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MATHEMATICAL MODELING OF MORPHOLOGICAL CHANGE AT A RIVER MOUTH

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Abstract: A mathematical model for calculating river mouth width is introduced, by considering sediment intrusion by waves, as well as sediment flushing by river and tidal discharge. Behavior of an analytical solution is discussed for various combinations of opposing forces acting at a river mouth. Furthermore, the model is extended for predicting migration process of a river mouth. Both of the models are applied to the Nanakita River mouth, Japan.

Key words: River mouth, Sand spit, Migration, Mathematical model

1. INTRODUCTION

For solving flows in a channel, there are various levels of complexity, from a simple 1-D model to a very complicated 3-D model, and most suitable model will be chosen considering objectives, accuracy and computing time, etc. Similarly, there are several types of models with different complexity for river mouth morphology. De Vriend (1996a, 1996b) overviewed existing models such as empirical, semi-empirical and process-based models. In recent years, Cayocca (2001) has proposed a model for long-term morphology change, in which wave- and tide-induced forcing are considered. Work, et al. (2001) also proposed similar numerical model for tidal inlets. Similar to flow calculations, however, there are practical situations in which considerably simplified model yields results of reasonable accuracy for engineering purposes.

In this paper, one-dimensional model for river mouth morphology is introduced. Furthermore, an extension of the model for calculating migration process of river mouth opening will be shown.

2. MATHEMATICAL MODEL FOR RIVER MOUTH WIDTH

2.1 MATHEMATICAL MODELING

A mathematical model has been proposed by Ogawa, et al. (1984) for predicting closing process at a river mouth owing to predominant wave motion. It is assumed that the wave motion is responsible for the intrusion of sediment into the mouth, while the river discharge is effective for flushing sediment out of the mouth as illustrated in Fig.1. Thus, the corresponding governing equation is expressed as follows (Ogawa, et al., 1984).

$$(1-\lambda)Lh\frac{dB}{dt} = e_r q_r B - e_w (1-\lambda) Q_w \tag{1}$$

where λ the porosity of sand, L the width of sand spit, h the water depth at a mouth, B the width of a river mouth, and e_r and e_w the efficiency of sediment inflow by waves and that of sediment outflow by current, respectively. Similar model was used by Kraus (1998) to reproduce well-know empirical formula for a tidal inlet between tidal prism and equilibrium cross-sectional area.

The sediment transport rate due to current, q_r, and that induced by wave motion, Q_w, can be evaluated by means of conventional formulae, Eq.(2) (Brown, 1950) and Eq.(3) (CERC, 1984), respectively.

$$q_r = K\left(\frac{u_*}{sgd}\right)^m u_* d \tag{2}$$

$$Q_W = \alpha(Ec_g)_b \sin\theta_b \cos\theta_b \tag{3}$$

where K is the constant (=10.0), u_* the shear velocity, s the immersed specific weight of sediment, g the gravitational acceleration, d the grain diameter, m the power (=2.0), α the empirical coefficient, $(Ec_g)_b$ the incident wave energy flux evaluated at the breaker line, and θ_b the breaking wave angle to the shoreline. The coefficient α in Eq.(3) has been evaluated to be 0.05 for our study area by Tanaka and Shuto (1991) by applying the one-line model to shoreline change adjacent to the Nanakita River mouth.

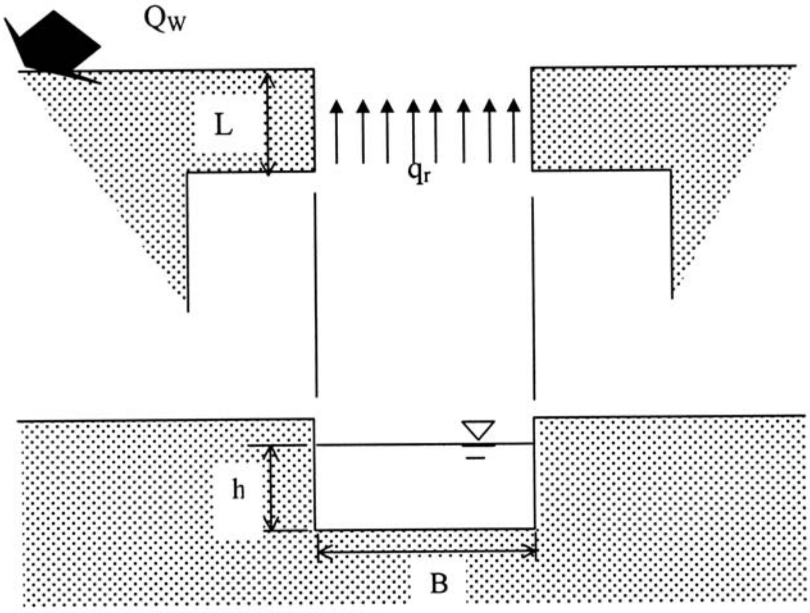


Fig. 1 Schematic explanation for the model

2.2 ANALYTICAL SOLUTION

The following exact solution can be derived from Eq.(1), assuming the wave condition and the river discharge are constant.

$$(B^* - 1) + \frac{A_1^*}{4}\log\left|\frac{(B^* - A_1^*)(1 + A_1^*)}{(B^* + A_1^*)(1 - A_1^*)}\right| - \frac{A_1^*}{2}\left\{\tan^{-1}\left(\frac{B^*}{A_1^*}\right) - \tan^{-1}\left(\frac{1}{A_1^*}\right)\right\} = -t^* \tag{4}$$

where the dimensionless quantities in Eq.(4) are defined as

$$B^* = \frac{B}{B_0}, A_1^* = \sqrt[4]{\frac{A_1 / A_2}{B_0}}, t^* = \frac{A_2 t}{B_0} \tag{5}$$

$$A_1 = \frac{e_r K d(\sqrt{g} nQ)^{2m+1}}{(1-\lambda)Lh(sgd)^m h^{7(2m+1)/6}} \tag{6}$$

$$A_2 = \frac{e_w Q_w}{Lh} \tag{7}$$

in which B_0 denotes the initial river width, n the Manning's friction coefficient and Q the effluent discharge consists of fresh water and tidal prism.

Some interesting asymptotic behaviors of a river width can be derived from Eq.(4). First of all, if the river discharge is zero, substitution of $A_1^*=0$ into Eq.(4) yields the following simplification.

$$B^* = 1 - t_* \tag{8}$$

Namely, the dimensionless river width shows linear reduction with the passage of time until it closes completely at $t_*=1$. Secondly, the width of equilibrium stage can be easily obtained by substituting $dB/dt=0$ in Eq.(1). The dimensionless form normalized by B_0 is

$$B_\infty^* = \frac{B_\infty}{B_0} = A_1^* \tag{9}$$

According to Eq.(4), it takes infinite duration to reach equilibrium width, though, t_∞ will be herewith defined as the duration when B becomes $0.99B_\infty$. A dimensionless form for t_∞ can be derived from Eq.(4).

$$t_\infty^* = A_1^* \left\{ a^* - \frac{1}{4}\log\left|\frac{1+A_1^*}{1-A_1^*}\right| - \frac{1}{2}\tan^{-1}\left(\frac{1}{A_1^*}\right) \right\} + 1 \tag{10}$$

where $a^*=0.722$ for $A_1^*>1$ and $a^*=0.711$ for $0<A_1^*<1$. As A_1^* approaches infinity, Eq.(10) can be approximated by the simple expression,

$$t_\infty^* = 0.723A_1^* \tag{11}$$

Time-variation of dimensionless width, B^*, computed from Eq.(4) is depicted in Fig.2. The parameter A_1^* denotes the ratio of sediment transport rate out of a river mouth due to river discharge to sediment intrusion into a river mouth caused by wave motion, as defined by Eq.(5). Thus, the horizontal line for $A_1^*=1.0$ in Fig.2 corresponds to dynamic equilibrium state, in which sediment transport rates out of and into a river mouth are balanced, whereas the family of curves for $A_1^*>1.0$ and $A_1^*<1.0$ represents widening and reduction processes of river mouth, respectively. In case of $A_1^*=0.0$, the river mouth closes completely at $t^*=1.0$ as mentioned earlier because of no sediment flushing out of a river mouth. According to the existing theory for river mouth closure without sediment outflow from a river mouth, the solution is given in terms of exponential function, denoting that infinite duration is needed for completion of closure (Tsujimoto, et al., 1989). It seems that the present solution, in which closure can be completed within finite duration, is more realistic.

2.3 FIELD OBSERVATION

Very severe draught occurred in Japan during the dry season in 1994. Due to considerably reduced river discharge, complete closure was induced five times at the Nanakita River mouth in Japan during this period. Detailed field observations were carried out immediately before and after the closures. By analyzing hydraulic characteristics measured at the mouth, date of the closure occurrence can be estimated. An analytical solution shown above will be compared with the field observation in 1994.

There have been a lot of field measurements of topography change at a river mouth in connection with practical problems such as flood control and maintenance of a navigation channel. Most of them are, however, made at relatively large rivers due to their practical importance, and, consequently, very few attention has been paid to small ones.

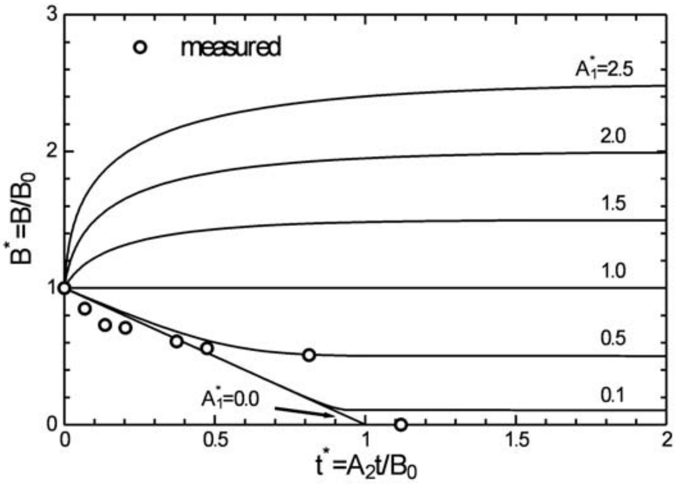


Fig. 2 Time-variation of river mouth width

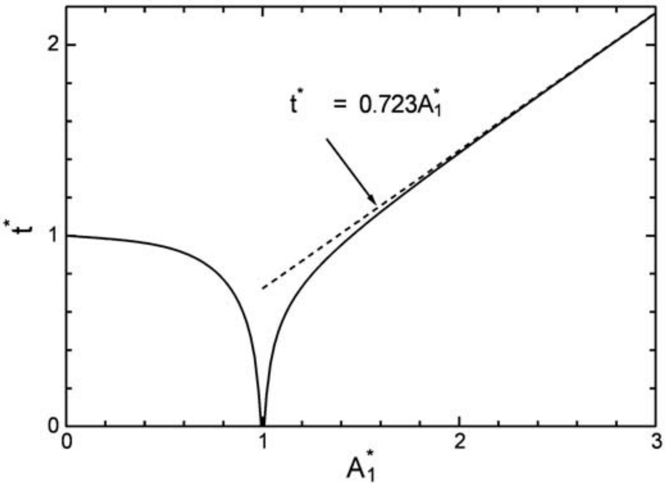


Fig. 3 Relationship between t^*_∞ and A_1^*

The authors have been conducting field measurement of topography change at the mouth of the Nanakita River, one of typical small rivers in Japan, since 1988 (Tanaka and Shuto, 1989, 1991, 1992; Tanaka, et al., 1995, 1996, 2000; Tanaka and Ito, 1996). The Nanakita River originates at the northern part of Sendai and pours into the Pacific Ocean as shown in Fig.4. The catchment area and the length of the river are 229km^2 and 45km, respectively.

From 1988 to 1993, the complete closure at the Nanakita River mouth has been observed only twice (Tanaka and Shuto, 1991), whereas in 1994, the closure occurred five times in contrast, probably due to remarkable reduction of the fresh water discharge during the dry season. In this section, results of detailed field surveying of the topography change will be firstly shown along with corresponding external forces acting at the river mouth. Furthermore, a mathematical model, considering littoral drift by waves and sediment flushing due to river discharge, will be applied to the Nanakita River in order to reproduce the change in the river mouth width.

Fig. 5 shows the significant wave height H_0, the wave period T, the wave incident direction θ measured clockwise from the north, and the river discharge Q. The wave characteristics are measured 4 km offshore from the river mouth at the mean depth of 20m, while the fresh water discharge is measured at a gauge station 9km upstream. In Fig.5, the dates when the river mouth closure was firstly observed are also drawn by thick arrows. It should be noted that these are not the dates when each closure occurred, but the dates when the authors firstly noticed the closure. It is seen that the river discharge is considerably small due to draught as compared with other years (Tanaka and Shuto, 1989, 1991) and that very high waves immediately before each closure are common in all events.

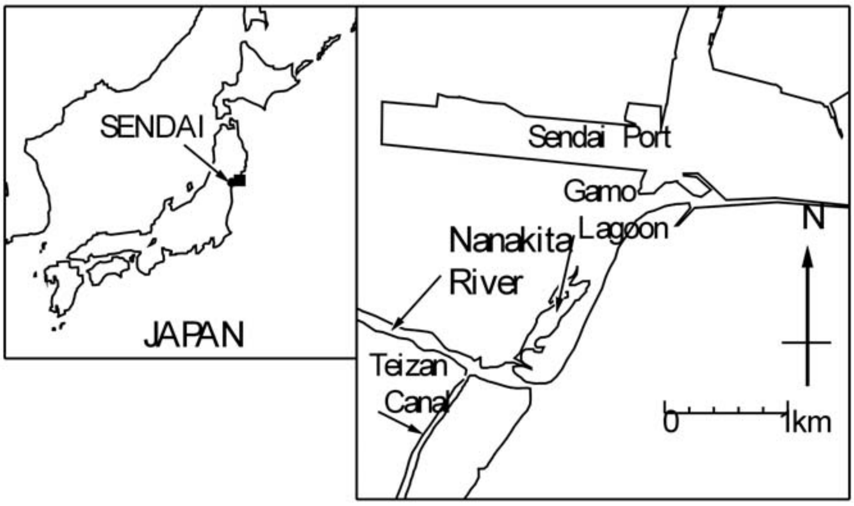


Fig. 4 Location map of the Nanakita River mouth

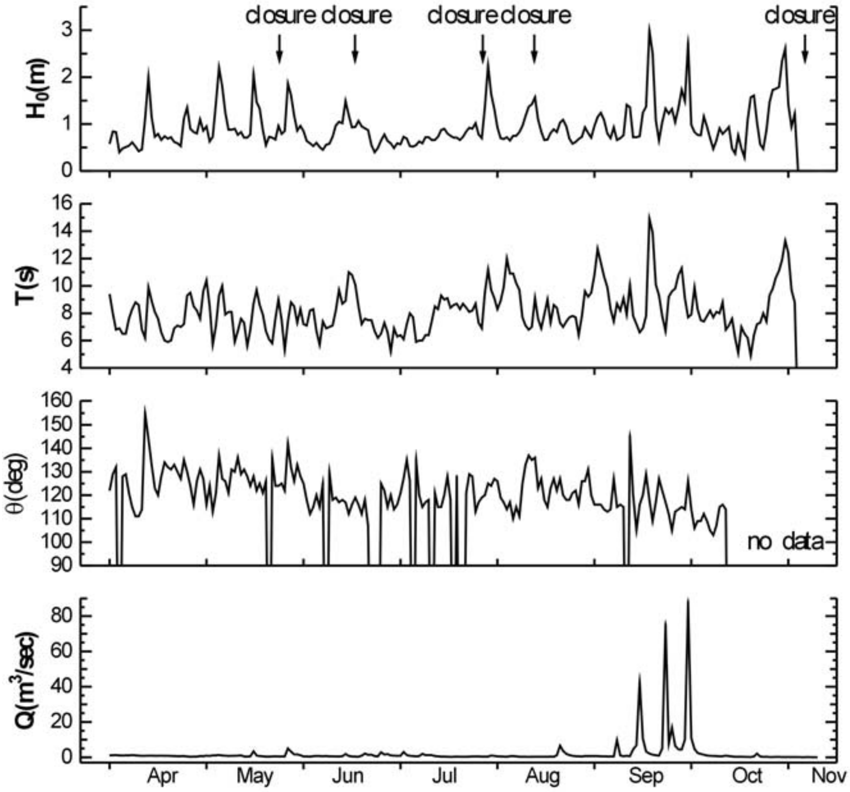


Fig. 5 Time-variation of waves and river discharge

According to Tanaka and Shuto's (1991) study on river mouth closure at the Nanakita River, the incoming waves with incident angle of about 100 deg. are the most unfavorable for keeping the mouth open, suggesting that onshore sediment transport is more effective for the river mouth closure than the longshore sediment movement. In 1994 data set shown in Fig.5, though, such a relationship between the occurrence of closure and the wave incident angle cannot be observed.

Water level variation in the mouth is shown in Fig.6 and Fig. 7 for June and October & November respectively, along with the tidal variation measured in the Sendai Port. The location of the measuring stations can be found in Fig.4.

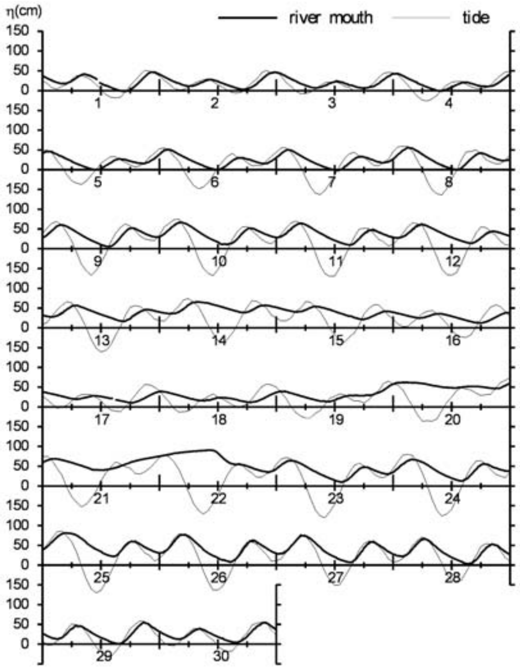
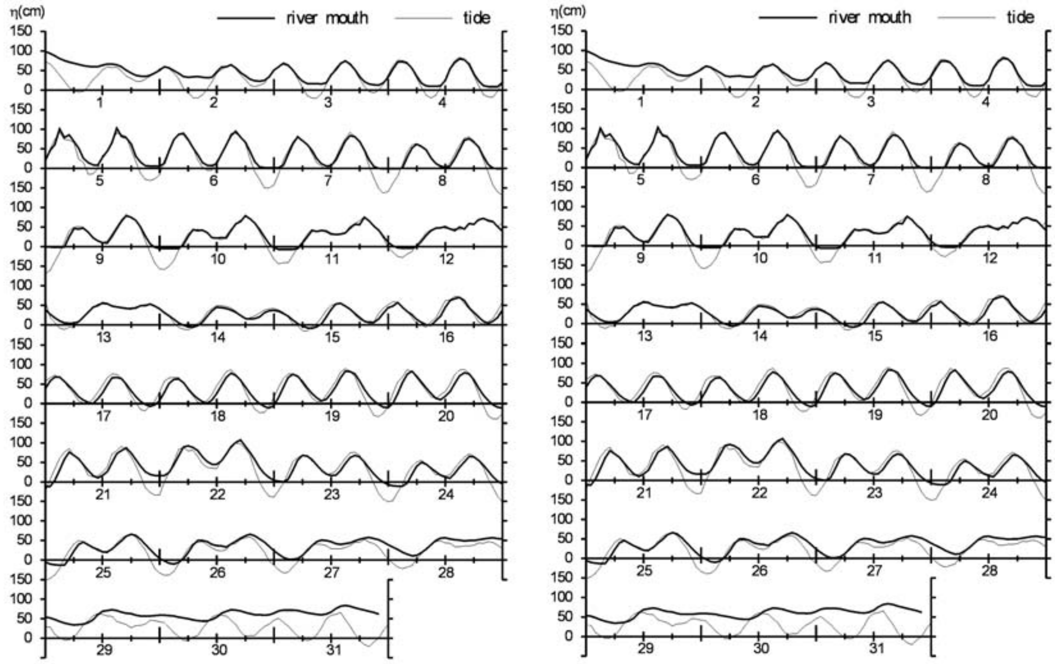


Fig. 6 Water level variation in June 1994


October

November

Fig. 7 Water level variation in October and November 1994

In June, the closure was firstly observed on 17th, and artificial excavation was carried out on 20th. The water level in the mouth shows less variation before and after 17th and is always higher than the tidal level. Even after the closure, slight time-variation can be seen in the Nanakita River mouth. This is due to the tide propagated through the Teizan Canal shown in Fig.4, which connects the Nanakita and Natori River mouths.

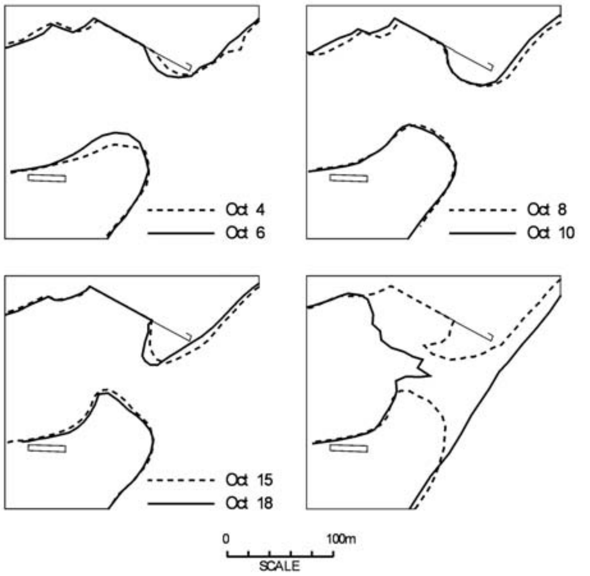


Fig. 8 River mouth closure

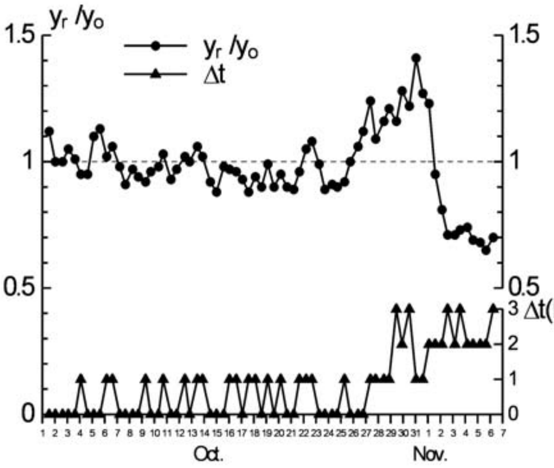


Fig. 9 Water level ratio and lag time

The closure in November was firstly observed on 6th. However, judging from Fig.7, it seems that the closure of the mouth progressed gradually from the end of October. Fig. 8 shows a temporal variation of the shoreline in the process of river mouth closure in November. Since it can be confirmed that the topography change affected much the water level inside the mouth (Tanaka and Shuto, 1991), the ratio y_R/y_O and the lag time Δt are plotted in Fig.9, where y_R and y_O the daily highest water levels in the river mouth and the sea, respectively, and Δt the time difference between y_R and y_O. At first, the water level ratio shows gradual increase from 1.0 on October 26th to 1.4 on November 1st, and then distinctly decreased to 0.6 on November 3rd, while Δt shows an increase from 0 hour up to 3 hours. Thus, this figure suggests that the river mouth closure was completed on November 3rd. The water level rise in the mouth immediately before the closure in Fig.3 was induced by wave set-up due to wave breaking in front of the river mouth (Tanaka and Shuto, 1992). The wave set-up height measured in the mouth shows close relationship with the wave height (Tanaka, et al., 2000).

2.4 COMPARISON BETWEEN THEORY AND FIELD DATA

The measured shoreline change at the river mouth shown in Fig.8 can be now compared with the present theory. The open circles in Fig.2 denote measured data at the Nanakita River mouth in 1994. Since the wave characteristics and the fresh water discharge are assumed to be constant in the theory, the quantities shown in Fig.5 except wave direction are averaged from October 4th through November 7th, whereas the averaging of wave direction is done from October 4th through November 12th due to lack of the data after November 13th as depicted in Fig.5. As for the coefficients, e_r and e_w in Eq.(1), the values determined by one of the authors (Tanaka, et al., 1995) are used for the present computation. It is observed that the measured change in the river mouth width shows reasonable agreement with the present theory.

The mathematical model introduced in **2.1** has been extended to a river mouth with non-rectangular cross-section by Ogawa, et al. (1984) and Shimizu, et al.(1986), and has been applied river mouths in Japan.

3. MATHEMATICAL MODEL FOR RIVER MOUTH MIGRATION

3.1 RIVER MOUTH MIGRATION MODEL

When a river discharge cannot maintain the position and the sectional area of its river mouth which are made by precedent floods, the river meanders between sand bars developed by wind waves. In this section, a mathematical method will be introduced to predict the longshore position of the river mouth, as an extension of the method to predict the cross-sectional area shown in the previous section.

There have been numerous studies of topography change at river mouth, in these several decades, based on both field observations and laboratory experiments. However, it is in very recent years that hydrodynamic models for predicting topography change at a river mouth have been developed and applied. To the knowledge of the author, Shuto and Aota (1980) was the first who proposed a predictive model for the change in cross-sectional area of a river mouth, considering sediment transport due to waves and river discharge. After their study, the model has been modified (Ogawa, et al., 1984; Tanaka, et al., 1996), as described earlier. From the practical view point, this model still has a defect that it cannot provide any information on the migration of a river mouth, which is closely related to coastal problems such as sand spit flushing during a flood and salinity intrusion into the river mouth due to tidal variation.

In this section, a mathematical model for predicting the migration of river mouth is firstly introduced. Secondly, the model will be applied to the Nanakita River, where field observation of seasonal migration of the river mouth has been carried out with very short interval.

In the model explained in previous for predicting river mouth width, it is assumed that the wave motion is responsible for sand transport into the mouth, while the unidirectional current which consists of fresh water discharge and tidal discharge is effective for flushing sand out of the mouth as illustrated in Fig.1. Here, we assume that the growth of the right (left) sand spit is attributed to sediment movement due to waves coming from the right- (left-) hand side. A schematic explanation for the model is given in Fig.10. The corresponding equation of conservation of sediment mass is given by Eq.(12) and Eq.(13) for right and left sand spits, respectively.

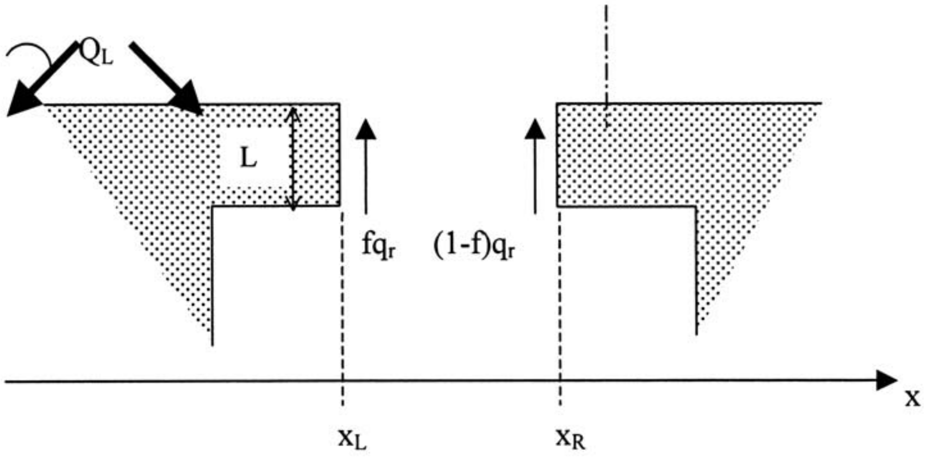


Fig. 10 Schematic explanation for the model

for right sand spit:

$$(1-\lambda)Lh\frac{dx_R}{dt} = e_r q_r (1-f)B - e_w(1-\lambda)Q_R \tag{12}$$

for left sand spit:

$$(1-\lambda)Lh\frac{dx_L}{dt} = -e_r q_r fB + e_w(1-\lambda)Q_L \tag{13}$$

where x_R and x_L are the coordinates of the tip of the right and left sand spits, respectively, and f the weight function for sand movement due to current. Sediment movement due to wave motion is dependent on the incident angle of waves θ as follows.

$$\text{If } \theta<0 \text{ then } Q_R=Q_w,\ Q_L=0 \tag{14}$$

$$\text{If } \theta>0 \text{ then } Q_R=0,\ Q_L= Q_w \tag{15}$$

Subtraction of Eq.(12) from Eq.(13) exactly reduces to Eq.(1). Hence, it can be said that Eqs.(12) & (13) are the generalized expressions of Eq.(1). In the equations above, we have three unknown coefficients : e_r, e_w and f. The first two coefficients of sand transport out of and into river mouth, can be calculated as done by Ogawa, et al. (1984). The last coefficient, f, divides the river discharge into two, one affecting the right bank and another the left bank. This coefficient may reflect the course of river flow and tidal current in the river mouth and its neighborhood. At the beginning of the present study, there was no rational method to determine the value of this coefficient. It is only estimated by minimizing the difference between the measured and computed locations of a river mouth.

The sediment transport rates, q_B and Q_w in Eq.(12) and Eq.(13) are estimated by means of Eq.(2) and Eq.(3), respectively. The wave characteristics at the breaking point in Eq.(6) is computed by use of Snell's law in combination with Goda's wave breaking criterion (Goda, 1970).

3.2 MIGRATION OF THE NANAKITA RIVER MOUTH

The location of the study area is already given in Fig.4. As seen in Fig.4, the river mouth is located in the area sheltered by land against the waves coming from the northeast. The waves from the southeast are usually predominant, resulting in the longshore sediment movement from the south to the north.

The field observation was initiated in 1988, and has been being conducted. Based on the seven year field data, the seasonal change of the topography at the Nanakita river mouth can be summarized as follows (see Fig.11).

(i) Winter: Since the river discharge is considerably small, sand is transported into the mouth due to wave motion, resulting in shoals around the mouth.

(ii) From spring to summer: The right sand spit develops due to predominant waves from the southeastern direction.

(iii) Early autumn: Sand spits are sometimes flushed by floods induced by typhoons. After that, the right sand spit develops again in the northward direction.

(iv) From autumn to early winter: Due to the reduction of the river discharge and due to high waves, the sand spits are intruded into the river.

As briefed above, the migration of the Nanakita River mouth is dependent on the relative magnitude of the river discharge and incoming waves.

3.3 COMPARISON BETWEEN MEASUREMENT AND MODEL CALCULATION

The present model is applied to the Nanakita River mouth in Japan (Fig.4), where we have been measuring the topography change since 1988. Figure 12 shows the temporal variation of the width of the river mouth in 1990. Since the present model cannot be applied to sand spit flushing due to overflowing flood, the computation restarts after the occurrence of big floods. The predicted width of the river mouth (curves) agrees reasonably well with the measured (circles).

Fig. 13 shows a comparison of the measured and computed longshore locations of the tip of right (south) sand spit and those of left (north) sand spit. The optimized values of f in each period are used in the computation. The x-axis is almost parallel to the shoreline and taken positive southward with the origin located on the left-hand side of the mouth. This means that upper lines and corresponding marks give the location of the tip of right sand spit. The overall trend of the migration is well predicted, e.g. southward movement in April and May and northward migration in July and August.

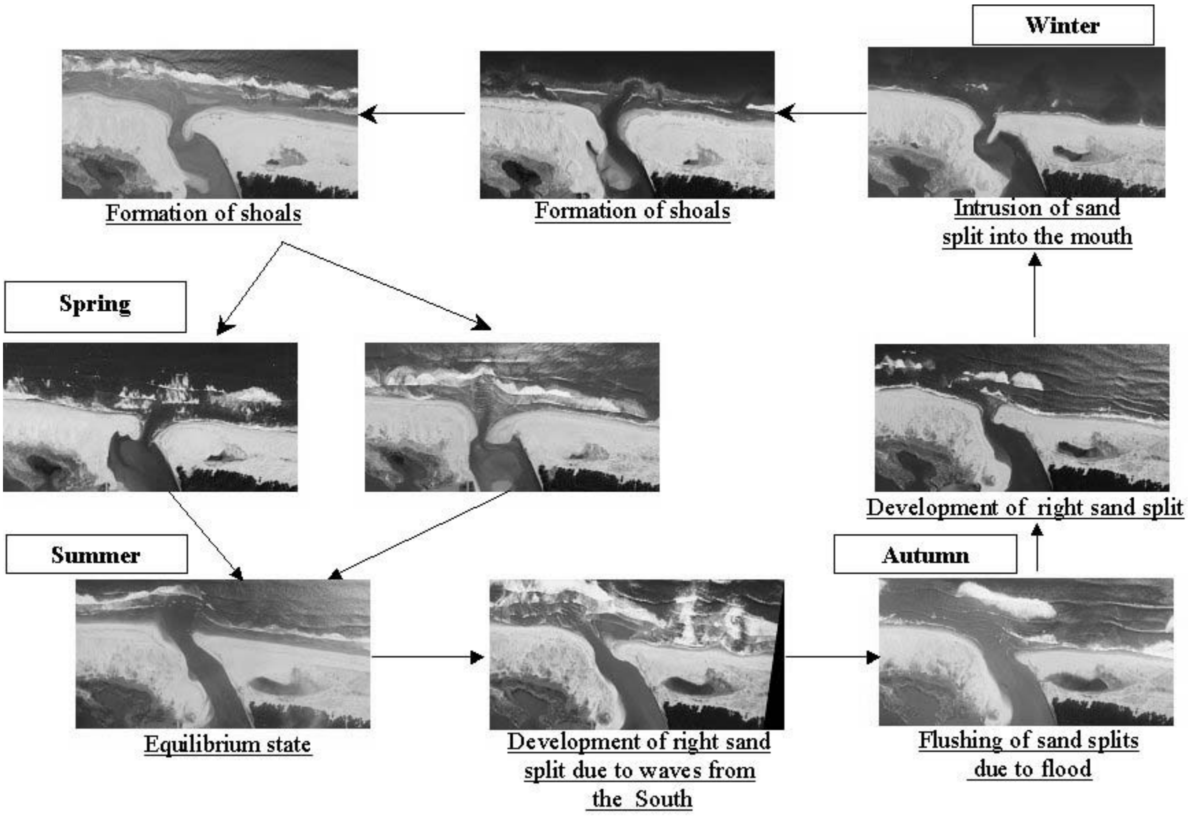


Fig. 11 Seasonal topography change at the Nanakita river mouth

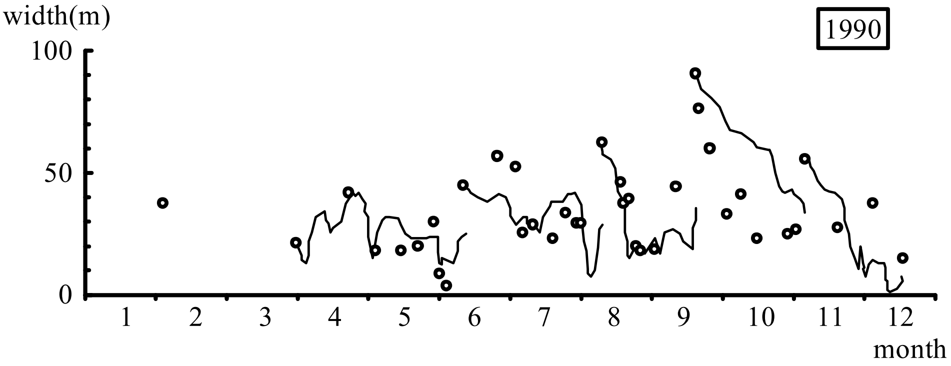


Fig. 12 Width of the Nanakita River mouth

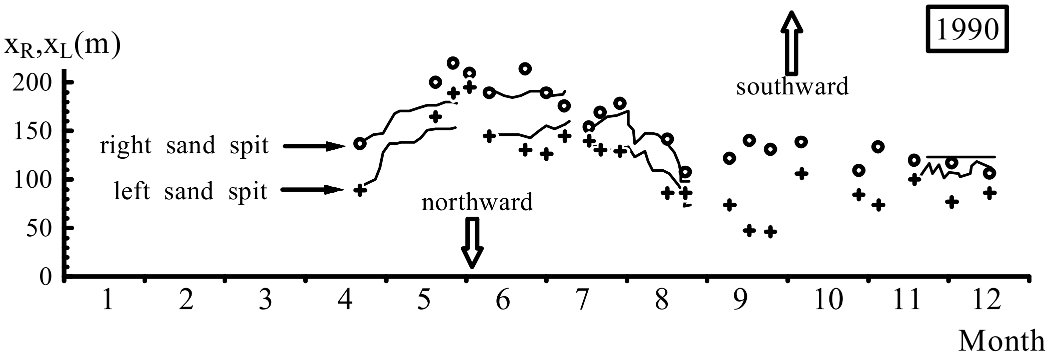


Fig. 13 Comparison of the river mouth migration based on optimized weight function

The characteristics of the weight function, f, is examined in Fig.14 in terms of the river discharge, Q, and the coordinate of the center of the mouth, x_c. The function f may reflect the course of river flow approaching the river mouth. There is a revetment on the right bank slightly upstream and the river always flows down along this bank revetment. Therefore, the river course is quantitatively expressed by the coordinate of the center of the mouth. The numbers close to each group of open circles connected by solid lines refer the optimized result for each set of the measured data from 1989 through 1992. Note that x≒80m is the location of the river mouth when the river pours into the ocean straightly without meandering. When Q is larger than 50m^3/s, f is about 0.6, independent of x_c, i.e., the right and the left sand spits are almost equally eroded by floods. This result is reasonable, because the river course is almost straight when the flood discharge is big. In case of the discharge less than 50m^3/s, no distinct trend can be seen in the figure. One of the reasons of this result might be as follows. The discharge becomes very small in winter, during which the sand spits tend to intrude into the river as depicted in Fig.11. This means that sand movement in the onshore direction is predominant. However, only longshore sediment transport is considered in the present model. As a tentative criterion, the values in the squares are selected and applied to the condition classified by the dotted lines in Fig.14.

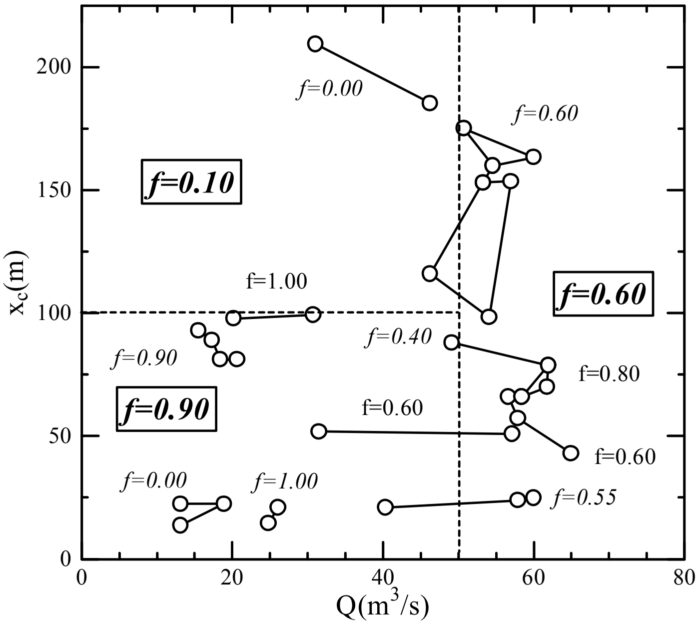


Fig. 14 Dependence of weight function, f, on Q and x_c

4. CONCLUSIONS

A mathematical model is shown for predicting time-variation of width at a river mouth under the combined influence of incoming waves and effluent river discharge. Analytical solution can be obtained, assuming constant wave characteristics and fresh water discharge. The measurements and the theory showed reasonable agreement.

Conservation equations of sediment mass are derived for right and left sand spits at a river mouth, and are applied to the Nanakita River mouth in Japan. When distinct changes in the topography appear, the prediction shows reasonable agreements with the measured. A further study is needed, if we want to select values of the weight function, f, a priori.

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