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OCEANIC DIFFUSION VIEWED THROUGH THE DYE DIFFUSION STUDY

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Abstract: Slug dye diffusion study to obtain the horizontal diffusivity was often carried out in Japan in 1960s to 1970s. The data of these studies are reviewed. Also, several methods to obtain the oceanic diffusivity are reviewed. Especially, a new method generalizing the Gifford's method, which is quite effective if the maximum dye area is known, is proposed and tested with the available data. The overall characteristics of horizontal diffusivity in the coastal area is reviewed.

Key words: Oceanic diffusion, Dye diffusion, Horizontal diffusivity, Hydraulics

1. INTRODUCTION

Slug dye diffusion study to obtain the horizontal diffusivity was often carried out in Japan in 1960s to 1970s. Generally at present, two methods to obtain the horizontal diffusivity of the ocean are employed. One is to analyze the turbulent velocity component of the current-meter data to obtain what is equivalent to the horizontal diffusivity. The other is use of slug dye release study.

Slug dye diffusion study, though not often practiced recently because of the environmental concern, has an advantage as a method to measure diffusivity because what is observed is certainly the material diffusion.

Dye cloud released on the sea surface is transported in the three dimension as well as suspect to the three dimensional diffusion, making the analysis to obtain diffusivity difficult. If the dye release study is carried out on the fairly calm sea without prominent divergence or convergence of sea water, then the horizontal diffusion most likely to be predominant and the rate of the diffusion of the dye cloud supposedly gives a good measure of horizontal diffusivity. The problem is how to obtain the rate of diffusion that is directly related to the horizontal diffusivity. And this paper tries to address just this problem, first reviewing the past works and then proposing the new generalized method.

2. SLUG DYE RELEASE TUDY AND EQUIVALENT CONCENTRIC SPREAD

In the slug dye release study, small amount of dye solution, 20 to 30 liter of 5 percent solution of Rhodamine B or Fluorescein Sodium, is released off a small boat onto the sea surface and the resulting spread of dye cloud is observed. Observation method of dye cloud is usually to take an aerial photographs from an airplane. It is very rare that the distribution of dye concentrations measured at the time interval that the apparatus allows. Fig. 1 shows such an example in which the concentration of dye is measured with a towed in situ colorimeter. Aerial photograph of dye cloud only gives the picture of dye cloud whose area is the only data in such a case. The area of the dye cloud generally increases in time reaching the maximum and then fading because of dilution eventually vanishing from the sea surface. Fig. 2 shows such a data showing the dye cloud area versus time.

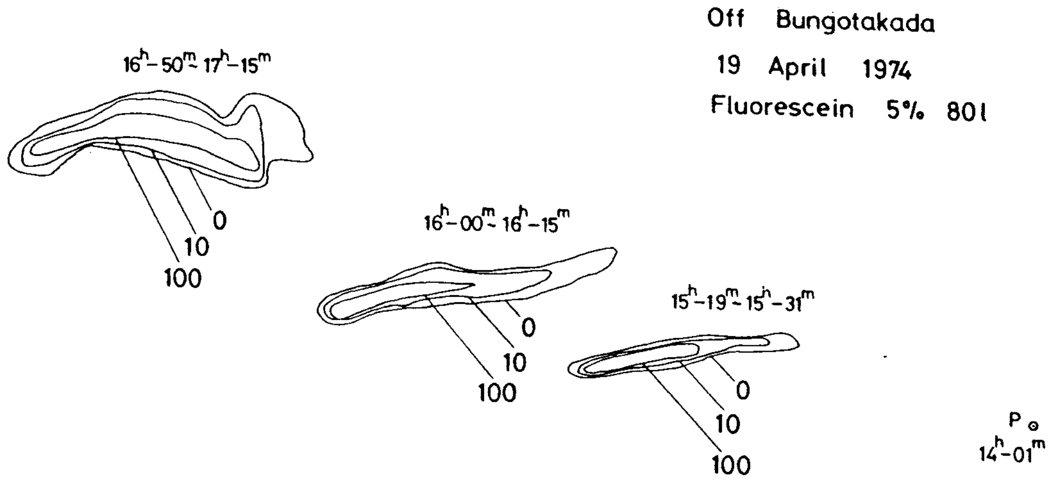

Fig. 1 Example of dye diffusion with measured concentration distribution

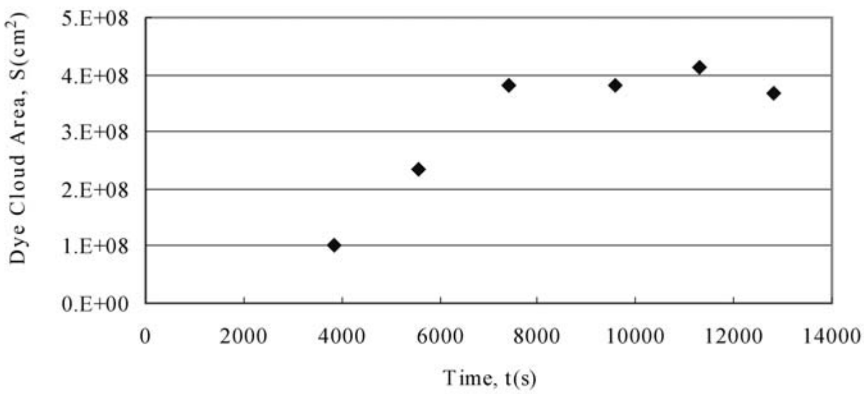

Fig. 2 Dye cloud area versus time

The dye cloud, as shown in Fig. 1, is transported riding the ocean current and is dispersed in irregular shape due to in-homogeneity of turbulence and local shear. To analyze such a detailed behavior of the dye cloud would obviously be very difficult to say the least and impossible unless detailed measurement of dye concentration as well as current velocity is made. Frequent practice is then to assume that the dye diffusion can be replaced by the fictitious diffusion into horizontally concentric shape to obtain the measure of the horizontal diffusivity. Such a distribution is expressed by the following equation.

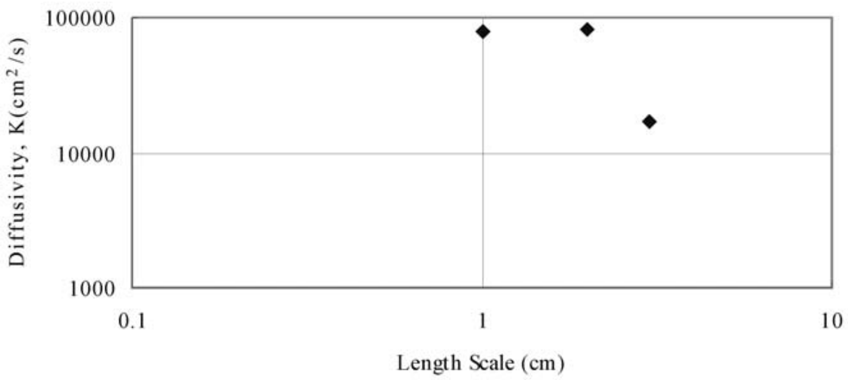

Fig. 3 Horizontal diffusivity calculated with the simple method Eq. (2).

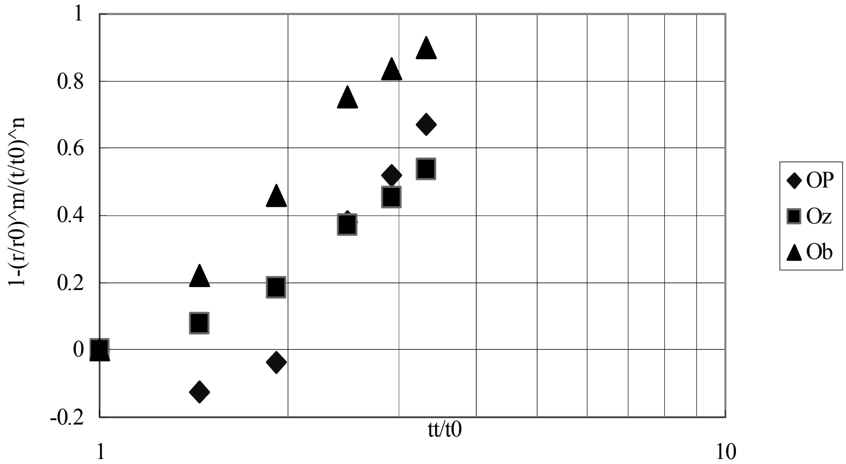

Fig. 4 Fitting of dye cloud spread with the ocean diffusion theories

$$C = \frac{A}{(kt)^{2n/m}} \exp\left(-\frac{nr^m}{m^2 kt^n} \right) \tag{1}$$

where C is dye concentration, A is the value related to the total dye quantity, r and t are radius from center of gravity and time, respectively, and k, m and n are constants depending upon the oceanic diffusion theory adopted.

Table 1, originally due to Okubo(1962), gives the parameters and some important values related to the diffusion depending on the theory proposed in the past. This table gives the lifetime for dye cloud, T, and the time the dye cloud attains the maximum t_m.

3. REVIEW OF THE METHODS OF ANALYSIS

With the obtained experimental data as exemplified in Fig. 2, two techniques to obtain horizontal diffusivity can be mentioned. They are (1) calculating horizontal diffusivity as temporal rate of increase in dye cloud area, and (2) fitting the equivalent radius of the dye cloud to the oceanic diffusion law.

The first method is quite simple and summarized as follows. Given data of dye cloud area S_i at time t_i, i=1,2,…, the estimated value of the horizontal diffusivity is calculated as follows

$$K_i = \frac{S_i - S_{i-1}}{t_i - t_{i-1}} \tag{2}$$

This value is referenced with the equivalent radius of the cloud r_i, which is calculated as

$$r_i = \frac{\sqrt{S_i} + \sqrt{S_{i-1}}}{2\sqrt{\pi}} \tag{3}$$

Fig. 3 shows the plot of K_i versus r_i calculated from the data on Fig. 1. In this particular example, it can be seen that because of the small number of data, Fig. 3 does not show a dependence of K on r as expected from the oceanic diffusion theory.

In the second method, peripheral dye concentration of a dye cloud is regarded as constant and the data of the equivalent dye cloud radius r, in time t is fitted directly with Eq. (1). Eq. (1), however, forms a difficult curve to be fitted with the data and the data can be more easily fitted with the reduced equation, as proposed by Adachi et al. (1974), which is given as

$$1 - \frac{(r_t / r_0)^m}{(t_t / t_0)^n} = \frac{2mk(t_0^n / r_0^m)}{\log e} \log \frac{t_t}{t_0} \tag{4}$$

Fig. 4 gives the example of the plot of the left-hand-side of Eq. (4) versus t/t_0 for OP, Oz and Ob. The oceanic theory is tested whether or not the plotted points follow a straight line in this semi-logarithmic figure. It can be seen from Fig. 4 that Ob best fits the data although this plotting appears to depend on the initial value.

Table 1 Various Theories of Oceanic Diffusion

Theory	Horizontal Diffusivity	t_m	m	n	r_o- versus t, where r-r_o at a const. C	$\bar{r}-\bar{t}$	Dispersive radius σ_r^2
Fick	$K = k$	T/e	2	1	$r_o = 2\sqrt{K}\left(t\ln T/t\right)^{1/2}$	$\bar{r} = \left(e\bar{t}\ln\left(1/\bar{t}\right)\right)^{1/2}$ $\bar{r} = \left(\bar{t}\left(1-\ln\bar{t}\right)\right)^{1/2}$	$16\pi kt$
Joseph & Sendner	$K = pr$ *diffusion* p; *velocity*	T/e	1	1	$r_o = pt\ln T/t$	$\bar{r} = e\bar{t}\left(\ln 1/\bar{t}\right)$ $\bar{r} = \bar{t}\left(l-\ln\bar{t}\right)$	$6p^2t^2$
Ozmidov	$K = \gamma r^{3/4}$ *energy* γ; *dissipation* *parameter*	T/e	2/3	1	$r_o = \left(\gamma t\ln T/t\right)^{3/2}$	$\bar{r} = \left(e\bar{t}\left(\ln - l/\bar{t}\right)\right)^{3/2}$ $\bar{r} = \left(\bar{t}\left(l-\ln\bar{t}\right)\right)^{3/2}$	$60\gamma^3t^3$
Okudo & Pritchard	$K = \omega^2 t$ *diffusion* ω, *velocity*	$T/e^{1/2}$	2	2	$r_o = \left(\omega^2t^2\ln T/t\right)^{1/2}$	$\bar{r} = \left(2e\bar{t}\left(\ln l/\bar{t}\right)\right)^{1/2}$ $\bar{r} = \bar{t}\left(l-2\ln\bar{t}\right)/\sqrt{2}$	ω^2t^2
Okubo	$K = \alpha^2 r^{2/3}t$ *energy* a; *dissipation* *parameter*	$T/e^{1/2}$	4/3	2	$r_o = \left(\alpha^2t^2\ln T/t\right)^{3/4}$	$\bar{r} = \left(2e\overline{t^2}\left(\ln l/\bar{t}\right)\right)^{3/4}$ $\bar{r} = \left(\bar{t}\left(l-2\ln\bar{t}\right)\right)^{3/4}$	$4\alpha^3t^3/\sqrt{\pi}$
Obukhov	$K = \beta^3t^2$ *energy* β; *dissipation* *parameter*	$T/e^{1/3}$	2	3	$r_o = (\beta^3t^{\bar{3}}\ln T/t)^{1/2}$	$\bar{r} = \left(3e\overline{t^3}inl/\bar{t}\right)^{1/2}$ $\bar{r} = \left(\bar{t}\left(1-3\ln\bar{t}\right)\right)^{1/2}$	β^3t^3

T: Lifetime of a dye cloud, tm: time to attain the max. radius rm. $\bar{r} = r/rm$, $\bar{t} = t/tm$

Table 2 Constants in Eq. (7)

Theory	*a*	*b*
JS	6	3/2
OZ	60	20/9
O-P	1	1
OK	$4/\sqrt{\pi}$	$8\sqrt{2/3\pi}$
Ob	1	1

4. METHOD TO CALCULATE THE DYE DISPERSION RADIUS

With the data of dye cloud area versus time, a new method of analysis to obtain the horizontal diffusivity is proposed herein. This method was originally proposed for the atmospheric diffusion by Gifford(1957). But his method was applicable for the simple Fickian diffusion law and this paper attempts to generalize this method to include various theories of oceanic diffusion. In this method, the concentric horizontal dye distribution given by Eq. (1)

is also assumed. Assuming the peripheral concentration discernible by an areal photograph is constant, the dispersion radius of the dye concentration distribution is calculated by its definition. given as,

$$\sigma_r^2 = \int_0^\infty r^2 C 2\pi r dr / \int_0^\infty C 2\pi r dr \tag{5}$$

The result is given in Table 1 for various theories of oceanic diffusion. Eliminating time dependence from the solution of the diffusion equation (1) using the result of Eq. (5), and noting that the radius *r* vanished at the lifetime *T*, one obtains the expression for dispersion radius, o_r^2 in terms of the maximum radius r_m,

$$\sigma_r^2 = ar^2 \left[\ln\left(br_m^2 e^{2/m}\right) - \ln \sigma_r^2\right]^{-2/m} \tag{6}$$

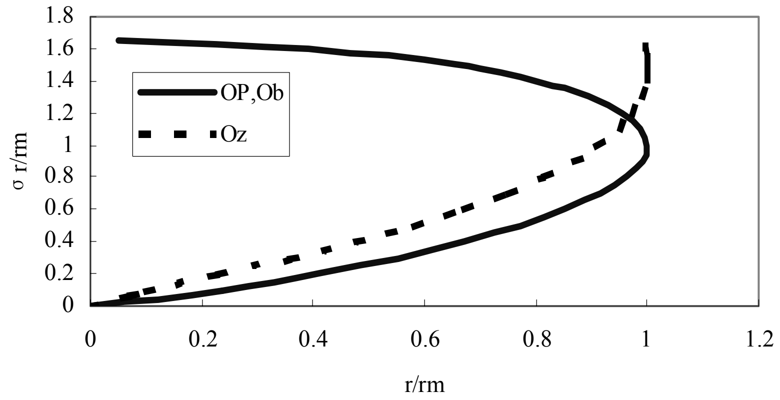

Fig. 5 Dye dispersion radius versus dye cloud radius

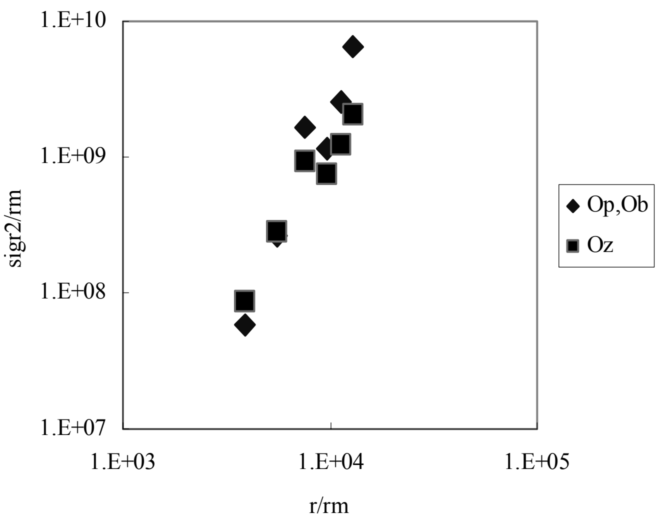

Fig. 6 Dye dispersion radius versus time

where constants *a* and *b* are given in Table 2. Fig. 5 gives the relationship between dye dispersion radius and dye cloud radius as given by Eq. (6) for two typical cases of Okubo-Pritchard(OP)and Obukyov(Ob)(two cases give the identical relationship), and Ozmidov(Oz). It shows that the dispersion radius uniformly increases as the dye cloud radius increases, hits the maximum and then decreases to null. Fig. 6 gives the typical application of this method by plotting the dispersion radius calculated from Eq. (6) out of the data of Fig. 2 versus time. Comparing with the time dependence of the dispersion radius given in Table 1, the theory of either OP or Ob appears to have an edge over that of Ob.

5. GENERAL TENDENCY OF HORIZONTAL DIFFUSIVITY

It can be claimed that the method to calculate the dispersion radius out of the dye cloud radius is an excellent method in that the time dependency of the dispersion radius is simple and straightforward. It has a drawback, however, in that the maximum dye cloud radius has to be measured to apply this method. If one looks for data available today covering a large number of fields, one has to be content with the dye cloud data sans the maximum value. In such a case, the only available method seems to be the simple method applying Eq. (2) and (3). A large number of field data is available in the Seto Inland Sea of Japan and the dye cloud area obtained is shown in Fig. 7. From this data, the simple method is applied to obtain the horizontal diffusivity versus the length scale as shown in Fig. 8. Fig. 8 should be regarded as the basic data for the coastal, calm sea and one can perhaps draw a conclusion one pleases to apply any of the oceanic diffusion theory given in Table 1.

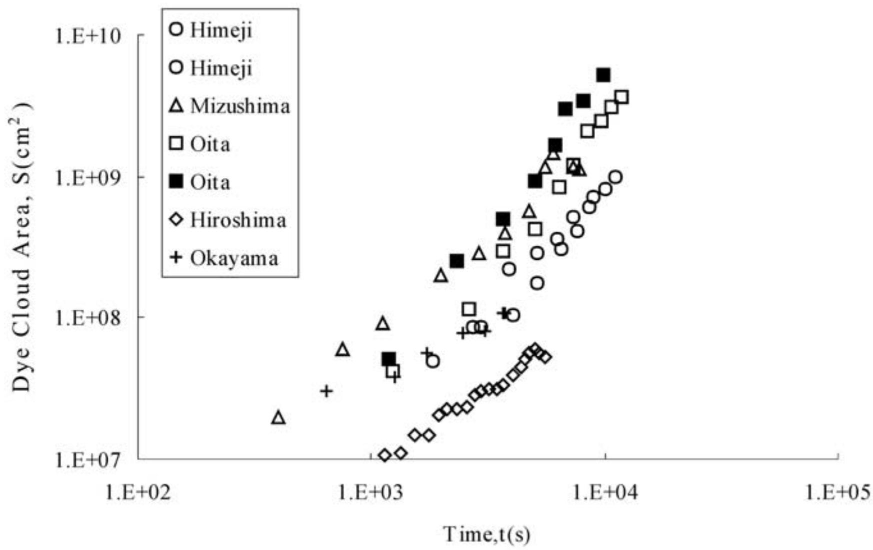

Fig. 7 Dye cloud area versus time obtained at various coastal seas

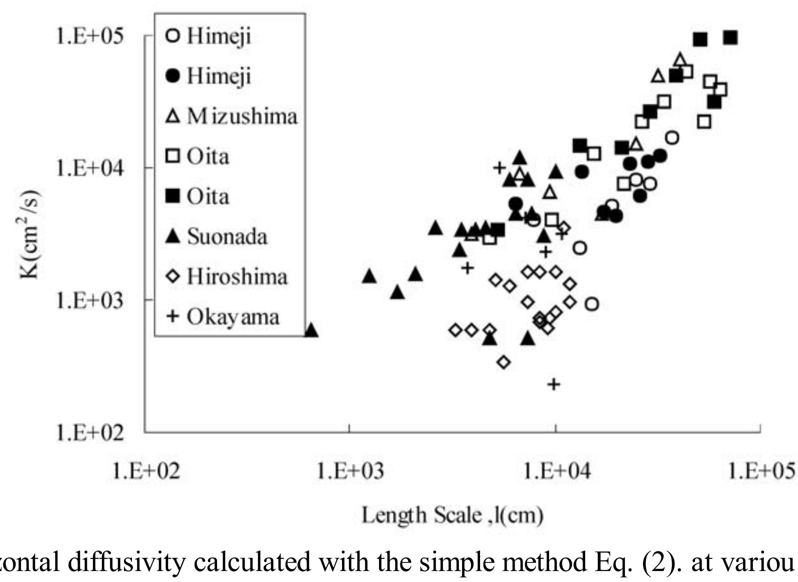

Fig. 8 Horizontal diffusivity calculated with the simple method Eq. (2). at variou coastal seas

6. CONCLUDING REMARKS

The study on the horizontal diffusion in the ocean is not popular today. The reason for this, other than the environmental concern, probably lies on the fact that so much concern is paid to accuracy and efficiency of the calculation method of the ocean current and little concern over the calculated pollutant concentration value in the ocean. The author believes that the review and proposal of a new method placed in this paper of obtaining the horizontal diffusivity and especially overall result of Fig. 8 should someday be deemed as invaluable.

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