2D DEPTH-AVERAGED SEDIMENT TRANSPORT MODEL TAKEN INTO ACCOUNT OF BEND FLOWS

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ABSTRACT

A numerical experiment is carried out to investigate methods for depth-averaged two-dimensional (2D) modeling of suspended sediment transport in river bends. The hydrodynamic simulation includes the commonly-used dispersion stress terms arisen from secondary current by integration of the product of the discrepancy between the mean and the actual vertical velocity distribution.

Two existing 2D depth-averaged models for lateral convection of suspended sediment induced by secondary current are investigated in this study. One is the bend flow model which includes the lateral convection term by incorporating the assumption of secondary current velocity profile and mean sediment concentration profile. The other is the conventional model neglects the lateral convection term induced by secondary current and use the Schmidt number as a calibration parameter for lateral dispersion. Both models are applied to actual rivers. Analysis of the simulation results confirms the importance of lateral convective transport of suspended sediment.

The vertical sediment concentration profile is very important for simulating lateral convection of suspended sediment. Numerical experiments reveal that using common concentration profile sometimes leads to unreasonable lateral convection for coarse particles. For the falling velocity of suspended sediment particles is small, the vertical velocity of secondary current affects the concentration profile considerably. Considering vertical velocity of secondary current, the model can be improved.

1. INTRODUCTION

Flow pattern in curved channels is three-dimensional and many 3D numerical models have been developed to simulate the complicated spiral flow motion and sediment transport in river bends. However hydraulic engineers in practice often adopt 2D depth-averaged models because of their simplicity and less computation time. The conventional 2D depth-averaged models assume that vertical velocity and sediment concentration are uniform and ignore secondary-current effect. The

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bend-flow 2D depth-averaged model takes into account the secondary-current effect in momentum equation and sediment transport equation.

When using a conventional model to simulate a bend flow, many researchers (Bui Minh Duc et al. 2004; Jian Ye and J.A. McCorquodale 1997) increase the coefficient of exchange of momentum in the horizontal direction, i.e., the effective eddy viscosity, to account the effects of the secondary motion. For the same reason, when using conventional models to simulate mass transport, it is necessary to reduce the Schmidt number to correct for the effect of dispersion (Jian Ye and J.A. McCorquodale 1997; Jennifer G. Duan 2004). Although these simulations showed good agreement as compared with experiment data, conventional models are not adequate for mass transport in curved channels because the Schmidt number varies in a wide range and needs calibration.

Flokstra (1977) indicated the need of dispersion stress terms for bend-flow simulation. Finnie et al. (1999), Lien H.C. et al. (1999) and Yee-Chung Jin (1993) later followed Flokstra's concept to solve a transport equation for streamwise vorticity and incorporated the so-called associated acceleration terms, i.e., dispersion stress terms, to the depth-averaged equations. T.Y. Hsieh and J. C. Yang studied the suitability of 2D models for bend flow simulation, by using a conventional model and a bend-flow model. The analysis of simulation results indicated that the maximum relative difference in longitudinal velocity is mainly related to the relative strength of the secondary current and the relative length of the channel. Empirical relations between the maximum relative difference in longitudinal velocities, the relative strength of the secondary current, and the relative length of the channel, were proposed to be used as a guideline for model users to determine the proper approach to simulate the bend-flow problem by either using a conventional model or a bend-flow model.

Syunsuke Ikeda and Tatsuya Nishimura (1985) first included lateral convective transport of suspended sediment induced by secondary flows in their model for defining the lateral bed topography in bends. The vertical concentration distribution of suspended sediment was assumed to be the commonly used exponential distribution. Lateral convective transport of suspended sediment was obtained by integration of the product of the vertical secondary current velocity distribution and vertical concentration distribution. Their study reveals that the lateral convective transport of suspended sediment induced by secondary flow modifies considerably the bed profile at the outer region of bends. Fang C.M. (2003), Zhong D.R. and Zhang H.W. (2004) introduced Syunsuke Ikeda's concept into bend-flow 2D sediment transport models. Compared with traditional models, which reduce the Schmidt number to correct for the effect of dispersion, a bend-flow model simulates lateral sediment transport effect directly.

2. FLOW SIMULATION

2.1 Governing Differential Equations

The governing equations for flow simulation are the depth-averaged Reynolds approximation of momentum equations and continuity equation, written in orthogonal curvilinear coordinates as follows:

$$g_\xi \cdot g_\eta \cdot \frac{\partial z}{\partial t} + \frac{\partial}{\partial \xi}\left(h\bar{u}g_\eta\right) + \frac{\partial}{\partial \eta}\left(h\bar{v}g_\xi\right) = 0 \qquad (1)$$

$$\frac{\partial \bar{u}}{\partial t} + \frac{\bar{u}}{g_\xi}\frac{\partial \bar{u}}{\partial \xi} + \frac{\bar{v}}{g_\eta}\frac{\partial \bar{u}}{\partial \eta} + \frac{\bar{u}\bar{v}}{g_\eta}\frac{\partial g_\xi}{\partial \eta} - \frac{\bar{v}^2}{g_\xi g_\eta}\frac{\partial g_\eta}{\partial \xi} + \frac{g}{g_\xi}\frac{\partial z}{\partial \xi} = \varepsilon\left(\frac{1}{g_\xi}\frac{\partial A}{\partial \xi} - \frac{1}{g_\eta}\frac{\partial B}{\partial \eta}\right) + T_\xi + F_\xi \qquad (2)$$

$$\frac{\partial \bar{v}}{\partial t} + \frac{\bar{u}}{g_\xi}\frac{\partial \bar{v}}{\partial \xi} + \frac{\bar{v}}{g_\eta}\frac{\partial \bar{v}}{\partial \eta} + \frac{\bar{u}\bar{v}}{g_\xi}\frac{\partial g_\eta}{\partial \xi} - \frac{\bar{u}^2}{g_\xi g_\eta}\frac{\partial g_\xi}{\partial \eta} + \frac{g}{g_\eta}\frac{\partial z}{\partial \eta} = \varepsilon\left(\frac{1}{g_\eta}\frac{\partial A}{\partial \eta} + \frac{1}{g_\xi}\frac{\partial B}{\partial \xi}\right) + F_\eta \qquad (3)$$

in which：

$$A=\left[\frac{\partial}{\partial\xi}\left(\bar{u}g_{\eta}\right)+\frac{\partial}{\partial\eta}\left(\bar{v}g_{\xi}\right)\right]/\,g_{\eta}g_{\xi}$$

$$B=\left[\frac{\partial}{\partial\xi}\left(\bar{v}g_{\eta}\right)-\frac{\partial}{\partial\eta}\left(\bar{u}g_{\xi}\right)\right]/\,g_{\eta}g_{\xi}$$

$$T_{\xi}=\frac{1}{g_{\xi}g_{\eta}h}\frac{\partial}{\partial\eta}\int_{Z_b}^{Z_s}g_{\xi}\cdot(\bar{u}-u)\cdot(\bar{v}-v)dz \qquad (4)$$

in which ξ and η =orthogonal curvilinear coordinates in streamwise direction and transverse direction respectively; g_{ξ} and g_{η} =metric coefficients in ξ and η directions respectively; u, and v =velocity in ξ and η directions respectively; g =gravitational acceleration; Z_b and Z_s =bed elevation and water surface elevation respectively; h =water depth; F_{ξ} and F_{ξ} flow resistance in ξ and η directions respectively; ε =turbulent kinematical viscosity; overbar ($\bar{\ }$)=depth averaged;

In momentum equations, only the dispersion stress in $\xi-$direction (T_{ξ}) is concerned. T_{ξ} is evaluated explicitly, using assumed shape functions for the velocity profile. The exponential velocity profile in streamwise is adopted:

$$u=\bar{u}\,\frac{m+1}{m}\left(\frac{Z-Z_b}{h}\right)^{1/m} \qquad (5)$$

The velocity profile in transverse direction proposed by Odgaard (1986) is adopted:

$$v=\frac{\bar{u}h}{k^2 r}\frac{(2kC/\sqrt{g}+1)(kC/\sqrt{g}+1)}{2(kC/\sqrt{g})^2+kC/\sqrt{g}+1}\left(2\frac{Z-Z_b}{h}-1\right)=\frac{\bar{u}h}{r}f_m\left(2\frac{Z-Z_b}{h}-1\right) \qquad (6)$$

2.2 Verification

Using laboratory data to verify bend flow simulation was conducted by many researchers (Finnie et al. 1999; Lien H.C. et al. 1999; Yee-Chung Jin 1993; Jennifer G. Duan 2004). Here a river reach of the Zhangjiang is selected for verification of the model. The ground plan of the river reach is almost an arc with radius of 2500m. The U-shaped channel is about 800m wide and 8km long.

A mesh of 220×60 and the no-slip boundary at the banks were used in the simulation. The upstream boundary condition was the inflow discharge per unit width, and the downstream boundary condition was the measured water level. Figure 1 shows the velocity redistribution across the channel width along the bend considering the dispersion stress, which clearly demonstrates a shift of the maximum main velocity along the channel bend from the inner-bank region toward the outer-bank. Figure 2 shows the comparison between measured transverse velocity distributions across A-A section and computed distributions. The velocity distribution lines considering the dispersion stress are more close to the measured than computed without the dispersion stress. The fact that the maximum difference between the two computed velocity lines is about 15% indicates that the secondary flow is important here.

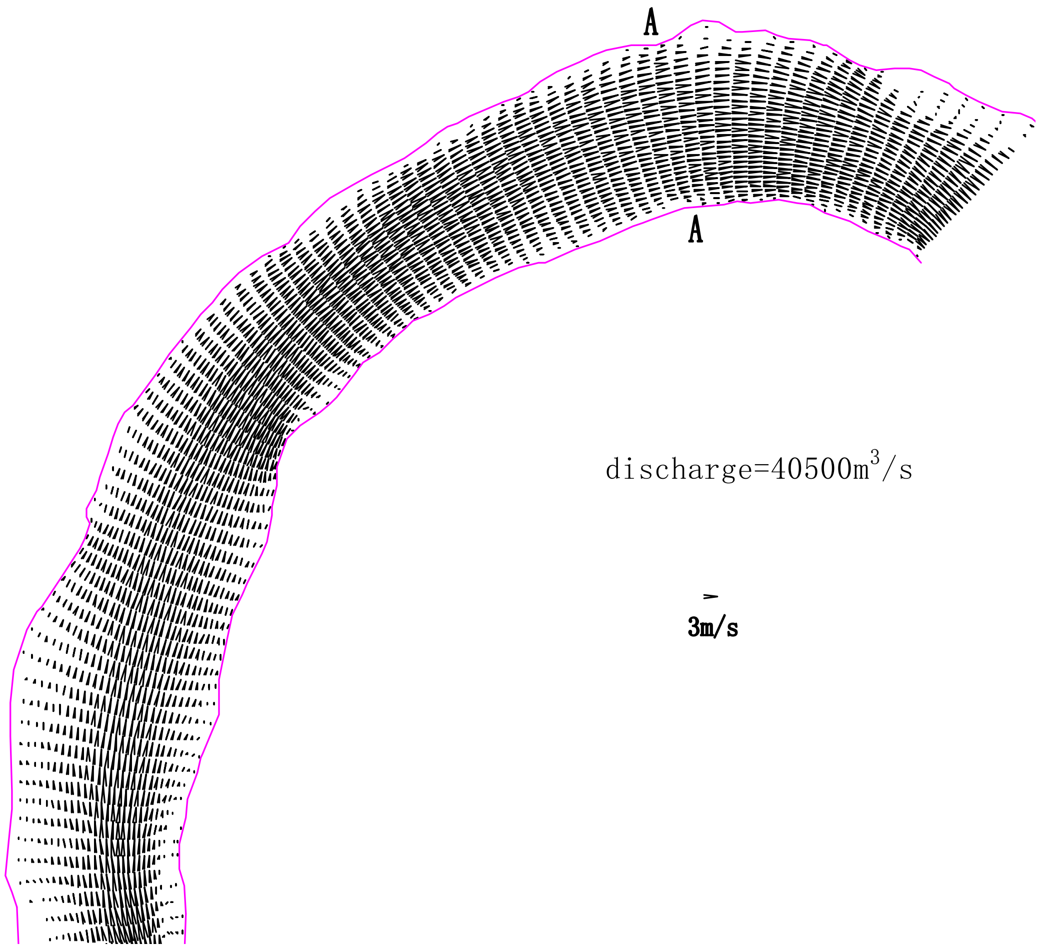
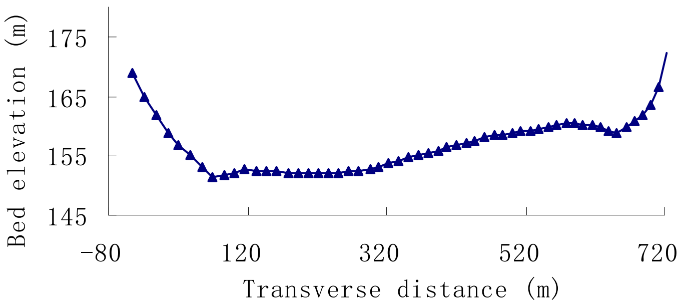
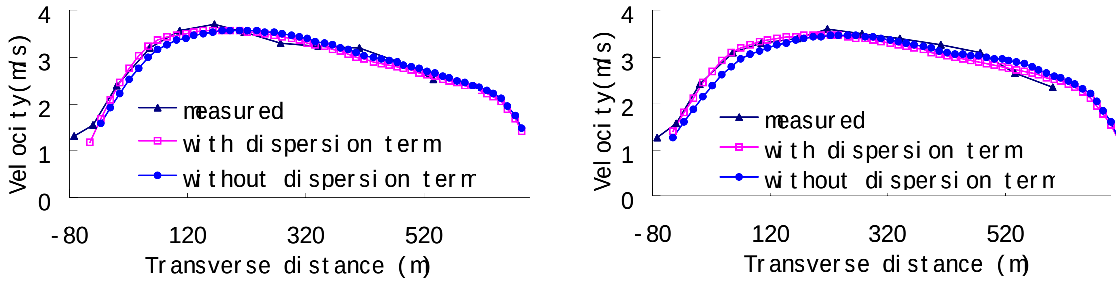


Figure 1 Velocity vector field considering the dispersion stress



Section



At discharge =31400m^3/s

At discharge =40500m^3/s

Figure 2 Cross section and velocity distributions

3. SEDIMENT TRANSPORT SIMULATION

The suspended-load transport calculation is based on the following depth-averaged mass conservation equation:

$$\frac{\partial(h\bar{s})}{\partial t}+\frac{\partial(h\overline{us})}{\partial x}+\frac{\partial(h\overline{vs})}{\partial y}+\frac{\partial}{\partial y}\int_{Z_b}^{z_s}(v-\bar{v})(s-\bar{s})\cdot dz = \frac{\partial}{\partial x}(\varepsilon_s\frac{\partial(hs)}{\partial x})+\frac{\partial}{\partial y}(\varepsilon_s\frac{\partial(hs)}{\partial y})+\alpha\omega\cdot(s^*-\bar{s}) \tag{7}$$

in which s =suspended sediment concentration; s^* =suspended sediment carrying capacity; ω =falling velocity of sediment particle; ε_s =eddy diffusivity for turbulent particle transport. $\frac{\partial}{\partial y}\int_{Z_b}^{z_s}(v-\bar{v})(s-\bar{s})\cdot dz$ =lateral convection term caused by secondary flow.

3.1 Simulation and Verification

The vertical concentration distribution of suspended sediment s is unknown. Firstly, it is supposed to be described by the familiar equilibrium equation:

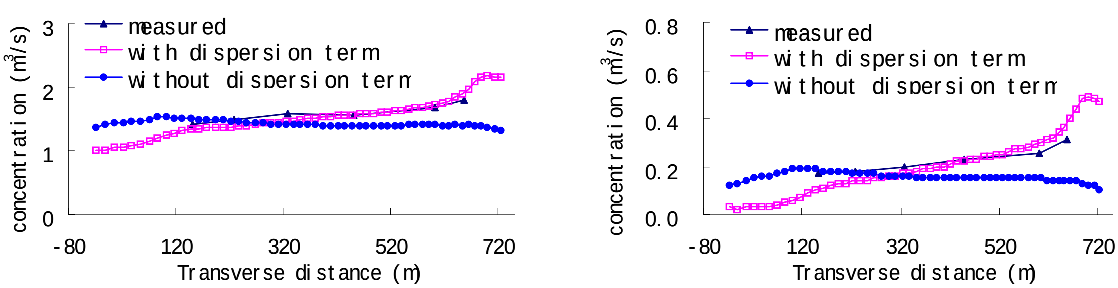
$$\omega\cdot\frac{\partial s}{\partial z}+\frac{\partial}{\partial z}(\varepsilon_z\frac{\partial s}{\partial z})=0 \tag{8}$$

Considering boundary conditions, s solved from (8) as:

$$s=\frac{\bar{s}h}{e^{\frac{-\omega h}{\varepsilon}}-1}\frac{\omega}{\varepsilon}e^{-\frac{\omega z}{\varepsilon}} \tag{9}$$

(9), (6) and (7) form the bend flow suspended sediment transport model.

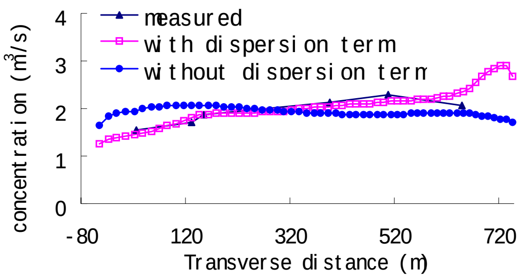
Figure 3 shows the comparison of computed transverse concentration distribution across A-A with the convection term and without the convection term. It is obvious that the model with the convection term transports sediment from the outer bank (left) to the inner bank (right), and gives concentration distribution much closer to the measured than the model without the convection term. The average difference between the two computed lines is about 20%.



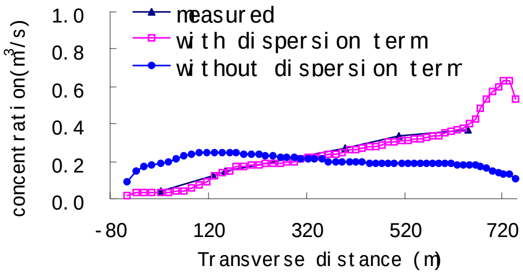
Total sediment

Group of 0.1mm～0.25mm grain diameter

(a) Discharge =31400m^3/s



Total sediment



Group of 0.1mm～0.25mm grain diameter

(b) Discharge =40500m^3/s)

Figure 3 Transverse concentration distributions

3.2 Analysis and Improvement

In Figure 3, the sediment concentration near the inner bank computed by the bend flow model is much lager than the measured, especially for coarse particles. The difference should be induced by the vertical sediment concentration distribution (9). Because the falling velocity of suspended sediment particles is much small, the vertical sediment concentration distribution can be affected by vertical velocity of the secondary flow. Considering the vertical velocity of the secondary flow, (8) becomes:

$$(\omega - w)\cdot\frac{\partial s}{\partial z} + \frac{\partial}{\partial z}(\varepsilon_z \frac{\partial s}{\partial z}) = 0 \qquad (10)$$

For existing formula describing w is hardly available, it can be derived from the continuity equation for the secondary flow.

$$\frac{\partial v}{\partial r} + \frac{\partial w}{\partial z} = 0$$

$$w = -\int_{Z_b}^{Z} \frac{\partial v}{\partial r}\cdot dz \qquad (11)$$

Substituting (6) into (11), w is solved as:

$$w = -f_m h \cdot \frac{\partial}{\partial r}(\frac{\bar{u}h}{r}) \cdot \frac{Z - Z_b}{h} \cdot (\frac{Z - Z_b}{h} - 1) = f_c \cdot \frac{Z - Z_b}{h} \cdot (\frac{Z - Z_b}{h} - 1) \qquad (12)$$

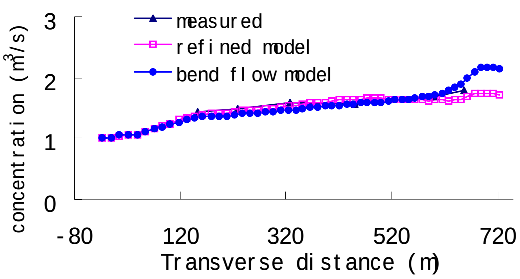
Substituting (12) into (10), after simplifying, obtained s as:

If $f_c \le -6\omega$:
$$s = \bar{s} \qquad (13)$$

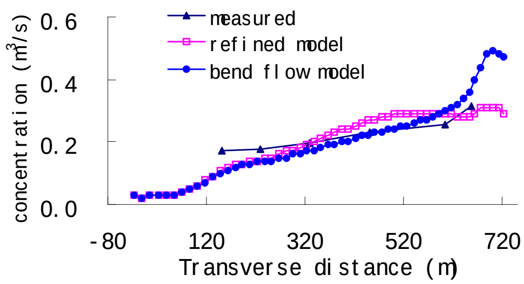
If $-6\omega \le f_c \le 0$:
$$s = \frac{\bar{s}h}{e^{\frac{-h(\omega + f_c/6)}{\varepsilon}} - 1}\frac{1}{\varepsilon}(\omega + f_c/6)e^{-\frac{(\omega + f_c/6)(z - z_b)}{\varepsilon}} \qquad (14)$$

If $f_c \geq 0$: s is defined by (6)

When (13), (14) and (6) are used, the bend flow suspended sediment transport model can be improved. Figure 4 shows the comparison of computed transverse concentration distribution considering vertical velocity and without considering vertical velocity. It is obvious that the model considering vertical velocity gives concentration distribution closer to the measured line than the model without considering vertical velocity near the inner bank.

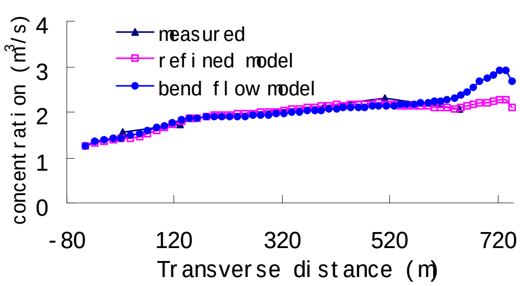


Total sediment

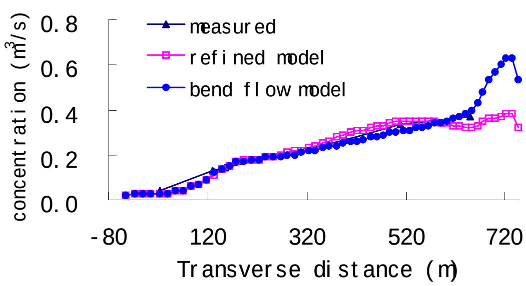


Group of 0.1mm～0.25mm grain diameter

(a) Discharge =31400m^3/s



Total sediment



Group of 0.1mm～0.25mm grain diameter

(b) Discharge =40500m^3/s)

Figure 4 Transverse concentration distributions

4. CONCLUSIONS

Both 2D depth-averaged bend flow model and suspended sediment transport model considering secondary current are compared with traditional models, and the importance of considering secondary current is confirmed. Model verifications indicate that the bend flow model simulated transverse velocity distribution agrees well with the measured river data, but transverse sediment concentration has considerable deviation from the measured data near the inner bank. An improved bend flow sediment transport model has been developed by taking the influence of secondary current vertical velocity on vertical sediment concentration distribution into account. But existing vertical velocity profiles of secondary current and vertical sediment concentration profiles in river bends are hardly available, further experiment study is necessary for improving bend flow sediment transport models.

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