Propagation of cylindrical vector beams in a turbulent atmosphere*

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Based on the extended Huygens–Fresnel principle, the propagation of cylindrical vector beams in a turbulent atmosphere is investigated. The intensity distribution and the polarization degree of beams on propagation are studied. It is found that the beam profile has a Gaussian shape under the influence of the atmospheric turbulence, and the polarization distribution shows a dip in the cross section as the beam propagates in the turbulent atmosphere. It is also found that the beam profile and the polarization distribution are closely related to beam parameter and atmospheric turbulence.

Keywords: cylindrical vector beams, turbulent atmosphere, propagation properties

PACC: 9265, 9265R, 9265M

1. Introduction

The propagation of various laser beams in a turbulent atmosphere was investigated extensively due to wide applications in connection with remote sensing, imaging, and optical communication.[1−6] It is well known that a partially coherent beam is less influenced by atmospheric turbulence than a fully coherent one, and therefore the propagation of partially coherent beams in the turbulent atmosphere has received much attention.[7−15]

On the other hand, owing to the unique properties and wide application in data storage, optical inspection and metrology, optical communication, etc., beams with spatially variant states of polarization become another interesting subject. Generation, propagation, and focusing properties of such beams were widely studied.[16−19] For the spatially variant states of polarized beam, cylindrical vector (CV) beams attracted more and more attention. The generation property and the focusing property of such beams have been discussed;[20−22] however, the propagation of CV beams in a turbulent atmosphere has not been reported to the best of our knowledge. In this paper, the propagation of CV beams in the turbulent atmosphere is discussed, and the intensity distribution and the polarization degree of beams on propagation are also investigated.

2. Theory

In the paraxial approximation, the field of a radially polarized beam and the field of an azimuthally polarized beam can be expressed, respectively, as[17,18]

$$E_r(x,y) = E_0\left[\frac{x}{w_0}\exp\left(-\frac{r^2}{w_0^2}\right)e_x + \frac{y}{w_0}\exp\left(-\frac{r^2}{w_0^2}\right)e_y\right], \tag{1}$$

$$E_\theta(x,y) = E_0\left[-\frac{y}{w_0}\exp\left(-\frac{r^2}{w_0^2}\right)e_x + \frac{x}{w_0}\exp\left(-\frac{r^2}{w_0^2}\right)e_y\right], \tag{2}$$

where w_0 is the beam width, E_0 is a constant, and $r^2 = x^2 + y^2$.

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As shown in Fig. 1, a CV beam can be regarded as a linear superposition of a radially polarized beam and an azimuthally polarized beam, which can be expressed as[22]

$$E_{\mathrm{CV}}(x,y)=E_r(x,y)+E_\theta(x,y)=E_0\left[\frac{x-y}{w_0}\exp\left(-\frac{r^2}{w_0^2}\right)e_x+\frac{x+y}{w_0}\exp\left(-\frac{r^2}{w_0^2}\right)e_y\right]. \tag{3}$$

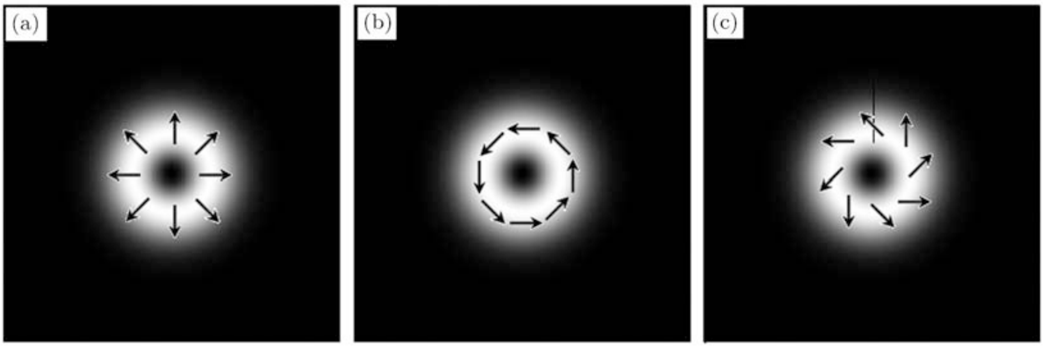

Fig. 1. Synthesis scheme of CV beams. A radially polarized beam (a) and an azimuthally polarized beam (b) are linearly superposed into a CV beam (c).

For such a beam, the polarization and the intensity in the $z>0$ plane can be obtained from the beam coherence-polarization (BCP) matrix, which can be defined as follows:[23]

$$\varGamma(\boldsymbol{r}_1,\boldsymbol{r}_2,z)=\begin{pmatrix}\varGamma_{11}(\boldsymbol{r}_1,\boldsymbol{r}_2,z) & \varGamma_{12}(\boldsymbol{r}_1,\boldsymbol{r}_2,z)\\ \varGamma_{21}(\boldsymbol{r}_1,\boldsymbol{r}_2,z) & \varGamma_{22}(\boldsymbol{r}_1,\boldsymbol{r}_2,z)\end{pmatrix}, \tag{4}$$

where

$$\varGamma_{\alpha\beta}(\boldsymbol{r}_1,\boldsymbol{r}_2,z)=\left\langle E_\alpha(\boldsymbol{r}_1,z)E_\beta^*(\boldsymbol{r}_2,z)\right\rangle,\quad(\alpha,\beta=1,2). \tag{5}$$

By substituting Eq. (3) into Eq. (4), the BCP matrix for a CV beam in the $z=0$ plane can be expressed as

$$\varGamma(\boldsymbol{r}_1,\boldsymbol{r}_2,0)=\frac{E_0^2}{w_0^2}\exp\left(-\frac{r_1^2+r_2^2}{w_0^2}\right)\begin{pmatrix}(x_1x_2+y_1y_2-x_2y_1-x_1y_2) & (x_1x_2-y_1y_2-x_2y_1+x_1y_2)\\ (x_1x_2-y_1y_2+x_2y_1-x_1y_2) & (x_1x_2+y_1y_2+x_2y_1+x_1y_2)\end{pmatrix}. \tag{6}$$

The BCP matrix of the beam in the output plane ($z>0$) can be determined from the knowledge of the BCP matrix in the source plane ($z=0$) by using the extended Huygens–Fresnel integral as follows:[7–15]

$$\begin{aligned}\varGamma(\boldsymbol{r}_1,\boldsymbol{r}_2,z) &= \frac{k^2}{4\pi^2z^2}\iint\varGamma(\boldsymbol{r}_1',\boldsymbol{r}_2',0)\exp\left[-\frac{\mathrm{i}k}{2z}(\boldsymbol{r}_1'-\boldsymbol{r}_1)^2+\frac{\mathrm{i}k}{2z}(\boldsymbol{r}_2'-\boldsymbol{r}_2)^2\right]\\ &\quad\times\langle\exp[\varPsi(\boldsymbol{r}_1',\boldsymbol{r}_1,z;\omega)+\varPsi^*(\boldsymbol{r}_2',\boldsymbol{r}_2,z;\omega)]\rangle\,\mathrm{d}\boldsymbol{r}_1'\mathrm{d}\boldsymbol{r}_2',\end{aligned} \tag{7}$$

where

$$\begin{aligned}\langle\exp[\varPsi(\boldsymbol{r}_1',\boldsymbol{r}_1,z;\omega)+\varPsi^*(\boldsymbol{r}_2',\boldsymbol{r}_2,z;\omega)]\rangle &= \exp[-0.5D_\varPsi(\boldsymbol{r}_1'-\boldsymbol{r}_2')]\\ &= \exp\left[-\frac{1}{\rho_0^2}[(\boldsymbol{r}_1'-\boldsymbol{r}_2')^2+(\boldsymbol{r}_1'-\boldsymbol{r}_2')(\boldsymbol{r}_1-\boldsymbol{r}_2)+(\boldsymbol{r}_1-\boldsymbol{r}_2)^2]\right],\end{aligned} \tag{8}$$

where $k=2\pi/\lambda$ is the wave number, with λ being the wavelength; $D_\varPsi(\boldsymbol{r}_1'-\boldsymbol{r}_2')$ is the wave structure function in Rytov's representation; $\rho_0=(0.545C_n^2k^2z)^{-3/5}$ is the coherence length of spherical wave propagating in the turbulent medium, with C_n^2 being the structure constant of turbulent medium. Here we have used a quadratic approximation for Rytov's phase structure function.

Substituting Eq. (8) into Eq. (7), we can obtain the BCP matrix in the output plane, which can be expressed as

$$\begin{aligned}\varGamma(\boldsymbol{r}_1,\boldsymbol{r}_2,z) &= \frac{k^2}{4\pi^2z^2}\iint\varGamma(\boldsymbol{r}_1',\boldsymbol{r}_2',0)\exp\left[-\frac{\mathrm{i}k}{2z}(\boldsymbol{r}_1'-\boldsymbol{r}_1)^2+\frac{\mathrm{i}k}{2z}(\boldsymbol{r}_2'-\boldsymbol{r}_2)^2\right]\\ &\quad\times\exp\left[-\frac{1}{\rho_0^2}[(\boldsymbol{r}_1'-\boldsymbol{r}_2')^2+(\boldsymbol{r}_1'-\boldsymbol{r}_2')(\boldsymbol{r}_1-\boldsymbol{r}_2)+(\boldsymbol{r}_1-\boldsymbol{r}_2)^2]\right]\mathrm{d}\boldsymbol{r}_1'\mathrm{d}\boldsymbol{r}_2'.\end{aligned} \tag{9}$$

Substituting Eq. (6) into Eq. (9), and according to Ref. [17], the element of BCP matrix for $\boldsymbol{r}_1 = \boldsymbol{r}_2 = \boldsymbol{r}$ in the $z > 0$ plane can be expressed as

$$\Gamma_{11}(\boldsymbol{r},\boldsymbol{r},z) = E_0^2 \frac{2k^2\rho_0^4 w_0^6 z^2}{A_1^2}\left[\frac{2}{\rho_0^2} + \frac{k^2\rho_0^2(k^2w_0^4+4z^2)}{2z^2A_1}(x-y)^2\right]\exp\left[-\frac{2k^2\rho_0^2w_0^2}{A_1}\boldsymbol{r}^2\right], \tag{10}$$

$$\Gamma_{12}(\boldsymbol{r},\boldsymbol{r},z) = \Gamma_{21}(\boldsymbol{r},\boldsymbol{r},z) = E_0^2 \frac{k^4\rho_0^6 w_0^6(k^2w_0^4+4z^2)}{A_1^3}(x^2-y^2)\exp\left[-\frac{2k^2\rho_0^2w_0^2}{A_1}\boldsymbol{r}^2\right], \tag{11}$$

$$\Gamma_{22}(\boldsymbol{r},\boldsymbol{r},z) = E_0^2 \frac{2k^2\rho_0^4 w_0^6 z^2}{A_1^2}\left[\frac{2}{\rho_0^2} + \frac{k^2\rho_0^2(k^2w_0^4+4z^2)}{2z^2A_1}(x+y)^2\right]\exp\left[-\frac{2k^2\rho_0^2w_0^2}{A_1}\boldsymbol{r}^2\right], \tag{12}$$

where

$$A_1 = k^2\rho_0^2w_0^4 + 4z^2(\rho_0^2+2w_0^2).$$

The average intensity distribution can be calculated from[23]

$$I(\boldsymbol{r},z) = \Gamma_{11}(\boldsymbol{r},\boldsymbol{r},z) + \Gamma_{22}(\boldsymbol{r},\boldsymbol{r},z), \tag{13}$$

and the polarization degree can be expressed as

$$P(\boldsymbol{r},z) = \sqrt{1 - \frac{4\det[\Gamma(\boldsymbol{r},\boldsymbol{r},z)]}{\{\mathrm{Tr}[\Gamma(\boldsymbol{r},\boldsymbol{r},z)]\}^2}}. \tag{14}$$

3. Numerical calculation

In this section, some numerical calculations are presented to show the changes of the intensity distribution and the polarization distribution of CV beams on propagation. In the following discussion, we choose $k = 10^7$. In Fig. 2, the normalized intensity distribution of CV beams propagating in the turbulent atmosphere is presented. The parameters for calculation are chosen as $w_0 = 0.01$ m and $C_n^2 = 10^{-13}$ m$^{-2/3}$. Figure 2(a) shows the change in intensity distribution of CV beams propagating in the atmospheric turbulence, and figures 2(b)–2(d) are the corresponding intensity distributions in the cross section with three propagation distances: $z = 100$ m, $z = 300$ m, and $z = 500$ m respectively. As shown in Fig. 2, as the CV beam propagates in the turbulent atmosphere, the intensity distribution turns into a Gaussian shape under the influence of atmospheric turbulence.

To study the influence of turbulence on the propagation of CV beams, the normalized beam profiles of CV beams propagating in the atmospheric turbulence with different structure constants are plotted in Fig. 3. The beam width in calculation is $w_0 = 0.01$ m. As shown in Fig. 3, the stronger the turbulence is, the shorter the propagation distance for the beam profile to change into a Gaussian shape will be, and the more obvious the beam spreading will be.

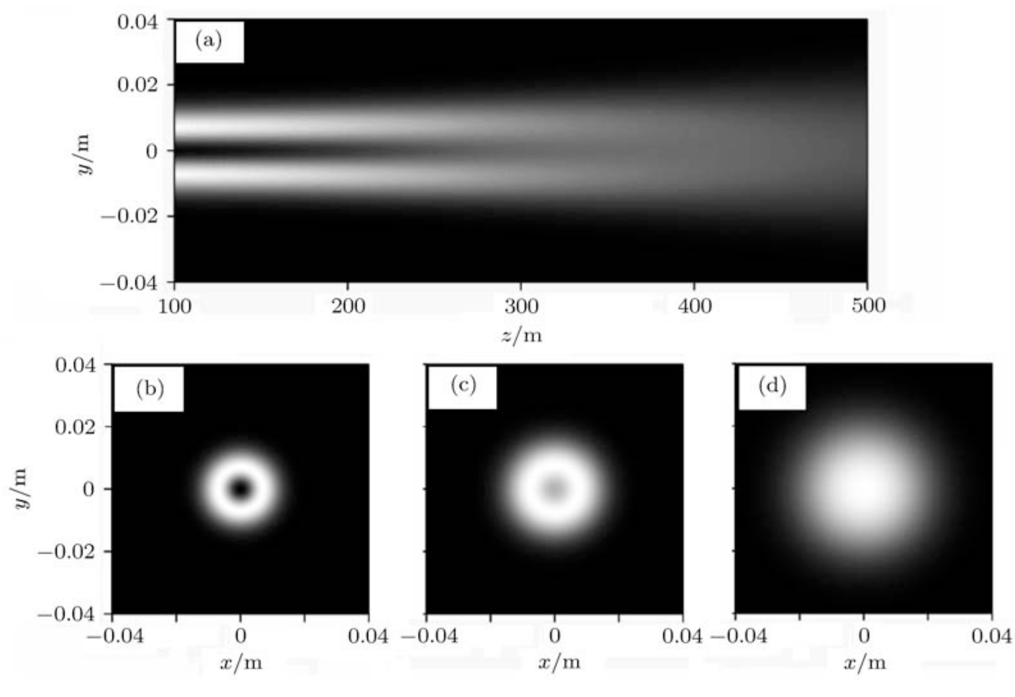

Fig. 2. Intensity distributions of CV beams propagates in the turbulent atmosphere.

The influence of the beam width of CV beams on beam profile is calculated, and the results are shown in Fig. 4. The structure constant in calculation is $C_n^2 = 10^{-14}$ m$^{-2/3}$. It is shown that the propagation distance for the beam profile to change into a Gaussian shape is shorter for a beam with smaller beam width, and the beam spreading is determined by beam width when the propagation distance is not very long. However, with the increase of the propagation distance, the beam spreading is not determined by beam parameters, but by atmospheric turbulence.

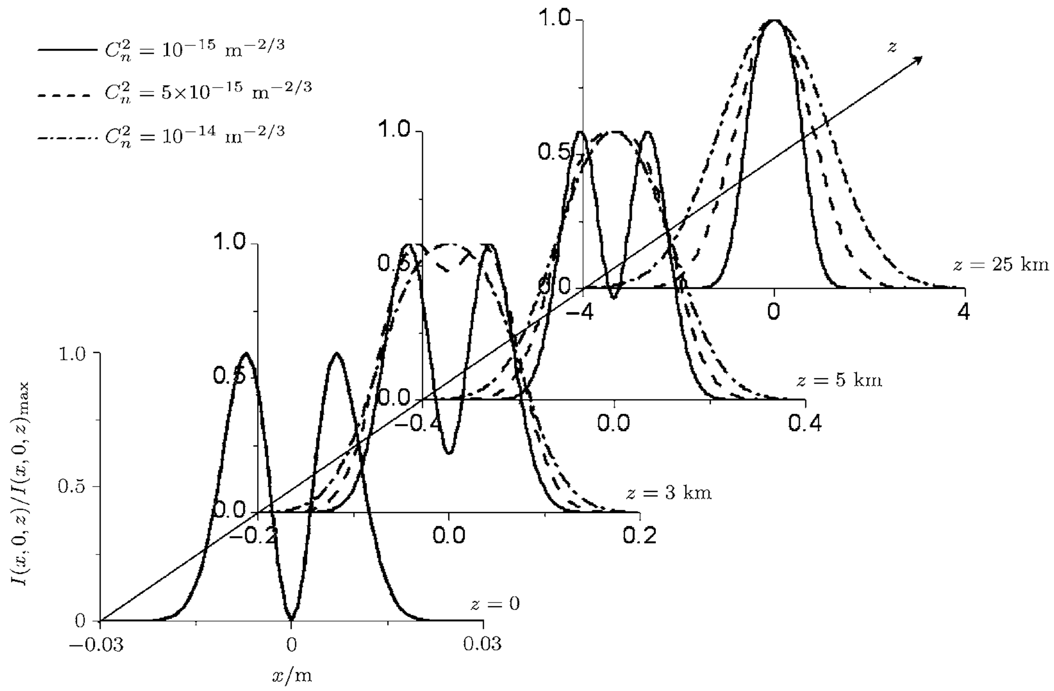

Fig. 3. Normalized beam profiles of CV beams propagating in atmospheric turbulence for three different structure constants of turbulence.

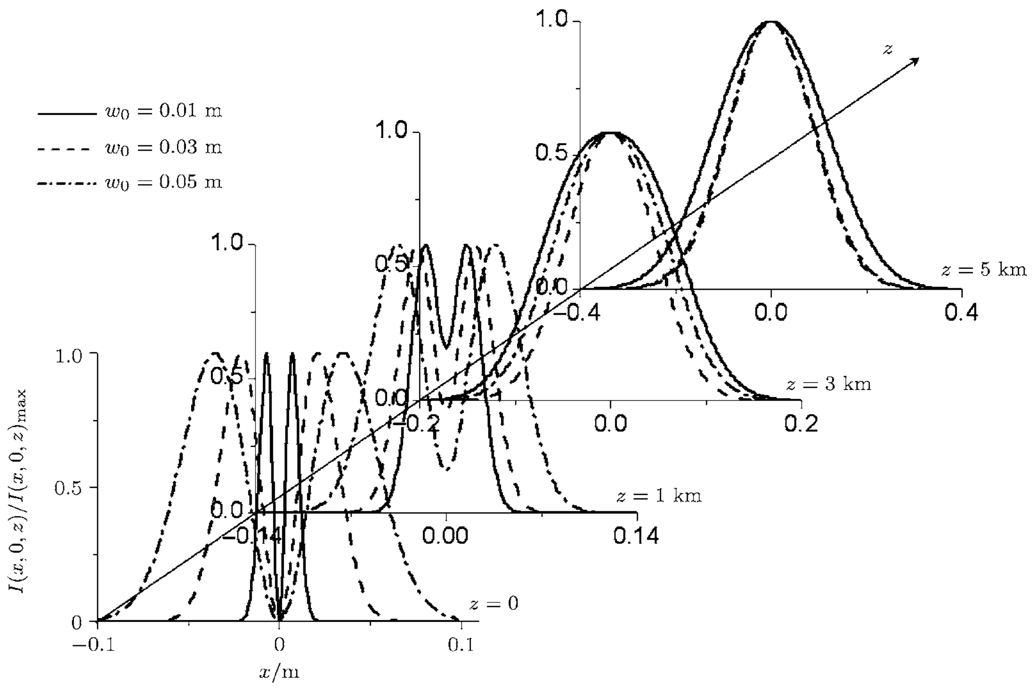

Fig. 4. Normalized beam profiles of CV beams propagating in atmospheric turbulence for three different beam widths.

Now we come to discuss the polarization degree of CV beams propagating in the turbulent atmosphere. First, the changes in polarization of a CV beam on propagation are plotted in Fig. 5. Figure 5(a) shows the changes in polarization of CV beams propagating in the atmospheric turbulence, and figures 5(b)–5(d) are the corresponding polarizations in the cross section with three propagation distances: $z = 100$ m, $z = 500$ m, and $z = 1000$ m respectively. The parameters in calculation are chosen as $w_0 = 0.01$ m, and $C_n^2 = 10^{-14}$ m$^{-2/3}$. As shown in Fig. 5, the polarization degree shows a dip as the CV beam propagates in the turbulent atmosphere, and the width of the dip increases as the propagation goes on.

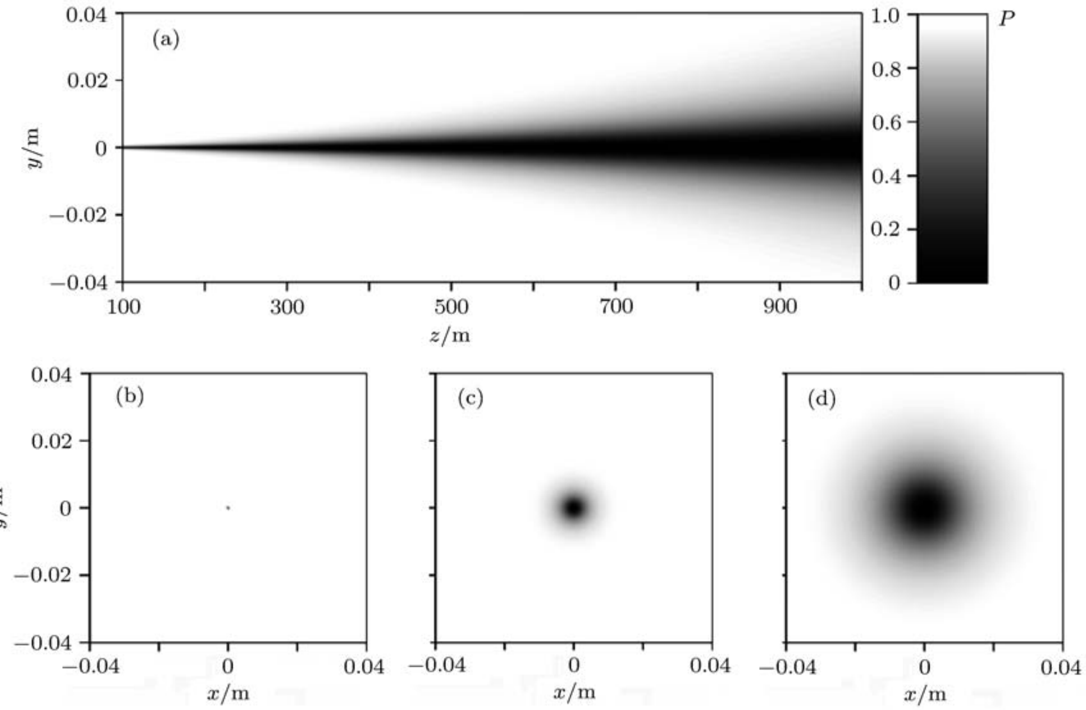

Fig. 5. Polarization distributions of CV beams propagating in the turbulent atmosphere.

To learn the influence of turbulence and beam parameter on polarization degree, the cross lines of the polarization of a CV beam propagating in the atmospheric turbulence with different structure constants and different beam widths are presented in Fig. 6 and in Fig. 7, respectively.

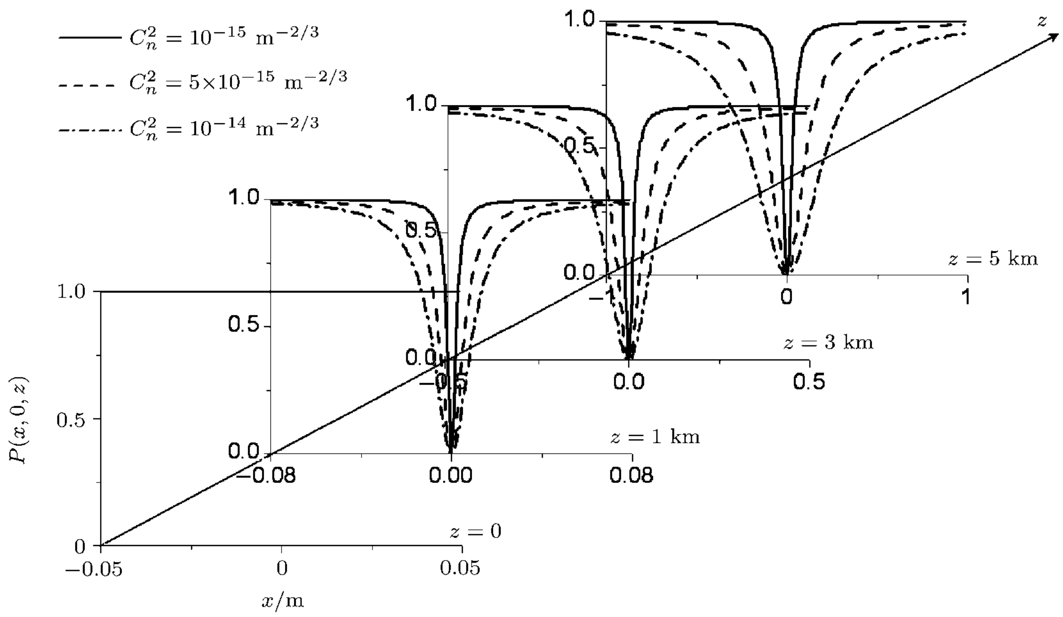

Fig. 6. Influence of turbulence on polarization distribution of CV beams on propagation.

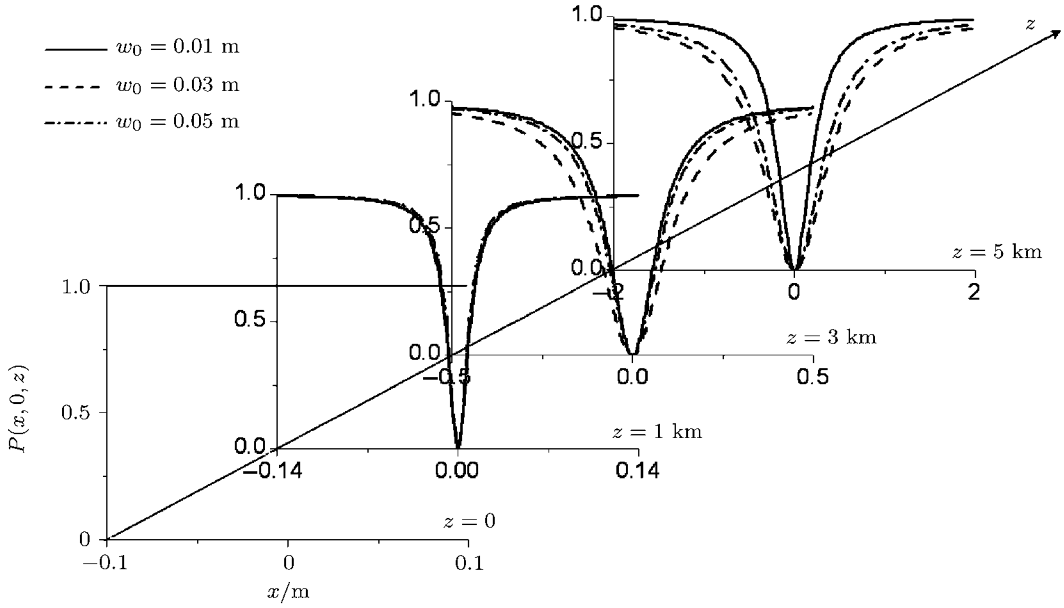

Fig. 7. Influence of beam width on polarization distribution of CV beam on propagation.

In Fig. 6, the influence of atmospheric turbulence on propagation property is investigated. The beam-width in calculation is $w_0 = 0.01$ m. It is shown that the stronger the turbulence is, the more obvious the dip of the polarization distribution will be. The influence of the beam width on beam propagation is shown in Fig. 7. The structure constant in calculation is $C_n^2 = 10^{-14}$ m$^{-2/3}$. One can find that when the beams propagate in the same atmospheric turbulence, the beam width plays an important role in polarization distribution, and the width of the dip first increases, and then decreases with the increase of the beam width.

4. Conclusion

The intensity distribution and the polarization degree of CV beams propagating in the turbulent atmosphere are investigated. The influence of the atmospheric turbulence and the beam parameter on propagation property is discussed. It is found that the beam profile can turn into a Gaussian shape under the influence of the atmospheric turbulence, and this change is closely related to the beam width and the extent of the turbulence. The numerical results also show that the polarization properties of beams are destroyed, and a dip appears as CV beams propagate in the turbulent atmosphere.

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