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General Solution of Generalized (2 + 1)-Dimensional Kadomtsev-Petviashvili (KP) Equation by Using the (G'/G)-Expansion Method

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Abstract

In this work, the (G'/G)-expansion method is proposed for constructing more general exact solutions of the (2 + 1)-dimensional Kadomtsev-Petviashvili (KP) equation and its generalized forms. Our work is motivated by the fact that the (G'/G)-expansion method provides not only more general forms of solutions but also periodic and solitary waves. If we set the parameters in the obtained wider set of solutions as special values, then some previously known solutions can be recovered. The method appears to be easier and faster by means of a symbolic computation system.

Keywords: (G'/G)-Expansion Method, Generalized Kadomtsev-Petviashvili (KP) Equation, Hyperbolic Function Solutions, Trigonometric Function Solutions

1. Introduction

Nonlinear evolution equations (NLEEs) have been the subject of study in various branches of mathematical-physical sciences such as physics, biology, chemistry, etc. The analytical solutions of such equations are of fundamental importance since a lot of mathematical-physical models are described by NLEEs. Among the possible solutions to NLEEs, certain special form solutions may depend only on a single combination of variables such as traveling wave variables. In the literature, there is a wide variety of approaches to nonlinear problems for constructing traveling wave solutions. Some of these approaches are the Jacobi elliptic function method [1], inverse scattering method [2], Hirotas bilinear method [3], homogeneous balance method [4], homotopy perturbation method [5], Weierstrass function method [6], symmetry method [7], Adomian decomposition method [8], sine/cosine method [9], tanh/coth method [10], the Exp-function method [11-16] and so on. But, most of the methods may sometimes fail or can only lead to a kind of special solution and the solution procedures become very complex as the degree of nonlinearity increases.

Recently, the (G'/G)-expansion method, firstly introduced by Wang *et al.* [17], has become widely used to search for various exact solutions of NLEEs [17-27]. The value of the (G'/G)-expansion method is that one treats nonlinear problems by essentially linear methods. The method is based on the explicit linearization of NLEEs for traveling waves with a certain substitution which leads to a second-order differential equation with constant coefficients. Moreover, it transforms a nonlinear equation to a simple algebraic computation.

The generalized (2 + 1)-dimensional Kadomtsov-Petviashivilli (gKP) equation given by

$$\left(u_t + u^n u_x + \delta u_{xxx}\right)_x + \frac{\gamma}{2} u_{yy} = 0, |n| > 1$$

The objectives of this work are twofold. First, we describe the (G'/G)-expansion method. Second, we aim to implement the present method to obtain general exact travelling wave solutions of governing equation.

2. Description of the (G'/G)-Expansion Method

The objective of this section is to outline the use of the (G'/G)-expansion method for solving certain nonlinear partial differential equations (PDEs). Suppose we have a nonlinear PDE for $u(x, y, t)$, in the form

$$P\left(u, u_x, u_t, u_{xx}, u_{xy}, \cdots\right) = 0, \tag{1}$$

where P is a polynomial in its arguments, which includes nonlinear terms and the highest order derivatives. The transformation $u(x, y, z, t) = U(\xi)$, $\xi = kx + \ell y + \omega t$, reduces Equation (4) to the ordinary differential equation (ODE)

$$P\left(U, kU', \omega U', k^2 U'', k\ell U'', \cdots\right) = 0, \tag{2}$$

where $U = U(\xi)$, and prime denotes derivative with respect to ξ. We assume that the solution of Equation (2) can be expressed by a polynomial in (G'/G) as follows:

$$U(\xi) = \sum_{i=1}^{n} \alpha_i \left(G'/G\right)^i + \alpha_0, \alpha_n \neq 0. \tag{3}$$

where α_0, and α_i, are constants to be determined later, $G(\xi)$ satisfies a second order linear ordinary differential equation (LODE):

$$\frac{\mathrm{d}^2 G(\xi)}{\mathrm{d}\xi^2} + \lambda \frac{\mathrm{d}G(\xi)}{\mathrm{d}\xi} + \mu G(\xi) = 0. \tag{4}$$

where λ and μ are arbitrary constants. Using the general solutions of Equation (4), we have

$$\frac{G'(\xi)}{G(\xi)} = \begin{cases} \dfrac{\sqrt{\lambda^2 - 4\mu}}{2} \left(\dfrac{C_1 \sinh\left(\dfrac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right) + C_2 \cosh\left(\dfrac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right)}{C_1 \cosh\left(\dfrac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right) + C_2 \sinh\left(\dfrac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right)} \right) - \dfrac{\lambda}{2}, \lambda^2 - 4\mu > 0, \\ \dfrac{\sqrt{4\mu - \lambda^2}}{2} \left(\dfrac{-C_1 \sin\left(\dfrac{\sqrt{4\mu - \lambda^2}}{2}\xi\right) + C_2 \cos\left(\dfrac{\sqrt{4\mu - \lambda^2}}{2}\xi\right)}{C_1 \cos\left(\dfrac{\sqrt{4\mu - \lambda^2}}{2}\xi\right) + C_2 \sin\left(\dfrac{\sqrt{4\mu - \lambda^2}}{2}\xi\right)} \right) - \dfrac{\lambda}{2}, \lambda^2 - 4\mu < 0, \end{cases} \tag{5}$$

and it follows, from (3) and (4), that

$$\begin{aligned} U' &= -\sum_{i=1}^{n} i\alpha_i \left[\left(G'/G\right)^{i+1} + \lambda \left(G'/G\right)^i + \mu \left(G'/G\right)^{i-1} \right], \\ U'' &= \sum_{i=1}^{n} i\alpha_i \left[(i+1)\left(G'/G\right)^{i+2} + (2i+1)\lambda \left(G'/G\right)^{i+1} + i(\lambda^2 + 2\mu)\left(G'/G\right)^i + (2i-1)\lambda\mu \left(G'/G\right)^{i-1} + (i-1)\mu^2 \left(G'/G\right)^{i-2} \right], \end{aligned} \tag{6}$$

and so on, here the prime denotes the derivative with respective to ξ.

To determine u explicitly, we take the following four steps:

Step 1. Determine the integer n by substituting Equation (3) along with Equation (4) into Equation (2), and balancing the highest order nonlinear term(s) and the highest order partial derivative.

Step 2. Substitute Equation (3) give the value of n determined in Step 1, along with Equation (4) into Equation (2) and collect all terms with the same order of (G'/G) together, the left-hand side of Equation (2) is converted into a polynomial in (G'/G). Then set each coefficient of this polynomial to zero to derive a set of algebraic equations for k, ω, α_0 and α_i.

Step 3. Solve the system of algebraic equations obtained in *Step* 2, for $a, b, c, \omega, \alpha_0$ and α_i by use of Maple.

Step 4. Use the results obtained in above steps to derive a series of fundamental solutions $u(\xi)$ of Equation (2) depending on (G'/G), since the solutions of Equation (4) have been well known for us, then we can obtain exact solutions of Equation (1).

3. Application

In this section, we will demonstrate the (G'/G)-expansion method on the generalized (2 + 1)-dimensional Kadomtsev-Petviashvili (KP) equation given by

$$\left(u_t + u^n u_x + \delta u_{xxx}\right)_x + \frac{\gamma}{2} u_{yy} = 0, |n| > 1, \tag{7}$$

where δ, and γ are constants. Using the wave variable $\xi = kx + \ell y + \omega t$, in (7) and integrating the resulting equation and neglecting the constant of integration, we find

$$\left(k\omega + \frac{\gamma \ell^2}{2}\right)U + \frac{k^2}{n+1}U^{n+1} + \delta k^4 U'' = 0, |n| > 1, \tag{8}$$

To achieve our goal, we use the transformation

$U(\xi)=V^{\frac{1}{n}}(\xi)$, that will carry (8) into the ODE

$$n^2(n+1)\left(k\omega+\frac{\gamma\ell^2}{2}\right)V^2+k^2n^2V^3+(n+1)\delta k^4\left(nVV''+(1-n)(V')^2\right)=0, \tag{9}$$

According to *Step* 1, we get $3m=2m+2$, hence $m=2$. We then suppose that Equation (9) has the following formal solutions:

$$V=\alpha_2(G'/G)^2+\alpha_1(G'/G)+\alpha_0, \alpha_2\neq 0, \tag{10}$$

where α_2, α_1, and α_0, are constants which are unknown to be determined later.

Substituting Equation (10) into Equation (9) and collecting all terms with the same order of (G'/G) together, the left-hand sides of Equation (9) are converted into a polynomial in (G'/G). Setting each coefficient of each polynomial to zero, we derive a set of algebraic equations for k, ℓ, ω, λ, $\mu\alpha_0$, α_1, and α_2, and solving them by use of Maple, we get the following general result:

$$\alpha_0=-\frac{2k^2\delta\mu(2+n^2+3n)}{n^2}, \alpha_1=-\frac{2k^2\delta\lambda(2+n^2+3n)}{n^2},$$
$$\alpha_2=-\frac{2k^2\delta(2+n^2+3n)}{n^2}, \omega=-\frac{2k^4\delta\lambda^2-8k^4\delta\lambda+\gamma\ell^2n^2}{2kn^2}, \tag{11}$$

Substitute the above general case in (10), we get

$$V=\frac{-2\delta k^2(2+n^2+3n)}{n^2}\left[(G'/G)^2+\lambda(G'/G)+\mu\right], \tag{12}$$

$|n|>1, n\neq -2.$

then use the transformation $U(\xi)=V^{1/n}(\xi)$, when $\lambda^2-4\mu>0$, the hyperbolic function solutions of Equation (7), becomes:

$$u(\xi)=\left(\frac{-2\delta k^2(2+n^2+3n)}{n^2}\right)^{\frac{1}{n}}\left[\left(\frac{\sqrt{\lambda^2-4\mu}\left[C_1\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)+C_2\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)\right]}{2\left[C_2\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)+C_1\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)\right]}-\frac{\lambda}{2}\right)^2\right.$$
$$\left.+\lambda\left(\frac{\sqrt{\lambda^2-4\mu}\left[C_1\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)+C_2\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)\right]}{2\left[C_2\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)+C_1\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right)\right]}-\frac{\lambda}{2}\right)+\mu\right]^{\frac{1}{n}}, \tag{13}$$

and when $\lambda^2-4\mu<0$, the trigonometric function solutions of Equation (7), will be:

$$u(\xi)=\left(\frac{-2\delta k^2(2+n^2+3n)}{n^2}\right)^{\frac{1}{n}}\left[\left(\frac{\sqrt{4\mu-\lambda^2}\left[-C_1\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)+C_2\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)\right]}{2\left[C_2\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)+C_1\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)\right]}-\frac{\lambda}{2}\right)^2\right.$$
$$\left.+\lambda\left(\frac{\sqrt{4\mu-\lambda^2}\left[-C_1\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)+C_2\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)\right]}{2\left[C_2\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)+C_1\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)\right]}-\frac{\lambda}{2}\right)+\mu\right]^{\frac{1}{n}}, \tag{14}$$

where $\xi=kx+\ell y-\frac{2k^4\delta\lambda^2-8k^4\delta\lambda+\gamma\ell^2n^2}{2kn^2}t$, and C_1, C_2, λ, and μ are arbitrary constants.

In particular, when $C_2=0$, then the general solutions

(13) and (14) reduces, respectively,

$$u(\xi)=\left(\frac{-2\delta k^2\left(2+n^2+3n\right)}{n^2}\right)^{\frac{1}{n}}\left[\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(kx+\ell y-\frac{2k^4\delta\lambda^2-8k^4\delta\lambda+\gamma\ell^2n^2}{2kn^2}t\right)\right)-\frac{\lambda}{2}\right)^2\right.$$
$$\left.+\lambda\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(kx+\ell y-\frac{2k^4\delta\lambda^2-8k^4\delta\lambda+\gamma\ell^2n^2}{2kn^2}t\right)\right)-\frac{\lambda}{2}\right)+\mu\right]^{\frac{1}{n}}, \quad (15)$$

$$u(\xi)=\left(\frac{-2\delta k^2\left(2+n^2+3n\right)}{n^2}\right)^{\frac{1}{n}}\left[\left(\frac{-\sqrt{\lambda^2-4\mu}}{2}\tan\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(kx+\ell y-\frac{2k^4\delta\lambda^2-8k^4\delta\lambda+\gamma\ell^2n^2}{2kn^2}t\right)\right)-\frac{\lambda}{2}\right)^2\right.$$
$$\left.+\lambda\left(\frac{-\sqrt{\lambda^2-4\mu}}{2}\tan\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(kx+\ell y-\frac{2k^4\delta\lambda^2-8k^4\delta\lambda+\gamma\ell^2n^2}{2kn^2}t\right)\right)-\frac{\lambda}{2}\right)+\mu\right]^{\frac{1}{n}}, \quad (16)$$

and when $C_1 = 0$, then we deduce from general solutions (13) and (14) that,

$$u(\xi)=\left(\frac{-2\delta k^2\left(2+n^2+3n\right)}{n^2}\right)^{\frac{1}{n}}\left[\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\coth\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(kx+\ell y-\frac{2k^4\delta\lambda^2-8k^4\delta\lambda+\gamma\ell^2n^2}{2kn^2}t\right)\right)-\frac{\lambda}{2}\right)^2\right.$$
$$\left.+\lambda\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\coth\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(kx+\ell y-\frac{2k^4\delta\lambda^2-8k^4\delta\lambda+\gamma\ell^2n^2}{2kn^2}t\right)\right)-\frac{\lambda}{2}\right)+\mu\right]^{\frac{1}{n}}, \quad (17)$$

$$u(\xi)=\left(\frac{-2\delta k^2\left(2+n^2+3n\right)}{n^2}\right)^{\frac{1}{n}}\left[\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\cot\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(kx+\ell y-\frac{2k^4\delta\lambda^2-8k^4\delta\lambda+\gamma\ell^2n^2}{2kn^2}t\right)\right)-\frac{\lambda}{2}\right)^2\right.$$
$$\left.+\lambda\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\cot\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(kx+\ell y-\frac{2k^4\delta\lambda^2-8k^4\delta\lambda+\gamma\ell^2n^2}{2kn^2}t\right)\right)-\frac{\lambda}{2}\right)+\mu\right]^{\frac{1}{n}}, \quad (18)$$

where $k, \ell, \lambda,$ and μ are arbitrary constants.

For important case $n=\frac{3}{2}$, the KP Equation (7) reduce to

$$\left(u_t+u^{3/2}u_x+\delta u_{xxx}\right)_x+(\gamma/2)u_{yy}=0, \quad (19)$$

where δ, and γ are constants, then according to results in (11), the general hyperbolic and trigonometric function solution of (19) will be

$$u(\xi)=\left[\frac{35k^2\delta\left[C_1^2\left(\lambda^2-4\mu\right)-C_2^2\left(\lambda^2-4\mu\right)\right]}{18\left[C_2\sinh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi\right)+C_1\cosh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi\right)\right]^2}\right]^{2/3}, \quad (20)$$

$$u(\xi)=\left[-\frac{35k^2\delta\left[C_1^2\left(4\mu-\lambda^2\right)+C_2^2\left(4\mu-\lambda^2\right)\right]}{18\left[C_2^2+\left(C_1^2-C_2^2\right)\cos^2\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)+2C_1C_2\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi\right)\right]}\right]^{2/3}, \quad (21)$$

where

$$\xi = kx + \ell y - \frac{4k^4\delta\lambda^2 - 16k^4\delta\lambda + (9/2)\gamma\ell^2}{9k}t,$$

and $C_1, C_2, k, \ell, \lambda,$ and μ are arbitrary constants. When $C_2 = 0,$ then the general hyperbolic and trigonometric function solution (20) and (21) reduce to

$$u(\xi) = \left[\frac{35k^2\delta(\lambda^2 - 4\mu)}{18\cos\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\left(kx + \ell y - \frac{4k^4\delta\lambda^2 - 16k^4\delta\lambda + \frac{9}{2}\gamma\ell^2}{9k}t\right)\right)}\right]^{\frac{2}{3}}, \tag{22}$$

$$u(\xi) = \left[-\frac{35k^2\delta(4\mu - \lambda^2)}{18\cos^2\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\left(kx + \ell y - \frac{4k^4\delta\lambda^2 - 16k^4\delta\lambda + \frac{9}{2}\gamma\ell^2}{9k}t\right)\right)}\right]^{\frac{2}{3}}, \tag{23}$$

and when $C_1 = 0,$ then the general solution (20)-(21) reduce to

$$u(\xi) = \left[-\frac{35k^2\delta(\lambda^2 - 4\mu)}{18\sinh^2\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\left(kx + \ell y - \frac{4k^4\delta\lambda^2 - 16k^4\delta\lambda + \frac{9}{2}\gamma\ell^2}{9k}t\right)\right)}\right]^{\frac{2}{3}}, \tag{24}$$

$$u(\xi) = \left[-\frac{35k^2\delta(4\mu - \lambda^2)}{18\sin^2\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\left(kx + \ell y - \frac{4k^4\delta\lambda^2 - 16k^4\delta\lambda + \frac{9}{2}\gamma\ell^2}{9k}t\right)\right)}\right]^{\frac{2}{3}}. \tag{25}$$

We would like to note that the obtained solutions with an explicit linear function in ξ have been checked with Maple by putting them back into the original Equations (7).

4. Conclusions and Future Work

This study shows that the (G'/G)-expansion method is quite efficient and practically well suited for use in finding exact solutions for the generalized (2 + 1)-dimensional Kadomtsev-Petviashvili (gKP) equation. The reliability of the method and the reduction in the size of computational domain give this method a wider applicability. Though the obtained solutions represent only a small part of the large variety of possible solutions for the equations considered, they might serve as seeding solutions for a class of localized structures existing in the physical systems. Furthermore, our solutions are in more general forms, and many known solutions to these equations are only special cases of them. With the aid of Maple, we have assured the correctness of the obtained solutions by putting them back into the original equation. We hope that they will be useful for further studies in applied sciences. According to Case 5, present method failed to obtain the general solution of gKP for $n=-1$, and $n=-2$, therefore the authors hope to extend the (G'/G)-expansion method to solve these especial type of gKP.

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