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A STUDY ON LENGTH AND THE MAXIMUM WIDTH OF THE CIRCULATING REGION CAUSED BY A DIKE

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Abstract : Based on 2-D depth averaged water flow differential equations and using some assumptions different from those made by other researchers on turbulent viscous shear stress, the lateral distribution of main flow velocity, and turbulent viscous coefficient, the formulas used to compute circulating flow length and the maximum circulating width are derived in this paper. The results are in reasonable agreement with experimental data. The research results indicate that the circulating flow length varies with the dike length, depth, roughness, area contraction ratio and river width contraction ratio, and the maximum circulating width varies with the river width, area contraction ratio and river width contraction ratio. The width contraction ratio equals to the area contraction ratio only for rectangular channel, and they are different from each other for natural river.

Key words: Dike, Circulating flow length, Maximum circulating width, Area contraction ratio, Width contraction ratio

1. INTRODUCTION

Studying the flow problems associated with dikes, which are widely applied to the river training works, is of great significance.

If a dike is not submerged, water flows around the dike and forms a large circulating domain in its downstream. The circulating flow length is the longitudinal distance where the flow is deflected and shielded by the dike. The width of the circulating region increases gradually from the dike section and approaches the maximum value at the contraction section. The width at that location is called the maximum circulating one. To estimate the circulating flow length and the maximum circulating width is favorable in knowing range shielded by the circulating flow caused by a dike, velocity field change after the construction of dike, determination of the dike spacing and its effects.

2. MATHEMATICAL FORMULATIONS

2.1 CIRCULATING FLOW LENGTH

From Fig.1, it can be seen that water flows around a dike and forms a contraction section, and its longitudinal distance equals to one-thirds of the circulating flow length downstream the dike. At this location the width of the main flow is the smallest and the circulating width is the widest. Then the main flow disperses and the circulating width decreases gradually and equals to zero at the end of circulating flow.

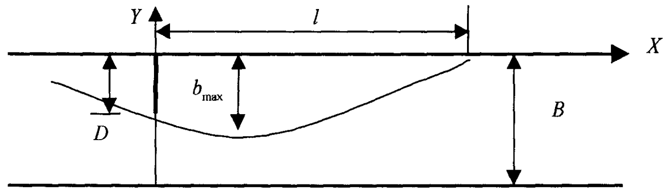

Fig. 1 Sketch of the circulating region caused by a dike

The flow around the dike can be described by the stable 2-D depth-averaged motion equations.

$$\frac{\partial hu}{\partial x}+\frac{\partial hv}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-g\frac{\partial z}{\partial x}+\frac{\partial}{\partial x}\left(v_t\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(v_t\frac{\partial u}{\partial y}\right)-\frac{u\left(u^2+v^2\right)^{1/2}}{C_0^2h} \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-g\frac{\partial z}{\partial y}+\frac{\partial}{\partial x}\left(v_t\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(v_t\frac{\partial v}{\partial y}\right)-\frac{v\left(u^2+v^2\right)^{1/2}}{C_0^2h} \tag{3}$$

Where h is the depth; z is the water level; u, v is depth-averaged velocity components in x *and* y directions, respectively; v_t is the turbulent kinematics viscosity; C_0 and C are dimensionless Chezy coefficient and Chezy coefficient, respectively, $C_0=\frac{C}{\sqrt{g}}$; g is the acceleration due to gravity. Considering that the lateral velocity is much less than the longitudinal one, so we can assume the lateral velocity equals to zero for studying the velocity fields downstream the dike, at the same time the turbulent shear stress $v_t\frac{\partial u}{\partial y}$ is much larger than $v_t\frac{\partial u}{\partial x}$ [3], so we can ignore Eq. (3), thus, Eq. (1) and (2) can be simplified into:

$$\frac{\partial hu}{\partial x}=0 \tag{4}$$

$$u\frac{\partial u}{\partial x}=-g\frac{\partial z}{\partial x}+\frac{\partial}{\partial y}\left(v_t\frac{\partial u}{\partial y}\right)-\frac{u^2}{C_0^2h} \tag{5}$$

Integrating Eq. (4) and (5) along y direction, and averaging along the river width Bx :

$$\frac{1}{Bx}\int_0^{Bx}\frac{\partial hu}{\partial x}dy=0 \tag{6}$$

$$\frac{1}{Bx}\int_0^{Bx}u\frac{\partial u}{\partial x}dy=-\frac{1}{Bx}\int_0^{Bx}g\frac{\partial z}{\partial x}dy+\frac{1}{Bx}\int_0^{Bx}\frac{\partial}{\partial y}\left(v_t\frac{\partial u}{\partial y}\right)dy-\frac{1}{Bx}\int_0^{Bx}\frac{u^2}{C_0^2h}dy \tag{7}$$

Eq. (6) and (7) can be rewritten.

$$Q=HBxU=AU=const \tag{8}$$

$$U\frac{dU}{dx}=-g\frac{dZ}{dx}-\frac{U^2}{C_0^2H}+\frac{1}{Bx}v_t\frac{\partial u}{\partial y}\bigg|_0^{Bx} \tag{9}$$

where Q is the discharge ; H is the cross-sectional average depth; B_x is the width of main flow; U *is* the cross-sectional average velocity; A is the cross-sectional area; Z is the cross-sectional average level ; u is the x-direction component of depth-averaged velocity; v_t is the turbulent kinematics viscosity; C_0 is the dimensionless Chezy coefficient computed by the cross-sectional average depth.

Eq. (9) is different from the ordinary flow equation for it involves the turbulent shear stress, which needs to be assumed in suitable manner.

The turbulent kinematics viscosity is directly proportional to the characteristic length and the characteristic velocity, which indicate large-scale turbulence. Turbulence is strong and its shear stress is large at the boundary between the main flow and circulating region. The experiments and computed results all indicate this, so we can assume

$$v_t = c_1 u y \qquad (0 \le y \le B_x) \tag{10}$$

Where c_1 is the coefficient; y is the lateral distance.

The experimental results show the lateral distribution of the depth-averaged velocity at the dike section, contraction section and all other sections downstream follows the elliptic law

$$\frac{u^2}{(u_{\max})^2} + \frac{y^2}{\left(\frac{1}{2}Bx\right)^2} = 1 \tag{11}$$

Where $u_{\max}$ is the maximum depth-averaged velocity at a cross section.

From Eq. (11) we can get

$$\frac{\partial u}{\partial y} = -\frac{y}{u}\left(\frac{u_{\max}}{\frac{1}{2}Bx}\right)^2 \tag{12}$$

Substituting Eq. (10) and (12) into Eq. (9), yields

$$U\frac{dU}{dx} = -g\frac{dZ}{dx} - \frac{U^2}{C_0^2 H} - \frac{4c_1 u_{\max}^2}{Bx} \tag{13}$$

And get the cross-sectional average velocity by integration of Eq. (11) along the width

$$U = \frac{1}{Bx}\int_0^{Bx} u dy = \frac{1}{4}u_{\max}\pi \tag{14}$$

$$\text{Namely, } u_{\max} = \frac{4U}{\pi} \tag{15}$$

Substituting Eq. (15) into Eq. (13) gives

$$U\frac{dU}{dx} = -g\frac{dZ}{dx} - \frac{U^2}{C_0^2 H} - \frac{64c_1 U^2}{\pi^2 Bx} \tag{16}$$

Let $c_2 = \dfrac{64c_1}{\pi^2}$, Eq. (16) can be rewritten as

$$U\frac{dU}{dx} = -g\frac{dZ}{dx} - \frac{U^2}{C_0^2 H} - \frac{c_2 U^2}{Bx} \tag{17}$$

Let

$$-g\frac{dZ}{dx} = \frac{U^2}{C_0^2 H}\left(c_3 + c_4\frac{x}{l}\right) \quad [5] \tag{18}$$

Where l is the circulating flow length downstream a dike.

Substituting Eq. (18) into Eq. (17), dividing each side of the equation by U^2, and integrating this equation, we can obtain

$$\int_0^l \frac{dU}{U} = \frac{1}{C_0^2 H}\int_0^l\left(-1 + c_3 + c_4\frac{x}{l}\right)dx - c_2\int_0^l \frac{1}{Bx}dx \tag{19}$$

In order to solve Eq. (19), it must be known that how B_x varies within the circulating flow length. As an approximation, assuming that B_x varies linearly between 0~*l*. When the distance varies from 0 to *l*/3, B_x varies from *B-D* to *B-b*max ; when the distance varies from *l*/3 to *l*, B_x varies *from B-*b_{max} to *B*. Namely,

$$Bx = \begin{cases} B - D + \frac{3}{l}(D - b_{\max})x & (0 \le x \le \frac{l}{3}) \\ \frac{1}{2}(2B - 3b_{\max}) + \frac{3}{2l}b_{\max}x & (\frac{l}{3} \le x \le l) \end{cases} \tag{20}$$

Where $b_{\max}$ is the maximum circulating width downstream a dike.

Substituting Eq. (20) into Eq. (19) and utilizing Eq. (8), we can obtain the following expression

$$\ln\frac{A - A'}{A} = \frac{1}{C_0^2 H}\left(-1 + c_3 + \frac{c_4}{2}\right)l - c_2\frac{l}{D}\ln\frac{B}{B - D} \tag{21}$$

Namely

$$l = \frac{C_0^2 H \ln\frac{A}{A - A'}}{1 - c_3 - \frac{c_4}{2} + c_2 C_0^2 \frac{H}{D}\ln\frac{B}{B - D}} \tag{22}$$

Let $1 - c_3 - \frac{c_4}{2} = k_b$, $c_2 = k_c$, Eq. (22) can be rewritten:

$$l = \frac{C_0^2 H \ln\frac{A}{A - A'}}{k_b + k_c C_0^2 \frac{H}{D}\ln\frac{B}{B - D}} \tag{23}$$

Eq. (23) indicates that the circulating flow length varies with dike length *D*, depth *H*, dimensionless Chezy coefficient C_0, the area contraction rate $(A-A')/A$ and the width contraction rate $(B-D)/B$. River section configuration is expressed by the area contraction rate in Eq. (23). So the equation not only can be used in the rectangular channels, but also can be used in the natural rivers. The coefficients in Eq. (23) may be slightly different if verified by different experimental data.

By use of data measured by Dou, the coefficients of Eq. (23) can be obtained:

$$l = \frac{C_0^2 H \ln\frac{A}{A - A'}}{0.146 + 0.073 C_0^2 \frac{H}{D}\ln\frac{B}{B - D}} = \frac{13.66 C_0^2 H \ln\frac{A}{A - A'}}{2 + C_0^2 \frac{H}{D}\ln\frac{B}{B - D}} \tag{24}$$

The comparison between the computed by use of Eq. (24) and the measured is shown in Fig.2. The computed values agree well with the measured ones.

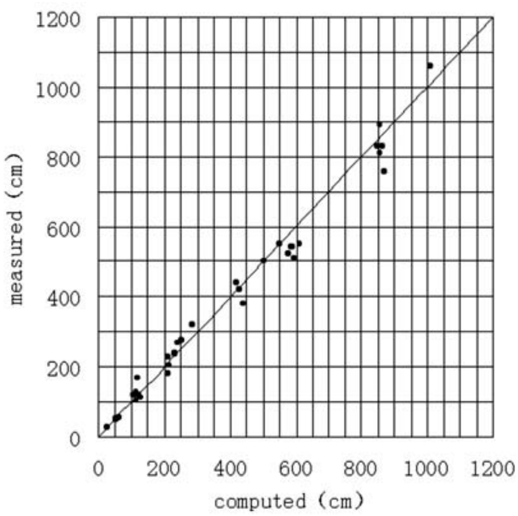

Fig. 2 Comparison between the computed and the measured circulating flow length

Different methods are adopted to compute the area contraction rate $\Delta A = \frac{A - A'}{A}$ according to the sectional configuration.

For a rectangular channel, $A=BH$, $A'=DH$, $\Delta A = \frac{B-D}{B}$, Eq. (24) becomes

$$l = \frac{13.66 C_0^2 H \ln \frac{B}{B-D}}{2 + C_0^2 \frac{H}{D} \ln \frac{B}{B-D}} \tag{25}$$

The sectional configuration of most natural rivers shapes as a parabola [6]. If the river breach is straight and nearly regular, the maximum value of depth is in the center, then the sectional configuration can be described as follows

$$h = h_{\max}\left(1 - \frac{4y^2}{B^2}\right) \qquad -\frac{1}{2}B \le y \le \frac{1}{2}B \tag{26}$$

In which $h_{\max}$ is the maximum value of depth at a cross section.

So the river area, the dike blocking area and area contraction rate ΔA follows respectively

$$A = \int_{-\frac{B}{2}}^{\frac{B}{2}} h_{\max}\left(1 - \frac{4y^2}{B^2}\right) dy = \frac{2}{3} B h_{\max} \tag{27}$$

$$A' = \int_{-\frac{B}{2}}^{-\frac{B}{2}+D} h_{\max}\left(1 - \frac{4y^2}{B^2}\right) dy = 2\frac{D^2 h_{\max}}{B} - \frac{4D^3 h_{\max}}{B^2} \tag{28}$$

$$\Delta A = \frac{A - A'}{A} = \frac{B^3 + 2(D^3 - 1.5BD^2)}{B^3} \tag{29}$$

Eq. (24) can be rewritten

$$l = \frac{13.66 C_0^2 H \ln \frac{B^3}{B^3 + 2(D^3 - 1.5BD^2)}}{2 + C_0^2 \frac{H}{D} \ln \frac{B}{B-D}} \tag{30}$$

Different formulas may be selected according to the sectional configuration when computing the circulating flow length of a dike. Eq. (24) or Eq. (25) may be selected for the rectangular channels. Eq. (24) or Eq. (30) may be selected for the natural rivers. If an equation only suitable for rectangular channels is used to compute the circulating flow length of a dike for the natural river, the length value computed will vary more or less with *D/B* .As to this, we can analyze as the following.

For a rectangular channel, Eq. (25) can be rewritten

$$C_l = \frac{13.66 C_0^2 H}{2 + C_0^2 \frac{H}{D} \ln \frac{B}{B-D}} \tag{31}$$

$$l = C_l \ln \frac{B}{B-D} \tag{32}$$

The dimensionless circulating flow length can be expressed

$$l_A = \frac{l}{C_l} = \ln \frac{B}{B-D} \tag{33}$$

Similarly, for a natural river, Eq. (30) can be rewritten

$$l_B = \frac{l}{C_l} = \ln \frac{B^3}{B^3 + 2(D^3 - 1.5BD^2)} \tag{34}$$

The variations of dimensionless lengths computed by Eq. (33) and Eq. (34) with D/B are shown in Fig.3. It is seen when $D/B<0.5$, $l_A>l_B$; when $D/B=0.5$, $l_A=l_B$; when $D/B>0.5$, $l_A<l_B$; When $D/B=(0.5\sim0.6)$, $l_A\approx l_B$.

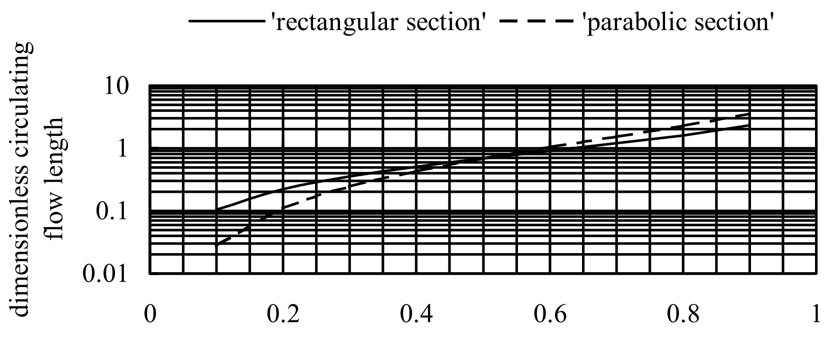

Fig. 3 Variations of length of the circulating region with sectional configuration

2.2 THE MAXIMUM CIRCULATING WIDTH DOWNSTREAM A DIKE

It is known by analyzing the experimental data that the width of main flow at contraction section is directly proportional to the width of flow at dike section and has a relation with the area contraction ratio. The equation for the maximum circulating width of a dike at the contraction section can be expressed as follows

$$B - b_{\max} = f\left(\frac{A - A'}{A}\right)(B - D) \tag{35}$$

Namely

$$b_{\max} = B - f\left(\frac{A - A'}{A}\right)(B - D) \tag{36}$$

The maximum circulating width downstream a dike can be determined by experimental data.

$$b_{\max} = B - \left(0.20 + 0.80\frac{A - A'}{A}\right)(B - D) \tag{37}$$

The comparison between the computed by Eq. (37) and the measured maximum circulating width is shown in Fig.4. The computed values agree well with the measured ones.

For a rectangular channel, $\frac{A - A'}{A} = \frac{B - D}{B}$, Eq. (37) becomes:

$$b_{\max} = B - \left(0.20 + 0.80\frac{B - D}{B}\right)(B - D) \tag{38}$$

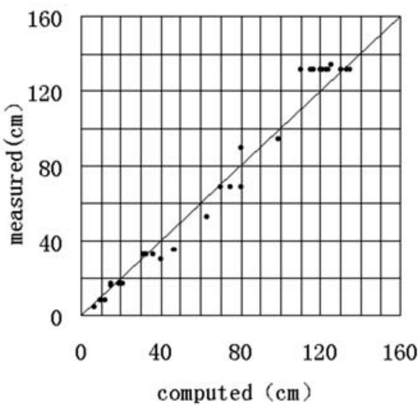

Fig. 4 Comparison between the computed and the measured maximum circulating width

For a natural river, substituting Eq. (29) into Eq. (37), yields

$$b_{\max} = B - (0.20 + 0.80\frac{B^3 + 2(D^3 - 1.5BD^2)}{B^3})(B - D) \tag{39}$$

Dividing each side of the equations by *B,* Eq. (38) and Eq. (39) can be transformed into dimensionless forms. Comparing the results computed by Eq. (38) and Eq. (39) respectively, the variations are shown in Fig.5. From Fig.5 we know that the difference between Eq. (38) and Eq. (39) is similar as the one between Eq. (33) and Eq. (34).

When computing the circulating flow length and the maximum circulating width for natural rivers, if the flow area and the blocking area of a dike are known, we should select Eq. (24) and Eq. (37) .If only the width of river and the length of a dike are known, we should select Eq. (30) and Eq. (39).

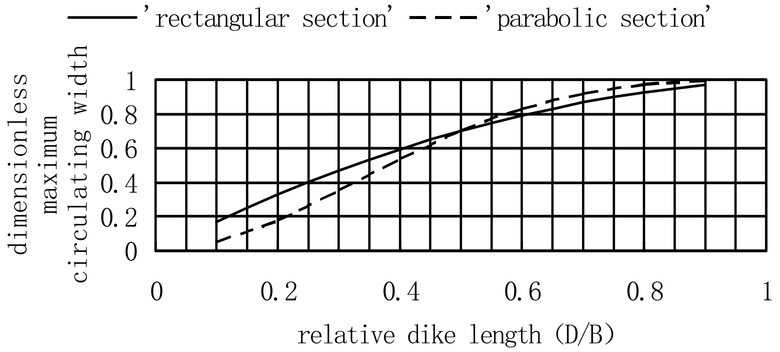

Fig. 5 Variations of maximum circulating width with sectional configuration

3. CONCLUSIONS

(1) The circulating flow length varies with the dike length, depth, roughness, area contraction ratio, river width contraction ratio, etc. and the maximum circulating width varies with the river width, area contraction ratio and river width contraction ratio, etc.

(2) The width contraction ratio equals to the area contraction ratio only for the rectangular channel and they are different from each other for natural rivers.

(3) The formulas that are applied to compute the circulating flow length and the maximum circulating width downstream a dike can be used for the natural rivers in theory. Whereas the natural river is very complex, so the formulas must be verified before they are used.

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