

Joint Decision of Pricing and Cross-Ruff Coupon Value under Substitute (Complement) Demand

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ABSTRACT

This paper focuses on the optimal decisions of pricing and cross-ruff coupon face value for two linked brands products. We develop the profit maximization models for two kinds of coupons: in-pack cross-ruff coupon and on-pack cross-ruff coupon under substitute (complement) demand, and conclude that 1) the coupon values, prices and profit under substitute demand are higher than under complement demand or independent demand; 2) the profit under on-pack coupon mode are higher than in-pack coupon mode. According to the results, we provide the managerial insights for choosing the linked brands. Through numerical example, we observe that the coupon values, target brand prices and carrier brand prices for on-pack cross-ruff coupon are more sensitive than in-pack cross-ruff coupon to reference price.

Keywords: Cross-Ruff Coupon; Carrier Brand; Target Brand; Pricing

1. Introduction

Coupon is a very typical method of promotion widely used by manufacturers and retailers. Coupons are superior to direct discount for the commodity price, because direct price discount will affect consumers' price expectations or form price discrimination in the market. According to the report issued by Marketing Services Company of NCH Group, coupon is the most powerful method to stimulate demand and encourage customers to try new brands. The report points out that in the case of issuing coupons, 27.2% customers who originally planned to buy other brands will decide to replace the brand temporarily, 64.3% customers who have no specific brand purchase plan will change the brand. There are many types of coupons, which are widely applied in practice or discussed among theoretical workers, such as direct mail coupons [1-3], coupons published in newspaper or magazine [4,5], and package coupons [6].

This study will mainly focus on the Cross-ruff Coupon. Cross-ruff Coupon refers to this kind of coupons, which consumers get another assigned product or brand discount coupons during the purchase process of a particular brand or product, resulting in the purchase of the assigned product/brand can get a discount. For example, a customer enters a beverage store to buy a bottle of Diet Coke (a Coca Cola Company's low-calorie cola soft drinks), at the same time, he received a Diet Sprite (a Coca Cola Company's low calorie lemon flavored soft drinks) discount coupons. This coupon gives an advantage in the lin-

kage between different brands/products consumers purchase activity. During September 28, 2008 to January 3, 2009, HP Company has provided coupons of type DJ D1560, DJ D1455 and DJ D4360 printers for purchasers who buy any type of HP notebook or desktop computers.

In order to the convenience of description, we define brand/product which issues coupons as Carrier Brand, and define promotion brand/product as Target Brand. According to the survey, cross coupon acceptance ratio is the double of coupons published in newspaper. The academic community' study on coupons mainly focuses on the influence on the choice of general coupons [2,6,7], influence on consumer behavior [8], the influence on social welfare [9] etc. However, cross coupons is different from direct mail coupons, coupons published in newspaper or magazine, consumers buy other products when they accept cross coupons, so it links different buy behavior. Cross coupons is also different from package coupons, the latter can only make consumers buy the same product, cross coupons achieve the aim that promote another product (*i.e.* target brand) by purchasing one brand product (*i.e.* carrier brand). Therefore, research model related to these types of coupons will be not suitable to cross coupons. Some researchers also studied on cross coupons, such as Dhar and Raju [10], they studied on the influences of cross coupons on customer choice behavior, got the condition to improve sales and profits, and further give the management implications to select suitable carrier brand and target brand. While this study only analyzed the influence degree of issuing coupons, didn't solve such que-

stions as the optimal coupon value, and how to determine the price of products of carrier brand and target brands respectively. Therefore, this paper will focus on resolving joint decision of pricing and cross-ruff coupon value under substitute (complement) demand conditions based on the literatures.

2. Analysis for Decision Model

Raju [6] proposed that there are different distribution forms when enterprise issues coupons with product package, according to cost, product characteristics, policy constraints and other actual conditions, including In-Pack coupons, which is packaged within the package, and On-Pack coupons, which is printed or pasted on the outer package. Accordingly, this paper will mainly constructs two kinds of models, *i.e.* optimal decision model for cross-ruff coupons within the package and cross-ruff coupons printed or pasted on the package. In order to the concision for writing, the following use “IPC coupons” instead of coupons which is packaged within the package, and use “OPC coupons” instead of coupons which is printed or pasted on the outer package. The desired symbol is defined as follows:

- $D =$ Demand for product
- $P =$ Price of product unit
- $R =$ Value of Cross-ruff Coupon
- $C =$ Cost of product unit
- $w =$ Cost of acceptance of coupon unit
- $\pi =$ Profit of enterprise

In the process of building model, we use subscript c represent carrier brand, use t represent target brand, use s represent demand condition, use m represent the demand is complementary, use d represent the demand is independent. And as for all symbols “ \pm ” or “ \mp ”, the upper part represents demand for substitutive conditions, the lower part represents demand for complementary conditions, such as “ \pm ” represent that when the demand for carrier brand and target brand demand is substitutive, we take “+”; when the demand for them is complementary, we take “-”. Firstly, we develop and analyze the IPC coupons optimal value decision model, and then, we give the same process of OPC coupons.

2.1. IPC Coupons

Assume that the demand is linear function, due to the substitutive (complementary) relationship between two

products demand, the change of one product’s a price will affect the demand for another product. At the same time, according to the definition of IPC coupon, we know that at the time of purchase, customers do not know whether there are coupons in the package, only when the purchase completed and the package is opened, they find coupons and determine whether exchange. So this kind of coupons has no effect on the needs of carrier brand no effect, it can only affects the demand for target brand. Therefore, the demand function of carrier brand and target brand can be expressed as follows respectively.

$$D_c = a_c - b_c P_c \pm c_c P_t, \quad D_t = a_t - b_t P_t \pm c_t P_c + r D_c$$

Among them, D_c represents the demand of carrier brand, D_t represents the demand of target brand, $a_c, b_c, c_c, a_t, b_t, c_t$ are parameters of demand function of carrier brand and target brand. r represents the acceptance rate of coupons, it is determined by the relative size of coupon value and reference price P_0 , and P_0 is the maximum value of coupons when there is 100% coupon acceptance ratio, function form as follows [11]:

$$r = \frac{R}{P_0}$$

Profit function of enterprise can be represented as follows:

$$\pi = D_c (P_c - C_c) + D_t (P_t - C_t) - r D_c (R + w) \quad (1)$$

The first order partial derivative of profit function with respect to variable R can be represented as follows:

$$\frac{\partial \pi}{\partial R} = \frac{a_c - b_c P_c \pm c_c P_t}{P_0} (P_t - C_t - 2R - w) \quad (2)$$

Appoint Equation (2) is equal to zero, we can get

$$R = \frac{P_t - C_t - w}{2} \quad (3)$$

Take Equation (3) into Equation (1), the first order partial derivative of profit function with respect to P_c, P_t can be represented as follows respectively:

$$\frac{\partial \pi}{\partial P_c} = a_c - 2b_c P_c \pm (c_c + c_t) P_t + b_c C_c \mp c_t C_t - \frac{b_c (P_t - C_t - w)^2}{4P_0} \quad (4)$$

$$\frac{\partial \pi}{\partial P_t} = a_t - 2b_t P_t \pm (c_c + c_t) P_c + b_t C_t \mp c_c C_c \pm \frac{c_c (P_t - C_t - w)^2}{4P_0} + \frac{(a_c - b_c P_c \pm c_c P_t)(P_t - C_t - w)}{2P_0} \quad (5)$$

Further get Hessian Matrices of profit function to P_c and P_t :

$$H = \begin{bmatrix} -2b_c & -\frac{b_c (P_t - C_t - w)}{2P_0} \pm c_c \pm c_t \\ -\frac{b_c (P_t - C_t - w)}{2P_0} \pm c_c \pm c_t & -2b_t \pm \frac{c_c (P_t - C_t - w)}{P_0} + \frac{a_c - b_c P_c \pm c_c P_t}{2P_0} \end{bmatrix} \quad (6)$$

The first and second order principle which can judge the characteristic of H is D_1 and D_2 respectively, and $D_1 = -2b_c$,

$$D_2 = 4b_c b_t \mp 4rb_c c_c - \frac{b_c D_c}{P_0} - (rb_c \mp c_t \mp c_c)^2.$$

As a concave function, profit function π ask $D_1 < 0$, and $D_2 > 0$, if

$$4b_c b_t \mp 4rb_c c_c - \frac{b_c D_c}{P_0} - (rb_c \mp c_t \mp c_c)^2 > 0$$

Simplified it, condition (7) can be obtained

$$4b_c b_t \mp 4rb_c c_c > \frac{b_c D_c}{P_0} + (rb_c \mp c_t \mp c_c)^2 \quad (7)$$

Because $r = \frac{P_t - C_t - w}{2P_0} \leq 1$, by a validated numerical

experiments indication, condition (7) is true at actual situation of enterprise operation.

Let (4), (5) are equal to zero, dissolve the equation system, we can get:

1) Under the substitutive conditions, the optimal product price of carrier brand and target brand P_{cs}^{ipc} and P_{ts}^{ipc} are

$$P_{cs}^{ipc} = \frac{a_c + b_c C_c + c_c P_{ts}^{ipc} + c_t (P_{ts}^{ipc} - C_t)}{2b_c} - \frac{(P_{ts}^{ipc} - C_t - w)^2}{8P_0} \quad (8)$$

$$P_{ts}^{ipc} = \frac{a_t + b_t C_t + c_t P_{cs}^{ipc} + c_c (P_{cs}^{ipc} - C_c)}{2b_t} + \frac{c_c (P_{ts}^{ipc} - C_t - w)^2}{8b_t P_0} + \frac{(a_c - b_c P_{cs}^{ipc} + c_c P_{ts}^{ipc})(P_{ts}^{ipc} - C_t - w)}{4b_t P_0} \quad (9)$$

2) Under the complementary conditions, the optimal product price of carrier brand and target brand P_{cm}^{ipc} and P_{tm}^{ipc} are

$$P_{cm}^{ipc} = \frac{a_c + b_c C_c - c_c P_{tm}^{ipc} - c_t (P_{tm}^{ipc} - C_t)}{2b_c} - \frac{(P_{tm}^{ipc} - C_t - w)^2}{8P_0} \quad (10)$$

$$P_{tm}^{ipc} = \frac{a_t + b_t C_t - c_t P_{cm}^{ipc} - c_c (P_{cm}^{ipc} - C_c)}{2b_t} + \frac{(a_c - b_c P_{cm}^{ipc} - c_c P_{tm}^{ipc})(P_{tm}^{ipc} - C_t - w)}{4b_t P_0} - \frac{c_c (P_{tm}^{ipc} - C_t - w)^2}{8b_t P_0} \quad (11)$$

Let $c_c = 0$, $c_t = 0$, the optimal product price of carrier brand and target brand under independent condition P_{cd}^{ipc} and P_{td}^{ipc} are

$$P_{cd}^{ipc} = \frac{a_c + b_c C_c}{2b_c} - \frac{(P_{td}^{ipc} - C_t - w)^2}{8P_0} \quad (12)$$

$$P_{td}^{ipc} = \frac{a_t + b_t C_t}{2b_t} + \frac{(a_c - b_c P_{cd}^{ipc})(P_{td}^{ipc} - C_t - w)}{4b_t P_0} \quad (13)$$

Take the optimal price of target brand P_{td}^{ipc} into (3), the optimal value of IPC coupons R^{ipc} can be obtained.

Compare expressions (8)-(13), such conclusion can be educed:

Conclusion 1: When IPC coupons are issued, the optimal price of carrier brand and target brand, the value of coupons is higher at the condition of substitutive than that of complementary, that is, $P_{ts}^{ipc} > P_{td}^{ipc} > P_{tm}^{ipc}$, and $R_s^{ipc} > R_d^{ipc} > R_m^{ipc}$.

2.2. OPC Coupons

According to the definition of OPC coupons, customers have known that there are coupons when they buy one product. So this kind of coupons can not only excite customers to buy the carrier brand/product, but also inspire them to buy the target brand/product, namely, there are influence on the demand for the two type of brands. The demand function of carrier brand and target brand can be represented as follows respectively.

$$D_c = a_c - b_c P_c \pm c_c P_t + d_c R, \quad D_t = a_t - b_t P_t \pm c_t P_c + r D_c$$

The profit function of enterprise is

$$\pi = D_c (P_c - C_c) + D_t (P_t - C_t) - r D_c (R + w) \quad (14)$$

Further changes to

$$\pi = \frac{(a_c - b_c P_c \pm c_c P_t + d_c R)[P_0 (P_c - C_c) + R(P_t - C_t - R - w)]}{P_0} + (a_t - b_t P_t \pm c_t P_c)(P_t - C_t) \quad (15)$$

Lemma: the enterprise profit is optimal, if and only if

$$R = \frac{P_t - C_t - w}{2} + \frac{d_c P_0}{2b_c} \quad (16)$$

The prove method refer Khouja [11].

Take (16) into profit function (15), the first order partial derivative of profit function with respect to variable P_c and P_t , we can get

$$\frac{\partial \pi}{\partial P_c} = a_c - 2b_c P_c + b_c C_c \pm (c_c + c_t) P_t \mp c_t C_t + d_c \left(\frac{P_t - C_t - w}{2} + \frac{d_c P_0}{2b_c} \right) - \frac{b_c}{P_0} \left[\left(\frac{P_t - C_t - w}{2} \right)^2 - \left(\frac{d_c P_0}{2b_c} \right)^2 \right] \quad (17)$$

$$\frac{\partial \pi}{\partial P_t} = a_t - 2b_t P_t + b_t C_t \pm (c_c + c_t) P_c \mp c_c C_c + \frac{d_c}{2} (P_c - C_c) + \frac{1}{P_0} \left(\frac{d_c}{2} \pm c_c \right) \left[\left(\frac{P_t - C_t - w}{2} \right)^2 - \left(\frac{d_c P_0}{2b_c} \right)^2 \right] + \frac{(P_t - C_t - w)}{2P_0} \left[a_c - b_c P_c \pm c_c P_t + d_c \left(\frac{P_t - C_t - w}{2} + \frac{d_c P_0}{2b_c} \right) \right] \tag{18}$$

Further get Hessian Matrices of profit function to P_c and P_t :

$$H = \begin{bmatrix} -2b_c & \frac{d_c}{2} - \frac{b_c (P_t - C_t - w)}{2P_0} \pm c_c \pm c_t \\ \frac{d_c}{2} - \frac{b_c (P_t - C_t - w)}{2P_0} \pm c_c \pm c_t & -2b_t + \frac{D_c}{2P_0} + \left(\frac{d_c}{2} \pm c_c \right) \left(\frac{P_t - C_t - w}{P_0} \right) \end{bmatrix} \tag{19}$$

The first and second order principle which can judge the characteristic of H is D_1 and D_2 respectively, and $D_1 = -2b_c$,

$$D_2 = 4b_c b_t - (d_c \pm c_c \pm c_t - b_c r)^2 - (2b_c r - d_c)(d_c \pm 2c_c) - \frac{b_c D_c}{P_0}$$

when $D_1 < 0$ and $D_2 > 0$, profit function π is concave function. So, profit function is concave function, if

$$4b_c b_t > (d_c \pm c_c \pm c_t - b_c r)^2 + (2b_c r - d_c)(d_c \pm 2c_c) + \frac{b_c D_c}{P_0} \tag{20}$$

By a validated numerical experiments indication, condition (20) is true at actual situation of enterprise opera-

tion.

Let (17) (18) are equal to zero, dissolve the equation systems, we can get

1) Under the substitutive conditions, the optimal product price of carrier brand and target brand P_{cs}^{opc} and P_{ts}^{opc} can be represented as follows:

$$P_{cs}^{opc} = \frac{a_c + b_c C_c + c_c P_{ts}^{opc} + c_t (P_{ts}^{opc} - C_t)}{2b_c} + \frac{d_c}{2b_c} \left(\frac{P_{ts}^{opc} - C_t - w}{2} + \frac{d_c P_0}{2b_c} \right) - \frac{1}{2P_0} \left[\left(\frac{P_{ts}^{opc} - C_t - w}{2} \right)^2 - \left(\frac{d_c P_0}{2b_c} \right)^2 \right] \tag{21}$$

$$P_{ts}^{opc} = \frac{a_t + b_t C_t + c_t P_{cs}^{opc} + c_c (P_{cs}^{opc} - C_c)}{2b_t} + \frac{d_c}{4b_t} (P_{cs}^{opc} - C_c) + \frac{1}{2b_t P_0} \left(\frac{d_c}{2} + c_c \right) \left[\left(\frac{P_{ts}^{opc} - C_t - w}{2} \right)^2 - \left(\frac{d_c P_0}{2b_c} \right)^2 \right] + \frac{1}{2b_t} \left(\frac{P_{ts}^{opc} - C_t - w}{2P_0} \right) \left[a_c - b_c P_{cs}^{opc} + c_c P_{ts}^{opc} + d_c \left(\frac{P_{ts}^{opc} - C_t - w}{2} + \frac{d_c P_0}{2b_c} \right) \right] \tag{22}$$

2) Under the complementary conditions, the optimal product price of carrier brand and target brand P_{cm}^{opc} and

P_{tm}^{opc} can be represented as follows:

$$P_{cm}^{opc} = \frac{a_c + b_c C_c - c_c P_{tm}^{opc} - c_t (P_{tm}^{opc} - C_t)}{2b_c} + \frac{d_c}{2b_c} \left(\frac{P_{tm}^{opc} - C_t - w}{2} + \frac{d_c P_0}{2b_c} \right) - \frac{1}{2P_0} \left[\left(\frac{P_{tm}^{opc} - C_t - w}{2} \right)^2 - \left(\frac{d_c P_0}{2b_c} \right)^2 \right] \tag{23}$$

$$P_{tm}^{opc} = \frac{a_t + b_t C_t - c_t P_{cm}^{opc} - c_c (P_{cm}^{opc} - C_c)}{2b_t} + \frac{d_c}{4b_t} (P_{cm}^{opc} - C_c) + \frac{1}{2b_t P_0} \left(\frac{d_c}{2} - c_c \right) \left[\left(\frac{P_{tm}^{opc} - C_t - w}{2} \right)^2 - \left(\frac{d_c P_0}{2b_c} \right)^2 \right] + \frac{1}{2b_t} \left(\frac{P_{tm}^{opc} - C_t - w}{2P_0} \right) \left[a_c - b_c P_{cm}^{opc} - c_c P_{tm}^{opc} + d_c \left(\frac{P_{tm}^{opc} - C_t - w}{2} + \frac{d_c P_0}{2b_c} \right) \right] \tag{24}$$

Let C_c , C_t , the optimal product price of carrier brand and target brand under independent condition P_{cd}^{opc} and

P_{td}^{opc} are

$$P_{cd}^{opc} = \frac{a_c + b_c C_c}{2b_c} + \frac{d_c}{2b_c} \left(\frac{P_{td}^{opc} - C_t - w}{2} + \frac{d_c P_0}{2b_c} \right) - \frac{1}{2P_0} \left[\left(\frac{P_{td}^{opc} - C_t - w}{2} \right)^2 - \left(\frac{d_c P_0}{2b_c} \right)^2 \right] \tag{25}$$

$$\begin{aligned}
 P_{id}^{opc} = & \frac{a_t + b_t C_t}{2b_t} + \frac{d_c}{4b_t} (P_{cd}^{opc} - C_c) \\
 & + \frac{d_c}{4b_t P_0} \left[\left(\frac{P_{id}^{opc} - C_t - w}{2} \right)^2 - \left(\frac{d_c P_0}{2b_c} \right)^2 \right] \\
 & + \frac{1}{2b_t} \left(\frac{P_{id}^{opc} - C_t - w}{2P_0} \right) \left[a_c - b_c P_{cd}^{opc} \right. \\
 & \left. + d_c \left(\frac{P_{id}^{opc} - C_t - w}{2} + \frac{d_c P_0}{2b_c} \right) \right]
 \end{aligned} \tag{26}$$

Take the optimal price of target brand $P_{t.}^{opc}$ into (16), the optimal value of OPC coupons R_s^{opc} can be obtained.

Compare expressions (21)-(26), such conclusion can be educed:

Conclusion 2: When OPC coupons are issued, the optimal price of carrier brand and target brand, the value of coupons is higher at the condition of substitutive than that of complementary, that is,

$$P_{s.}^{opc} > P_{d.}^{opc} > P_{m.}^{opc}, R_s^{opc} > R_d^{opc} > R_m^{opc}.$$

If there is no coupon to issue, the optimal price of two kinds of products can be represented as follows respectively

$$P_{c.}^{no} = \frac{a_c + b_c C_c \pm (c_c + c_t) P_{t.}^{no} \mp c_t C_t}{2b_c} \tag{27}$$

$$P_{t.}^{no} = \frac{a_t + b_t C_t \pm (c_c + c_t) P_{c.}^{no} \mp c_c C_c}{2b_t} \tag{28}$$

Next, we can further analyze the concrete decision process for IPC and OPC coupons by numerical examples, and validate the conclusions educed above.

3. Numerical Examples

Assume that the demand function of carrier brand is $D_c = 350,000 - 5000P_c \pm 3000P_t + d_c R$, demand function of target brand is

$$D_t = 350,000 - 7000P_t \pm 3000P_c + (R/P_0) D_c.$$

Unit cost of carrier brand $C_c = 10$, Unit cost of target brand $C_t = 10$, cost of acceptance of coupon unit $w = 0.25$, and the ratio of customer exchange coupons is a linear function of the value of coupons, reference price $P_0 = 90$. When enterprise does not issue coupons, namely $R = 0$, the optimal value of two brands are

$$P_{cs}^{no} = 72 (P_{cm}^{no} = 31.92; P_{cd}^{no} = 40) \text{ and}$$

$$P_{is}^{no} = 59 (P_{im}^{no} = 18.46; P_{id}^{no} = 30),$$

the maximum enterprise profit $\pi_s^{no} = 17,852,000$ ($\pi_m^{no} = 4,017,300$; $\pi_d^{no} = 7,300,000$). If enterprise issue coupons, but coupons does not affect the demand of carrier brand, namely IPC coupons, $d_c = 0$, then the optimal

price of two brands are $P_{cs}^{ipc} = 71.47$ ($P_{cm}^{ipc} = 31.43$; $P_{cd}^{ipc} = 39.39$) and $P_{is}^{ipc} = 64.18$ ($P_{im}^{ipc} = 19.10$; $P_{id}^{ipc} = 31.28$), the optimal value of coupons is $R_s^{ipc} = 26.97$ ($R_m^{ipc} = 4.43$; $R_d^{ipc} = 10.51$), the maximum enterprise profits $\pi_s^{ipc} = 21,519,000$ ($\pi_m^{ipc} = 4,044,600$; $\pi_d^{ipc} = 7,300,000$). Conspicuously, the profit is bigger when enterprise issue coupons than that of don't issue. When coupons has effect on the demand of carrier brand, namely OPC coupons, let $d_c = 2500$, then the optimal price of two brands are $P_{cs}^{opc} = 117.29$ ($P_{cm}^{opc} = 39.19$; $P_{cd}^{opc} = 50.69$) and $P_{is}^{opc} = 96.86$ ($P_{im}^{opc} = 18.94$; $P_{id}^{opc} = 35.10$), the optimal value of coupons is $R_s^{opc} = 87.32$ ($R_m^{opc} = 22.94$; $R_d^{opc} = 34.92$), the maximum enterprise profits $\pi_s^{opc} = 31,113,000$ ($\pi_m^{opc} = 4,843,400$; $\pi_d^{opc} = 9,380,500$), Conspicuously, the profit is bigger when enterprise issue coupons than that of don't issue, and is higher than that of issuing IPC coupons, the above rate is about 44.58%.

From the research results, we can see when enterprises use cross ruff coupons for promotions, the stronger the substitution between the target brand and carrier brand, the bigger the enterprise profit is, and the promotion effect is better. Conversely, the stronger the complementary the selected brand, the smaller the enterprise profit increment is, and the promotional effect is worse.

4. Conclusions

According to the enterprise management reality, *i.e.* during the sales process of best-selling products or mature product, enterprises often choose to release coupons to promote the unsalable goods or new product, this paper studied the determination problems of coupons value and product price. Through the construction of joint decision model for two most common cross-ruff coupons (IPC and OPC coupons) value and product price, analyzed the optimal coupon value and the optimal carrier brand and target brand product pricing strategies under different demand structure (substitutive, complementary, independent). According to the related strategy, this paper put forward the management implications for the choice of associated brand when enterprise chooses coupons to sales promotion, *i.e.* enterprise should choose the brands pair which have strong demand substitution as carrier brand and target brand as far as possible, while minimize the needs complementary between associated brands.

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