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Verifiable Data Streaming

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Abstract. In a *verifiable data streaming* protocol, the client streams a long string to the server who stores it in its database. The stream is verifiable in the sense that the server can neither change the order of the elements nor manipulate them. The client may also retrieve data from the database and update them. The content of the database is publicly verifiable such that any party in possession of some value s and a proof π can check that s is indeed in the database.

We introduce the notion of verifiable data streaming and present an efficient instantiation that supports an exponential number of values based on general assumptions. Our main technique is an authentication tree in which the leaves are not fixed in advanced such that the user, knowing some trapdoor, can authenticate a new element on demand *without* pre- or re-computing all other leaves. We call this data structure *chameleon authentication tree* (CAT). We instantiate our scheme with primitives that are secure under the discrete logarithm assumption. The algebraic properties of this assumption allow us to obtain a very efficient verification algorithm. As a second application of CATs, we present a new transformation from any one-time to many-time signature scheme that is more efficient than previously known solutions.

1 Introduction

In a verifiable data streaming protocol (VDS), the client $\mathcal{C}$ streams a long string $S = s[1], \ldots, s[m]$ to the server $\mathcal{S}$ who stores the string in its database DB. The length of S exceeds the client's memory such that $\mathcal{C}$ cannot read or store the entire string at once. Instead, $\mathcal{C}$ reads some substring $s[i] \in \{0,1\}^k$ and sends it to the server. The stream is verifiable in the sense that the server cannot change the order of the elements or manipulate them. The entries in the database are publicly verifiable such that any party in possession of $s[i]$ and a proof $\pi_{s[i]}$ can check that $\mathcal{C}$ stored $s[i]$ in DB hold by $\mathcal{S}$.

The client has also the ability to retrieve and update any element in the database. Whenever the client wishes to get some value $s[j]$ from DB, then the server appends a proof $\pi_{s[j]}$ that shows the authenticity of $s[j]$ with respect to some verification key PK. To update some element i in DB, the client retrieves $(s[i], \pi_{s[i]})$ from $\mathcal{S}$, checks its validity, and sends the updated value $s'[i]$ back to the server.

1.1 Trivial Approaches

It seems that a VDS can easily obtained by letting the client sign all values it streams to the server. This solution indeed works for append-only databases that do not consider the order of the elements (clearly, a stateful solution encodes the position into each element). What makes the problem interesting is that the client can update the elements in the database. In this setting, the trivial solution does not work anymore, because all previous entries in DB would still verify under the public key and it is unclear how to revoke previous signatures. One could let the client store all previous elements locally in order to keep track of all changes. This approach, however, would not only lead the problem *ad absurdum*, but it simply is impossible due to the limited storage capacities of the client.

1.2 Applications

Several companies offer already storage in the Internet such as, e.g., Google drive, Dropbox, Apple's iCloud, and many more. The basic idea is that users can outsource most of their data to a seemingly unbounded storage. In some cases, such as Google's Chromebook, the complete data are stored in the cloud while the client keeps only a small portion of the data. Google provides for the users of the Chromebook some free storage, but they have to pay a yearly fee for any additional space.

From a high-level point of view, this can be seen as data streaming, where a weak client streams a huge amount of data to a very powerful server. A crucial point here is the authenticity of the data. How can the client make sure that the server is not charging the client for space it is not using (e.g., by adding random data to the user's database)? Furthermore, the client needs to verify that the server keeps the current version of the data without modifying it, or switching back to an old version.

Taking the order of data into account is a very natural requirement in computer science. As an example consider a server that stores the DNA sequences for a health insurance. A difference in the sequence of the DNA usually means some sort of mutation which effects, e.g., in a disease. Thus, if a malicious server manages to change the order in a patient's DNA sequence, then the client might have to pay a higher fee due to some medical risks (such as genetic disease) that might be implied by this mutation. The website of the Museum of Paleontology of UC Berkley describes the affect of mutations [oCMoP12].

1.3 Our Contribution

We introduce the notion of verifiable data streaming and present an instantiation that supports an exponential number of values based on general assumptions. Our solution has efficient updates and the data in the database are publicly verifiable. Moreover, our construction is secure in the standard model. We summarize our contributions as follows:

- Our main technical contribution is an authentication tree that authenticates an exponential number of values, but where the leaves are not defined in advance. The owner of a trapdoor can add elements to the tree without pre- or re-computing all other elements. We call such a tree a chameleon authentication tree (CAT).

- We show the generality of our technique by applying it to two different problems: Firstly, we build a verifiable data streaming protocol based on CATs. This scheme supports an

exponential number of elements, efficient updates, and the items in the database are publicly verifiable. The second application of CATs is a new transformation from any one-time to a many-time signature scheme in the standard model that is more efficient than all previous approaches.

- We instantiate our construction with primitives that are secure under the discrete logarithm assumption in the standard model. This assumption is not only very mild, but the algebraic structure allows us to obtain a more efficient verification algorithm. The basic idea is to apply batch verification techniques to our verification algorithm.

- Concretely, this means that the verification algorithm of the many-time signature scheme is 50 times faster than the one obtained via the Goldwasser-Micali-Rivest (GMR) transformation. Batch verification techniques do not seem to be applicable to the GMR transformation, because every one-time signature scheme uses different keys, while our transformation does not. Since no direct construction of a signature scheme secure under the discrete logarithm assumption is known in the standard model, our construction is the most *efficient* one.

1.4 Related Work

Verifiable data streaming is related to verifiable databases (VDB) by Benabbas, Gennaro, and Vahlis [BGV11]. The main difference to their work is that the data during the setup phase in a streaming protocol are unknown. Moreover, our notion has an algorithm that allows adding elements to the database that consists of a single message from the user to the server (which does not change the verification key of the database). One might wonder if VDBs can be used to simulate verifiable data streaming protocols by generating a database of exponential size and adding the entries via the update algorithm. This idea, however, does not work because the update procedure usually requires interaction and the server updates the verification key afterwards. Another difference is that the data in a VDB are usually unordered. That is, the element d_i in the database DB is associated to some key x_i, i.e., $DB(x_i) = d_i$. But there is no explicit ordering of the elements.

The problem of VDBs has previously been investigated in the context of accumulators [Ngu05, CKS09, CL02] and authenticated data structures [NN00, MND+01, PT07, TT10]. These approaches, however, often rely on non-constant assumptions (such as the q-Strong Diffie-Hellman assumption) as observed in [BGV11]. More recent works, such as [BGV11] or [CF11], focus on storing specific values (such as polynomials) instead of arbitrary ones and they usually only support a polynomial number of values (instead of exponentially many). Moreover, the scheme of [BGV11] is not publicly verifiable.

Proofs-of-retrievability are also similar in the sense that the server proves to the client that it is actually storing all of the client's data [SW08, FB06, SM06]. The interesting research area of memory delegation [CKLR11] is also different, because it considers verifiable computation on (streamed) data. A more efficient solution has been suggested by Cormode, Mitzenmacher, and Thaler in [CMT12].

2 Verifiable Data Streaming

In a verifiable data streaming protocol (VDS), a client $\mathcal{C}$ reads some long string $S = s[1], \ldots, s[m] \in \{0,1\}^{mk}$ that $\mathcal{C}$ wishes to outsource to a server $\mathcal{S}$ in a streaming manner. Since the client cannot store and read the entire string at once, $\mathcal{C}$ reads a substring $s[i] \in \{0,1\}^k$ of S and sends $s[i]$ to the server who stores the value in its database DB. We stress that we are interested in a streaming protocol, i.e., the communication between the client and the server at this stage is unidirectional and the string is ordered. The data must be publicly verifiable in the sense that the server holds some public key PK and everybody in possession of some data $s[i]$ and a proof $\pi_{s[i]}$ can verify that $s[i]$ is stored in DB. Whenever the client wishes to retrieve some data $s[i]$ from the database DB, $\mathcal{C}$ sends i to the server who returns $s[i]$ together with a proof $\pi_{s[i]}$. This proof shows that s is the i^{th} element in DB and its authenticity with respect to PK. In addition, the client has the ability to update any value $s[i]$ to a new string $s'[i]$ which leads to a new verification key PK'. More formally:

Definition 2.1. *A* verifiable data streaming *protocol* $\mathcal{VDS} = (\mathsf{Setup}, \mathsf{Append}, \mathsf{Query}, \mathsf{Verify}, \mathsf{Update})$ *is a protocol between two PPT algorithms: a client* $\mathcal{C}$ *and a server* $\mathcal{S}$*. The server can store an exponential number* n *of elements in its database* DB *and the client keeps some small state* $\mathcal{O}(\log n)$*. The scheme consists of the following PPT algorithms:*

$\mathsf{Setup}(1^\lambda)$**:** *The setup algorithm takes as input the security parameter* 1^λ*. It returns a verification key* PK *and a secret key* SK*. The verification key* PK *is given to the server* $\mathcal{S}$ *and the secret key to the client* $\mathcal{C}$*. W.l.o.g.,* SK *always contains* PK*.*

$\mathsf{Append}(SK, s)$**:** *This algorithm appends the value* s *to the database* DB *hold by the server. The client sends a single message to the server who stores the element in* DB*. Adding elements to the database may change the private key to* SK'*, but it does not change the verification key* PK*.*

$\mathsf{Query}(PK, DB, i)$**:** *The interactive query protocol is defined as* $\langle \mathcal{S}(PK, DB), \mathcal{C}(i) \rangle$ *and is executed between* $\mathcal{S}(PK, DB)$ *and* $\mathcal{C}(i)$*. At the end of the protocol, the client either outputs the* i^{th} *entry* $s[i]$ *of* DB *together with a proof* $\pi_{s[i]}$*, or* $\perp$*.*

$\mathsf{Verify}(PK, i, s, \pi_{s[i]})$**:** *The verification algorithm outputs* $s[i]$ *if* $s[i]$ *is the* i^{th} *element in the database* DB*, otherwise it returns* $\perp$*.*

$\mathsf{Update}(PK, DB, SK, i, s')$**:** *The interactive update protocol* $\langle \mathcal{S}(PK, DB), \mathcal{C}(SK, i, s') \rangle$ *takes place between the server* $\mathcal{S}(PK, DB)$ *and the client* $\mathcal{C}(SK, i, s')$ *who wishes to update the* i^{th} *entry of the database* DB *to* s'*. At the end of the protocol the server sets* $s[i] \leftarrow s'$ *and both parties update* PK *to* PK'*.*

A verifiable data streaming protocol must fulfill the usual completeness requirements.

2.1 Efficiency and Security Evaluation of VDS

Verifiable data streaming protocols should fulfill both "system" and "crypto" criteria. System criteria usually require that a scheme must be as efficient as possible. In our setting, efficiency should be evaluated w.r.t. computational complexity, storage complexity, and communication complexity. The server in a VDS must be able to store an exponential number of elements and we require that

there is no a-priori bound on the number of queries to the server. The verifiers in the system should be stateless with public verifiability. Everybody in possession of a data s and a proof π_s should be able to verify that s is stored at position i and that s is valid w.r.t. the verification key PK.

The most important crypto criteria are the following: A malicious server $\mathcal{A}$ should not be able to add elements to the database outsourced by the client without its help. This means that $\mathcal{A}$ might ask the client to add q elements to its database DB (where q is adaptively determined by $\mathcal{A}$), but he is unable to add any further element that verifies under PK. A verifiable streaming protocol is order-preserving, i.e., the malicious server $\mathcal{A}$ cannot change the order of any element in the database. Furthermore, $\mathcal{A}$ should not be able to change any element in the database. Again, this property must hold even if $\mathcal{A}$ has the ability to ask the client to update q elements of its choice. Finally, the server should only be able to issue proofs that allow to recover the stored file. These criteria follow the ones that have been suggested in the context of proofs-of-retrievability [SW08].

2.2 Security of VDS

The security notion of VDS is similar to the one of verifiable databases [BGV11], but differs in many aspects: First, our model considers the case of public verifiability, while the one of [BGV11] does not. Second, we are dealing with a stream of data that has an explicit ordering. In contrast, the model of [BGV11] fixes the size of DB during the setup and guarantees authenticity only for these data. In particular, the adversary breaks the security in our model if he manages to output a data $s[i]$ with a valid proof $\pi_{s[i]}$, but where $s[i]$ is *not* the i^{th} value in DB.

We model this intuition in a game between a challenger and an adversary $\mathcal{A}$ that adaptively adds and updates elements to resp. in the database. At the end of the game, $\mathcal{A}$ tries to compute a false statement saying that a *different* data $s[i]$ is the i^{th} value in the database. This covers the different attack scenarios we have discussed so far. A successful attacker could (1) change the order of an element; (2) add an element to the database without the help of the user; (3) break the update mechanism. More formally:

Setup: The challenger runs $\mathsf{Setup}(1^\lambda)$ to generate a private key SK and a public key PK. It sets up an initially empty database DB and gives the public key PK to the adversary $\mathcal{A}$.

Queries: The challenger provides two interfaces for $\mathcal{A}$ that $\mathcal{A}$ may query adaptively and in an arbitrary order. If the adversary queries the *append interface* on some data s, then the challenger will run $\mathsf{Append}(SK, s)$ to append s to its database DB. Subsequently, it returns the corresponding proof π_s to $\mathcal{A}$. The second interface is an *update interface* that takes as input an index j and an element $s'[j]$. Whenever $\mathcal{A}$ invokes this interface, the challenger will run the protocol $\mathsf{Update}(PK, DB, SK, i, s')$ with $\mathcal{A}$. Notice that each call to this interface will update the verification key as well. By $DB = s[1], \ldots, s[q]$ we denote the state of the database after $\mathcal{A}$'s last query and PK^* is the corresponding verification key stored by the challenger.

Output: Eventually, the adversary outputs $(i^*, s^*, \pi^*_{s^*})$ and let $\hat{s} \leftarrow \mathsf{Verify}(PK^*, i^*, s^*, \pi^*_{s^*})$. The adversary is said to win the game if $\hat{s} \neq \perp$ and $\hat{s} \neq s[i^*]$.

We define $\mathbf{Adv}^{\mathsf{os}}_{\mathcal{A}}$ to be the probability that the adversary $\mathcal{A}$ wins in the above game.

Definition 2.2. *A verifiable data streaming protocol is* secure *if for any efficient adversary $\mathcal{A}$ the probability $\mathbf{Adv}^{\mathsf{os}}_{\mathcal{A}}$ is negligible (as a function of λ).*

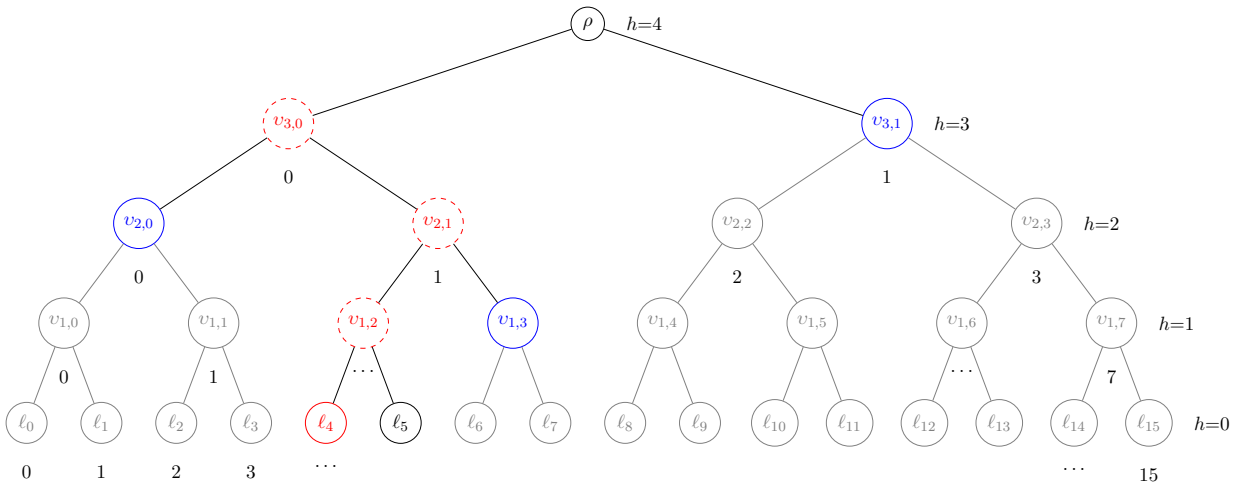

Figure 1: Given a binary tree CAT which consists of a root node ρ, a set of inner nodes υ, and some leaf nodes ℓ. The blue nodes define the authentication path of leaf ℓ_4. The gray nodes right-handed of ℓ_5 do not exist at this stage.

3 Preliminaries

Before describing our construction formally, we introduce the following basic notations for binary trees (c.f. consider the tree depicted in Figure 1). The algorithms using this tree will be described in the subsequent sections. Let CAT be a binary tree consisting of a root node ρ, a set of inner nodes υ, and some leaf nodes ℓ. The depth of the tree is defined by $D = \mathsf{poly}(\lambda)$ and the level of a node in the tree is denoted by $h = 0, \ldots, D-1$, where leaf nodes have level $h = 0$ and the root node in turn has level $h = D-1$. At each level h the nodes are defined by $v_{h,i}$, where $i = 0, \ldots, 2^{D-h}$ is the position of the node in the tree counted from left to right. Furthermore, inner nodes of the tree are computed according the following rule: $v_{h,i} \leftarrow \mathsf{Ch}(v_{h-1,\lfloor i/2^{h-1} \rfloor} || v_{h-1,\lfloor i/2^{h-1}-2 \rfloor}; r_{h,\lfloor i/2^h+1 \rfloor})$, if $\lfloor i/2^h \rfloor \equiv 1 \mod 2$ and $v_{h,i} \leftarrow H(v_{h-1,\lfloor i/2^{h-1} \rfloor - 2} || v_{h-1,\lfloor i/2^{h-1}-1 \rfloor})$, if $\lfloor i/2^h \rfloor \equiv 0 \mod 2$. Notice that H is a hash function, Ch a chameleon hash function, and r some randomness. By ℓ_i we denote the i^{th} leaf counted from left to right. The authentication path auth of a leaf ℓ consists of all siblings of the nodes on the path from ℓ to ρ. If a parent node is computed by a chameleon hash, then it also stores the corresponding randomness in a list R. E.g., the authentication path $\mathsf{auth}_{\ell_{15}}$ of the leaf ℓ_{15} is $\mathsf{auth}_{\ell_{15}} = (\ell_{14}, v_{1,6}, r_{2,3}, v_{2,2}, r_{3,1}, v_{3,0})$ and $R_{\ell_{15}} = (r_{1,7}, r_{2,3}, r_{3,1}, r_\rho)$.

3.1 Chameleon Hash Functions and their Security

A chameleon hash function is a randomized hash function that is collision-resistant but provides a trapdoor [KR00]. Given the trapdoor csk, a message x with some randomness r, and any additional message x', it is possible to efficiently compute a value r' such that the chameleon hash algorithm Ch maps to the same y, i.e., $\mathsf{Ch}(x; r) = \mathsf{Ch}(x'; r') = y$.

Definition 3.1. *A* chameleon hash function *is a tuple of PPT algorithms* $\mathcal{CH} = (\mathsf{Gen}, \mathsf{Ch}, \mathsf{Col})$*:*

Gen(1^λ)**:** *The key generation algorithm returns a key pair* (csk, cpk) *and we set* $\mathsf{Ch}(\cdot) := \mathsf{Ch}(cpk, \cdot)$.

$\mathsf{Ch}(x;r)$: *The input of the hash algorithm is a message $x \in \{0,1\}^{\mathsf{in}}$ and some randomness $r \in \{0,1\}^{\lambda}$ (which is efficiently sampable from some range $\mathcal{R}_{cpk}$). It outputs a hash value $h = \mathsf{Ch}(x;r) \in \{0,1\}^{\mathsf{out}}$.*

$\mathsf{Col}(csk, x, r, x')$: *The collision-finding algorithm returns a value r' such that $\mathsf{Ch}(x;r) = y = \mathsf{Ch}(x';r')$.*

Uniform Distribution: *The output of Ch is uniformly distributed, i.e., it also holds that for any cpk, x, r, x' the distribution of $\mathsf{Col}(csk, x, r, x')$ (over the choice of r) is the same as the distribution of r itself, also implying that a hash value $\mathsf{Ch}(x;r)$ (over the choice of r) is distributed independently of x.*

A chameleon hash function must be collision-resistant. This means that any malicious party should not be able to find two pairs (x_0, r_0) and (x_1, r_1) that map to the same image. More precisely is the following definition.

Definition 3.2. *A chameleon hash function $\mathcal{CH} = (\mathsf{Gen}, \mathsf{Ch}, \mathsf{Col})$ is* collision-resistant *if the advantage function $\mathbf{Adv}^{ch\text{-}col}_{\mathcal{CH},\mathcal{A}}$ is a negligible function in λ for all PPT adversaries $\mathcal{A}$, where*

$$\mathbf{Adv}^{ch\text{-}col}_{\mathcal{CH},\mathcal{A}} := \Pr\left[\begin{array}{c|c} \mathsf{Ch}(x;r) = \mathsf{Ch}(x';r') & (csk, cpk) \leftarrow \mathsf{Gen}(1^{\lambda}); \\ \textit{and } (x,r) \neq (x',r') & (x, x', r, r') \leftarrow \mathcal{A}(\mathsf{Ch}) \end{array}\right].$$

Observe that collision-resistance only holds as long as the adversary has not learned a collision. Indeed, some chameleon hash functions allow to recover the private key if a collision is known, such as, e.g., [KR00]. A comprehensive discussion about this problem is given in [Ad04].

Collision-resistance of hash functions is defined analogously and omitted here.

4 Chameleon Authentication Trees

The central building block that we use in our VDS protocol is a technique that we call *chameleon authentication tree* (CAT). A CAT is an authentication tree that has the ability to authenticate an exponential number of 2^D leaves that are not fixed in advanced, where $D = \mathsf{poly}(\lambda)$. Instead, the owner of a trapdoor can authenticate a new element on demand *without* pre- or re-computing all other leaves.

4.1 Formal Definition of CATs

We formalize CATs as a triple of efficient algorithms: A CAT generation algorithm catGen, a path generation algorithm addLeaf that adds a leaf to the tree and returns the corresponding authentication path, and a path verification algorithm catVrfy that checks if a certain leaf is part of the tree.

Definition 4.1. *A* chameleon authentication tree *is a tuple of PPT algorithms* $\mathsf{CAT} = (\mathsf{catGen}, \mathsf{addLeaf}, \mathsf{catVrfy})$*:*

$\mathsf{catGen}(1^{\lambda}, D)$: *The CAT generation algorithm takes as input a security parameter λ and an integer D that defines the depth of the tree. It returns a private key* sp *and verification key* vp*.*

$\mathsf{addLeaf}(\mathsf{sp}, \ell)$**:** *The path generation algorithm takes as input a private key* sp *and a leaf* $\ell \in \mathcal{L}$ *from some leaf space* $\mathcal{L}$*. It outputs a key* sp'*, the index* i *of* ℓ *in the tree, and the authentication path* auth*.*

$\mathsf{catVrfy}(\mathsf{vp}, i, \ell, \mathsf{auth})$**:** *The verification algorithm takes as input a public key* vp*, an index* i*, a leaf* $\ell \in \mathcal{L}$*, and a path* auth*. It outputs* 1 *iff* ℓ *is the* i^{th} *leaf in the tree, otherwise* 0*.*

A CAT is $\mathsf{complete}$ *if for each* $(\mathsf{sp}, \mathsf{vp})$ *outputted by* $\mathsf{catGen}(1^\lambda, D)$ *and for any* $\ell \in \mathcal{L}$*, the following holds: if* $(\mathsf{sp}', i, \mathsf{auth}) \leftarrow \mathsf{addLeaf}(\mathsf{sp}, \ell)$*, then* $\mathsf{catVrfy}(\mathsf{vp}, i, \ell, \mathsf{auth}) = 1$*.*

4.2 Security of CATs

We identify two security properties that a CAT should support. Loosely speaking, an adversary $\mathcal{A}$ should not be able to change the structure of the CAT. In particular, changing the sequence of the leaves, or substitute any leaf should be a successful attack. We call this property *structure-preserving*. Furthermore, an adversary should not be able to add further leaves to a CAT. We refer to this property as *one-wayness*.

Structure-Preserving. We formalize the first property by an interactive game between the challenger and an adversary $\mathcal{A}$. The challenger generates a key pair $(\mathsf{sp}, \mathsf{vp})$ and hands the verification key vp over to the adversary $\mathcal{A}$. The attacker may then send q leaves $\ell_1, \ldots, \ell_{q(\lambda)}$ (adaptively) to the challenger who returns the corresponding authentication paths $(\mathsf{auth}_1, \ldots, \mathsf{auth}_{q(\lambda)})$. Afterwards, the adversary $\mathcal{A}$ tries to break the structure of the CAT by outputting a leaf that has not been added to the tree at a particular position. More formally:

Setup: The challenger runs the algorithm $\mathsf{catGen}(1^\lambda, D)$ to compute a private key sp and a verification key vp. It gives vp to the adversary $\mathcal{A}$.

Streaming: Proceeding adaptively, the attacker $\mathcal{A}$ streams a leaf $\ell \in \mathcal{L}$ to the challenger. The challenger computes $(\mathsf{sp}', i, \mathsf{auth}) \leftarrow \mathsf{addLeaf}(\mathsf{sp}, \ell)$ and returns (i, auth) to $\mathcal{A}$. Denote by $Q := \{(\ell_1, 1, \mathsf{auth}_1), \ldots, (\ell_{q(\lambda)}, q(\lambda), \mathsf{auth}_{q(\lambda)})\}$ the ordered sequence of query-answer pairs.

Output: Eventually, $\mathcal{A}$ outputs $(\ell^*, i^*, \mathsf{auth}^*)$. The attacker $\mathcal{A}$ is said to win the game if: $1 \leq i^* \leq q(\lambda)$ and $(\ell^*, i^*, \mathsf{auth}^*) \notin Q$ and $\mathsf{catVrfy}(\mathsf{vp}, i^*, \ell^*, \mathsf{auth}^*) = 1$.

We define $\mathbf{Adv}^{\mathsf{sp}}_{\mathcal{A}}$ to be the probability that the adversary $\mathcal{A}$ wins in the above game.

Definition 4.2. *A chameleon authentication tree* CAT*, defined by the efficient algorithms* $(\mathsf{catGen}, \mathsf{addLeaf}, \mathsf{catVrfy})$ *with* n *leaves, is* $\mathsf{structure\text{-}preserving}$ *if for any* $q \in \mathbb{N}$*, and for any PPT algorithm* $\mathcal{A}$*, the probability* $\mathbf{Adv}^{\mathsf{sp}}_{\mathcal{A}}$ *is negligible (as a function of* λ*).*

One-Wayness. We model the second property in a game between a challenger and an adversary as follows:

Setup: The challenger runs the algorithm $\mathsf{catGen}(1^\lambda, D)$ to compute a private key sp and a verification key vp. It gives vp to the adversary $\mathcal{A}$.

Streaming: Proceeding adaptively, the attacker $\mathcal{A}$ streams a leaf $\ell \in \mathcal{L}$ to the challenger. The challenger computes $(\mathsf{sp}', i, \mathsf{auth}) \leftarrow \mathsf{addLeaf}(\mathsf{sp}, \ell)$ and returns (i, auth) to $\mathcal{A}$.

Output: Eventually, $\mathcal{A}$ outputs $(\ell^*, i^*, \mathsf{auth}^*)$. The attacker $\mathcal{A}$ is said to win the game if: $q(\lambda) < i^* \leq n$ and $\mathsf{catVrfy}(\mathsf{vp}, i^*, \ell^*, \mathsf{auth}^*) = 1$.

We define $\mathbf{Adv}^{\mathsf{ow}}_{\mathcal{A}}$ to be the probability that the attacker $\mathcal{A}$ wins in the above game.

Definition 4.3. *A chameleon authentication tree* CAT*, defined by the PPT algorithms* $(\mathsf{catGen}, \mathsf{addLeaf}, \mathsf{catVrfy})$ *with n leaves, is* one-way *if for any $q \in \mathbb{N}$, and for any PPT algorithm $\mathcal{A}$, the probability $\mathbf{Adv}^{\mathsf{ow}}_{\mathcal{A}}$ is negligible (as a function of λ).*

4.3 Weakly Secure CATs

We give two weaker security definitions for CATs that are sufficient for our compiler that turns any one-time signature scheme into a many-time signature scheme. The difference to the previous definition is that the attacker has to commit to its queries before seeing the public key.

Weak Structure-Preserving. The game between the challenge and the adversary $\mathcal{A}$ is defined as follows:

Queries: The attacker $\mathcal{A}$ outputs a sequence of queries $(\ell_1, \ldots, \ell_{q(\lambda)})$.

Responses: The challenger runs the algorithm $\mathsf{catGen}(1^\lambda, D)$ to compute a private key sp and a verification key vp. Let $\mathsf{sp}_0 := \mathsf{sp}$. It runs $(\mathsf{sp}_i, i, \mathsf{auth}_i) \leftarrow \mathsf{addLeaf}(\mathsf{sp}_{i-1}, \ell_i)$ for $i = 1, \ldots, q(\lambda)$ and returns $(\mathsf{vp}, (1, \mathsf{auth}_1), \ldots, (q(\lambda), \mathsf{auth}_{q(\lambda)}))$ to the adversary $\mathcal{A}$. Let $Q := \{(\ell_1, 1, \mathsf{auth}_1), \ldots, (\ell_{q(\lambda)}, q(\lambda), \mathsf{auth}_{q(\lambda)})\}$ be the ordered sequence of query/answer pairs.

Output: Eventually, $\mathcal{A}$ outputs a tuple $(\ell^*, i^*, \mathsf{auth}^*)$. The attacker $\mathcal{A}$ is said to win the game if: $1 \leq i^* \leq q(\lambda)$ and $(\ell^*, i^*, \mathsf{auth}^*) \notin Q$ and $\mathsf{catVrfy}(\mathsf{vp}, i^*, \ell^*, \mathsf{auth}^*) = 1$.

We define $\mathbf{Adv}^{\mathsf{w\text{-}sp}}_{\mathcal{A}}$ to be the probability that the adversary $\mathcal{A}$ wins in the above game.

Definition 4.4. *A chameleon authentication tree* CAT*, defined by the efficient algorithms* $(\mathsf{catGen}, \mathsf{addLeaf}, \mathsf{catVrfy})$ *with n leaves, is* weakly structure-preserving *if for any $q \in \mathbb{N}$, and for any PPT algorithm $\mathcal{A}$, the probability $\mathbf{Adv}^{\mathsf{w\text{-}sp}}_{\mathcal{A}}$ is negligible (as a function of λ).*

Weak One-Wayness. Analogously, we model this game between a challenger and an adversary as follows:

Queries: The attacker $\mathcal{A}$ outputs a sequence of queries $(\ell_1, \ldots, \ell_{q(\lambda)})$.

Responses: The challenger runs the algorithm $\mathsf{catGen}(1^\lambda, D)$ to compute a private key sp and a verification key vp. Let $\mathsf{sp}_0 := \mathsf{sp}$. It runs $(\mathsf{sp}_i, i, \mathsf{auth}_i) \leftarrow \mathsf{addLeaf}(\mathsf{sp}_{i-1}, \ell_i)$ for $i = 1, \ldots, q(\lambda)$ and returns $(\mathsf{vp}, (1, \mathsf{auth}_1), \ldots, (q(\lambda), \mathsf{auth}_{q(\lambda)}))$ to the adversary $\mathcal{A}$. Let $Q := \{(\ell_1, 1, \mathsf{auth}_1), \ldots, (\ell_{q(\lambda)}, q(\lambda), \mathsf{auth}_{q(\lambda)})\}$ be the ordered sequence of query/answer pairs.

Output: Eventually, $\mathcal{A}$ outputs $(\ell^*, i^*, \mathsf{auth}^*)$. The attacker $\mathcal{A}$ is said to win the game if: $q(\lambda) < i^* \leq n$ and $\mathsf{catVrfy}(\mathsf{vp}, i^*, \ell^*, \mathsf{auth}^*) = 1$.

We define $\mathbf{Adv}^{\mathsf{w\text{-}ow}}_{\mathcal{A}}$ to be the probability that the attacker $\mathcal{A}$ wins in the above game.

Definition 4.5. *A chameleon authentication tree* CAT*, defined by the PPT algorithms* $(\mathsf{catGen}, \mathsf{addLeaf}, \mathsf{catVrfy})$ *with n leaves, is* weakly one-way *if for any $q \in \mathbb{N}$, and for any PPT algorithm $\mathcal{A}$, the probability $\mathbf{Adv}^{\mathsf{w\text{-}ow}}_{\mathcal{A}}$ is negligible (as a function of λ).*

5 A Weakly Secure Scheme

In this section, we describe a weakly secure CAT and we give a general transformation from a weakly secure CAT to a fully secure one in Section 6. The basic idea of our construction is a careful combination of hash functions H and chameleon hash functions Ch. Recall that in a chameleon hash function the owner of the trapdoor can easily find collisions, i.e., for a given string x (and randomness r) there exists an efficient algorithm that computes a value r' such that $\mathsf{Ch}(x;r) = \mathsf{Ch}(x';r')$. We first discuss why obvious approaches do not seem to work. Subsequently, we explain the main ideas of our construction, define them formally, and give a proof of security.

5.1 Naïve Approaches do not Work

The first idea would be to build a Merkle tree, where the server stores the entire tree and the client keeps the last authentication path in its state. This idea, however, does not work, because all leaves are necessary to compute the root. Using dummy nodes does not solve the problem, because the root would change whenever a new leaf is authenticated. Thus, the second approach might be to store the outputs of chameleon hash functions as the leaves with the idea that whenever the client wishes to authenticate a new value, it simply applies the trapdoor such that the new leaf authenticates under the same root. This idea, however, does still not work. One reason is that the client would have to store all leaves (together with the corresponding randomness) in order to compute a collision. One might be temped to let the server store these values, but this does not work for several reasons: First of all, the data are streamed. This means that there is no communication from the server to the client at this stage. But even if we would allow bi-directional interaction, it would immediately lead to an attack: Suppose that the client wishes to append the leaf $\hat{\ell}$. To do so, $\mathcal{C}$ asks the server to send the dummy leaf ℓ with the corresponding randomness r. Then, the client applies the trapdoor to compute the matching randomness $\hat{r}$ and sends the updated values $\hat{\ell}, \hat{r}$ to the server. The problem is that the malicious server would learn a colluding pair $(\ell, r), (\hat{\ell}, \hat{r})$ such that $\mathsf{Ch}(\ell;r) = \mathsf{Ch}(\hat{\ell};\hat{r})$. In many schemes, this knowledge would allow the server to compute *another* pair (ℓ^*, r^*) such that $\mathsf{Ch}(\ell;r) = \mathsf{Ch}(\hat{\ell};\hat{r}) = \mathsf{Ch}(\ell^*;r^*)$ (some schemes even allow to recover the trapdoor *csk* if one knows a collision, such as, e.g., [KR00]).

5.2 Intuition of our Construction

As a warm up, we first illustrate the high-level idea of our instantiation with the tree shown in the left part of Figure 2. We stress that our actual construction is slightly different, because the catGen algorithm does not know the leaves ℓ_0 and ℓ_1.

We set up the tree such that the root and every right-handed node of the tree are computed by a chameleon hash function and all left-handed nodes with a collision-resistant hash function. The first step is to set up the tree by computing the hash value of the leaves ℓ_0 and ℓ_1 as $v_{1,0} \leftarrow H(\ell_0 \| \ell_1)$. Then, the algorithm picks two random values $x_{1,1}, r_{1,1}$ to compute the dummy right-handed node $v_{1,1} \leftarrow \mathsf{Ch}(x_{1,1}; r_{1,1})$. The next step is to compute the parent node $v_{2,0} \leftarrow H(v_{1,0} \| v_{1,1})$ and to pick two additional random values $x_{2,1}, r_{2,1}$. The algorithm then sets $v_{2,1} \leftarrow \mathsf{Ch}(x_{2;1}; r_{2,1})$ and $\rho \leftarrow \mathsf{Ch}(v_{2,0} \| v_{2,1}; r_\rho)$ using some randomness r_ρ. The authentication path of the leaves ℓ_0 and ℓ_1 only consists of the nodes $v_{1,1}, v_{2,1}$ and the randomness r_ρ. It does *not* contain the pre-images $x_{1,1}, r_{1,1}$ (resp. $x_{2,1}, r_{2,1}$). We stress that this is crucial for the security proof.

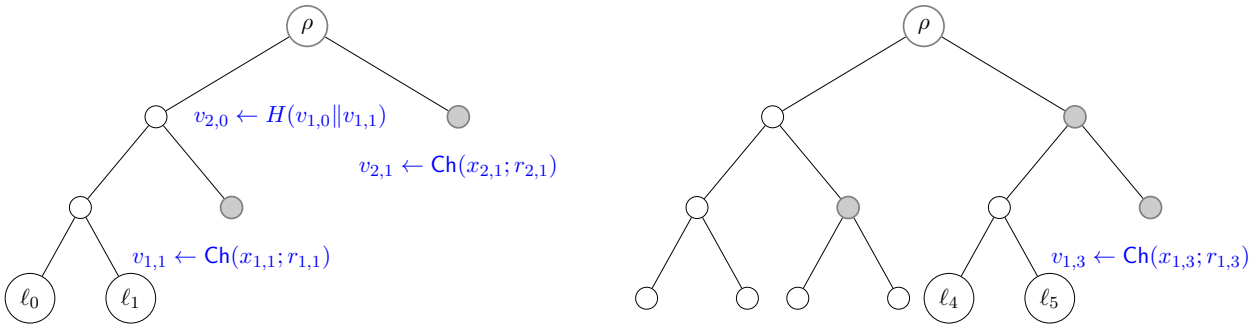

Figure 2: **Left**: A CAT of depth 3 that authenticates the leaves ℓ_0 and ℓ_1. The root and the right nodes are computed by a chameleon hash and the left nodes by a collision-resistant hash function. The leaves $\ell_2, \ldots, \ell_7$ are unknown. **Right**: Appending the leaves ℓ_4 and ℓ_5 to the CAT, requires the computation of a collision in node $v_{2,1}$.

To add the elements ℓ_2 and ℓ_3 to the tree, the algorithm sets $x'_{1,1} \leftarrow (\ell_2 \| \ell_3)$ and computes the collision for the node $v_{1,1}$ using its trapdoor csk and randomness $r_{1,1}$, i.e., $r'_{1,1} \leftarrow \mathsf{Col}(csk, x_{1,1}, r_{1,1}, x'_{1,1})$. This means that the chameleon hash function maps to the same value $v_{1,1} = \mathsf{Ch}(x_{1,1}; r_{1,1}) = \mathsf{Ch}(x'_{1,1}; r'_{1,1})$ and thus, the tree authenticates the leaves ℓ_2, ℓ_3 (using randomness $r'_{1,1}$). The authentication path of the leaves ℓ_2, ℓ_3 consists of $\mathsf{auth} = (v_{0,1}, v_{2,1})$ and $R = (r'_{1,1}, r_\rho)$. Thus, the attacker only learns $x'_{1,1}, r'_{1,1}$ and not the dummy values $x_{1,1}, r_{1,1}$ that has been used to compute $v_{1,1}$.

Now, assume that we would like to add two additional elements ℓ_4 and ℓ_5 to the CAT (c.f. right part of Figure 2). Observe that we are in the situation where all leaves in the left part of tree are used and the right part of the tree consists only of the element $v_{2,1}$. The complete right subtree with root node $v_{2,1}$ does not exist at this point as it was unnecessary to authenticate any of the previous leaves. In order to authenticate the leaves ℓ_4 and ℓ_5, our algorithm generates the skeleton of the right subtree that is needed for the corresponding authentication path. That is, the algorithm computes $v_{1,2} \leftarrow H(\ell_4 \| \ell_5)$, picks two random values $x_{1,3}, r_{1,3}$, and sets $v_{1,3} \leftarrow \mathsf{Ch}(x_{1,3}; r_{1,3})$. The last step is to apply the trapdoor to the chameleon hash function used in node $v_{2,1} = \mathsf{Ch}(x_{2,1}; r_{2,1})$, i.e., it sets $x'_{2,1} \leftarrow (v_{1,2} \| v_{1,3})$ computes $r'_{2,1} \leftarrow \mathsf{Col}(csk, x_{2,1}, r_{2,1}, x'_{2,1})$. The authentication path of the leaves ℓ_4, ℓ_5 consists of $\mathsf{auth} = (v_{1,3}, v_{2,0})$ and $R = (r_{2,1}, r_\rho)$.

We would like to draw the readers attention to the way we apply the trapdoor to the nodes. The idea is to apply the trapdoor to the first node on the path from the leaf ℓ_i to the root ρ that is computed by a chameleon hash. This way we guarantee that each trapdoor is applied to each node only once and therefore, the one who stores the tree never sees a collision.

5.3 Special Property of the Construction

One of the interesting properties of our construction is the amount of information that is needed to add an element to the tree. In fact, only the current authentication path, the pre-images of the chameleon hashes with the corresponding randomness, and the trapdoor are needed. More precisely, consider the tree shown in the left part of Figure 2. The CAT on the left side is basically the authentication path for the leaves ℓ_0, ℓ_1 and the missing nodes that are required to compute the authentication path for the leaves ℓ_4 and ℓ_5 are computed on the fly. This means that if the

tree has depth 2^D for some $D = \mathsf{poly}(\lambda)$, the clients stores only $\log(2^D) = \mathsf{poly}(\lambda)$ elements. As it turns out, this property will be very useful for our verifiable data streaming protocol, where the client essentially stores these elements and the sever the entire tree.

5.4 Construction

Although the high level idea of CATs is quickly graspable, the formal description is rather complicated. To simplify the exposition, we denote by $[\mathbf{a}]$ a vector of elements, i.e., $[\mathbf{a}] = (a_0, \ldots, a_{D-1})$ and $[(\mathbf{x}, \mathbf{r})] = ((x_0, r_0), \ldots, (x_{D-1}, r_{D-1}))$, resp.

Construction 5.1. Let $H : \{0,1\}^{2\mathsf{len}} \mapsto \{0,1\}^{\mathsf{len}}$ be a hash function and let $\mathcal{CH} = (\mathsf{Gen}, \mathsf{Ch}, \mathsf{Col})$ be a chameleon hash function such that $\mathsf{Ch} : \{0,1\}^{2\mathsf{len}} \mapsto \{0,1\}^{\mathsf{len}}$. We define the chameleon authentication tree $\mathsf{wCAT} = (\mathsf{wcatGen}, \mathsf{waddLeaf}, \mathsf{wcatVrfy})$ as follows:

$\mathsf{wcatGen}(1^\lambda, D)$**:** The key generation algorithm computes $(cpk, csk) \leftarrow \mathsf{Gen}(1^\lambda)$, it picks two random values $x_\rho \leftarrow \{0,1\}^{2\mathsf{len}}, r_\rho \leftarrow \{0,1\}^\lambda$, sets $\rho \leftarrow \mathsf{Ch}(x_\rho; r_\rho)$, sets the counter $c \leftarrow 0$, and the state to $\mathtt{st} \leftarrow (c, D, x_\rho, r_\rho)$. It returns the private key sp as $(csk, \mathtt{st})$ and the public key vp as (cpk, ρ).

$\mathsf{waddLeaf}(\mathsf{sp}, \ell)$**:** The path generation algorithm parses the private key sp as $(csk, \mathtt{st})$ and recovers the counter c from $\mathtt{st}$. Then, it parses the leaf $\ell \in \{0,1\}^{2\mathsf{len}}$ as (ℓ_c, ℓ_{c+1}) and distinguishes between two cases:

- $c = 0$**:** $\mathsf{waddLeaf}$ picks random values $x_{h,1} \leftarrow \{0,1\}^{2\mathsf{len}} r_{h,1} \leftarrow \{0,1\}^\lambda$ (for $h = 1, \ldots, D-2$), and sets $v_{h,1} \leftarrow \mathsf{Ch}(x_{h,1}; r_{h,1})$. Subsequently, it computes the authentication path for ℓ as defined in the algorithm $\mathsf{wcatVrfy}$ up to the level $D-2$. Denote by x'_ρ the resulting value. Then, $\mathsf{waddLeaf}$ applies the trapdoor csk to the root node ρ to obtain the matching randomness r'_ρ, i.e., $r'_\rho \leftarrow \mathsf{Col}(csk, x_\rho, r_\rho, x'_\rho)$. The algorithm computes the corresponding authentication path for the leaf ℓ as $\mathsf{auth} = ((v_{h+1,1}, \ldots, v_{D-1,1}), r'_\rho)$, it sets the counter $c \leftarrow 2$, and the state $\mathtt{st'} \leftarrow (c, D, x'_\rho, r'_\rho, [\mathbf{x}, \mathbf{r}], \ell_0, \ell_1)$. The algorithm returns $\mathsf{sp}' = (csk, \mathtt{st}')$, the index 0, and the authentication path auth.
- $c > 0$**:** The algorithm $\mathsf{waddLeaf}$ gets the counter c from the state st, creates a new list auth and proceeds as defined in Figure 3.

$\mathsf{wcatVrfy}(\mathsf{vp}, i, \ell, \mathsf{auth})$**:** The input of the path verification algorithm is a public key $\mathsf{vp} = (cpk, \rho)$, the index i of the leaf ℓ, and the authentication path $\mathsf{auth} = ((v_{1,\lfloor i/2 \rfloor}, \ldots, v_{D-2,\lfloor i/2^{D-2} \rfloor}), R)$, where R is a non-empty set that contains all randomness that are necessary to compute the chameleon hash functions. The verification algorithm sets $(\ell_i, \ell_{i+1}) \leftarrow \mathsf{Ch}_1(\ell; r)$ and computes the node $v_{h,i}$ for $h = 2, \ldots, D-2$ as follows:

If $\lfloor i/2^h \rfloor \equiv 1 \mod 2$**:**

$$x \leftarrow v_{h-1,\lfloor i/2^{h-1} \rfloor} || v_{h-1,\lfloor i/2^{h-1} \rfloor + 1},$$
$$v_{h,i} \leftarrow \mathsf{Ch}\left(x; r_{h,\lfloor i/2^h \rfloor}\right), \text{ with } r_{h,\lfloor i/2^h \rfloor} \in R.$$

If $\lfloor i/2^h \rfloor \equiv 0 \mod 2$**:**

$$x \leftarrow v_{h-1,\lfloor i/2^{h-1} \rfloor - 2} || v_{h-1,\lfloor i/2^{h-1} \rfloor - 1},$$
$$v_{h,i} \leftarrow H(x).$$

$c \leftarrow c+2$
for $h = 1$ to $D-2$ **do**

if $\lfloor c/2^h \rfloor$ is even **then**	**if** $\lfloor c/2^h \rfloor$ is odd **then**
if $(v_{h,\lfloor c/2^h \rfloor+1}) \notin \mathsf{st}$ **then**	**if** $(v_{h,\lfloor c/2^h \rfloor}) \in \mathsf{st}$ **then**
$x_{h,\lfloor c/2^h \rfloor+1} \leftarrow \{0,1\}^{2\mathsf{len}}$	$x'_{h,\lfloor c/2^h \rfloor} = (v_{h-1,\lfloor c/2^{h-1} \rfloor} \| v_{h-1,\lfloor c/2^{h-1} \rfloor+1})$
$r_{h,\lfloor c/2^h \rfloor+1} \leftarrow \{0,1\}^{\lambda}$	$r'_{h,\lfloor c/2^h \rfloor} \leftarrow \mathsf{Col}(csk, x_{h,\lfloor c/2^h \rfloor}, r_{h,\lfloor c/2^h \rfloor}, x'_{h,\lfloor c/2^h \rfloor})$
$v_{h,\lfloor c/2^h \rfloor+1} = \mathsf{Ch}(x_{h,\lfloor c/2^h \rfloor+1}; r_{h,\lfloor c/2^h \rfloor+1})$	$v_{h,\lfloor c/2^h \rfloor-1} = H(v_{h-1,\lfloor c/2^{h-1} \rfloor-2} \| v_{h-1,\lfloor c/2^{h-1} \rfloor-1})$
$\mathsf{st}.\mathsf{add}(x_{h,\lfloor c/2^h \rfloor+1}, r_{h,\lfloor c/2^h \rfloor+1})$	$R.\mathsf{add}(r'_{h,\lfloor c/2^h \rfloor})$
$\mathsf{auth}.\mathsf{add}(v_{h,\lfloor c/2^h \rfloor+1})$	$\mathsf{st}.\mathsf{add}(r'_{h,\lfloor c/2^h \rfloor})$
else	$\mathsf{auth}.\mathsf{add}(v_{h,\lfloor c/2^h \rfloor-1})$
$\mathsf{auth}.\mathsf{add}(v_{h,\lfloor c/2^h \rfloor+1})$	$\mathsf{st}.\mathsf{del}(v_{h-1,\lfloor c/2^{h-1} \rfloor-2}, v_{h-1,\lfloor c/2^{h-1} \rfloor-1}, x_{h,\lfloor c/2^h \rfloor}, r_{h,\lfloor c/2^h \rfloor})$
end if	**else**
end if	**end if**

end for
$R.\mathsf{add}(r)$
output $(\mathsf{sp}', c, (\mathsf{auth}, R))$

Figure 3: Algorithm to generate the authentication path.

Finally, the verifier computes the root node $\hat{\rho}$ as $\hat{\rho} \leftarrow \mathsf{Ch}(v_{D-2,0} \| v_{D-2,1}; r_\rho)$ (with $r_\rho \in R$). If $\hat{\rho} = \rho$, then the leaf is authenticated, and otherwise rejected.

5.5 Proof of Security

To explain the proof idea, consider an efficient adversary $\mathcal{A}$ that inserts at most $q := q(\lambda)$ leaves. Since the adversary is efficient, the number of leaves are polynomially bounded. The idea is to store the q leaves $\ell_1, \ldots, \ell_q$ in the tree and then to choose dummy nodes such that the entire tree has polynomial depth $D = \mathsf{poly}(\lambda)$. Notice that the entire tree does not exist at any time (cf. the tree shown in Figure 4), i.e., the subtree consists of the leaves $\ell_1, \ldots, \ell_q$, but the gray nodes, and the dotted nodes in the tree are dummy nodes. Now, recall that the adversary wins if it outputs a tuple $(\ell^*, i^*, \mathsf{auth}^*) \notin Q$. We distinguish between the case where $1 \leq i^* \leq q$ and where $q+1 \leq i^* \leq 2^D$.

In the first part of the proof, where $1 \leq i^* \leq q$, we show how to find a collision in either (1) the hash function or (2) the chameleon hash function. In the second case where we assume that $q+1 \leq i^* \leq 2^D$, we further distinguish between the cases where either (2.1) the adversary inverts the chameleon hash function or (2.2) it finds a collision in it.

The main observation in the second part is that the path from the leaf ℓ^* to the root ρ must contain a *right-handed* node on the authentication path of the node ℓ_q. In particular, this node must be one of dummy nodes. Since we only create a polynomial number of dummy nodes, we can guess which of these nodes is contained in the path. If the reduction guesses this index correctly, then it can embed the challenge of the one-wayness game. Notice, that the adversary might compute a different pre-image. In this case, however, we break the collision-resistance of the chameleon hash function. Notice that embedding this challenge is only possible, because of the careful construction of the tree as discussed in Section 5.2. In particular, this technique would *not* work if we would use the trapdoor of the chameleon hash function in a different way. Observe, that the tree still authenticates an exponential number of leaves (even if $\mathcal{A}$ is only capable of adding polynomial number of leaves to the tree).

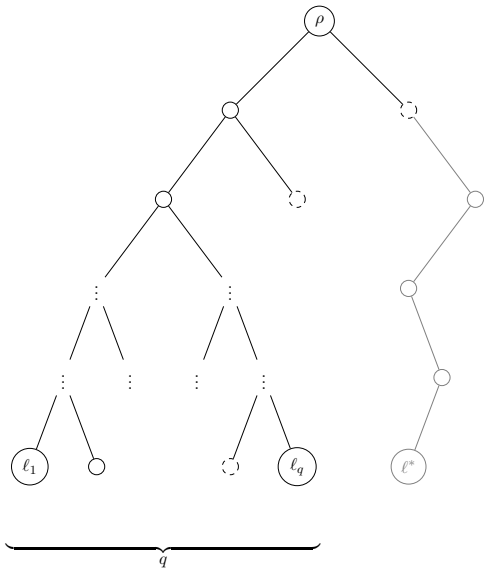

Figure 4: This figure shows how we set up the tree in the proof. The gray path corresponds to the case where the adversary outputs a pair $(\ell^*, i^*, \mathsf{auth}^*)$ such that $i^* > q$.

Theorem 5.2. *If H is a collision-resistant hash function and $\mathcal{CH}$ a one-way collision-resistant chameleon hash function, then Construction 5.1 is a chameleon authentication tree with depth $D = \mathsf{poly}(\lambda)$ and that is weakly structure-preserving and weakly one-way.*

We prove this theorem with the following two propositions.

Proposition 5.3. *If H is a collision-resistant hash function and $\mathcal{CH}$ a collision-resistant chameleon hash function, then Construction 5.1 is weakly structure-preserving.*

Proof. Suppose towards contradiction that Construction 5.1 is *not* weakly structure-preserving. Then, there exists an efficient attacker $\mathcal{A}$ that wins the security game defined in Definition 4.4 with non-negligible probability. In the following we show how to construct an algorithm $\mathcal{B}$ that either finds a collision in the hash function H (denoted by $\mathcal{B}_H$) or in the chameleon hash function Ch (denoted by $\mathcal{B}_{\mathsf{Ch}}$). Since the reduction is in both cases roughly the same, we only describe the algorithm $\mathcal{B}_H$.

Setup: The algorithm $\mathcal{B}_H$ runs a black-box simulation of $\mathcal{A}_H$, who outputs a sequence of leaves $\ell_1, \ldots, \ell_q$. The reduction $\mathcal{B}_H$ sets up a CAT as follows: It generates a key pair of the chameleon hash function $(csk, cpk) \leftarrow \mathsf{Gen}(1^\lambda)$ and picks dummy nodes $y_i \leftarrow \{0,1\}^{\mathsf{len}}$ uniformly at random such that the entire tree has depth $D = \mathsf{poly}(\lambda)$. It then computes the root ρ of the tree from the bottom to the top. Whenever a computation of a chameleon hash is involved, then $\mathcal{B}$ picks a fresh randomness uniformly at random. It stores all nodes x and the corresponding randomness r in a table T. Furthermore, it computes the authentication path auth_i for the leaf ℓ_i (for $i = 1, \ldots, q$) and sets $\mathsf{vp} \leftarrow (cpk, \rho)$. Finally, $\mathcal{B}$ runs a black-box simulation of $\mathcal{A}_H$ on $(\mathsf{vp}, \{(\ell_i, i, \mathsf{auth}_i)\}_{i=1}^q)$.

Computing the collision: Eventually, $\mathcal{A}_H$ stops outputting a pair $(\ell^*, i^*, \mathsf{auth}^*) \notin Q$ with $\mathsf{auth}^* = ((v^*_{1,\lfloor i/2 \rfloor}, \ldots, v^*_{D-2,\lfloor i/2^{D-2} \rfloor}), R^*)$ and $1 \leq i^* \leq q$. Let $\mathsf{mPath}^* = (v'^*_0, \ldots, v'^*_{D-1})$ denote the path from the leaf ℓ^* to the root. If $\mathcal{A}_H$ succeeds, then the nodes auth^* authenticates the leaf ℓ^*, but $\mathcal{A}_H$ has not received the tuple $(\ell^*, i^*, \mathsf{auth}^*)$, i.e., $(\ell^*, i^*, \mathsf{auth}^*) \notin Q$ where $Q = ((\ell_1, 1, \mathsf{auth}_1), \ldots, (\ell_{q(\lambda)}, q, \mathsf{auth}_q))$. Now consider the leaf ℓ_{i^*} together with the corresponding authentication path $\mathsf{auth} = ((v_{1,\lfloor i/2 \rfloor}, \ldots, v_{D-2,\lfloor i/2^{D-2} \rfloor}), R)$ and with its path $\mathsf{mPath} = (v'_0, \ldots, v'_{D-1}))$ to the root. Let $\mathsf{auth}_{i^*} := (v'_0, \ldots, v'_{i^*})$ be the sub-path and let $\mathsf{mPath}_{i^*} = (i, \ell, (v'_0, \ldots, v'_{i^*}))$, respectively. Then, we distinguish two cases:

Case 1: Suppose that $\mathsf{mPath} \neq \mathsf{mPath}^*$. Since both paths have the same root ρ, there must exist an index $0 \leq i < D-1$ with $\mathsf{mPath}_{i+1} = \mathsf{mPath}^*{}_{i+1}$ and $\mathsf{mPath}_i \neq \mathsf{mPath}^*{}_i$. Now, a collision is found since $\mathsf{mPath}_{i+1} = H(\mathsf{auth}_i || \mathsf{mPath}_i)$ and since $\mathsf{mPath}^*{}_{i+1} = H(\mathsf{aPath}^*{}_i || \mathsf{mPath}^*{}_i)$.

Case 2: Suppose that $\mathsf{mPath} = \mathsf{mPath}^*$. If $\ell_i \neq \ell^*$, then a collision is found. On the other hand, if $\ell_i = \ell^*$, then auth_i and auth^* are distinct. Suppose that $\mathsf{auth}_i \neq \mathsf{aPath}^*{}_i$ for an index $i < D-1$. Since $\mathsf{mPath}_{i+1} = H(\mathsf{auth}_i || \mathsf{mPath}_i)$ and because $\mathsf{mPath}^*{}_{i+1} = H(\mathsf{aPath}^*{}_i || \mathsf{mPath}^*{}_i)$ a collision is found.

It follows easily from the reduction that $\mathcal{B}_H$ performs a perfect simulation from $\mathcal{A}_H$'s point of view, and that both algorithms are efficient. Thus, $\mathcal{B}_H$ finds a collision whenever $\mathcal{A}_H$ succeeds. Denote by ϵ_H the corresponding probability. Assuming hat ϵ_H is non-negligible, however, contradicts the assumption that the hash function is collision-resistant.

The algorithm $\mathcal{B}_{\mathsf{Ch}}$ that finds a collision in the chameleon hash function Ch works analogously to $\mathcal{B}_H$. We denote by $\mathcal{A}_{\mathsf{Ch}}$ the underlying adversary and by ϵ_{Ch} its success probability. The algorithm $\mathcal{B}$ then simply guesses if it has access to $\mathcal{A}_H$ or $\mathcal{A}_{\mathsf{Ch}}$. Thus, we calculate the overall success probability $\epsilon'(\lambda)$ of $\mathcal{B}$ as

$$\epsilon'(\lambda) := \mathrm{Prob}[\mathcal{B}\ \mathsf{succ}] = \frac{1}{2}(\epsilon_H(\lambda) + \epsilon_{\mathsf{Ch}}(\lambda)),$$

where $X\ \mathsf{succ}$ denotes the event that the algorithm X wins its security game. □

Proposition 5.4. *If $\mathcal{CH}$ is a one-way collision-resistant chameleon hash function, then Construction 5.1 is weakly one-way.*

Proof. Assume to the contrary that $\mathcal{A}$ is an adversary that returns a pair $(\ell^*, i^*, \mathsf{auth}^*)$ such that $q+1 \leq i^* \leq 2^D$. Then, we construct an attacker $\mathcal{B}$ against the one-wayness of Ch.

Setup: The input of $\mathcal{B}$ is an image $y^* \leftarrow \mathsf{Ch}(x; r)$ and the public key cpk of the chameleon hash function. It runs $\mathcal{A}$ in a black-box way obtaining q leaves $\ell_1, \ldots, \ell_q$. It sets up the tree by picking $t-1$ dummy nodes $y_i \leftarrow \{0,1\}^{\mathsf{len}}$ and guessing a random index $j = 1, \ldots, t$. Then, $\mathcal{B}$ computes the tree from the bottom to the top using where the first q leaves are $\ell_1, \ldots, \ell_q$. It uses the nodes $y_1, \ldots, y^*, \ldots, y_{t-1}$ as dummy nodes such that the entire tree has depth $D = \mathsf{poly}(\lambda)$ and let ρ be the resulting root node. Furthermore, $\mathcal{B}$ computes the authentication paths auth_i for the leaves ℓ_i (for $i = 1, \ldots, q$). The attacker $\mathcal{B}$ then sets $\mathsf{vp} \leftarrow (cpk, \rho)$ and runs a black-box simulation of $\mathcal{A}$ on input $(\mathsf{vp}, \{(\ell_i, i, \mathsf{auth}_i)\}_{i=1}^q)$.

Inverting Ch: At some point, $\mathcal{A}$ might stop, outputting a pair $(\ell^*, i^*, \mathsf{auth}^*)$ with $\mathsf{auth}^* = ((v^*_{1,\lfloor i/2 \rfloor}, \ldots, v^*_{D-2,\lfloor i/2^{D-2} \rfloor}), R^*)$ and $q+1 \leq i^* \leq 2^D$. Let $\mathsf{mPath}^* = (v'^*_0, \ldots, v'^*_{D-1})$ denote the path

from the leaf ℓ^* to the root. If there exists an index j such that $v'_j = y^*$, then $\mathcal{B}$ computes the authentication path auth^* up to y^*. Then, the algorithm $\mathcal{B}$ outputs the resulting pre-image $x^* = \mathsf{auth}^*$ together with the corresponding randomness r^*. Otherwise, it aborts.

For the analysis first note that $\mathcal{B}$ performs a perfect simulation from $\mathcal{A}$'s point of view (applying the same arguments as in the previous proof). Now, assume that $\mathcal{A}$ succeeds with non-negligible probability. Then, it returns a valid pair $(\ell^*, i^*, \mathsf{auth}^*)$ with $\mathsf{auth}^* = ((v^*_{1,\lfloor i/2 \rfloor}, \ldots, v^*_{D-2,\lfloor i/2^{D-2} \rfloor}), R^*)$ such that $q+1 \leq i^* \leq 2^D$. Since the authentication path verifies, it follows from our construction that one node in auth^* is a right-handed node on the authentication path of ℓ_q. Since it is a right node, it follows that this node is the output of a chameleon hash function. Now, assume that $\mathcal{B}$ has guessed this node correctly. Then, it follows from our construction that $\mathcal{B}$ computes a pre-image (x^*, r^*) of y^*. We have to show that $(x^*, r^*) = (x, r)$. This, however, follows from the collision-resistance of Ch. If we assume towards contradiction that $(x^*, r^*) \neq (x, r)$, then we can easily build an adversary that finds a collision in Ch.

Assume further that $\mathcal{A}$ succeeds with non-negligible probability $\epsilon_O(\lambda)$. Then, it is easy to see that $\mathcal{B}$ wins with probability $\delta_O(\lambda) := \epsilon_O(\lambda)/t$. This, however, either contradicts the one-wayness or the collision-resistance of Ch. □

Remark. It is well-known that chameleon hash function can be build under the discrete logarithm assumption [KR00] and also assuming the hardness of the short integer solution (SIS) that is related to worst-case lattice-based assumptions such as approximating the shortest vector problem [GPV08, CHKP10]. Thus, we obtain the following result.

Proposition 5.5. *Chameleon authentication trees exist in the standard model if the discrete logarithm problem is hard (according to Theorem 8.1) and also under the SIS assumption.*

6 Adaptively Secure CATs

In this section, we present a general transformation that turns any weakly secure CAT into one that satisfies our strong notions of security according to Definitions 4.2 and 4.3. Our construction follows the transformation that turns any signature scheme secure against existential unforgeability with respect to weak chosen-message attacks into an adaptively secure one [KR00, ST01, BB04, HW09a]. The basic idea is to store the output of a chameleon hash with an independent key in the leaves.

Construction 6.1. Let $\mathsf{wCAT} = (\mathsf{wcatGen}, \mathsf{waddLeaf}, \mathsf{wcatVrfy})$ be a CAT and $\mathcal{CH} = (\mathsf{Gen}, \mathsf{Ch}, \mathsf{Col})$ a chameleon hash function. We define the chameleon authentication tree $\mathsf{CAT} = (\mathsf{catGen}, \mathsf{addLeaf}, \mathsf{catVrfy})$ as follows:

$\mathsf{catGen}(1^\lambda, D)$**:** The key generation algorithm runs $(\mathsf{wsp}, \mathsf{wvp}) \leftarrow \mathsf{wcatGen}(1^\lambda)$ and $(csk, cpk) \leftarrow \mathsf{catGen}(1^\lambda)$, it sets (wsp, csk) as sp and (wvp, cpk) as vp.

$\mathsf{addLeaf}(\mathsf{sp}, \ell)$**:** The path generation algorithm parses sp as (wsp, csk). It picks $r \leftarrow \{0,1\}^\lambda$ uniformly at random, sets $x \leftarrow \mathsf{Ch}(\ell; r)$, computes $(\mathsf{wsp}', i, \mathsf{wauth}) \leftarrow \mathsf{waddLeaf}(\mathsf{wsp}, x)$ and returns $\mathsf{auth} \leftarrow (\mathsf{wauth}, i, r)$.

$\mathsf{catVrfy}(\mathsf{vp}, i, \ell, \mathsf{auth})$**:** The path verification algorithm parses vp as (wvp, cpk) and auth as (wauth, i, r). It sets $x \leftarrow \mathsf{Ch}(\ell; r)$ and returns $\mathsf{wcatVrfy}(\mathsf{wvp}, i, \mathsf{wauth}, x)$.

We prove the security of our construction with the following theorem.

Theorem 6.2. *If* $\mathsf{wCAT} = (\mathsf{wcatGen}, \mathsf{waddLeaf}, \mathsf{wcatVrfy})$ *is a weakly structure-preserving (resp. weakly one-way) chameleon authentication tree and* $\mathcal{CH} = (\mathsf{Gen}, \mathsf{Ch}, \mathsf{Col})$ *a one-way collision-resistant chameleon hash function, then Construction 6.1 is structure-preserving (resp. one-way).*

The proof follows the one of [HW09b], but differs in one aspect. In the case of signature schemes, the attacker succeeds if he forges a fresh message. In our case, however, this might not be the case. In particular, an attacker succeeds if he manages to change the order of two streamed values. Thus, we have to cover the case where $\mathcal{A}$ re-uses some values *without* violating the collision-resistance property of $\mathcal{CH}$.

Proof. Let $(\ell^*, i^*, \mathsf{auth}^*)$ be the output by the adversary $\mathcal{A}$ against the structure-preserving (resp. one-wayness) property, where $\mathsf{auth}^* = (\mathsf{wauth}^*, i^*, r^*)$ and $x^* = \mathsf{Ch}(\ell^*; r^*)$. We construct an algorithm $\mathcal{B}$ that constructs the CAT and records the values of all leaves and their corresponding authentication paths. Let $((x_1, 1, \mathsf{auth}_1), \ldots, (x_q, q, \mathsf{auth}_q))$ be the CAT after $\mathcal{A}$ added q leaves (adaptively) and consider the tuple $(\ell_{i^*}, i^*, \mathsf{auth}_{i^*})$ that was initially stored in the CAT at position i^*. The key observation is that we can distinguish between two cases: We say that the adversary $\mathcal{A}$ is a type-1 attacker, if $x_{i^*} = x^*$. Otherwise, if $x_i \neq x^*$, then we say that $\mathcal{A}$ is of type-2. In the first case, we construct an attacker against the collision-resistance of $\mathcal{CH}$. Otherwise, if $x_{i^*} \neq x^*$, then we build an attacker breaking the weak structure-preserving (resp. one-wayness) property of wCAT. The algorithm $\mathcal{B}$ initially guesses which type of attacker $\mathcal{A}$ is.

Type-1 Attacker. If $\mathcal{B}$ has black-box access to a type-1 attacker, then it breaks the collision-resistance of $\mathcal{CH}$ as follows:

Setup: The input of $\mathcal{B}$ is the public key cpk of the chameleon hash $\mathcal{CH}$. It generates a key pair of the CAT $(\mathsf{wsp}, \mathsf{wvp}) \leftarrow \mathsf{wcatGen}(1^\lambda, D)$ and runs $\mathcal{A}$ on $\mathsf{vp} = (\mathsf{wvp}, cpk)$.

Queries: Whenever $\mathcal{A}$ wishes to add a leaf ℓ to the tree, then $\mathcal{B}$ picks a value $r \leftarrow \{0,1\}^\lambda$ uniformly at random, sets $x \leftarrow \mathsf{Ch}(\ell; r)$, computes the path as $(\mathsf{wsp}', i, \mathsf{wauth}) \leftarrow \mathsf{waddLeaf}(\mathsf{wsp}, x)$, and returns $\mathsf{auth} = (\mathsf{wauth}, i, r)$. In addition, it records the pair (ℓ, r) in a table T.

Output: Eventually, $\mathcal{A}$ stops, outputting a pair $(\ell^*, i^*, \mathsf{auth}^*)$ with $\mathsf{auth}^* = (\mathsf{wauth}^*, r^*)$, $x^* = \mathsf{Ch}(\ell^*; r^*)$. The reduction $\mathcal{B}$ returns $(\ell^*, r^*), (\ell_{i^*}, r_{i^*})$.

For the analysis observe, that $\mathcal{B}$ is efficient, because $\mathcal{A}$ runs in polynomial time. Now, assume that $\mathcal{A}$ succeeds with non-negligible probability and that $\mathcal{B}$ guessed correctly. In this case, $\mathcal{B}$ returns a collision. To see this, note that ℓ^* and ℓ_{i^*} must be distinct, because $\mathcal{A}$ wins the game. Since $\mathcal{A}$ is a type-1 attacker, it follows that $x_i = x_i^*$. This, however, contradicts the assumption that $\mathcal{CH}$ is collision-resistant.

Type-2 Attacker. We use a type-2 attacker $\mathcal{A}$ in a black-box way to construction a reduction $\mathcal{B}$ against the weak structure-preserving property as follows:

Setup: The algorithm $\mathcal{B}$ picks q values $\ell_1, \ldots, \ell_q$ uniformly at random from $\{0,1\}^{\mathsf{in}}$ and q strings $r_1, \ldots, r_q$ at random from $\{0,1\}^\lambda$. It stores these values in a table T and generates a key pair of the chameleon hash $(csk, cpk) \leftarrow \mathsf{Gen}(1^\lambda)$ and outputs $x_i \leftarrow \mathsf{Ch}(\ell_i; r_i)$ for $i = 1, \ldots, q$. The

challenger answers with the public key wvp and the list of corresponding authentication paths $((1, \mathsf{wauth}_1), \ldots, (q, \mathsf{wauth}_q))$. Then, $\mathcal{B}$ sets $\mathsf{vp} \leftarrow (\mathsf{wvp}, cpk)$ and runs a black-box simulation of $\mathcal{A}$ on vp.

Queries: Whenever $\mathcal{A}$ wishes to add a leaf ℓ' to the tree, then $\mathcal{B}$ computes a collision in the chameleon hash $r' \leftarrow \mathsf{Col}(csk, \ell, r, \ell')$, and returns $\mathsf{auth} = (\mathsf{wauth}, i, r')$.

Output: Eventually, $\mathcal{A}$ stops, outputting a pair $(\ell^*, i^*, \mathsf{auth}^*)$ with $\mathsf{auth}^* = (\mathsf{wauth}^*, r^*)$, $x^* = \mathsf{Ch}(\ell^*; r^*)$. The algorithm $\mathcal{B}$ returns $(x^*, i^*, \mathsf{wauth}^*)$.

It follows from our construction that $\mathcal{B}$ is efficient, because $\mathcal{A}$ runs in polynomial-time. Let us assume that $\mathcal{A}$ succeeds with non-negligible probability and that $\mathcal{B}$ guessed the type of attacker correctly. This means, however, that $x^* \neq x_{i^*}$ which contradicts the assumption that the underlying CAT is weakly structure-preserving. □

The proof for (weakly) one-wayness is the same and is omitted.

6.1 Comparison to Mercurial Commitments

Building authentication trees like structures from other primitives than collision-resistant hash functions has been done before. For example Chase et al. introduce a new flavor of commitment schemes that are called mercurial commitments [CHL+05]. Loosely speaking, a mercurial commitment is a commitment scheme that provides a hard and a soft decommitment algorithm. The hard decommitment behaves like a regular decommitment. But soft decommitment are different in the sense that binding does not hold anymore. In some sense a soft decommitment provides a chameleon hash functionality to the commitment scheme. In [CHL+05] the author also suggest a tree based construction where they combine hard and soft commitments in the tree. This construction, however, is not comparable to our solution. The main reason is that the efficiency of our construction depends heavily on an exact combination of hash functions and chameleon hash functions. Moreover, we designed our construction in such a way that only a single inversion of the chameleon hash is needed. The tree designed in [CHL+05] depends on the database. Thus, the structure of both trees is simply not comparable. Moreover, it is easy to show that mercurial commitments are a strictly stronger primitive.

7 Construction of a Verifiable Data Streaming Protocol

Our VDS is not a completely black-box construction from a CAT, because updating leaves is not supported by a CAT in general. Instead, we use the algorithms of a CAT whenever it is possible and exploit the concrete structure of our scheme when we describe the update mechanism and also in the proof.

The main idea of our construction is to split the data in the CAT between the server $\mathcal{S}$ and the client $\mathcal{C}$. That is, the client basically stores the trapdoor and the authentication path of the current value. As discussed in Section 5.3, this information is sufficient to authenticate the next leaf. The server, however, stores the entire tree (as it has been specified so far) and the randomness of all chameleon hashes learned so far. As an example, consider the tree in Figure 1. The client stores the blue authentication path including the values $(x_{3,1}, r_{3,1})$ and $(x_{1,3}, r_{1,3})$ of the two "unused" inner nodes and the trapdoor csk, while the sever stores the entire tree.

To retrieve any element from the database, the client sends the index i to the server who returns the element $s[i]$ together with the corresponding authentication path $\pi_{s[i]} = \mathsf{auth}_{s[i]}$. Verifying works straightforwardly by checking the authentication path.

Updating an element $s[i]$ to $s'[i]$ in DB work as follows: First, $\mathcal{C}$ runs the query algorithm to obtain the element $s[i]$ and the corresponding authentication path $\pi_{s[i]} = \mathsf{auth}_{s[i]}$. If the verification algorithm $\mathsf{Verify}(PK, i, s, \pi_{s[i]})$ evaluates to 1, then $\mathcal{C}$ updates the leaf $\ell_i = s[i]$ to $s[i]'$. Subsequently, it updates the authentication path $\mathsf{auth}_{\ell[i]}$ to $\mathsf{auth}_{s'[i]}$ analogously to the algorithm $\mathsf{wcatVrfy}$ as defined in Construction 5.1. Denote by ρ' the resulting value. The client $\mathcal{C}$ sets $\rho \leftarrow \rho'$ in its public key PK and sends the new authentication path $\mathsf{auth}_{s'[i]}$ to $\mathcal{S}$. The server updates the entry in DB, all leaves, and the root, which results in a new public key PK'.

Construction 7.1. Let $\mathsf{CAT} = (\mathsf{catGen}, \mathsf{addLeaf}, \mathsf{catVrfy})$ be the chameleon authentication tree as defined in Construction 6.1. We define the verifiable data streaming protocol $\mathcal{VDS} = (\mathsf{Setup}, \mathsf{Append}, \mathsf{Query}, \mathsf{Verify}, \mathsf{Update})$ as follows:

$\mathsf{Setup}(1^\lambda)$**:** The setup algorithm picks some $D = \mathsf{poly}(\lambda)$ and generates the CAT $(\mathsf{sp}, \mathsf{vp}) \leftarrow \mathsf{catGen}(1^\lambda, D)$ as defined in Construction 6.1. In particular, the private key is $SK = \mathsf{sp} = (csk, csk_1, \mathtt{st})$ and the public key is $PK = \mathsf{vp} = (cpk, cpk_1, \rho)$, where ρ is the root of the initially empty tree. The client $\mathcal{C}$ gets the private key SK and the server the public key PK. The server also sets up an initially empty database DB.

$\mathsf{Append}(SK, s)$**:** To append an element s to DB, the client $\mathcal{C}$ runs the algorithm $\mathsf{addLeaf}(\mathsf{sp}, s)$ locally which returns a key sp', an index i, and an authentication path auth_i. It sends i, s and auth_i to the sever $\mathcal{S}$. The server appends s to DB, it adds the unknown nodes from $\mathsf{auth}_i = ((v_{1,\lfloor i/2 \rfloor}, \ldots, v_{D-2,\lfloor i/2^{D-2} \rfloor}), R)$ to its tree, and stores the new randomness from R.

$\mathsf{Query}(PK, DB, i)$**:** The client sends the index i to the server who responses with $s[i]$ and the corresponding authentication path $\pi_{s[i]} = \mathsf{auth}_i$, or with $\perp$ if the i^{th} entry in DB is empty.

$\mathsf{Verify}(PK, i, s, \pi_{s[i]})$**:** The verification algorithm parses PK as vp and $\pi_{s[i]}$ as $\mathsf{auth}_{s[i]}$, it returns s if the algorithm $\mathsf{catVrfy}(\mathsf{vp}, i, s, \pi_i)$ outputs 1. Otherwise, it outputs $\perp$.

$\mathsf{Update}(PK, DB, SK, i, s')$**:** To update the i^{th} element in DB to s', the clients parses SK as the trapdoor sp of the CAT, its state $\mathtt{st}$, and the pairs $(x_{i,j}, r_{i,j})$ of the "unused" nodes computed via the chameleon hash function ("unused" means that the trapdoor has not been applied to these nodes). The first move in the protocol is by the client who sends the index i to $\mathcal{S}$. The server returns $s[i]$ and the corresponding proof $\pi_{s[i]}$ (which is the authentication path auth_i). The client $\mathcal{C}$ runs $\mathsf{Verify}(PK, i, s[i], \pi_{s[i]})$ to check the validity of $s[i]$. If Verify returns $\perp$, then $\mathcal{C}$ aborts. Otherwise, it sets the leaf $\ell_i = s[i]$ to s' and re-computes the authentication path with the new leaf (as described in Construction 5.4). The output of this algorithm is a new root ρ'. Subsequently, the client updates all nodes that are stored in its state $\mathtt{st}$, but that have been updated by re-computing the authentication path with the new leaf (this includes at least the root ρ and thus, the verification key PK). Notice that the randomness used for the chameleon hash functions remain the same. Then, $\mathcal{C}$ sends the new authentication path auth'_i, the updated leaf s', and the updated verification key PK' to the server. The server first verifies the authentication path. If it is valid, then $\mathcal{S}$ updates the stored value $s[i]$ to $s[i]'$, the corresponding nodes in the CAT, as well as its verification key PK'. Otherwise, it aborts.

Regarding security, we prove the following theorem:

Theorem 7.2. *If H is a collision-resistant hash function and $\mathcal{CH} = (\mathsf{Gen}, \mathsf{Ch}, \mathsf{Col})$ a chameleon hash function that is one-way and collision-resistant, then Construction 7.1 is a secure verifiable data streaming protocol w.r.t. Definition 2.2.*

Proof. The following proof is nonblack-box in the sense that we present a reduction to the collision-resistance (resp. one-wayness) of the chameleon hash (resp. of the hash function). The proof idea builds up on both, the proofs of Theorems 5.2 and 6.2. That is, the proof consists of three main parts:

- First, we build a weakly secure CAT from a (chameleon) hash function. Recall that weakly secure means that the adversary commits to his queries *before* learning the public key. This part follows from Theorem 5.2.
- Second, we store the output of a different chameleon hash in the leaves of our weakly secure CAT. The basic idea of this transformation is that the simulator commits to random values in order to obtain the public key. Then it uses the trapdoor of the chameleon hash in order to answer the adaptively chosen queries by the adversary (see Theorem 6.2).
- Third, the problem with the adaptivity of CATs is that leaves cannot be changed after-the-fact. In a VDS, however, the client is allowed to change previously streamed values whenever it wishes. Recall that updating means in our construction to re-compute the tree with the fresh leaf. Since this operation does not affect "unused" leaves (in particular, this operation does not leak any information about the randomness of "future" leaves), the simulator can safely perform this operation locally.

Recall that the public key $PK = (cpk, cpk_1, \rho)$ and we set $\mathsf{Ch}(\cdot) := \mathsf{Ch}(cpk, \cdot)$ and $\mathsf{Ch}_1(\cdot) := \mathsf{Ch}(cpk_1, \cdot)$. Suppose that $\mathcal{A}$ is a PPT adversary against the VDS according to Definition 2.2 that streams at most $q := q(\lambda)$ elements to the challenger. Eventually, $\mathcal{A}$ returns a tuple (i^*, s^*, π^*), such that (i^*, s^*, π^*) is a valid tuple w.r.t. PK^*, but $s^* \neq s[i^*]$. In the following, we distinguish between two cases: we say that $\mathcal{A}$ is a type-1 attacker if $1 \leq i^* \leq q$ and he is of type-2 if $q < i^* \leq n$ (recall that n is the size of the database). In what follows, we are building a reduction that initially guesses which type of adversary $\mathcal{A}$ is.

Type-1 Attacker: Consider the output (i^*, s^*, π^*) of the attacker $\mathcal{A}$, where the proof $\pi^* = \mathsf{auth}^* = (\mathsf{wauth}^*, i^*, r^*)$ and $x^* = \mathsf{Ch}_1(s^*; r^*)$. Now, similar to the proof of Theorem 5.2, we distinguish between two cases: If the value x^* is stored at position i^*, then we say that $\mathcal{A}$ is of type-1a. Otherwise, if $x^* \neq x[i^*]$, we say that $\mathcal{A}$ is of type-1b. If $\mathcal{B}$ has access to an attacker of type-1a, then $\mathcal{B}$ finds a collision in $\mathcal{CH}_1$ and if $\mathcal{A}$ is of type-1b he finds in a collision in either $\mathcal{CH}$ or in H. The algorithm $\mathcal{B}$ initially guesses what type $\mathcal{A}$ is.

Type-1a Attacker: The algorithm $\mathcal{B}$ with access to an adversary $\mathcal{A}$ of type-1a works as follows:

Setup: The input of $\mathcal{B}$ is a public key cpk_1 and recall that $\mathsf{Ch}_1(\cdot) := \mathsf{Ch}(cpk_1, \cdot)$. The algorithm $\mathcal{B}$ picks another key pair of a chameleon hash function $(cpk, csk) \leftarrow \mathsf{Gen}(1^\lambda)$ (with $\mathsf{Ch}(\cdot) := \mathsf{Ch}(cpk, \cdot)$) and picks two values (x_ρ, r_ρ) uniformly at random. Then, $\mathcal{B}$ sets $\rho \leftarrow \mathsf{Ch}(x_\rho; r_\rho)$, $PK \leftarrow (cpk, cpk_1, \rho)$, it creates an initially empty database DB, sets the counter $c \leftarrow 0$, and runs $\mathcal{A}$ on input PK.

Insert Query: Whenever $\mathcal{A}$ asks the challenger to insert an element s into DB, then $\mathcal{B}$, increases the counter $c \leftarrow c+1$, picks a fresh value r_c uniformly at random, sets $x_c \leftarrow \mathsf{Ch}(s_c; r_c)$ records the tuple (c, s_c, r_c) in DB, and simulates the algorithm $(\mathsf{sp}', i, \mathsf{wauth}) \leftarrow \mathsf{waddLeaf}(csk, x_c)$ as defined in Construction 5.1. It returns $\pi_{s[c]} = \mathsf{auth}_c = (\mathsf{wauth}_c, c, r_c)$ to $\mathcal{A}$.

Update Query: The algorithm $\mathcal{B}$ answers update queries of the form (i, s') in the following way. It sets the i^{th} entry of DB to s' and updates the nodes that lay on the authentication path auth' of s'. Denote the updated root node by ρ'. Then, it updates the public key to $PK' \leftarrow (cpk, cpk_1, \rho')$, and returns the proof $\pi_{s'}$ to $\mathcal{A}$.

Output: At some point, the attacker $\mathcal{A}$ may stop, outputting a tuple (i^*, s^*, π^*), such that $\mathsf{Verify}(PK^*, i^*, s^*, \pi_{s^*}) = 1$. Notice that PK^* is the public key hold by the challenger. The algorithm $\mathcal{B}$ parses $\pi^* = \mathsf{auth}^* = (\mathsf{wauth}, r^*)$, it recovers (s_{i^*}, r_{i^*}) from DB and returns $(s^*, r^*), (s_{i^*}, r_{i^*})$.

It follows from the construction that $\mathcal{B}$ is efficient and that it performs a perfect simulation from $\mathcal{A}$'s point of view. In the following assume that $\mathcal{A}$ is of type-1a and that it succeeds with non-negligible probability. But if both conditions hold, then $s^* \neq s_{i^*}$ (because $\mathcal{A}$ succeeds) and $\mathsf{Ch}(s^*; r^*) = \mathsf{Ch}(s_{i^*}; r_{i^*})$ (because $\mathcal{A}$ is of type-1a). This, however, contradicts the assumption that $\mathcal{CH}$ is collision-resistant.

Type-1b Attacker: This part of the proof is the same as the proof of Theorem 6.2. The main observation is that both elements $(s[i^*], \mathsf{auth}[i^*])$ and the adversaries output (s^*, auth^*) verify under the same public key PK^* at the same position i^* in tree. Thus, there must be a collision in either the hash function or the chameleon hash function. Since the proof is roughly the same as the one of Theorem 6.2, we omit it here.

Type-2 Attacker: This part of the proof is analogously to the one of Proposition 5.4. The main observation is that the authentication path corresponding to $\mathcal{A}$'s output, must contain at least one right-handed node on the authentication path of the last inserted value s_q. Since it must be a right-handed node, it is computed by a chameleon hash function. Furthermore, the update queries do not change this fact as they only involve "old" values. We build an algorithm $\mathcal{B}$ that either breaks the one-wayness or the collision-resistance of $\mathcal{CH}$. We describe the reduction to the one-wayness, but we stress that the one against collision-resistance is roughly the same.

Setup: The input of $\mathcal{B}$ is a public key cpk and a challenge image y^*. Let $\mathsf{Ch}(\cdot) := \mathsf{Ch}(cpk, \cdot)$. It generates an additional key pair $(csk_1, cpk_1) \leftarrow \mathsf{Gen}(1^\lambda)$, q pairs (x, r) uniformly at random, and t random dummy nodes $y \in \{0,1\}^{\mathsf{len}}$ such that the resulting tree has depth D. Let $\mathsf{Ch}_1(\cdot) := \mathsf{Ch}(cpk_1, \cdot)$. Then, $\mathcal{B}$ sets $\ell_i \leftarrow \mathsf{Ch}_1(x_i; r_i)$ (for $i = 1, \ldots, q$), it stores the pairs (x_i, r_i) in a table T, and creates a binary authentication tree as follows. The first q leaves of the tree are $\ell_1, \ldots, \ell_q$. Then, it sets up a list $y_1, \ldots, y^*, \ldots, y_t$ in which the challenge y^* is embedded at a randomly chosen position i. These dummy nodes are the right nodes on the authentication paths in the tree that are needed to obtain a tree of depth D (see Figure 4). It sets $PK \leftarrow (cpk, cpk_1, \rho)$ and runs $\mathcal{A}$ in a black-box way on PK.

Insert Queries: Whenever $\mathcal{A}$ wishes to insert an element s_i in the database, then $\mathcal{B}$ retrieves the pair (x_i, r_i) from T and computes a collision in the chameleon hash for which $\mathcal{B}$ knows the

trapdoor: $r'_i \leftarrow \mathsf{Col}(csk_1, x_i, r_i, s_i)$. It returns the corresponding authentication path auth_i as the proof π_i to $\mathcal{A}$.

Update Queries: At some point, $\mathcal{A}$ sends (i, s') to the challenger to update the i^{th} entry in DB to s'. The algorithm $\mathcal{B}$ replaces s by s' and updates the nodes that lay on the authentication path auth' of s'. Denote the updated root node by ρ'. Then, it updates the public key to $(PK' \leftarrow (cpk, cpk_1, \rho'))$, and returns the proof $\pi_{s'}$ to $\mathcal{A}$.

Output: Eventually, $\mathcal{A}$ stops, outputting a tuple (i^*, s^*, π^*), such that $\mathsf{Verify}(PK, i^*, s^*, \pi_{s^*}) = 1$. The algorithm $\mathcal{B}$ inverts Ch as follows. It first checks that y^* lies on the authentication path $\pi_{s^*} = \mathsf{auth}_{s^*}$. If this is not the case, it aborts. Otherwise, if y^* lies on the authentication path, then $\mathcal{B}$ re-computes the authentication path up to the point where y^* is the root node (following Construction 5.1) and denote by x^*, r^* the resulting values. The algorithm $\mathcal{B}$ stops, outputting x^*, r^*.

For the analysis, observe that $\mathcal{B}$ performs a perfect simulation from $\mathcal{A}$'s point of view and that $\mathcal{B}$ is efficient, because $\mathcal{A}$ runs in polynomial time. Furthermore, assume that $\mathcal{A}$ succeeds with non-negligible probability ϵ and that $q < i^* \leq n$ because $\mathcal{A}$ is a type -2 attacker. If $q < i^* \leq n$, then at least one node of $y_1, \ldots, y^*, \ldots, y_t$ lies on the authentication path of s^*. Since t is polynomially bounded, $\mathcal{B}$ can guess the index with non-negligible probability $1/t$. Assuming that $\mathcal{B}$ guessed the index correctly, implies that x^*, r^* is a valid pre-image of the challenge y^*. Finally, it might be the case that $\mathcal{B}$ computes a *different* pre-image, i.e., $y^* \leftarrow \mathsf{Ch}(\hat{x}, \hat{r})$, but $(x^*, r^*) \neq (\hat{x}, \hat{r})$. If this would be the case, then we could easily build a reduction against the collision-resistance of $\mathcal{CH}$ by initially choosing $(\hat{x}, \hat{r})$ uniformly at random and performing the same simulation. Thus, we now assume that $(x^*, r^*) = (\hat{x}, \hat{r})$, and therefore $\mathcal{B}$ inverts the chameleon hash function with non-negligible probability. This, however, contradicts our initial assumption that $\mathcal{CH}$ is one-way (resp. collision-resistant). □

8 Concrete Instantiation With Faster Verification

Our construction can be instantiated with any chameleon hash function, but we choose the one of Krawczyk and Rabin (in the following called KR chameleon hash function) [KR00]. The scheme is secure under the discrete logarithm assumption (in the standard model)[1], which is a very appealing and mild assumption. Furthermore, the algebraic properties allow us to construct a very efficient path verification algorithm by applying batch verification techniques.

Assumption 8.1. *Let q be a prime and let $p = 2q + 1$ be a strong prime. Let $\mathbb{G}$ be the unique cyclic subgroup of $\mathbb{Z}_p^*$ of order q and let g be a generator of $\mathbb{G}$. Then, the* discrete logarithm problem *holds if for all efficient algorithms $\mathcal{A}$ the probability*

$$Prob\left[\ y \leftarrow \mathbb{G}\,; \alpha \leftarrow \mathcal{A}(p, q, g, y) :\ 0 \leq \alpha \leq q - 1 \quad \wedge g^{\alpha} \equiv y \mod p\ \right] = \nu(\lambda)$$

is negligible (as a function of λ).

[1] To the best of our knowledge, this instantiation is currently the most efficient one based on the discrete logarithm assumption in the standard model.

8.1 Building Block

The KR chameleon hash function $\mathcal{CH} = (\mathsf{Gen}, \mathsf{Ch}, \mathsf{Col})$ is defined as follows:

Gen(1^λ)**:** The key generation algorithm picks a prime q such that $p = 2q + 1$ is also prime. It also chooses a random generator g and a value $\alpha \in \mathbb{Z}_q^*$. It returns $(csk, cpk) \leftarrow ((\alpha, g), (X, g, p, q))$ with $X = g^\alpha \mod p$.

Ch(cpk, x)**:** The input of the hash algorithm is a key $cpk = (X, g, p, q)$ and a message $x \in \mathbb{Z}_q^*$. It picks a random value $r \in \mathbb{Z}_q^*$ and outputs $g^x X^r \mod p$.

Col(csk, x, r, x')**:** The collision finding algorithm returns $r' \leftarrow \alpha^{-1}(x - x') + r \mod q$.

8.2 Batch Verification of Chameleon Hash Functions

The most expensive operation in a CAT of depth D is the verification of an authentication path. This computation involves (in the worst case) D computations of the chameleon hash function (which is the authentication of the last leaf). For our concrete instantiation this means that the verification algorithm verifies D times equations of the form $h_i = g^{x_i} X^{r_i}$. This step is rather expensive as it involves many modular exponentiations. Instead of verifying all equations straightforwardly, we apply *batch verification* techniques as introduced by Bellare, Garay, and Rabin [BGR98]. The basic idea is to verify sequences of modular exponentiations significantly faster than the naïve re-computation method. In what follows, let λ_b be the security parameter such that the probability of accepting a batch that contains an invalid hash is at most $2^{-\lambda_b}$. Note, that it is necessary to test that all elements belong to the group $\mathbb{G}$ as discussed comprehensively by Boyd and Pavlovski in [BP00]. The size of λ_b is a trade off between efficiency and security. Therefore, it depends heavily on the application. Camenisch, Hohenberger, and Pedersen suggest $\lambda_b = 20$ bits for a rough check and $\lambda_b = 64$ bit for higher security [CHP07].

Definition 8.2. *A* batch verifier *for a relation R is a probabilistic algorithm V that takes as input (possibly a description of R) a batch instance $X = (inst_1, \ldots, inst_D)$ for R, and a security parameter λ. It satisfies:*

(1) If X is correct, then V outputs 1.

(2) If X is incorrect, then the probability that V outputs 1 *is at most* $2^{-\lambda_b}$.

The naïve batch verifier re-computes all instances, i.e., it consists of computing $R(inst_i)$ for each $i = 1, \ldots, D$, and checking that each of these D values is 1.

8.2.1 Small Exponent Test for Chameleon Hash Functions

Bellare, Garay, and Rabin suggest three different methods of computing batch verification for modular exponentiations [BGR98]. Here, we focus only on the small exponent test because it is the most efficient one for the verification of *up to* 200 elements. In our scenario, 200 elements means that we can authenticate 2^{200} elements. The authors consider equations of the form $y_i = g^{x_i}$. The naïve approach would be to check if $\prod_D y_i = g^{\sum_D x_i}$. This, however, is not sufficient as it is easy to produce two pairs (x_1, y_1) and (x_2, y_2) that pass the verification but each individual does not. One example of such a pair is $(x_1 - \beta, y_1)$ and $(x_2 + \beta, y_2)$ for any β.

According to [FGHP09], the small exponent test works as follows: Pick D exponents δ_i of a small number of $\{0,1\}^{\lambda_b}$ at random and compute $x \leftarrow \sum_D x_i\delta_i \mod q$ and $y \leftarrow \prod_D y_i^{\delta_i}$. Output 1 iff $g^x = y$. The probability of accepting a bad pair is $2^{-\lambda_b}$ and the value λ_b is a trade off between efficiency and security.

8.3 Batch Verification of KR Chameleon Hash Function

The algorithm Batch that performs the batch verification of D chameleon hash values $h_1, \ldots, h_D$ on messages $x_1, \ldots, x_D$ using the randomness $r_1, \ldots, r_D$ works as follows: It first checks that all elements are in the group. If not, then it rejects the query. Otherwise, it picks D random elements $\delta_1, \ldots, \delta_D$ where $\delta_i \in \{0,1\}^{\lambda_b}$ and checks that

$$g^{\sum_D x_i\delta_i} X^{\sum_D r_i\delta_i} = \prod_D h_i^{\delta_i}.$$

It outputs 1 if the equation holds and otherwise 0.

Theorem 8.3. *The algorithm* Batch *is a batch verifier for the KR chameleon hash function.*

The following proof follows the proofs of [CHP07, BGR98].

Proof. We first show that if all hash values have the desired form, then our batch verification algorithm accepts with probability 1. Keeping in mind that the validation of the hash function checks that $h_i = g^{x_i} X^{r_i}$, then we can easily show that

$$\begin{aligned} g^{\sum_D x_i\delta_i} X^{\sum_D r_i\delta_i} &= \prod_D h_i^{\delta_i} = \prod_D (g^{x_i} X^{r_i})^{\delta_i} \\ &= \prod_D g^{x_i\delta_i} X^{r_i\delta_i} = g^{\sum_D x_i\delta_i} X^{\sum_D r_i\delta_i}. \end{aligned}$$

The next step is to show that the other direction is also true. To do so, we apply the technique for proving small exponents test as in [BGR98]. Since our batch verification algorithm accepts, it follows that $h_i \in \mathbb{G}$. This allows us to write $h_i = g^{\rho_i}$ for some $\rho_i \in \mathbb{Z}_q$. Moreover, we know that $X = g^x$ for some $x \in \mathbb{Z}_q$. We then can re-write the above equation as

$$\begin{aligned} \prod_D h_i^{\delta_i} = g^{\rho_i\delta_i} &= g^{\sum_D \delta_i(m_i + \alpha r_i)} \\ \Rightarrow \sum_D \rho_i\delta_i &= \sum_D \delta_i(x_i + \alpha r_i) \\ \Rightarrow \sum_D \rho_i\delta_i - \sum_D \delta_i(x_i + \alpha r_i) &\equiv 0 \mod q. \end{aligned}$$

Setting $\beta_i = \rho_i - (x_i + \alpha r_i)$ this is equivalent to:

$$\prod_D \delta_i\beta_i \equiv 0 \mod q. \tag{1}$$

Now, assume that $\mathsf{Batch}((h_1, x_1, r_1), \ldots, (h_D, x_D, r_D))$ outputs 1, but there exists an index $i = 1$ (this holds w.l.o.g.) such that $g^{x_1\delta_1} X^{r_1\delta_1} \neq h_1^{\delta_1}$. In particular, this means that $\beta_1 \neq 0$. Since q is

prime, then β_1 is the inverse of γ_1 such that $\beta_1\gamma_1 \equiv 1 \mod q$. Taking this and Equation (1), we obtain

$$\delta_1 = -\gamma_1 \sum_{i=2}^{D} \delta_i\beta_i \mod q. \tag{2}$$

Now, given the elements $((h_1, x_1, r_1), \ldots, (h_D, x_D, r_D))$ such that $\mathsf{Batch}((h_1, x_1, r_1), \ldots, (h_D, x_D, r_D)) = 1$ and let bad denote the event that we break the batch verification, i.e., $g^{x_1\delta_1}X^{r_1\delta_1} \neq h_1^{\delta_1}$. Observe that we do not make any assumptions about the remaining values. Let $\Delta' = \delta_2, \ldots, \delta_D$ and let $|\Delta'|$ be the number of possible values for this vector. It follows from Equation 2 and from the fact that Δ' is fixed that there exists exactly one value δ_1 that will make bad happen. This means, however, that the probability that bad occurs is $\mathrm{Prob}[\,\mathsf{bad} \mid \Delta'] = 2^{-\lambda_b}$. Choosing the value δ_1 at random and summing over all possible choices of Δ', we get $\mathrm{Prob}[\,\mathsf{bad}] \leq \sum_{i=1}^{\Delta'}(\mathrm{Prob}[\,\mathsf{bad} \mid \Delta'] \cdot \mathrm{Prob}[\,\Delta'])$. Thus, we can calculate the overall probability as $\mathrm{Prob}[\,\mathsf{bad}] \leq \sum_{i=1}^{2^{\lambda_b(D-1)}}(2^{\lambda_b} \cdot 2^{-\lambda_b(D-1)}) = 2^{-\lambda_b}$. □

8.4 Efficiency

We analyze the efficiency of our batch verifier for the KR chameleon hash using the following notation. By $\mathsf{exp}(k_1)$ we denote the time to compute g^b in the group $\mathbb{G}$ where $|b| = k_1$. The efficiency is measured in number of multiplications. First, we have to compute $\prod_a h_i^{\delta_i}$. Instead of computing this product straightforwardly, we apply the algorithm $\mathsf{FastMult}((h_1, \delta_1), \ldots, (h_D, \delta_D))$ obtaining a total number of $\lambda_b + D\lambda_b/2$ multiplications on the average [BGR98]. In addition we have to compute $2D$ multiplications and finally $2\mathsf{exp}(k_1)$ exponentiations. Thus, the total number of multiplications is $\lambda_b + D(2 + \lambda_b/2) + 2\mathsf{exp}(k_1)$.

8.5 Benchmarking Results

We estimate the performance of our scheme by analyzing the most expensive component of our construction. That is, we have implemented the KR chameleon hash function and we use the implementation of SHA1 provided by the Java security package. These are the two main components of our construction. The additional overhead determining the nodes should add only a negligible overhead to the overall computational costs (recall that computing a chameleon hash involves modular exponentiations, which we believe is the most expensive step). We have implement the KR chameleon hash in Java 1.6 on a Intel Core i5 using 4GB 1333MHz DDR3 RAM. We have conducted two different experiments where we executed each algorithm 500 times with a CAT of depth 80 (thus it authenticates 2^{80} elements). The bit length of the primes in the first experiment is 1024 bits and in the second 2048 bits. The following values are the average computational costs. Adding a leaf to the tree in the worst case (this happens when the tree is empty) takes on average 283ms for 1024 bits and 1400ms for 2048 bits. Each of these executions involves 40 evaluations of the chameleon hash (including the generation of randomness), 40 SHA1 computations, and the computation of a collision. The timings to verify a path and to update it are slightly faster, because both operations do neither include the generation of random values, nor the computation of a collision. In the full version of this paper, we will include running times of the full implementation.

9 From One-Time to Many-Time Signature Schemes Using CATs

The second application of CATs is a new transformation that turns any one-time signature (OTS) scheme into a many-time signature (MTS) scheme. A one-time signature scheme allows a user to sign a *single* message. This primitive is well-known and has been introduced by Lamport [Lam79] and Rabin [Rab79]. One-time signature schemes are interesting from both, a theoretical and a practical point of view. Theoreticians study the primitive because it is used as a building block in realizing secure signature schemes based on one-way functions (in a black-box way). On the other hand, several extremely efficient instantiations have been suggested in practice. However, the "one-timeness" of such schemes regrettably causes a complex key-scheduling process since the signer has to generate a new key pair whenever it wishes to sign a new message. Motivated by this drawback – and also by the hope of obtaining an efficient many-time signature scheme – several publications investigate efficient transformations from a OTS to a MTS scheme.

9.1 Related Work

The related work covers the main transformations from OTS to MTS schemes. Furthermore, since we are interested in constructing a signature scheme based on the discrete logarithm assumption in the standard model, we also review the relevant publications.

Known Transformations. The two main transformation from OTS to MTS are Merkle trees and the one by Goldwasser-Micali-Rivest (GMR). Both schemes are very different in nature: Merkle trees are very efficient, but yield a stateful scheme where the number of signatures is polynomially bounded. The GMR transformation is not very efficient, but provides a stateless solution where the number of signatures is not a-priori bounded.

Merkle Tree. Merkle suggested a hash tree technique to authenticate many one-time keys [Mer88, Mer90]. Roughly speaking, in such a tree a user authenticates up to 2^D (for a small D) public one-time signature keys with a single element. The idea is to store all one-time public keys in the leaves of a complete binary tree and to build up the tree storing in each father node the hash value of the concatenation of its children nodes. To authenticate an element, stored in a leaf, the user publishes all hash values that are adjacent to the nodes along the path from the leaf node to the root node. Although this construction is very elegant and it offers very efficient signing and verification algorithm, it has several drawbacks:

Key generation: The key generation algorithm has to pre-compute the entire tree. Thus, this method is extremely expensive and the total number of signatures is bounded in advance and depends heavily on the storage and computational power. Many follow-up papers reduce the needed storage by adding a pseudorandom number generator (PRNG) to the construction [Nao91]. The basic idea is to store an initially seed in the first leaf and to use this seed to compute the values of all following leaves. It is clear that this idea reduces the needed storage, but it increases the computational cost. In particular, if D is large, this algorithm has to perform more than 2^D computations during the key generation process.

Number of signatures: The overall number of signatures is bounded by the key generation and it therefore supports only a fixed number of signatures. This follows from the fact that the key generation algorithm has to pre-compute all elements in the leaves in advance. Further

follow-up works also focus on improving the efficiency of the computation of the verification path, e.g., [Szy04, BDK+07, RLB+08].

Goldwasser-Micali-Rivest (GMR). A different approach, which can be seen as a generalization of the Naor-Yung construction [NY89], is the transformation due to Goldwasser, Micali, and Rivest [GMR88]. This method also relies on a tree-based structure in which the binary representation of the message defines the path from the root to the leaf. Each node consists of a one-time signature of the bit associated to the node and the public key of the next node. More precisely, let (SK_ρ, PK_ρ) be the key pair of the one-time signature scheme. This pair is associated to the root ρ of the tree. To sign a bit $m_b \in \{0,1\}$, the signing algorithm sets up the tree from the root to the leaves by generating two fresh key pairs $(SK_{0,0}, PK_{0,0})$, $(SK_{0,1}, PK_{0,1})$ of the one-time signature scheme. Subsequently, it signs the public keys by computing $\sigma_\rho \leftarrow \mathsf{Sign}(SK_\rho, PK_{0,0} \| PK_{0,1})$. The signing algorithm now uses SK_b to sign the message m_b by generating $\sigma_b \leftarrow \mathsf{Sign}(SK_{0,b}, m_b)$. Finally, it outputs the signature $\sigma = (PK_{0,b}, \sigma_b)$. The verification algorithm checks the validity of all individual one-time signatures. To extend this idea to an η-bit message, one parses the binary representation of the message $m = (m_0, m_1, \ldots, m_\eta)$ as the sequence from the top of the tree to the bottom. The signature itself consists of $\eta + 1$ public-keys and the verification algorithm has to verify $\eta + 1$ one-time signatures.

From a theoretical point of view, this construction is very interesting because the entire construction relies only on the assumption that one-way functions exists. Moreover, it handles an exponential number of signatures. In doing so, however, there is additional work required since the signing algorithm has to store *all keys* in its state to answer consistently. This means that we have to store a very large number of keys, which is again, impossible. To overcome this weakness, and also to remove the state, Goldreich suggested to apply a pseudo-random function (PRF) to the private key of the root and to the message in order to derive the randomness for all following one-time signature key pairs. Now, the main drawbacks of this construction are:

Signature Computation and Verification: The computation of a single signature involves η calls to the pseudo-random function, η key generations of the one-time signature scheme, and η signing operations of the one-time signature scheme. The verification algorithm performs $\eta + 1$ verification calls to the one-time signature scheme verification algorithm.

Signature Size: The signature consists of $\eta + 1$ public keys and $\eta + 1$ one-time signatures.

It is understood that the main contribution of the GMR construction is a transformation from one-time signature to many-time signature under the only assumption that one-time signature schemes exist.

Discrete Logarithm Based Signature Schemes. The construction of signature schemes is in general very complicated. Many instantiations rely on random oracles (RO) such as [ElG85, Sch90, Oka93, BR93, BLS04, GJKW07, GPV08], or they require strong assumptions, e.g., Strong RSA [CS99, Fis03], q-Strong Diffie-Hellman [BB08], and LRSW [CL04]. The situation is even worse if we consider only signature schemes based on the discrete logarithm assumption. Currently, there exists no security proof for the DSA signature scheme – neither in the RO model, nor under any other assumption. Even the security proof of the twin Nyberg-Rueppel [NPS01] signature scheme holds only in the generic group model. This model, however, suffers from the same weaknesses as the random oracle model as shown by Dent [Den02]. Moreover, Paillier and Vergnaud [PV05], Garg

et al. [GBL08], and Seurin [Seu12] give evidence that many prominent signature schemes based on the dlog assumption in the RO model may *not* be reducible to the dlog *without* the RO model. More precisely, the main results are:

Result I: The reduction of discrete logarithm based signature schemes in the standard model is still unknown;

Result II: The discrete logarithm problem and the forgeability of Schnorr signatures are unlikely to be equivalent in the standard model.

All signature schemes based on the discrete logarithm assumption secure in the standard model instantiate general transformations from one-time signature (resp. collision-resistant hash functions) to many-time signature schemes. These transformations, however, are either designed to rely on weak assumptions (such as the existence of one-way functions) or they only support a limited number of signatures. Therefore, we ask the following question:

> *Assuming that the discrete logarithm problem is hard, can we turn a one-time signature scheme into a many-time signature scheme more efficiently?*

The answer to this question not only leads to new signature schemes, but it also might help to find a non-generic (more efficient) signature scheme based on the discrete logarithm in the standard model.

9.2 Construction

The basic idea of our construction is to store the keys of the one-time signature scheme in the leaves of the CAT. Since the keys are generated uniformly at random (and in particular, independent of the message to be signed) a weakly secure CAT is sufficient.

Construction 9.1. Let $\mathsf{Sig} = (\mathsf{Gen}, \mathsf{Sign}, \mathsf{Vrfy})$ be a signature scheme defined over the message space $\mathcal{M} = \{0,1\}^\lambda$ and let $\mathsf{wCAT} = (\mathsf{wcatGen}, \mathsf{waddLeaf}, \mathsf{wcatVrfy})$ be a chameleon authentication tree. We define the signature scheme $\mathsf{cSig} = (\mathsf{cGen}, \mathsf{cSign}, \mathsf{cVrfy})$ as follows:

$\mathsf{cGen}(1^\lambda)$**:** The key generation algorithm runs $(\mathsf{wsp}, \mathsf{wvp}) \leftarrow \mathsf{wcatGen}(1^\lambda)$. It returns the private key $SK \leftarrow \mathsf{wsp}$, and the corresponding public key $PK \leftarrow \mathsf{wvp}$.

$\mathsf{cSign}(SK, m)$**:** To sign a message $m \in \{0,1\}^\lambda$, the signing algorithm generates a key pair $(SK', PK') \leftarrow \mathsf{Gen}(1^\lambda)$, signs the message $\sigma_0 \leftarrow \mathsf{Sign}(SK', m)$, and adds the public key to the CAT by computing $(\mathsf{wsp}', i, \mathsf{wauth}) \leftarrow \mathsf{waddLeaf}(\mathsf{wsp}, PK')$. It sets $\sigma_1 \leftarrow (i, \mathsf{wauth})$ and returns the signature $\sigma \leftarrow (\sigma_0, \sigma_1, PK')$.

$\mathsf{cVrfy}(PK, m, \sigma)$**:** The verification algorithm parses $PK = \mathsf{wvp}$ and $\sigma = (\sigma_0, \sigma_1, PK')$ and $\sigma_1 = (i, \mathsf{wauth})$. It outputs 1 iff $\mathsf{catVrfy}(\mathsf{wvp}, i, PK', \mathsf{auth}) = 1$ and $\mathsf{Vrfy}(PK', m, \sigma_0) = 1$.

9.3 Proof of Security

As already discussed in Section 5.5, the security of our construction holds for a tree that authenticates an exponential number of 2^D signatures. Again, this follows from our construction (as we never store all elements at the same time). Let $q := q(\lambda)$ be an upper bound on the number of signing queries from the adversary. Then, we distinguish the case where the adversary outputs a

forgery for a leaf $1 \leq i^* \leq q$ and the case where $q+1 \leq i^* \leq 2^D$. In the first case, where $1 \leq i^* \leq q$ is, we build an adversary that either breaks the structure-preserving property of the CAT or that forges the one-time signature scheme. In the second case, where $q+1 \leq i^* \leq 2^D$ is, we show how to beak the one-wayness the CAT.

Theorem 9.2. *If* Sig *is a secure one-time signature scheme and* CAT *a weakly structure-preserving and weakly one-way chameleon authentication tree, then Construction 9.1 is unforgeable under adaptive chosen message attacks.*

The proof idea is as follows: Fix an arbitrary adversary that asks at most q queries to its signing oracle and that outputs a valid signature $\sigma^* = (\sigma_0^*, \sigma_1^*, PK^*)$ on a fresh message m^*. The authentication path verifies that PK^* is a leaf in the tree and the key is used to verify that σ_0^* is a signature on the message m^*. Then, we guess if the index of the key of its forgery is smaller or bigger than q. If it is smaller, then we further distinguish the case where there exits an index i such that (1) $PK^* = PK_i$ and (2) $PK^* \neq PK_i$ (for all $i \in \{1, \ldots, q\}$). The first case, where $PK^* = PK_i$, implies that the adversary forgers the underlying one-time signature scheme. The second case, where $PK^* \neq PK_i$, means that there exists a path in the tree that does not correspond to one of the previously generated keys. This, however, yields a contradiction to the structure-preserving property of the CAT. In the case that $q+1 \leq i^* \leq 2^D$, we build an algorithm that break the one-wayness of the CAT.

Proof. Suppose towards contradiction that Construction 9.1 is not secure. Then there exists a PPT adversary $\mathcal{A}$ that queries its signing oracle at most $q := q(\lambda)$ and forges a signature on a fresh message of its choice. Namely, the algorithm $\mathcal{A}$, giving access to a signing oracle, outputs a valid signature $\sigma^* = (\sigma_0^*, \sigma_1^*, PK^*)$ on a message m^* that $\mathcal{A}$ has never queried before to its oracle. In the following we distinguish between the class of adversaries that outputs an authentication path such that $1 \leq i^* \leq q$ (denoted by $\mathcal{A}_{\leq}$) and the class for which $q+1 \leq i^* \leq 2^D$ (denoted by $\mathcal{A}_{\geq}$).

To begin with, we consider the first case in that we further distinguish if $\mathcal{A}_{\leq}$ returns a forgery that satisfies $PK^* = PK_i$ for some $i \in \{1, \ldots, q\}$ if $PK^* \neq PK_i$ for all $i \in \{1, \ldots, q)\}$. We refer to the first type of attacker as $\mathcal{A}_{\mathsf{Sig}}$ and to the latter one as $\mathcal{A}_{\mathsf{CAT}}$. We show how to construct an efficient adversary $\mathcal{B}$ that uses $\mathcal{A}_{\mathsf{Sig}}$ against the underlying one-time signature scheme, or $\mathcal{A}_{\mathsf{CAT}}$ to break the structure-preserving property of of the CAT. Algorithm $\mathcal{B}$ guesses initially whether it gets access to an attacker $\mathcal{A}_{\mathsf{Sig}}$ or $\mathcal{A}_{\mathsf{CAT}}$.

Attacking the Signature Scheme. The algorithm $\mathcal{B}$ obtains as input a public-key PK (challenge key) of a one-time signature scheme and has access to a signing oracle. It guesses an index $i^* \in \{1, \ldots, q\}$ uniformly at random, it generates the keys of the chameleon authentication tree with depth 2^D running $(\mathsf{sp}, \mathtt{st}, \mathsf{vp}) \leftarrow \mathsf{catGen}(1^\lambda, D)$ and it runs a black-box simulation of $\mathcal{A}_{\mathsf{Sig}}$ on input $PK' \leftarrow \mathsf{vp}$.

Whenever $\mathcal{A}_{\mathsf{Sig}}$ invokes its signing oracle on a message m_i, then $\mathcal{B}$ answers all queries $i \neq i^*$ as follows: It first computes a fresh key-pair of the one-time signature scheme $(PK_i, SK_i) \leftarrow \mathsf{Gen}(1^\lambda)$, signs the message $\sigma_0^i \leftarrow \mathsf{Sign}(SK_i, m_i)$ locally, adds the key to the CAT $(\mathsf{sp}', i, \mathsf{auth}) \leftarrow \mathsf{addLeaf}(\mathsf{sp}, PK_i)$, sets $\sigma_1^i \leftarrow (i, \mathsf{auth})$, and returns the signature $\sigma_i \leftarrow (\sigma_0^i, \sigma_1^i, PK_i)$. In the case that $i = i^*$, then $\mathcal{B}$ queries its one-time signing oracle exactly once on the message m_i obtaining the corresponding signature $\sigma_0^{i^*}$. It then adds the key to the CAT $(\mathsf{sp}', i, \mathsf{auth}) \leftarrow \mathsf{addLeaf}(\mathsf{sp}, PK)$, sets $\sigma_1^i \leftarrow (i, \mathsf{auth})$, and returns the signature as $\sigma_i \leftarrow (\sigma_0^{i^*}, \sigma_1^{i^*}, PK)$.

Eventually, $\mathcal{A}_{\mathsf{Sig}}$ stops, outputting a forgery (m^*, σ^*) with $\sigma^* = (\sigma_0^*, \sigma_1^*, PK^*)$. If $PK^* = PK$, then $\mathcal{B}$ stops outputting (m^*, σ_0^*). Otherwise, $\mathcal{B}$ aborts.

For the analysis it is easy to see that $\mathcal{B}$ performs a perfect simulation from $\mathcal{A}_{\mathsf{Sig}}$'s point of view and that $\mathcal{B}$ is efficient. Assume that $\mathcal{A}_{\mathsf{Sig}}$ outputs a forgery for which there exists an index i such that $PK^* = PK$ with non-negligible probability $\epsilon_1(\lambda)$. The probability that $\mathcal{B}$ guesses this index is $1/q(\lambda)$. We denote this event with hit and compute the success probability $\delta_1(\lambda)$ of $\mathcal{B}$ as

$$\delta_1(\lambda) := \mathrm{Prob}[\,\mathsf{hit}] \cdot \epsilon_1(\lambda) = \frac{\epsilon_1(\lambda)}{q(\lambda)},$$

which is non-negligible.

Attacking the Structure-Preserving Property. The algorithm $\mathcal{B}$ generates q key pairs of the one-time signature scheme uniformly at random $(PK_i, SK_i) \leftarrow \mathsf{Gen}(1^\lambda)$ and outputs $PK_1, \ldots, PK_q$. Subsequently, it obtains the public key vp and the corresponding authentication paths $\{(i, \mathsf{auth}_i))\}_{i=1}^q$. The algorithm $\mathcal{B}$ initializes the counter $c = 0$ and runs a black-box simulation of $\mathcal{A}_{\mathsf{Ch}}$ on input $PK = \mathsf{vp}$.

Whenever the attacker $\mathcal{A}_{\mathsf{Ch}}$ asks to see a signature on some message m_c, then $\mathcal{B}$ increments c, it signs the message $\sigma_0 \leftarrow \mathsf{Sign}(SK_c, m)$, and returns the signature $\sigma \leftarrow (\sigma_0, \sigma_1, PK_c)$, where $\sigma_1 = (c, \mathsf{auth}_c)$.

At a certain point $\mathcal{A}_{\mathsf{Ch}}$ may stop outputting a forgery (m^*, σ^*) with $\sigma^* = (\sigma_0^*, \sigma_1^*, PK^*)$, where $\sigma_1^* = (i^*, \mathsf{auth}^*)$. If $1 \leq i^* \leq q$ and $PK^* \neq PK_{i^*}$, then $\mathcal{B}$ returns (PK^*, σ_1^*) and stops (otherwise, $\mathcal{B}$ aborts).

It follows from our construction that $\mathcal{B}$ performs a perfect simulation of $\mathcal{A}$'s point of view and that both algorithm are efficient. Moreover, whenever $\mathcal{A}_{\mathsf{Ch}}$ succeeds, then $\mathcal{A}_{\mathsf{Ch}}$ returns a valid pair (m^*, σ^*) such that $1 \leq i^* \leq q$ such that $PK^* \neq PK_i$ for all $i = 1, \ldots, q$. Now, let assume that $\mathcal{A}_{\mathsf{Ch}}$ succeeds with non-negligible probability $\epsilon_2(\lambda)$. Then it is easy to see that $\mathcal{B}$ break structure-preserving property of the CAT with the same probability, i.e., $\delta_2(\lambda) = \epsilon_2(\lambda)$.

Attacking One-wayness. Suppose that $\mathcal{B}$ has access to an attacker $\mathcal{A}_{\geq}$ that outputs a valid forgery (m^*, σ^*) such that $q + 1 \leq i^* \leq 2^D$. We invert the CAT as follows: $\mathcal{B}$ generates q key pairs of the one-time signature scheme uniformly at random $(PK_i, SK_i) \leftarrow \mathsf{Gen}(1^\lambda)$ and outputs $PK_1, \ldots, PK_q$. Subsequently, it obtains the public key vp and the corresponding authentication paths $\{(i, \mathsf{auth}_i))\}_{i=1}^q$. The algorithm $\mathcal{B}$ initializes the counter $c = 0$ and runs a black-box simulation of $\mathcal{A}_{\geq}$ on input $PK = \mathsf{vp}$. If $\mathcal{A}_{\geq}$ wants to see a signature σ_i on some message m_i, then $\mathcal{B}$ sets $\sigma_0 \leftarrow \mathsf{Sign}(SK_i, m_i)$, it increments c, returns the signature $\sigma \leftarrow (\sigma_0, \sigma_1, PK_c)$, with $\sigma_1 = (c, \mathsf{auth}_c)$. Eventually, $\mathcal{A}_{\geq}$ outputs a valid forgery (m^*, σ^*) with $\sigma^* = (\sigma^*, \sigma^*, PK^*)$, where $\sigma_1^* = (i^*, \mathsf{auth}^*)$. If $q + 1 \leq i^* \leq 2^D$, then $\mathcal{B}$ returns (PK^*, σ_1^*) (otherwise $\mathcal{B}$ aborts).

It follows from our construction that $\mathcal{B}$ runs in PPT because the algorithm $\mathcal{A}_{\geq}$ is efficient. Moreover, it follows from our construction that $\mathcal{B}$ performs a perfect simulation from $\mathcal{A}_{\geq}$'s point of view. But then, we conclude that $\mathcal{B}$ succeeds whenever $\mathcal{A}_{\geq}$ wins. Denote by δ_O the corresponding probability.

We calculate the overall success probability (denoted with the event win) of $\mathcal{B}$ putting all cases together:

$$\mathrm{Prob}[\,\mathsf{win}] = \frac{1}{2}\left(\delta_O + \frac{1}{2}\left(\delta_1(\lambda) + \delta_2(\lambda)\right)\right) = \frac{\delta_O}{2} + \frac{\epsilon_1(\lambda)}{4q} + \frac{\epsilon_2(\lambda)}{4}.$$

	Merkle	GMR	**here 1**	GMR-Groth	**here 2**
no. of signatures	$\mathsf{poly}(\lambda)$	exp	exp	2^{160}	2^{160}
KeyGen	$2^{\eta}\cdot\mathsf{oKg}+\eta\cdot t_h$	t_{oKg}	$t_{\mathsf{Gen}}+2\eta\cdot t_r+\eta/2(t_h+t_{\mathsf{Ch}})$	$4\mathsf{rnd}+4\mathsf{exp}$	$160\mathsf{rnd}+80\mathsf{exp}$
signature size	$\eta\cdot\ell_h+\ell_{\mathsf{ots}}$	$\ell_m\cdot(\ell_{\mathsf{ots}}+\ell_{\mathsf{oSig}})$	$\ell_{\mathsf{ots}}+\eta/2\cdot(\ell_h+\ell_{\mathsf{Ch}})$	650 group elements	82 group elements
signing	$t_{\mathsf{oSig}}+\mathcal{O}(\eta)$	$\ell_m\cdot(t_{\mathsf{PRF}}+t_{\mathsf{oKg}}+t_{\mathsf{oSig}})$	$t_{\mathsf{oKg}}+t_{\mathsf{Col}}+\mathcal{O}(\eta)$	$640\mathsf{rnd}+640\mathsf{exp}+320\mathsf{mult}$	$82\mathsf{rnd}+84\mathsf{exp}+2\mathsf{mult}+80\mathsf{hash}$
verification	$\eta\cdot t_h+t_{\mathsf{oVf}}$	$\ell_m\cdot t_{\mathsf{oVf}}$	$\eta/2\cdot(t_h+t_{\mathsf{Ch}})+t_{\mathsf{oVf}}$	$480\mathsf{exp}+160\mathsf{mult}$	$(320+161\lambda_b)\mathsf{mult}+2\mathsf{exp}(k_1)$

Table 1: Let ℓ_m be the bit length of the message and assume that 2^{η} is the number of signatures. By $(SK_{\mathsf{ots}}, PK_{\mathsf{ots}})$ we denote a key pair of a one-time signature scheme where the length of each key is $\ell_{\mathsf{ots}} = |SK_{\mathsf{ots}}| = |PK_{\mathsf{ots}}|$, the length of a one-time signature is ℓ_{oSig}, and k is the key of a PRF. We assume that each randomness r has length ℓ_r and the generation takes time t_r. Let h be a hash function with output length ℓ_h and Ch is a chameleon hash with output length ℓ_{Ch}. By $t_{\mathsf{oKg}}, t_{\mathsf{oSig}}, t_{\mathsf{Ch}}, t_{\mathsf{Col}}, t_h$, and t_{PRF} we denote the running time of the key generation, the signing algorithm, the chameleon hash Ch, the collision finding algorithm, the hash function h, and of the PRF. The value λ_b is the security parameter such that the probability of accepting a batch that contains an invalid hash is at most $2^{-\lambda_b}$.

Since this probability is non-negligible it follows easily that this contradicts the assumption that either the one-time signature scheme is unforgeable or that the chameleon authentication tree is not structure-preserving or one-way. □

9.4 Comparison to Merkle Trees and GMR

We compare our transformation in terms of efficiency with previous solutions.

First of all, a CAT can handle an exponential number of leaves, thus it's comparable with the GMR transformation (using a PRF) and improves the Merkle tree. Recall, that the GMR transformation computes for each node (from the root to the leave) a new key pair as well as a signature of the OTS. The final signature consists of all "intermediate" signatures and keys. In our construction, however, calculating a new key pair for each new node is not necessary. Consequently, our signature size is at least a logarithm factor smaller than the GMR signature size and the signing algorithm is also more efficient because it does not need to run the key generation at each inner node.

Key Generation: The key generation algorithm of our solution generates the keys of a chameleon hash and computes the root node on dummy values x_ρ using randomness r_ρ. Thus, the key generation is comparable to the one of the GMR construction, but much more efficient than the Merkle tree.

Signature Size: If ℓ_m is the bit length of the message, then the size of a signature in our solution is much smaller than the one of the GMR transformation. The main reason is that the signature in the case of GMR consists of ℓ_m public keys plus ℓ_m one-time signatures. While in our case,

it only consists (in the worst case) of D chameleon hash values (resp. the corresponding randomness) and a single one-time signature.

Signing: Signing an (ℓ_m) bit message in the case of GMR involves ℓ_m evaluations of the PRF plus ℓ_m key generations plus ℓ_m signing operations. In our case, however, it only involves (in the worst case!) the generation of $D-1$ random values, $D-1$ hash evaluations, a single key-generation of the one-time signature scheme, and a single inversion of the trapdoor. All other elements are pre-computed and/or inductively updated. Again, our construction is slightly less efficient than the Merkle tree but it improves the GMR transformation.

Verification: The GMR transformation runs *for each bit of the message* the verification algorithm of the underlying OTS scheme. The verification of a CAT based construction is more efficient as involves (in the worst case) D chameleon hash executions and a single execution of the OTS verification algorithm.

Our transformation achieves the best of both worlds: The transformation supports an exponential number of 2^D signatures while simultaneously storing only D elements. Therefore, it can handle more signatures than the transformation based on the Merkle tree. On the other hand, the computation of authentication path and its verification is less efficient than in the Merkle tree, but is much more efficient than the GMR construction.

9.5 Applying Batch Verification Techniques to Chameleon Hash Functions

The right column of Table 1 describes the efficiency improvement by batch verification techniques. Concretely, it shows that verifying is about 50 times faster than the one of the GMR construction. Note, that batch verification techniques do not seem to be applicable to the Groth one-time signature scheme, because each node uses different keys.

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