variable velocity are given. And the computations show the correctness of our algorithm. Moreover, 3D shot profile prestack depth migration for /SEG/EAGE benchmark model are accomplished and its imaging result show that the traditional ADI hybrid or FFD method can yield very good images for complexly geological structures.

## 2 Theory

### 2.1 FD four-way splitting

The 3D one-way wave equation in the frequency-space domain is written as

$$\frac{\partial P}{\partial z} = \pm \frac{i\omega}{v} \sqrt{1 + \frac{v^2}{\omega^2} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)} P, \quad (1)$$

where  $P(\omega, x, y, z)$  is the wavefield in the frequency-space domain,  $\omega$  is the frequency,  $x$  is the coordinate along the inline direction,  $y$  is the coordinate along the crossline direction,  $z$  is the depth, and  $v(x, y, z)$  is the medium velocity. The plus and minus signs represent downgoing wave and upgoing wave respectively. For simplicity here follows, we take the positive sign. In the frequency-wavenumber domain, equation (1) becomes

$$\frac{\partial P}{\partial z} = \frac{i\omega}{v} \sqrt{1 - \frac{v^2}{\omega^2} (k_x^2 + k_y^2)} P. \quad (2)$$

The square-root in equation (2) can be approximated as

$$\begin{aligned} & \sqrt{1 - \frac{v^2}{\omega^2} (k_x^2 + k_y^2)} \\ & \approx \sqrt{1 - \frac{v^2}{\omega^2} k_x^2} + \sqrt{1 - \frac{v^2}{\omega^2} k_y^2} - 1 \\ & \approx \frac{1}{2} \left[ \sqrt{1 - \frac{v^2}{\omega^2} k_x^2} + \sqrt{1 - \frac{v^2}{\omega^2} k_y^2} + \sqrt{1 - \frac{v^2}{\omega^2} k_{x'}^2} + \sqrt{1 - \frac{v^2}{\omega^2} k_{y'}^2} \right] - 1, \end{aligned} \quad (3)$$

where  $k_x$  and  $k_y$  are wavenumber along  $0^\circ$  and  $90^\circ$  directions respectively, whereas  $k_{x'}$  and  $k_{y'}$  are the wavenumber along  $45^\circ$  and  $135^\circ$  directions. Substituting the four-way approximation into equation (2) and transforming it back to the frequency-space domain, we can obtain the following four-ways migration equation:

$$\begin{aligned} \frac{\partial P}{\partial z} & \approx -\frac{i\omega}{v} P + \frac{i\omega}{2v} \left[ \sqrt{1 + \frac{v^2}{\omega^2} \frac{\partial^2}{\partial x^2}} + \sqrt{1 + \frac{v^2}{\omega^2} \frac{\partial^2}{\partial y^2}} \right] P \\ & \frac{i\omega}{2v} \left[ \sqrt{1 + \frac{v^2}{\omega^2} \frac{\partial^2}{\partial x'^2}} + \sqrt{1 + \frac{v^2}{\omega^2} \frac{\partial^2}{\partial y'^2}} \right] P, \end{aligned} \quad (4)$$

where  $x'$  and  $y'$  are the position of  $45^\circ$  and  $135^\circ$  directions respectively and can be represented in terms of  $x$  and  $y$  by the following expression

$$x' = \frac{\sqrt{2}}{2}(x + y), \quad y' = \frac{\sqrt{2}}{2}(-x + y). \quad (5)$$

Wavefield extrapolation with equation (4) is called four-way splitting scheme. Here, we implement it on a data line. Such implementation is suitable to either the conventional finite-difference (FD) or the so-called Fourier finite-difference (FFD) method. For the FD method, at each extrapolating step, there are the following five equations successively in the frequency-space domain

$$\frac{\partial P}{\partial z} = -i\frac{\omega}{v} P, \quad (6)$$

$$\begin{aligned} \frac{\partial P}{\partial z} & = i\frac{\omega}{2v} \sqrt{1 + \frac{v^2}{\omega^2} \frac{\partial^2}{\partial x^2}} P, & \frac{\partial P}{\partial z} & = i\frac{\omega}{2v} \sqrt{1 + \frac{v^2}{\omega^2} \frac{\partial^2}{\partial x'^2}} P, \\ \frac{\partial P}{\partial z} & = i\frac{\omega}{2v} \sqrt{1 + \frac{v^2}{\omega^2} \frac{\partial^2}{\partial y^2}} P, & \frac{\partial P}{\partial z} & = i\frac{\omega}{2v} \sqrt{1 + \frac{v^2}{\omega^2} \frac{\partial^2}{\partial y'^2}} P. \end{aligned} \quad (7)$$

The numerical examples below show that wavefield extrapolation based on equations (6) to (7) can eliminate azimuthal anisotropic errors and thus improve migration precision.

## 2.2 splitting error

In order to analysis splitting errors of the FD method, we define the following relative error of four-way splitting

$$error_4 = \left| 1 - \frac{\frac{1}{2}[\sqrt{1 - \frac{v^2}{\omega^2}k_x^2} + \sqrt{1 - \frac{v^2}{\omega^2}k_y^2} + \sqrt{1 - \frac{v^2}{\omega^2}k_{x'}^2} + \sqrt{1 - \frac{v^2}{\omega^2}k_{y'}^2}] - 1}{\sqrt{1 - \frac{v^2}{\omega^2}(k_x^2 + k_y^2)}} \right|. \quad (8)$$

If replacing  $k_{x'}$  and  $k_{y'}$  by  $k_x$  and  $k_y$ , this equation is that of two-way splitting

$$error_2 = \left| 1 - \frac{\sqrt{1 - \frac{v^2}{\omega^2}k_x^2} + \sqrt{1 - \frac{v^2}{\omega^2}k_y^2} - 1}{\sqrt{1 - \frac{v^2}{\omega^2}(k_x^2 + k_y^2)}} \right|. \quad (9)$$

The accuracy of four-way splitting compared to the two-way splitting is shown in the numerical calculations below.

## 2.3 Hybrid or FFD four-way splitting

Like the derivation of four-war FD splitting scheme, we outline the derivation of hybrid or FFD four-way splitting scheme as follows. Introducing a reference velocity  $v_0$  and basing on the integral-differential expression of the square-root (Zhang G., 1985, 1993), in the frequency domain, equation (1) can be decompose in the following precise form operator

$$\frac{\partial P}{\partial z} = (A_1 + A_2 + A_3)P, \quad (10)$$

where

$$\begin{aligned} A_1 &= \frac{i\omega}{v_0} \sqrt{1 + \frac{v_0^2}{\omega^2} \left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right]}, & A_2 &= i\omega \left( \frac{1}{v} - \frac{1}{v_0} \right), \\ A_3 &= \frac{i\omega}{\pi} \int_{-1}^{+1} \left[ \frac{v^2 \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)}{\omega^2 + (sv)^2 \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)} - \frac{v_0^2 \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)}{\omega^2 + (sv_0)^2 \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)} \right] \sqrt{1 - s^2} ds \\ &\approx i \frac{\alpha \frac{v}{\omega} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)}{1 + \beta \frac{v^2}{\omega^2} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)}, \end{aligned} \quad (11)$$

where  $v_0(z)$  is the reference velocity. In the expression of  $A_3$  in (11) and the rational fraction approximation are used. Such approximation is necessary in numerical calculations. The coefficients  $\alpha$  and  $\beta$  can be written as follows (Ristow and Ruhül, 1995; Zhang w. et al., 1999)

$$\begin{aligned} a &= \frac{1}{2} \left( 1 - \frac{v_0}{v} \right), & b &= \frac{1}{4} \left[ \left( \frac{v_0}{v} \right)^2 + \frac{v_0}{v} + 1 \right], \text{ or} \\ a &= 0.47824 \left( 1 - \frac{v_0}{v} \right), & b &= 0.37637 \left( 1 + \frac{v_0^2}{v^2} \right). \end{aligned} \quad (12)$$

In the case of small dip angle,  $A_3$  can be neglected. And for the media with large velocity variations,  $A_3$  should be included. Operators  $A_1$ ,  $A_2$  and  $A_3$  are termed the phase-shift operator, time-shift operator and difference operator (Stoffa et al., 1990; Ristow and Ruhül, 1995). With the last approximation expression in equation (11), the FFD four-way wavefield extrapolation can be approximated as

$$\frac{\partial P}{\partial z} \approx (A_1 + A_2 + A_{31} + A_{32} + A_{41} + A_{42})P, \quad (13)$$

where

$$A_1 = \frac{i\omega}{v_0} \sqrt{1 + \frac{v^2}{\omega^2} \left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right]}, \quad A_2 = i\omega \left( \frac{1}{v} - \frac{1}{v_0} \right), \quad (14)$$

$$\begin{aligned} A_{31} &= i \frac{\omega}{2v} \frac{\alpha v^2 \frac{\partial^2}{\partial x^2}}{1 + \beta v^2 \frac{\partial^2}{\partial x^2}}, & A_{32} &= i \frac{\omega}{2v} \frac{\alpha v^2 \frac{\partial^2}{\partial y^2}}{1 + \beta v^2 \frac{\partial^2}{\partial y^2}}, \\ A_{33} &= i \frac{\omega}{2v} \frac{\alpha v^2 \frac{\partial^2}{\partial x^2}}{1 + \beta v^2 \frac{\partial^2}{\partial x^2}}, & A_{34} &= i \frac{\omega}{2v} \frac{\alpha v^2 \frac{\partial^2}{\partial y^2}}{1 + \beta v^2 \frac{\partial^2}{\partial y^2}}. \end{aligned} \quad (15)$$

Generally, for wavefield extrapolation of the following equation

$$\frac{\partial P}{\partial z} = LP, \quad (16)$$

where  $L$  is the linear operator with the form of  $L = \sum_{i=1}^n L_i$ , and  $n$  is the splitting number, the wavefield extrapolation can be completed by the following equations

$$\frac{\partial P}{\partial z} = L_i P, \quad i = 1, \dots, n. \quad (17)$$

With the above extrapolation equations of the four-way FD scheme (i.e. equations (6) and (7)) and the four-way hybrid or FFD scheme (i.e. equations (13) to (15)), the wavefield extrapolation can be implemented. We implement the four-way wavefield extrapolation on a data line.

## 2.4 Four-way wavefield extrapolation on a data line

We outline the implementation of wavefield extrapolation on a data line with the FD method and the FFD method as follows. When wavefield extrapolation is implemented along  $45^\circ$  and  $135^\circ$  two directions, it should be noted that the program complexity arises. However, introduce an ideal of data line, the difficulty can be overcome. The basic ideal is to transform the 2D data into the 1D data along a specific direction to form a data line. The wavefield extrapolation based on equations (6) to (7) on a data line can be implemented according to the following steps. First of all, equation (6) is used, which is the phase-shift extrapolation. Then do wavefield extrapolation along  $0^\circ$  and  $90^\circ$  directions with equation (7) which contributes to the traditional ADI wavefield extrapolation. And then, arrange the result data along  $45^\circ$  direction and do wavefield extrapolation with the third expression in equation (7). Finally, arrange the newly result data along  $135^\circ$  direction and then do extrapolate wavefield with the last expression in equation (7). By now, the wavefield extrapolation of one depth step is completed. For wavefield extrapolation of hybrid or FFD method based on equations (13) to (15), after finishing the wavefield extrapolation for operators  $A_1$  and  $A_2$ , which are the actions of phase-shift operator and time-shift operator respectively, the rest steps of wavefield extrapolation for operators  $A_{31}$ ,  $A_{32}$ ,  $A_{33}$  and  $A_{34}$  are similarly.

## 2.5 Wavefield extrapolation with ADI hybrid scheme

For the importance of traditional ADI scheme, we present briefly its difference scheme. The difference equation of the ADI hybrid scheme can be derived from the approximated difference operator  $A_3$  in equation (11). The corresponding difference scheme is

$$[1 + (\alpha_1 - i\beta_1)\delta_x^2 + (\alpha_2 - i\beta_2)\delta_y^2]P_{kl}^{n+1} = [1 + (\alpha_1 + i\beta_1)\delta_x^2 + (\alpha_2 + i\beta_2)\delta_y^2]P_{kl}^n, \quad (18)$$

where  $P_{kl}^n$  represents  $P(\omega, k\Delta x, l\Delta y, n\Delta z)$ ,  $\delta_x^2$  and  $\delta_y^2$  are the second-order central differences of  $x$  and  $y$  respectively. And  $\Delta x$ ,  $\Delta y$  and  $\Delta z$  are the spatial steps of  $x$ ,  $y$  and  $z$  respectively. The coefficients  $\alpha_1$ ,  $\alpha_2$ ,  $\beta_1$  and  $\beta_2$  can be written as

$$\alpha_1 = \frac{\beta v^2}{\omega^2 \Delta x^2}, \quad \alpha_2 = \frac{\beta v^2}{\omega^2 \Delta y^2}, \quad \beta_1 = \frac{\alpha \Delta z v}{2\omega \Delta x^2}, \quad \beta_2 = \frac{\alpha \Delta z v}{2\omega \Delta y^2}. \quad (19)$$

Equation (18) can be solved by the well-known alternately directional implicit scheme as follows

$$\begin{aligned} [1 + (\alpha_1 - i\beta_1)\delta_x^2]P_{ij}^{n+1/2} &= [1 + (\alpha_1 + i\beta_1)\delta_x^2]P_{ij}^n, \\ [1 + (\alpha_2 - i\beta_2)\delta_y^2]P_{ij}^{n+1} &= [1 + (\alpha_2 + i\beta_2)\delta_y^2]P_{ij}^{n+1/2}. \end{aligned} \quad (20)$$

Our numerical computations show that this traditional ADI hybrid FFD scheme in 3D shot profile prestack depth migration can yield very good images for complex structures.

# 3 Numerical calculations

## 3.1 Four-way FD and hybrid 3D postack depth migration

In order to demonstrate effects of the schemes in this paper, the migration for an impulse response is presented first. The grid number for  $x$ ,  $y$  and  $z$  is 64, the spatial step for  $x$  and  $y$  is  $15m$ . The extrapolation step is  $15m$ . The time sampling step is  $4ms$ . The medium velocity is  $3000m/s$ . It is well known that the theoretical 3D migration result in homogeneous media is a half sphere. The impulse is the Ricker wavelet with  $20Hz$  main frequency which located at the position of

$(x, y, z, t) = (480m, 480m, 500ms)$ . Figure 2 are the horizontal slices of migration result by the traditional FD two-way scheme. This figure shows that the migration errors caused by different azimuthal angles reach maximum along  $45^\circ$  and  $135^\circ$  directions. Figure 2(b) is the same slice but splitting along  $45^\circ$  and  $135^\circ$  directions, which shows the migration errors reach maximum along  $0^\circ$  and  $90^\circ$  directions. And figure 2(c) is that splitting along  $0^\circ$ ,  $90^\circ$ ,  $45^\circ$  and  $135^\circ$  four directions, which shows a perfect circle like theoretical predication.

For the media with constant velocity, due to the reference velocity used in hybrid method, there is only the action of phase-shift operator in the FFD extrapolation equation. That is to say, there is no any actual extrapolation for the rest time-shift operator  $A_2$  and difference operators  $A_2$ ,  $A_{31}$ ,  $A_{32}$ ,  $A_{33}$  and  $A_{34}$ . In this case, The migration result is a precise hemisphere because the Laplacian is an isotropic operator. Thus there is no azimuthal errors when the four-way scheme is used in a constant velocity media. Let's see an example with a variable velocity. Suppose the velocity is  $v(x, y, z) = 1600 + 3x + 3y + z(m/s)$ . The spatial steps and other parameters are the same with those in the example with constant velocity above. Figure 3 are the vertical slices of the 3D migration result at  $y = 660m$ , which calculated by the hybrid method of the two-way splitting scheme and the four-way splitting scheme respectively. Comparisons between figure 3a and figure 3b show that the numerical anisotropic errors of four-way FFD scheme is not very obvious and is much less than that of FD because the Laplacian in phase-shift operator  $A_1$  is an isotropic operator and can reduce those errors. Therefore, multiway splitting scheme for FFD method is not necessary generally. This is also proved by the migration result presented in figure 4 for a field data with variable velocity. In figure 4, wavefield extrapolation with the two-way FFD scheme (a) and the four-way FFD scheme (b) are used respectively, and comparisons show that there is no obvious difference actually.

### 3.2 ADI 3D prestack depth migration

3D SEG/EAGE salt model is an international dimensional benchmark model. The data used here has the 50 shot line with 160m line space. Each line has 96 shots with 80m shot space. Each shot has  $68 \times 6$  receivers. The grid element is  $40m \times 40m$ . The record length is  $4992ms$  with  $8ms$  time step. The model data amounts 6.23 Gbytes. In this large scale computation, the MPI parallel is used to improve computational efficiency. Here,  $x$  is the inline direction and  $y$  is the crossline direction. Figure 5 is the 3D shot profile prestack depth migration result by the ADI FFD scheme. Figure 5(a) is the vertical section of model at  $y = 6740m$  and figure 5(b) is the same sclice of the migration result. Figures 5 shows that the ADI FFD or hybrid method can actually yield precise images of the complicate structures with large velocity variations. In all calculations, the MPI parallel algorithm is adopted. And shot number is chosen as parallelization parameter for single-shot profile migration. The parallel efficiency is very high because the problem itself has very high parallel features.

## 4 Conclusion

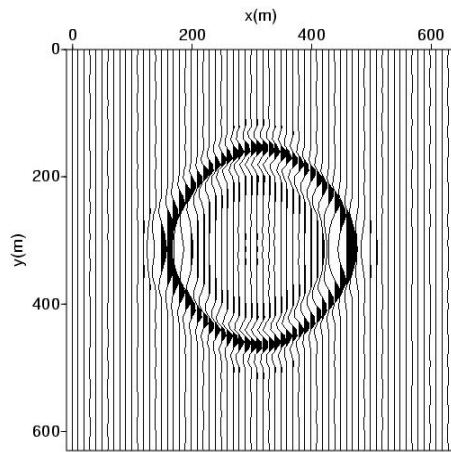
Based on the ideal of data line, the four-way splitting schemes and extrapolation equations for FD and hybrid or FFD methods are derived. The advantage of wavefield extrapolation on a data line is unconditionally stable. Numerical calculations show that the four-way FD algorithm can eliminate numerical anisotropic errors effectively. Moreover, the numerical anisotropic errors of FFD method is much less than that of FD because the Laplacian in phase-shift operator is an isotropic operator and can reduce those errors. A numerical migration for a field data with variable velocity show the imaging difference between the traditional ADI FD method and the four-way FFD method is very small. Thus, the traditional ADI FFD method is preferred in 3D prestack depth migration in order to save computational time. The ADI hybrid or FFD 3D shot profile prestack depth migration for SEG/EAGE salt model is implemented and fine imaging results are obtained. Calculations show that shot profile can yield precise images for 3D complex structures and has much more potential practical values. The MPI parallel algorithm is adopted to improve computational efficiency further.

## Acknowledgments

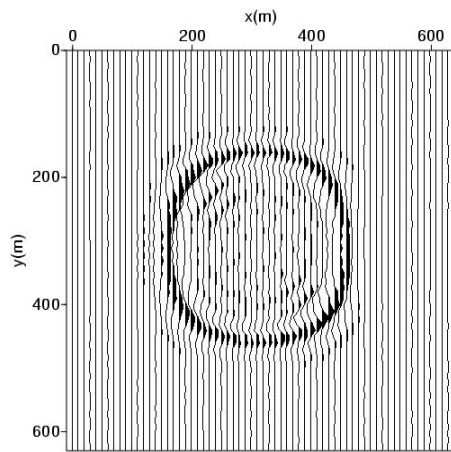
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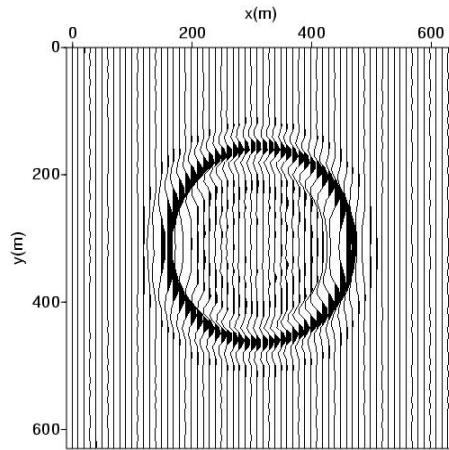
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(a)

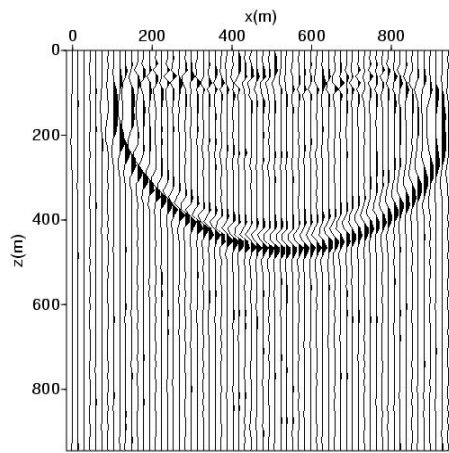


(b)



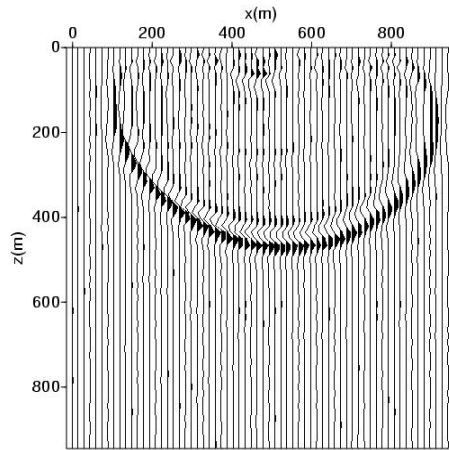
(c)

Figure 2. Horizontal slices of 3D post-stack depth migration for a impulse response with constant velocity. FD method with (a) traditional two-way splitting, (a)  $45^\circ$  and  $135^\circ$  two-way splitting, (c) Four-way splitting.



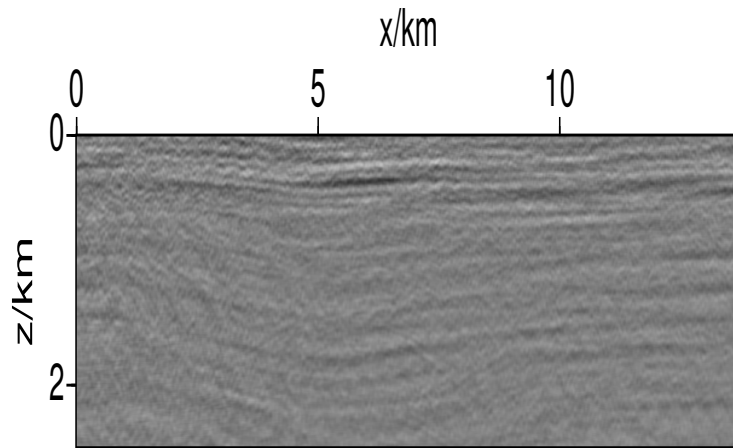
(a)



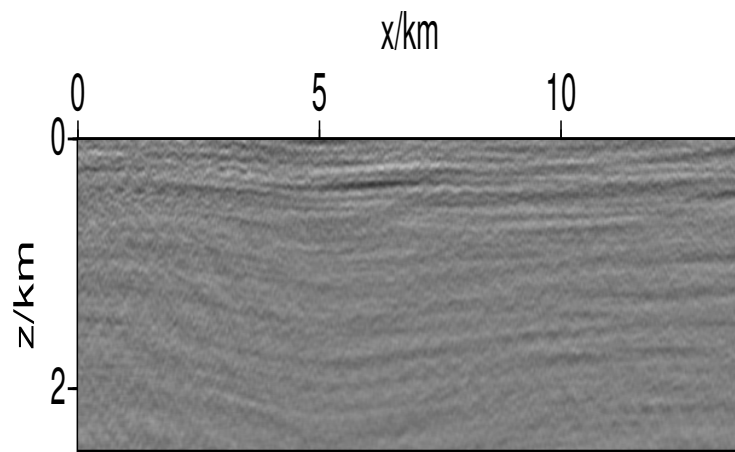


(b)

Figure 3. 3D post-stack depth migration for a impulse response with variable velocity. FFD method with (a) traditional two-way splitting, (b) Four-way splitting.

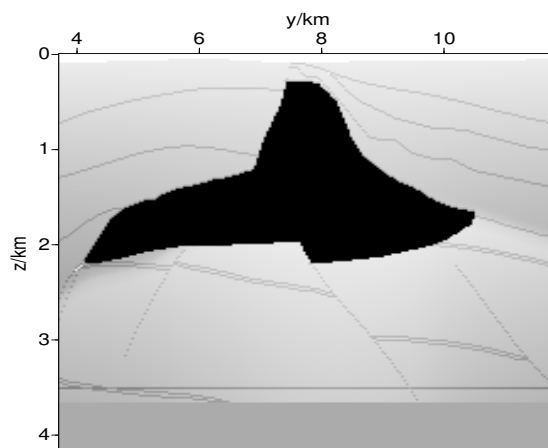


(a)

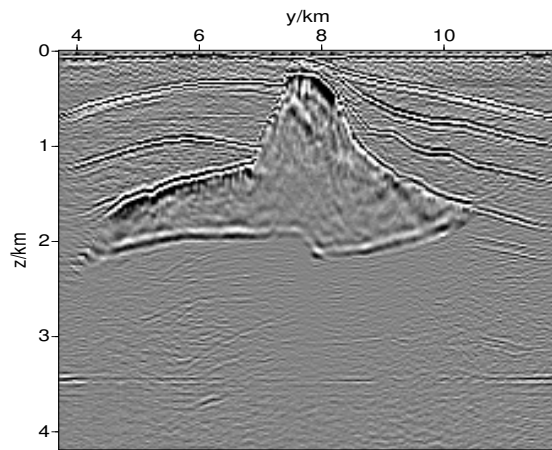


(b)

Figure 4. 3D post-stack depth migration for a field data with variable velocity. FFD method with (a) traditional two-way splitting, (b) Four-way splitting.



(a)



(b)

Figure 5. The vertical slice at  $x = 5400m$  along crossline of (a) velocity model, (b) 3D shot profile migration result with ADI splitting scheme.