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横观各向同性饱和地基中埋置荷载的非轴对称瞬态响应

王小岗

台州学院建筑工程系,台州 318000

摘 要 基于孔隙介质的 Biot 理论,首先利用 Laplace 变换,给出圆柱坐标系下横观各向同性饱和弹性多孔介质在变换域上的波动方程;将波动方程解耦后,根据方位角的 Fourier 展开和径向 Hankel 变换,求解了 Biot 波动方程,得到以土骨架位移、孔隙水压力和土介质总应力分量的积分形式的一般解;借助一般解,建立了有限厚度饱和土层和饱和半空间的精确动力刚度矩阵,并由土层的层间界面连续条件建立三维非轴对称层状饱和地基的总刚度方程;在此基础上,系统研究了横观各向同性饱和半空间体在内部集中荷载激励下的动力响应,并给出了问题的瞬态解答.该研究为运用边界元法求解饱和地基动力响应奠定了理论基础.

关键词 横观各向同性,饱和地基,Biot 波动方程,瞬态解

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The non-axial symmetrical transient response of transversely isotropic saturated soils to internal loading

WANG Xiao-Gang

Department of Civil Engineering and Architecture, Taizhou University, Taizhou 318000, *China*

Abstract Based on Biot′s theory for fluid-saturated porous media, the non-axial symmetrical transient response for transversely isotropic saturated soils under internal loading is studied in this paper. First, the governing differential equations for saturated soils are solved by Laplace transform and operator theory. Then, the general solutions of soil skeleton displacements and pore pressure as well as the total stresses for saturated soils are presented by mean of Fourier expanding and Hankel integral transform. Furthermore, using general solutions, the dynamic stiffness matrix for a layered saturated soils and a saturated half-space are derived exactly, and the global stiffness equation of a multi-layered half-space is assembled by using stiffness matrices and the continuity of tractions and fluid flow at layer interfaces. Finally, the transient foundation solution of transversely isotropic saturated half-space to impulsive concentrated loading is numerically presented. This study provides an effective method to analyze dynamical response between saturated soils and structures by BEM.

Keywords Transversely isotropic, Saturated soils, Biot′s wave equations, Transient solution

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作者简介 王小岗,男,1967 年生,教授,博士,从事土-结构动力相互作用研究工作.E-mail:wangxg824@tzc.edu.cn

1 引　言

饱和地基的瞬态动力响应问题在地震工程学、地球物理学以及土力学等领域有着广泛的应用. Paul[1], Kaynia[2], Chen[3,4] 及 Philippacopoulos[5,6] 讨论了饱和介质在集中力作用下的动力响应问题;黄义等[7]忽略了流体相对于固体骨架的惯性项,忽略土颗粒和孔隙流体可压缩性,研究了饱和多孔介质的非轴对称动力问题;Zhou 等[8,9]研究了饱和半空间体在内部集中荷载作用下的瞬态动力响应问题,并给出了问题的基本解;陈胜立等[10,11]利用一组简化的弹性饱和土波动方程,给出了埋置力源下的 Lamb 问题解答,但该解答仅适用于饱和半空间地基,无法处理层状地基,不具备一般性. 以上对饱和地基研究多限于各向同性饱和土介质. 由于问题的复杂性,针对横观各向同性饱和土动力响应的研究则相对较少,而后者的地基模式可能更符合天然海相或湖相沉积饱和土的物理力学特性. Tanguchi 等[12]给出了横观各向同性饱和半空间内部作用一阶跃点荷载时的基本解. 张引科等[13]给出了横观各向同性饱和介质三维非轴对称稳态响应解,但在处理 Bessel 函数的数值积分时存在不足. 黄义[14]、王小岗[15]系统地研究了横观各向同性饱和多孔介质的三维非轴对称 Lamb 问题,分析了介质的各向异性参数对位移、应力响应和空隙流体压力的影响. 何芳社等[16]借助连续介质理论,讨论了横观各向同性饱和多孔介质的基本方程组的形式,对计算参数的选取做了理论上的说明,但未涉及地基的动力响应问题. 蔡袁强等[17]忽略了土颗粒和孔隙流体的可压缩性,分析了上覆弹性土层横观各向同性饱和地基竖向振动特性,但仅限于轴对称情形. 针对埋置力源下,横观各向同性饱和地基的非轴对称瞬态动力响应问题的一般解法的研究,则尚未见文献报道,需要进一步研究.

本文研究内部激励下横观各向同性饱和地基的非轴对称瞬态动力响应问题的一般解法. 基于 Laplace 变换域上的横观各向同性饱和介质的三维波动方程,借助算子理论、Fourier 展开和 Hankel 变换技术,求解波动方程,得到方程的一般解,及有限厚度饱和土层和饱和半空间的精确动力刚度矩阵,并结合层间连续条件,建立饱和地基的总刚方程,进而系统研究了横观各向同性饱和地基的三维瞬态动力响应问题. 本文方法,可以方便地处理土层内各类荷载的影响,因而具有普遍性.

2 横观各向同性饱和土动力方程及通解

在圆柱坐标系下,取 z 轴沿介质对称轴方向,r-θ 平面平行于介质的各向同性平面,则横观各向同性饱和土介质的 Biot 动力方程表示为[14,15]

$$C_{66}\left[\nabla^2 u_r-\frac{1}{r}\left(2\frac{\partial u_\theta}{r\partial\theta}+\frac{u_r}{r}\right)\right]+C_{44}\frac{\partial^2 u_r}{\partial z^2}+(C_{66}+C_{12})\frac{\partial e}{\partial r}+(C_{13}+C_{44})\frac{\partial}{\partial r}\frac{\partial u_z}{\partial z}+\chi_1\frac{\partial p_{\mathrm f}}{\partial r}=\rho\ddot u_r+\rho_{\mathrm f}\ddot w_r\ , \tag{1}$$

$$C_{66}\left[\nabla^2 u_\theta-\frac{1}{r}\left(-2\frac{\partial u_r}{r\partial\theta}+\frac{u_\theta}{r}\right)\right]+C_{44}\frac{\partial^2 u_\theta}{\partial z^2}+(C_{66}+C_{12})\frac{\partial e}{r\partial\theta}+(C_{13}+C_{44})\frac{\partial}{r\partial\theta}\frac{\partial u_z}{\partial z}+\chi_1\frac{\partial p_{\mathrm f}}{r\partial\theta}=\rho\ddot u_\theta+\rho_{\mathrm f}\ddot w_\theta, \tag{2}$$

$$C_{44}\nabla^2 u_z+(C_{13}+C_{44})\frac{\partial e}{\partial z}+C_{33}\frac{\partial^2 u_z}{\partial z^2}+\chi_3\frac{\partial p_{\mathrm f}}{\partial z}=\rho\ddot u_z+\rho_{\mathrm f}\ddot w_z. \tag{3}$$

孔隙流体运动方程表示为

$$-\frac{\partial p_{\mathrm f}}{\partial r}=\rho_{\mathrm f}\ddot u_r+\frac{\rho_{\mathrm f}}{\phi}\ddot w_r+\frac{\rho_{\mathrm f}g}{k_{\mathrm{d1}}}\dot w_r, \tag{4}$$

$$-\frac{\partial p_{\mathrm f}}{r\partial\theta}=\rho_{\mathrm f}\ddot u_\theta+\frac{\rho_{\mathrm f}}{\phi}\ddot w_\theta+\frac{\rho_{\mathrm f}g}{k_{\mathrm{d1}}}\dot w_\theta, \tag{5}$$

$$-\frac{\partial p_{\mathrm f}}{\partial z}=\rho_{\mathrm f}\ddot u_z+\frac{\rho_{\mathrm f}}{\phi}\ddot w_z+\frac{\rho_{\mathrm f}g}{k_{\mathrm{d3}}}\dot w_z. \tag{6}$$

土骨架本构关系表示为

$$\begin{Bmatrix}\sigma_r\\ \sigma_\theta\\ \sigma_z\\ \tau_{\theta z}\\ \tau_{rz}\\ \tau_{r\theta}\end{Bmatrix}=\begin{bmatrix}C_{11}&C_{12}&C_{13}&0&0&0\\ C_{12}&C_{11}&C_{13}&0&0&0\\ C_{13}&C_{13}&C_{33}&0&0&0\\ 0&0&0&2C_{44}&0&0\\ 0&0&0&0&2C_{44}&0\\ 0&0&0&0&0&2C_{66}\end{bmatrix}\begin{Bmatrix}e_r\\ e_\theta\\ e_z\\ e_{\theta z}\\ e_{rz}\\ e_{r\theta}\end{Bmatrix}. \tag{7}$$

考虑有效应力原理及土骨架的几何方程后,饱和土介质的连续性方程可表示为

$$\vartheta p_{\mathrm f}=\chi_1 e+\chi_2\left(\frac{\partial u_z}{\partial z}\right)-\mathrm{div}\boldsymbol{w}, \tag{8}$$

式中 $\chi_1=-[1-(C_{11}+C_{12}+C_{13})/(3K_{\mathrm s})]$,

$\chi_3=-[1-2(C_{13}+C_{33})/(3K_{\mathrm s})]$,

$\vartheta=(1-\phi)/K_{\mathrm s}+\phi/K_{\mathrm l}-(2C_{11}+2C_{12}+4C_{13}+C_{33})/(9K_{\mathrm s}^2)$,

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{\partial}{r\partial r}+\frac{\partial^2}{r^2\partial\theta^2},\ e=\frac{\partial u_r}{\partial r}+\frac{1}{r}\frac{\partial u_\theta}{\partial\theta}+\frac{u_r}{r}.$$

u_r,u_θ 和 u_z 分别为饱和土骨架的径向、周向和竖向的位移分量；w_r,w_θ 和 w_z 分别为孔隙水的相对径向、周向和竖向位移分量；p_f 是孔隙水压力；ρ 是饱和介质固-液混合密度，$\rho=(1-\phi)\rho_s+\phi\rho_f$，$\rho_f$ 是孔隙流体密度，ρ_s 是土骨架材料密度，ϕ 是多孔介质的孔隙率；$k_{dj}(j=1,\ 3)$分别表示土体平行和垂直于各向同性面的动力渗透系数；g 是重力加速度；C_{ij} 是横观各向同性土介质的弹性常数，且满足关系 $C_{66}=(C_{11}-C_{12})/2$；K_s 为固体颗粒体积模量，K_l 为孔隙水体积模量.

引入无量纲参数和无量纲变量，即

$$r=r_0\tilde r,\ z=r_0\tilde z,\ u_i=r_0\tilde u_i,\ w_i=r_0\tilde w_i,$$

$$\tilde e=\frac{\partial\tilde u_r}{\partial\tilde r}+\frac{1}{\tilde r}\frac{\partial\tilde u_\theta}{\partial\theta}+\frac{\tilde u_r}{\tilde r},\ \rho_f=\rho\tilde\rho_f,$$

$$p_f=C_{44}\tilde p_f,\ \sigma_{ij}=C_{44}\tilde\sigma_{ij},\ t=r_0\tilde t\sqrt{\frac{\rho}{C_{44}}},$$

$$k_{d1}^2=\frac{r_0^2g^2}{\tilde k_1^2}\frac{\rho}{C_{44}},\ k_{d3}^2=\frac{r_0^2g^2}{\tilde k_3^2}\frac{\rho}{C_{44}},$$

$$\Lambda_1=C_{33}/C_{44},\ \Lambda_2=C_{11}/C_{44},$$

$$\Lambda_3=C_{13}/C_{44}+1,\ \Lambda_4=C_{12}/C_{44}.$$

定义变量 $\tilde t$ 的 Laplace 变换，即

$$\hat f(\tilde r,\tilde z,s)=\int_0^\infty f(\tilde r,\tilde z,\tilde t)\mathrm e^{-s\tilde t}\mathrm d\tilde t,\tag{9}$$

对方程(1)～(8)无量纲化，并施加变换(9)，得到

$$\frac12(\Lambda_2-\Lambda_4)\left[\nabla^2\hat u_r-\frac{1}{\tilde r}\left(2\frac{\partial\hat u_\theta}{\tilde r\partial\theta}+\frac{\hat u_r}{\tilde r}\right)\right]+\frac{\partial^2\hat u_r}{\partial\tilde z^2}+\frac12(\Lambda_2+\Lambda_4)\frac{\partial\hat e}{\partial\tilde r}+s^2(-1+\tilde\rho_f c_1)\hat u_r+\Lambda_3\frac{\partial}{\partial\tilde r}\frac{\partial\hat u_z}{\partial\tilde z}+b_6\frac{\partial\hat p_f}{\partial\tilde r}=0,\tag{10}$$

$$\frac12(\Lambda_2-\Lambda_4)\left[\nabla^2\hat u_\theta-\frac{1}{\tilde r}\left(-2\frac{\partial\hat u_r}{\tilde r\partial\theta}+\frac{\hat u_\theta}{\tilde r}\right)\right]+\frac{\partial^2\hat u_\theta}{\partial\tilde z^2}+\frac12(\Lambda_2+\Lambda_4)\frac{\partial\hat e}{\tilde r\partial\theta}+s^2(-1+\tilde\rho_f c_1)\hat u_\theta+\Lambda_3\frac{\partial}{\tilde r\partial\theta}\frac{\partial\hat u_z}{\partial\tilde z}+b_6\frac{\partial\hat p_f}{\tilde r\partial\theta}=0,\tag{11}$$

$$\nabla^2\hat u_z+\Lambda_1\frac{\partial^2\hat u_z}{\partial\tilde z^2}+s^2(-1+\tilde\rho_f c_3)\hat u_z+\Lambda_3\frac{\partial\hat e}{\partial\tilde z}+b_7\frac{\partial\hat p_f}{\partial\tilde z}=0,\tag{12}$$

$$b_6\hat e+b_7\frac{\partial\hat u_z}{\partial\tilde z}+\frac{1}{s^2\tilde\rho_f}\left(c_1\nabla^2\hat p_f+c_3\frac{\partial^2\hat p_f}{\partial\tilde z^2}\right)-b_8\hat p_f=0,\tag{13}$$

以及孔隙水相对位移关系，即

$$\hat w_r=c_1\left(-\hat u_r-\frac{1}{s^2\tilde\rho_f}\frac{\partial\hat p_f}{\partial\tilde r}\right),\tag{14}$$

$$\hat w_\theta=c_1\left(-\hat u_\theta-\frac{1}{s^2\tilde\rho_f}\frac{\partial\hat p_f}{\tilde r\partial\theta}\right),\tag{15}$$

$$\hat w_z=c_3\left(-\tilde u_z-\frac{1}{s^2\tilde\rho_f}\frac{\partial\hat p_f}{\partial\tilde z}\right),\tag{16}$$

式中 $b_6=\chi_1+c_1$，$b_7=\chi_3+c_3$，$b_8=C_{44}\vartheta$，

$$c_1=\frac{\phi s}{\phi\tilde k_1+s},\ c_3=\frac{\phi s}{\phi\tilde k_3+s}.$$

对方程(10)和(11)做运算 $\left(\frac{\partial}{\partial\tilde r}+\frac{1}{\tilde r}\right)(10)+\frac{\partial}{\tilde r\partial\theta}(11)$，同时，定义以下算子：

$$\nabla_e^2=\Lambda_2\tilde\nabla^2+\frac{\partial^2}{\partial\tilde z^2}+s^2(-1+\tilde\rho_f c_1),$$

$$\nabla_u^2=\tilde\nabla^2+\Lambda_1\frac{\partial^2}{\partial\tilde z^2}+s^2(-1+\tilde\rho_f c_3),$$

$$\nabla_p^2=\frac{1}{s^2\tilde\rho_f}\left(c_1\tilde\nabla^2+c_3\frac{\partial^2}{\partial\tilde z^2}\right)-b_8,$$

$$\tilde\nabla^2=\frac{\partial^2}{\partial\tilde r^2}+\frac{\partial}{\tilde r\partial\tilde r}+\frac{\partial^2}{\tilde r^2\partial\theta^2},$$

则方程(10)～(12)可简化为如下形式：

$$\boldsymbol D\begin{Bmatrix}\hat e\\ \hat u_z\\ \hat p_f\end{Bmatrix}=0,\tag{17}$$

式中，$\boldsymbol D$ 为算子矩阵，可表示为

$$\boldsymbol D=\begin{bmatrix}\nabla_e^2 & \Lambda_3\frac{\partial}{\partial\tilde z}\tilde\nabla^2 & b_6\tilde\nabla^2\\ \Lambda_3\frac{\partial}{\partial\tilde z} & \nabla_u^2 & b_7\frac{\partial}{\partial\tilde z}\\ b_6 & b_7\frac{\partial}{\partial\tilde z} & \nabla_p^2\end{bmatrix},$$

同时

$$|\boldsymbol D|=\nabla_e^2\nabla_u^2\nabla_p^2-b_6^2\nabla_u^2\tilde\nabla^2+2b_6b_7\Lambda_3\frac{\partial^2}{\partial\tilde z^2}\tilde\nabla^2-\Lambda_3^2\frac{\partial^2}{\partial\tilde z^2}\nabla_p^2\tilde\nabla^2-b_7^2\frac{\partial^2}{\partial\tilde z^2}\nabla_e^2.\tag{18}$$

利用算子理论，可以得到如下形式通解：

$$\hat e=A_{i1}\hat F,\ \hat u_z=A_{i2}\hat F,\ \hat p_f=A_{i3}\hat F\quad(i=1,2,3)\tag{19}$$

其中，A_{ij} 为矩阵 $\boldsymbol D$ 的代数余子式，且函数 $\hat F$ 满足下列方程

$$|\boldsymbol D|\hat F=0.\tag{20}$$

经研究，如果 i 取 1 或 2，当 $\hat p_f=0$，即对于单相横观各向同性土介质，此时只能得到 $\hat e$ 和 $\hat u_z$ 的零解，因此，不失一般性，取 $i=3$. 得到方程(17)的通解，即

$$\hat e=\left(\Lambda_3b_7\frac{\partial^2}{\partial\tilde z^2}-b_6\nabla_u^2\right)\hat F,\tag{21}$$

$$\hat{u}_z = (b_6\Lambda_3\tilde{\nabla}^2 - b_7\nabla_e^2)\frac{\partial \hat{F}}{\partial \tilde{z}}, \tag{22}$$

$$\hat{p}_f = \left(\nabla_e^2\nabla_u^2 - \Lambda_3^2\nabla^2\frac{\partial^2}{\partial \tilde{z}^2}\right)\hat{F}. \tag{23}$$

对相关变量沿周向进行 Fourier 展开，并记

$$[\hat{F},\hat{e},\hat{u}_z,\hat{u}_r,\hat{p}_f]^T = \sum_{n=0}^{\infty}[\hat{F}_n,\hat{e}_n,\hat{u}_{zn},\hat{u}_{rn},\hat{p}_{fn}]^T\cos n\theta$$

$$\hat{u}_\theta = \sum_{n=0}^{\infty}\hat{u}_{\theta n}\sin n\theta\ , \tag{24}$$

同时引入以下 Hankel 变换：

$$[\bar{F}_n,\bar{e}_n,\bar{u}_{zn},\bar{p}_{fn}]^T = \int_0^\infty[\hat{F}_n,\hat{e}_n,\hat{u}_{zn},\hat{p}_{fn}]^T J_n(k\tilde{r})\tilde{r}d\tilde{r}, \tag{25a}$$

$$\bar{U}_n = \int_0^\infty(\hat{u}_{rn}+\hat{u}_{\theta n})J_{n+1}(k\tilde{r})\tilde{r}d\tilde{r}, \tag{25b}$$

$$\bar{V}_n = \int_0^\infty(\hat{u}_{rn}-\hat{u}_{\theta n})J_{n-1}(k\tilde{r})\tilde{r}d\tilde{r}, \tag{25c}$$

式中，$J_n(k\tilde{r})$为第一类 n 阶 Bessel 函数.

将式(24)代入式(20)，并代以(25a)式,得到以下方程：

$$\left(\frac{\partial^6}{\partial\tilde{z}^6}+a_1\frac{\partial^4}{\partial\tilde{z}^4}+a_2\frac{\partial^2}{\partial\tilde{z}^2}+a_3\right)\bar{F}_n = 0, \tag{26}$$

式中

$$a_1 = [\beta_3(b_2+\Lambda_3^2k^2+\Lambda_1b_1)-\Lambda_1b_3-b_7^2]/(\beta_3\Lambda_1),$$
$$a_2 = \{b_1b_2\beta_3 - b_3[\Lambda_1b_1+b_2+\Lambda_3^2k^2] + b_6^2k^2\Lambda_1 - 2b_6b_7\Lambda_3k^2 - b_1b_7^2\}/(\beta_3\Lambda_1),$$
$$a_3 = (b_2b_6^2k^2 - b_1b_2b_3)/(\beta_3\Lambda_1).$$

其中 $\beta_1 = c_1/(s^2\tilde{\rho}_f)$，$\beta_3 = c_3/(s^2\tilde{\rho}_f)$，

$b_1 = f_1 - k^2\Lambda_2$，$b_2 = f_3 - k^2$，$b_3 = \beta_1k^2 + b_8$，

$f_1 = s^2(-1+\tilde{\rho}_fc_1)$，$f_3 = s^2(-1+\tilde{\rho}_fc_3)$.

方程(26)的解可表示为

$$\bar{F}_n = \sum_{s=1}^{3}(A_{sn}e^{\lambda_s\tilde{z}}+B_{sn}e^{-\lambda_s\tilde{z}}), \tag{27}$$

式中 A_{in}、$B_{in}(i=1,2,3)$为与 z 无关的待定常数，$\lambda_i(\mathrm{Re}[\lambda_i]\geqslant 0, i=1,2,3)$是满足如下代数方程的根，

$$\lambda^6 + a_1\lambda^4 + a_2\lambda^2 + a_3 = 0. \tag{28}$$

将式(24)代入式(21)～(23)，并代以(25a)式，再代以式(27),得到

$$\bar{e}_n = \sum_{s=1}^{3}\alpha_s(A_{sn}e^{\lambda_s\tilde{z}}+B_{sn}e^{-\lambda_s\tilde{z}}), \tag{29}$$

$$\bar{u}_{zn} = \sum_{s=1}^{3}u_s(A_{sn}e^{\lambda_s\tilde{z}}-B_{sn}e^{-\lambda_s\tilde{z}}), \tag{30}$$

$$\bar{p}_{fn} = \sum_{s=1}^{3}(A_{sn}e^{\lambda_s\tilde{z}}+B_{sn}e^{-\lambda_s\tilde{z}}), \tag{31}$$

式中系数记为

$$\alpha_s = [(\Lambda_3b_7-\Lambda_1b_6)\lambda_s^2 - b_2b_6]/\Delta_s,$$
$$u_s = -(b_1b_7+\Lambda_3b_6k^2+b_7\lambda_s^2)\lambda_s/\Delta_s,$$
$$\Delta_s = \Lambda_1\lambda_s^4 + (b_1\Lambda_1+b_2+\Lambda_3^2k^2)\lambda_s^2 + b_1b_2.$$

再将式(24)代入式(10)～(11),并代以(25b, 25c)式,同时注意到 $\hat{e}_n$ 的 n 阶 Hankel 变换式

$$\bar{e}_n = (k\bar{U}_n - k\bar{V}_n)/2, \tag{32}$$

得到

$$\bar{U}_n = \sum_{s=1}^{3}d_s(A_{sn}e^{\lambda_s\tilde{z}}+B_{sn}e^{-\lambda_s\tilde{z}}) + (A_{4n}e^{\lambda_0\tilde{z}}+B_{4n}e^{-\lambda_0\tilde{z}}), \tag{33}$$

$$\bar{V}_n = -\sum_{s=1}^{3}d_s(A_{sn}e^{\lambda_s\tilde{z}}+B_{sn}e^{-\lambda_s\tilde{z}}) + (A_{4n}e^{\lambda_0\tilde{z}}+B_{4n}e^{-\lambda_0\tilde{z}}), \tag{34}$$

式中 $\lambda_0^2 = -f_1 + (\Lambda_2-\Lambda_4)k^2/2$，

$$d_s = \frac{k(\Lambda_2+\Lambda_4)\alpha_s + 2\Lambda_3u_s\lambda_sk + 2b_6k}{2(\lambda_s^2-\lambda_0^2)}.$$

同上做法,利用式(14)～(16) 可得到孔隙水在 Hankel 变换域上的相对位移分量,即

$$\bar{w}_{zn} = \sum_{s=1}^{3}w_s(A_{sn}e^{\lambda_s\tilde{z}}-B_{sn}e^{-\lambda_s\tilde{z}}), \tag{35}$$

$$\bar{U}_n^w = \sum_{s=1}^{3}U_s(A_{sn}e^{\lambda_s\tilde{z}}+B_{sn}e^{-\lambda_s\tilde{z}}) - c_1(A_{4n}e^{\lambda_0\tilde{z}}+B_{4n}e^{-\lambda_0\tilde{z}}), \tag{36}$$

$$\bar{V}_n^w = -\sum_{s=1}^{3}U_s(A_se^{\lambda_s\tilde{z}}+B_se^{-\lambda_s\tilde{z}}) - c_1(A_{4n}e^{\lambda_0\tilde{z}}+B_{4n}e^{-\lambda_0\tilde{z}}), \tag{37}$$

式中变量定义为

$$\bar{w}_{zn} = \int_0^\infty\hat{w}_{zn}J_n(k\tilde{r})\tilde{r}d\tilde{r},$$

$$\bar{U}_n^w = \int_0^\infty(\hat{w}_{rn}+\hat{w}_{\theta n})J_{n+1}(k\tilde{r})\tilde{r}d\tilde{r},$$

$$\bar{V}_n^w = \int_0^\infty(\hat{w}_{rn}-\hat{w}_{\theta n})J_{n-1}(k\tilde{r})\tilde{r}d\tilde{r},$$

系数 $w_s = -c_3u_s - \beta_3\lambda_s$，

$U_s = \beta_1k - c_1d_s\ (s=1,2,3)$.

对相关应力分量进行 Fourier 展开，记

$$[\hat{\sigma}_z,\hat{\tau}_{rz}]^T = \sum_{n=0}^{\infty}[\hat{\sigma}_{zn},\hat{\tau}_{rzn}]^T\cos n\theta, \tag{38}$$

$$\hat{\tau}_{\theta z} = \sum_{n=0}^{\infty}\hat{\tau}_{\theta zn}\sin n\theta, \tag{39}$$

施加相应阶 Hankel 变换后,得到垂直于 z 轴平面上的应力分量的一般解，即

$$\bar{\sigma}_{zn} = \sum_{s=1}^{3}\xi_s(A_{sn}e^{\lambda_s\tilde{z}}+B_{sn}e^{-\lambda_s\tilde{z}}), \tag{40}$$

$$\overline{T}_{1n} = \sum_{s=1}^{3}\zeta_s(A_{sn}e^{\lambda_s\tilde{z}} - B_{sn}e^{-\lambda_s\tilde{z}}) + \lambda_0(A_{4n}e^{\lambda_0\tilde{z}} - B_{4n}e^{-\lambda_0\tilde{z}}), \quad (41)$$

$$\overline{T}_{2n} = -\sum_{s=1}^{3}\zeta_s(A_{sn}e^{\lambda_s\tilde{z}} - B_{sn}e^{-\lambda_s\tilde{z}}) + \lambda_0(A_{4n}e^{\lambda_0\tilde{z}} - B_{4n}e^{-\lambda_0\tilde{z}}). \quad (42)$$

式(40)～(42)中变量定义为

$$\bar{\sigma}_{zn} = \int_0^{\infty}\hat{\sigma}_{zn}J_n(k\tilde{r})\tilde{r}d\tilde{r},$$

$$\overline{T}_{1n} = \int_0^{\infty}(\hat{\tau}_{rzn} + \hat{\tau}_{r\theta n})J_{n+1}(k\tilde{r})\tilde{r}d\tilde{r},$$

$$\overline{T}_{2n} = \int_0^{\infty}(\hat{\tau}_{rzn} - \hat{\tau}_{r\theta n})J_{n-1}(k\tilde{r})\tilde{r}d\tilde{r}.$$

另外$\xi_s = (\Lambda_3 - 1)k\alpha_s + \Lambda_1 u_s\lambda_s + \chi_3$,

$\zeta_s = d_s\lambda_s - ku_s \ (s = 1,2,3).$

至此,得到横观各向同性饱和土三维非轴对称动力响应的积分变换解.相应表达式中的待定常数A_m、$B_m(m = 1,2,3,4)$可由适当的边界条件和连续性条件求得.

特别地,对于各向同性饱和土介质,

$C_{12} = C_{13} = \lambda,\ C_{11} = C_{33} = \lambda + 2\mu,$

$C_{44} = C_{66} = \mu,$

其中,λ、μ为土介质Lame系数.若忽略孔隙水和固体颗粒的压缩性,则$\chi_1 = \chi_3 = -1, \vartheta = 0, \mathrm{div}\boldsymbol{u} = \mathrm{div}\boldsymbol{w}$. 进一步,若只考虑饱和土的低频振动,即忽略孔隙水相对于固体骨架的惯性运动,令$\ddot{\boldsymbol{w}} = 0$,同时注意到$\eta/k' = \rho_f g/k_d$,$k'$为饱和土介质渗透系数,$\tilde{k}_1 = \tilde{k}_3 = \tilde{k}$,经过较为繁复的运算,方程(28)的根$\lambda_i$及$\lambda_0$可表示为

$$\lambda_1^2 = k^2,\ \lambda_2^2 = \lambda_0^2 = k^2 + s^2,$$

$$\lambda_3^2 = k^2 + \frac{\mu}{\lambda + 2\mu}(\tilde{\rho}_f\tilde{k}s - \tilde{\rho}_f s^2 + s^2).$$

注意到各参数的无量纲转换关系,以及$s = \mathrm{i}\omega$,ω为谐振圆频率,则以上3式与文献[7]结论完全一致.

3 饱和土层的动力刚度矩阵和刚度方程

3.1 有限厚度饱和土层的动力刚度矩阵

建立有限厚度饱和土层的动力刚度矩阵,是分析内部激励荷载作用下饱和地基动力响应的有效途径.为推导方便,同时便于区分地基平面内和平面外运动,令

$$\bar{u}_{rn} + \bar{u}_{\theta n} = \overline{U}_n, \bar{u}_{\theta n} - \bar{u}_{rn} = \overline{V}_n, \quad (43)$$

$$\bar{\tau}_{rzn} + \bar{\tau}_{\theta zn} = \overline{T}_{1n}/C_{44}, \bar{\tau}_{\theta zn} - \bar{\tau}_{rzn} = \overline{T}_{2n}/C_{44}, \quad (44)$$

可以得到

$$\bar{u}_{rn} = \sum_{s=1}^{3}d_s(A_{sn}e^{\lambda_s\tilde{z}} + B_{sn}e^{-\lambda_s\tilde{z}}), \quad (45)$$

$$\bar{u}_{\theta n} = A_{4n}e^{\lambda_0\tilde{z}} + B_{4n}e^{-\lambda_0\tilde{z}}, \quad (46)$$

$$\bar{\tau}_{rzn} = \sum_{s=1}^{3}\zeta_s(A_{sn}e^{\lambda_s\tilde{z}} - B_{sn}e^{-\lambda_s\tilde{z}}), \quad (47)$$

$$\bar{\tau}_{\theta zn} = \lambda_0(A_{4n}e^{\lambda_0\tilde{z}} - B_{4n}e^{-\lambda_0\tilde{z}}). \quad (48)$$

以位于$z \geqslant 0$的横观各向同性饱和地基为研究对象,设地基内第i土层的上、下界面坐标分别为$\tilde{z} = \tilde{z}_i$和$\tilde{z} = \tilde{z}_{i+1}$,层厚$\tilde{h}_i = \tilde{z}_{i+1} - \tilde{z}_i$.

定义上、下界面的土介质位移和孔隙水相对位移向量$\{\bar{\boldsymbol{d}}\}_i$,即

$$\{\bar{\boldsymbol{d}}\}_i = [\{\bar{\boldsymbol{d}}\}_i^{\mathrm{u}}, \{\bar{\boldsymbol{d}}\}_i^{\mathrm{b}}]^{\mathrm{T}},$$

$$\{\bar{\boldsymbol{d}}\}_i^{\mathrm{u}} = [\bar{u}_{rn}(k,s,\tilde{z}_i), \bar{u}_{zn}(k,s,\tilde{z}_i), \bar{w}_{zn}(k,s,\tilde{z}_i)]^{\mathrm{T}},$$

$$\{\bar{\boldsymbol{d}}\}_i^{\mathrm{b}} = [\bar{u}_{rn}(k,s,\tilde{z}_{i+1}), \bar{u}_{zn}(k,s,\tilde{z}_{i+1}), \bar{w}_{zn}(k,s,\tilde{z}_{i+1})]^{\mathrm{T}}.$$

应力和孔隙水压力向量为$\{\bar{\boldsymbol{\sigma}}\}_i$,

$$\{\bar{\boldsymbol{\sigma}}\}_i = [\{\bar{\boldsymbol{\sigma}}\}_i^{\mathrm{u}}, \{\bar{\boldsymbol{\sigma}}\}_i^{\mathrm{b}}]^{\mathrm{T}},$$

$$\{\bar{\boldsymbol{\sigma}}\}_i^{\mathrm{u}} = [-\bar{\tau}_{rzn}(k,s,\tilde{z}_i), -\bar{\sigma}_{zn}(k,s,\tilde{z}_i), \bar{p}_{fn}(k,s,\tilde{z}_i)]^{\mathrm{T}},$$

$$\{\bar{\boldsymbol{\sigma}}\}_i^{\mathrm{b}} = [\bar{\tau}_{rzn}(k,s,\tilde{z}_{i+1}), \bar{\sigma}_{zn}(k,s,\tilde{z}_{i+1}), -\bar{p}_{fn}(k,s,\tilde{z}_{i+1})]^{\mathrm{T}}.$$

这样,i土层上、下界面的位移和应力可表示为

$$\{\bar{\boldsymbol{d}}\}_i = [\overline{\boldsymbol{D}}]_i\{\boldsymbol{C}\}_i, \quad (49)$$

$$\{\bar{\boldsymbol{\sigma}}\}_i = [\overline{\boldsymbol{S}}]_i\{\boldsymbol{C}\}_i, \quad (50)$$

这里

$$[\overline{\boldsymbol{D}}]_i = \begin{bmatrix}[\overline{\boldsymbol{D}}_{11}]_i & [\overline{\boldsymbol{D}}_{12}]_i \\ [\overline{\boldsymbol{D}}_{21}]_i & [\overline{\boldsymbol{D}}_{22}]_i\end{bmatrix},$$

$$[\overline{\boldsymbol{S}}]_i = \begin{bmatrix}[\overline{\boldsymbol{S}}_{11}]_i & [\overline{\boldsymbol{S}}_{12}]_i \\ [\overline{\boldsymbol{S}}_{21}]_i & [\overline{\boldsymbol{S}}_{22}]_i\end{bmatrix},$$

$$\{\boldsymbol{C}\}_i = \{A_{1n}t_1^1, B_{1n}t_{-1}^1, A_{2n}t_1^2, B_{2n}t_{-1}^2, A_{3n}t_1^3, B_{3n}t_{-1}^3\}^{\mathrm{T}},$$

$$t_1^s = e^{\lambda_s\tilde{z}_i},\ t_{-1}^s = e^{-\lambda_s\tilde{z}_i} \quad (s = 1,2,3)$$

$$[\overline{\boldsymbol{D}}_{11}]_i = \begin{bmatrix}d_1 & d_1 & d_2 \\ u_1 & -u_1 & u_2 \\ w_1 & -w_1 & w_2\end{bmatrix},$$

$$[\overline{\boldsymbol{D}}_{12}]_i = \begin{bmatrix}d_2 & d_3 & d_3 \\ -u_2 & u_3 & -u_3 \\ -w_2 & w_3 & -w_3\end{bmatrix},$$

$$[\overline{\boldsymbol{D}}_{21}]_i = \begin{bmatrix}d_1e^{\lambda_1\tilde{h}_i} & d_1e^{-\lambda_1\tilde{h}_i} & d_2e^{\lambda_2\tilde{h}_i} \\ u_1e^{\lambda_1\tilde{h}_i} & -u_1e^{-\lambda_1\tilde{h}_i} & u_2e^{\lambda_2\tilde{h}_i} \\ w_1e^{\lambda_1\tilde{h}_i} & -w_1e^{-\lambda_1\tilde{h}_i} & w_2e^{\lambda_2\tilde{h}_i}\end{bmatrix},$$

$$[\overline{\boldsymbol{D}}_{22}]_i = \begin{bmatrix}d_2e^{-\lambda_2\tilde{h}_i} & d_3e^{\lambda_3\tilde{h}_i} & d_3e^{-\lambda_3\tilde{h}_i} \\ -u_2e^{-\lambda_2\tilde{h}_i} & u_3e^{\lambda_3\tilde{h}_i} & -u_3e^{-\lambda_3\tilde{h}_i} \\ -w_2e^{-\lambda_2\tilde{h}_i} & w_3e^{\lambda_3\tilde{h}_i} & -w_3e^{-\lambda_3\tilde{h}_i}\end{bmatrix},$$

$$[\bar{S}_{11}]_i = \begin{bmatrix} -\zeta_1 & \zeta_1 & -\zeta_2 \\ -\xi_1 & -\xi_1 & -\xi_2 \\ 1 & 1 & 1 \end{bmatrix},$$

$$[\bar{S}_{12}]_i = \begin{bmatrix} \zeta_2 & -\zeta_3 & \zeta_3 \\ -\xi_2 & -\xi_3 & -\xi_3 \\ 1 & 1 & 1 \end{bmatrix},$$

$$[\bar{S}_{21}]_i = \begin{bmatrix} \zeta_1 e^{\lambda_1 \tilde{h}_i} & -\zeta_1 e^{-\lambda_1 \tilde{h}_i} & \zeta_2 e^{\lambda_2 \tilde{h}_i} \\ \xi_1 e^{\lambda_1 \tilde{h}_i} & \xi_1 e^{-\lambda_1 \tilde{h}_i} & \xi_2 e^{\lambda_2 \tilde{h}_i} \\ -e^{\lambda_1 \tilde{h}_i} & -e^{-\lambda_1 \tilde{h}_i} & -e^{\lambda_2 \tilde{h}_i} \end{bmatrix},$$

$$[\bar{S}_{22}]_i = \begin{bmatrix} -\zeta_2 e^{-\lambda_2 \tilde{h}_i} & \zeta_3 e^{\lambda_3 \tilde{h}_i} & -\zeta_3 e^{-\lambda_3 \tilde{h}_i} \\ \xi_2 e^{-\lambda_2 \tilde{h}_i} & \xi_3 e^{\lambda_3 \tilde{h}_i} & \xi_3 e^{-\lambda_3 \tilde{h}_i} \\ -e^{-\lambda_2 \tilde{h}_i} & -e^{\lambda_3 \tilde{h}_i} & -e^{-\lambda_3 \tilde{h}_i} \end{bmatrix}.$$

将式(49)代入式(50)，得到

$$\{\bar{\boldsymbol{\sigma}}\}_i = [\bar{\boldsymbol{K}}]_i \{\bar{\boldsymbol{d}}\}_i, \tag{51}$$

$$[\bar{\boldsymbol{K}}]_i = [\bar{\boldsymbol{S}}]_i [\bar{\boldsymbol{D}}]_i^{-1}. \tag{52}$$

式(51)即为有限厚度的横观各向同性饱和土层第 i 层的动力方程，$[\bar{\boldsymbol{K}}]_i$ 为刚度矩阵. 借助计算软件，可以方便地计算 $[\bar{\boldsymbol{K}}]_i$，限于篇幅，本文不再列出.

对平面外运动，则有

$$\begin{Bmatrix} -\bar{\sigma}_{\theta zn}(k,s,z_i) \\ \bar{\sigma}_{\theta zn}(k,s,z_{i+1}) \end{Bmatrix} = \frac{\lambda_0}{\mathrm{sh}\lambda_0 \tilde{h}_i} \times \begin{bmatrix} \mathrm{ch}\lambda_0 \tilde{h} & -1 \\ -1 & \mathrm{ch}\lambda_0 \tilde{h} \end{bmatrix} \begin{Bmatrix} \bar{u}_{\theta n}(k,s,z_i) \\ \bar{u}_{\theta n}(k,s,z_{i+1}) \end{Bmatrix}. \tag{53}$$

3.2　饱和半空间的动力刚度矩阵

设饱和地基中，第 N 层为半空间体，其上界面坐标为 z_N. 应用波能辐射条件，得到

$$\{\bar{\boldsymbol{\sigma}}\}_N = [\bar{\boldsymbol{K}}]_N \{\bar{\boldsymbol{d}}\}_N, \tag{54}$$

其中

$$\{\bar{\boldsymbol{d}}\}_N = [\bar{u}_{rn}(k,s,\tilde{z}_N), \bar{u}_{zn}(k,s,\tilde{z}_N), \bar{w}_{zn}(k,s,\tilde{z}_N)]^{\mathrm{T}},$$

$$\{\bar{\boldsymbol{\sigma}}\}_N = [\bar{\tau}_{rzn}(k,s,\tilde{z}_N), \bar{\sigma}_{zn}(k,s,\tilde{z}_N), \bar{p}_{fn}(k,s,\tilde{z}_N)]^{\mathrm{T}},$$

$[\bar{\boldsymbol{K}}]_N$ 为 3×3 矩阵，其元素可表示为

$$\bar{k}_{11} = \zeta_1 l_1 + \zeta_2 l_2 + \zeta_3 l_3,$$

$$\bar{k}_{12} = -\zeta_1 m_1 + \zeta_2 m_2 + \zeta_3 m_3,$$

$$\bar{k}_{13} = -\zeta_1 n_1 - \zeta_2 n_2 - \zeta_3 n_3,$$

$$\bar{k}_{21} = -\xi_1 l_1 - \xi_2 l_2 - \xi_3 l_3,$$

$$\bar{k}_{22} = \xi_1 m_1 + \xi_2 m_2 + \xi_3 m_3,$$

$$\bar{k}_{23} = \xi_1 n_1 + \xi_2 n_2 + \xi_3 n_3,\ \bar{k}_{31} = -l_1 - l_2 - l_3,$$

$$\bar{k}_{32} = -m_1 - m_2 - m_3,\ \bar{k}_{33} = n_1 + n_2 + n_3.$$

其中

$$l_1 = (u_3 w_2 - u_2 w_3)/\Delta,\ l_2 = (u_1 w_3 - u_3 w_1)/\Delta,$$

$$l_3 = (u_2 w_1 - u_1 w_2)/\Delta,\ m_1 = (d_2 w_3 - d_3 w_2)/\Delta,$$

$$m_2 = (d_3 w_1 - d_1 w_3)/\Delta,\ m_3 = (d_1 w_2 - d_2 w_1)/\Delta,$$

$$n_1 = (d_3 u_2 - d_2 u_3)/\Delta,\ n_2 = (d_1 u_3 - d_3 u_1)/\Delta,$$

$$n_3 = (d_2 u_1 - d_1 u_2)/\Delta,\ \Delta = -d_1 l_1 - d_2 l_2 - d_3 l_3.$$

特别地，如对 $[\bar{\boldsymbol{K}}]_N$ 求逆矩阵，并假设半空间表面完全排水，只受竖向谐振荷载作用，令 $s = \omega \mathrm{i}$，同时注意到上文的无量纲参数关系，则可得到

$$\bar{u}_{zn}(k,s,0) = \frac{\gamma^d}{\Delta_d} \bar{\sigma}_{zn}(k,s,0), \tag{55}$$

其中 $\gamma^d = u_1(\zeta_3 - \zeta_2) + u_2(\zeta_1 - \zeta_3) + u_3(\zeta_2 - \zeta_1)$，

$$\Delta_d = \zeta_1(\xi_3 - \xi_2) + \zeta_2(\xi_1 - \xi_3) + \zeta_3(\xi_2 - \xi_1).$$

式(55)与文献[13]的(63)式完全相同.

3.3　饱和土层的总刚方程

由于相邻饱和土层在内界面上应力和孔隙水压力连续，以第 $i-1$ 和 i 层为例，连续条件表示为

$$\{\bar{\sigma}(k,s,z_i)\}_{i-1} - \{\bar{\sigma}(k,s,z_i)\}_i = \{\bar{\boldsymbol{P}}(k,s,z_i)\}_i^{\mathrm{T}}, \tag{56}$$

这里，

$$\{\bar{\boldsymbol{P}}(k,s,z_i)\}_i = \{\bar{P}_r(k,s,z_i), \bar{P}_z(k,s,z_i), \bar{p}_f(k,s,z_i)\}, \tag{57}$$

$\bar{P}_r(k,s,z_i)$，$\bar{P}_z(k,s,z_i)$ 和 $\bar{p}_f(k,s,z_i)$ 表示作用在土层第 $i-1$ 和 i 层之间内界面上，经 Laplace 变换、Fourier 展开及 Hankel 变换后的水平、竖向外荷载及孔隙水压力. 若该内界面透水且无荷载作用，则 $\{\bar{\boldsymbol{P}}\}_i^{\mathrm{T}}$ 为零矢量；若界面不透水，则 $\frac{\partial \bar{p}_f}{\partial z}\big|_{z=z_i} = 0$，需对式(50)做相应调整，即，将矩阵 $[\bar{\boldsymbol{S}}_{11}]_i$ 的第 3 行元素自第 1 列依次改为 λ_1、$-\lambda_1$ 和 λ_2；将矩阵 $[\bar{\boldsymbol{S}}_{12}]_i$ 的第 3 行依次改为 $-\lambda_2$、λ_3 和 $-\lambda_3$.

对每一个内界面运用式(56)，(51)和(54)，得到饱和土层的总刚度方程，即

$$\begin{bmatrix} \boldsymbol{K}_1 & & & & \\ & \boldsymbol{K}_2 & & & \\ & & \ddots & & \\ & & & \boldsymbol{K}_{N-1} & \\ & & & & \boldsymbol{K}_N \end{bmatrix} \begin{Bmatrix} \boldsymbol{d}_1 \\ \boldsymbol{d}_2 \\ \vdots \\ \boldsymbol{d}_{N-1} \\ \boldsymbol{d}_N \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{P}_1 \\ \boldsymbol{P}_2 \\ \vdots \\ \boldsymbol{P}_{N-1} \\ \boldsymbol{P}_N \end{Bmatrix}. \tag{58}$$

总刚度方程(58)包含共 $3N$ 个未知位移，若考虑土层的面外运动，则包含 $4N$ 个未知量. 另外，总刚方程(58)不仅可以很方便地处理土层内各类荷载作用的影响，还可对半空间上卧土层和下卧基层作灵活处理. 因此，本文方法较文献[8～10,17]更具一般性.

4　数值算例

为说明本文方法的有效性，考虑横观各向同性

饱和半空间体内作用埋置力源情形.设地表完全排水,在半空间体内 $z=h$ 处,存在力源

$$\boldsymbol{P} = P_r(r,\theta,h,t)\boldsymbol{e}_r + P_\theta(r,\theta,h,t)\boldsymbol{e}_\theta + P_z(r,\theta,h,t)\boldsymbol{e}_z.$$

以荷载所在的水平面为界面,将半空间体划分为 D_1 和 D_2 两个土层,其中 D_1 是厚度为 h 的有限饱和土层,$0\leqslant z\leqslant h$;D_2 为饱和半空间区域 $h\leqslant z\leqslant \infty$.地表边界条件表示为

$$\sigma_z(r,\theta,0,t) = 0,\ \tau_{z\theta}(r,\theta,0,t) = 0,$$
$$\tau_{zr}(r,\theta,0,t) = 0,\ p_{\mathrm{f}}(r,\theta,0,t) = 0. \quad (59)$$

变换后,得到 D_1 土层上界面荷载向量

$$\{\overline{\boldsymbol{P}}\}_1 = \{\overline{P}_{rn}(k,s,0),\overline{P}_{zn}(k,s,0),\overline{p}_{\mathrm{f}n}(k,s,0)\}^{\mathrm{T}} = \{0,0,0\}^{\mathrm{T}},$$
$$\overline{P}_{\theta n}(k,s,0) = 0,$$

D_1 土层下界面(也就是 D_2 土层上界面)荷载向量

$$\{\overline{\boldsymbol{P}}\}_2 = \{\overline{P}_{rn}(k,s,0),\overline{P}_{zn}(k,s,0),\overline{p}_{\mathrm{f}n}(k,s,0)\}^{\mathrm{T}},$$

其中

$$\overline{P}_{zn} = \int_0^\infty \hat{P}_{zn}\mathrm{J}_n(kr)r\mathrm{d}r,$$
$$\overline{P}_{rn} = \frac{1}{2}\int_0^\infty(\hat{P}_{rn}+\hat{P}_{\theta n})\mathrm{J}_{n+1}(kr)r\mathrm{d}r - \frac{1}{2}\int_0^\infty(\hat{P}_{rn}-\hat{P}_{\theta n})\mathrm{J}_{n-1}(kr)r\mathrm{d}r,$$
$$\overline{P}_{\theta n} = \frac{1}{2}\int_0^\infty(\hat{P}_{rn}+\hat{P}_{\theta n})\mathrm{J}_{n+1}(kr)r\mathrm{d}r + \frac{1}{2}\int_0^\infty(\hat{P}_{rn}-\hat{P}_{\theta n})\mathrm{J}_{n-1}(kr)r\mathrm{d}r.$$

特别地,当半空间体在 $z=h$ 处作用瞬态水平点力源时,地表位移可表示为

$$u_z(r,\theta,0,t) = \frac{1}{2\pi\mathrm{i}}\int_0^\infty\int_{a-\mathrm{i}\infty}^{a+\mathrm{i}\infty}[\overline{u}_z^{(1)}(k,s,0) \times k\mathrm{J}_1(kr)\mathrm{e}^{st}\mathrm{d}k\mathrm{d}s]\cos\theta,$$

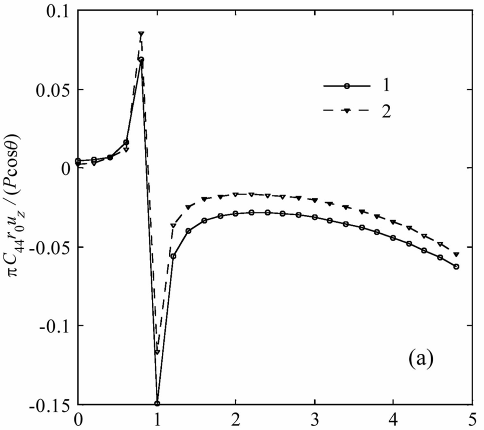
$$u_r(r,\theta,0,t) = \frac{1}{4\pi\mathrm{i}}\int_0^\infty\int_{a-\mathrm{i}\infty}^{a+\mathrm{i}\infty}\Big\{\overline{u}_r^{(1)}(k,s,0) \times\left[\frac{\mathrm{J}_1(kr)}{kr}-\mathrm{J}_0(kr)\right]+\overline{u}_\theta^{(1)}(k,s,0)\frac{\mathrm{J}_1(kr)}{kr}\Big\} \times \mathrm{e}^{st}k\mathrm{d}k\mathrm{d}s\Big\}\cos\theta,$$

$$u_\theta(r,\theta,0,t) = \frac{1}{4\pi\mathrm{i}}\int_0^\infty\int_{a-\mathrm{i}\infty}^{a+\mathrm{i}\infty}\Big\{\overline{u}_r^{(1)}(k,s,0) \times\frac{\mathrm{J}_1(kr)}{kr}+\overline{u}_\theta^{(1)}(k,s,0)\left[\frac{\mathrm{J}_1(kr)}{kr}-\mathrm{J}_0(kr)\right]\Big\} \times \mathrm{e}^{st}k\mathrm{d}k\mathrm{d}s\Big\}\sin\theta.$$

由于积分解的被积函数中含有 Bessel 函数,属广义振荡积分.本文进行数值计算时,按被积函数的零点,将积分区间分为各子区间,在每个子区间内采用16点 Gauss 数值积分,并对各区间积分结果采用 Euler 变换,加速收敛速度.对 Laplace 逆变换采用 Durbin 法[18]实现.

作为算例,考虑某横观各向同性饱和半空间体,其物理力学参数为 $C_{11}=10$ MPa,$C_{12}=2$ MPa,$C_{13}=2.4$ MPa,$C_{33}=8$ MPa,$C_{44}=3.2$ MPa;$K_{\mathrm{s}}=30$ MPa,$K_{\mathrm{l}}=2$ MPa;$\rho_{\mathrm{s}}=2600\ \mathrm{kg/m^3}$,$\rho_{\mathrm{l}}=1000\ \mathrm{kg/m^3}$,$\eta=10^{-3}$ Pa·s,$\phi=0.4$,$k_{\mathrm{d1}}=10^{-6}$ m/s,$k_{\mathrm{d3}}=10^{-7}$ m/s,$r_0=1$ m.力源埋深 h 处作用强度为 P_0 沿 $\theta=0°$ 方向水平集中激振力 $P(t)=P_0H(t)\delta(r)/(2\pi r)$,$H(t)$ 为 Heaviside 函数,$\delta(r)$ 为 Dirac-δ 函数,$P_0=1000$ N.

图1给出了地表水平激振力作用下,土骨架分别采用横观各向同性和各向同性模型时,地表 r 点的竖向和径向位移反应.图中曲线横坐标 $\tau=\tilde{t}$,即 $t=r_0\tilde{t}\sqrt{\rho/C_{44}}$.计算时假设地基竖向和水平向渗透系数相等,各向同性模型参数为 $C_{11}=C_{33}=8.4$ MPa,



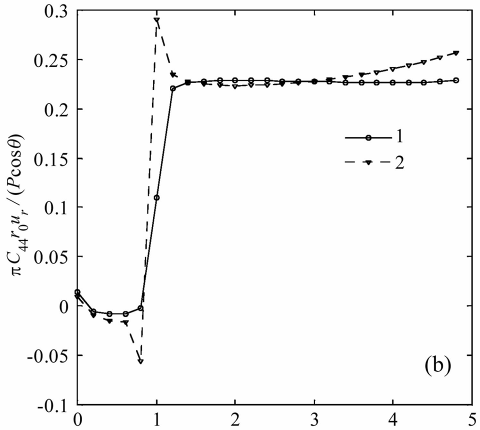


图1 不同土骨架参数的饱和地基地表位移

(a) 竖向位移 u_z;(b) 径向位移 u_r.1 各向同性;2 横观各向同性.

Fig.1 Surface displacement components of saturated soils with different parameters varying with time τ

(a) Vertical displacement u_z; (b) Radial displacement u_r. 1 Isotropic saturated soils; 2 Transversely isotropic saturated soils.

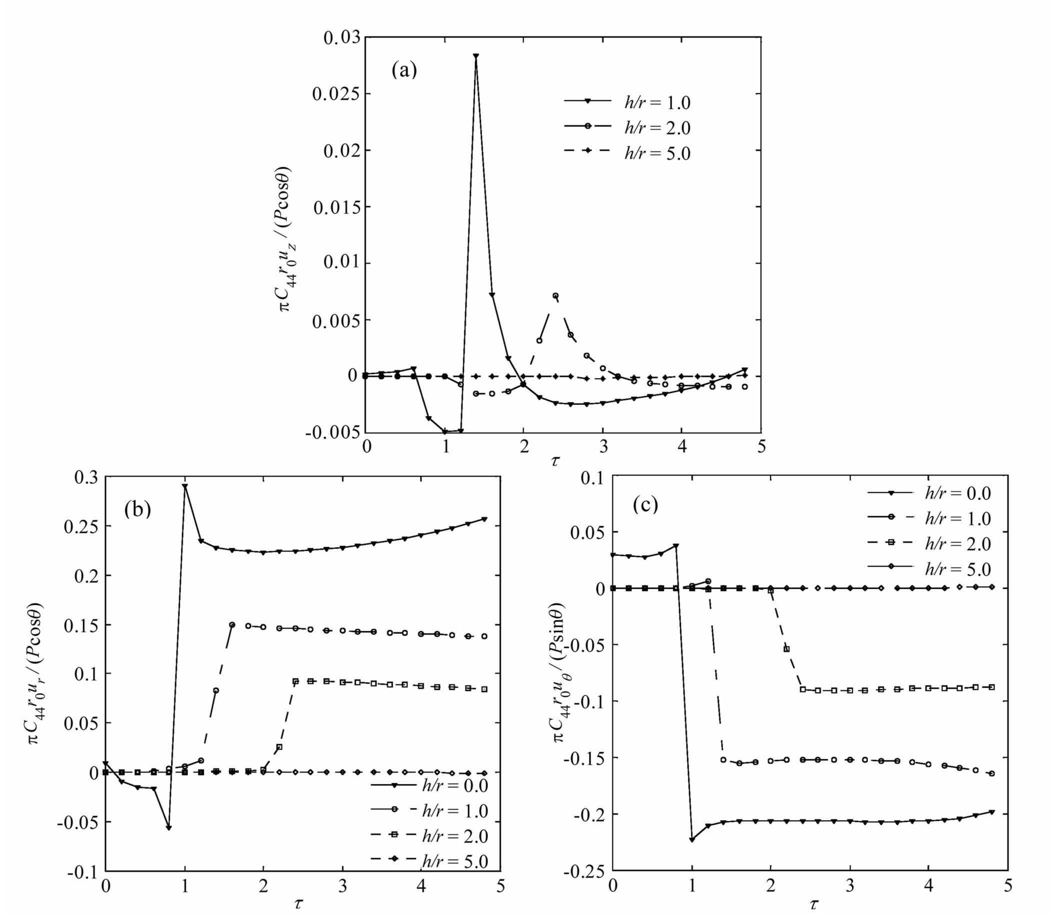


图 2　力源埋深不同时饱和地基地表位移

(a) 竖向位移 u_z；(b) 径向位移 u_r；(c) 环向位移 u_θ.

Fig. 2　Surface displacement components for saturated soils subjected to different buried horizontal impulsive loads vs. time τ

(a) Vertical displacement u_z; (b) Radial displacement u_r; (c) Tangential displacement u_θ.

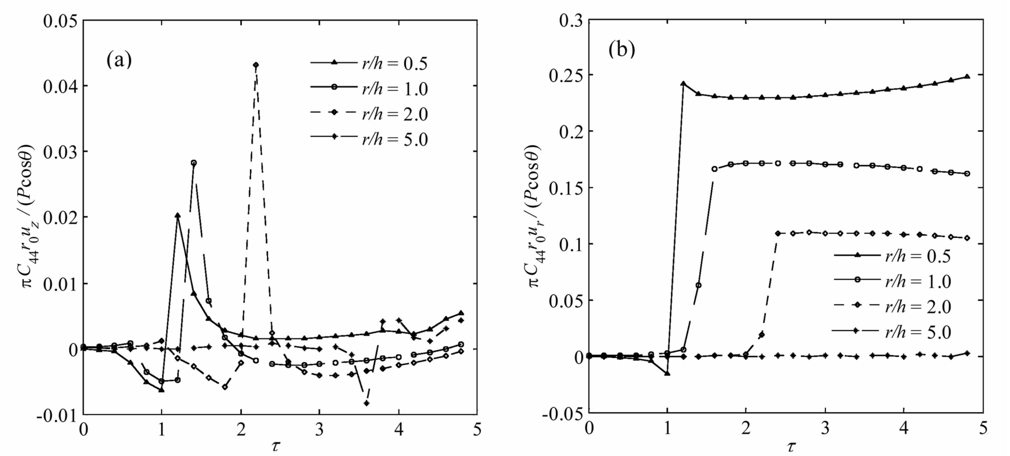


图 3　震中距 r 不同时饱和地基地表位移

(a) 竖向位移 u_z；(b) 径向位移 u_r.

Fig. 3　Surface displacement components for saturated soils at different distance r subjected to buried horizontal impulsive loads vs. time τ

(a) Vertical displacement u_z; (b) Radial displacement u_r.

$C_{12}=C_{13}=2$ MPa, $C_{44}=3.2$ MPa. 结果表明,各向同性地基模型不能准确描述具有明显各向异性特点的天然饱和地基.

图 2(a,b,c)分别给出了地基在埋深不同的水平激振力作用下,地表 r 点竖向、径向和环向位移时程曲线. 可以看出,荷载埋置越深,地表位移越小,当 $h/r=5$ 时,地表位移基本上接近于 0. 同时可看出,埋置水平激振力作用下,地表径向和环向位移明显大于竖向位移.

图 3(a,b)分别给出位于地表不同观测点上,竖向和径向位移的时程曲线. 曲线表明,地表位移随 r 的增大而逐渐减小,当 $r/h=5$ 时,地表位移接近于 0.

5 结 论

基于 Biot 波动理论,给出了埋置力源作用下,横观各向同性饱和地基三维非轴对称瞬态动力响应问题的一般解法. 借助 Laplace 变换和算子理论,得到波动方程的通解;采用 Fourier 分解和 Hankel 积分变换,给出了问题的一般解;基于一般解,建立横观各向同性饱和土层的精确动力刚度矩阵和地基的刚度方程,并对埋置水平荷载进行了数值分析. 本文方法为运用边界元法求解饱和地基动力响应奠定了理论基础.

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