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APPLICATION OF A BOUSSINESQ WAVE MODEL

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Abstract: The numerical wave generating and wave breaking are introduced in simulation of wave transformation based on fully non-linear Boussinesq equations. Non-reflection wave generating is achieved by adding a source function term into a continuity equation in the wave making region. The value of δ in source function with variation of relative water depth and wave height is found. Then the linear superposition method is used to simulate irregular wave, and is verified by the autocorrelation method of wave spectrum estimation. The relationship among the parameter Δt , m and N is discussed in spectrum estimation. In the point of wave breaking transforming turbulence energy, wave breaking terms expressing this transform are added into momentum equations to simulate wave transformation in the surf zone and the improved wave breaking model is tested by experimental data.

Key words: Boussinesq equation， Wave breaking， Source function wave generator， Unidirection irregular wave, Spectrum analysis

1. INTRODUCTION

In recent decades, much wave motion issue in coastal engineering is simulated by the Boussinesq type equations. After the standard Boussinesq equations derived by Peregrine（1967）, which had two major limitations, many researchers such as Witting 1984, Madsen 1992, Nwogu 1993, Ge Wei and Kirby 1995, Agnon 1999, Zou Zhili, Lin Jianguo, Zhang Yonggang have improved the Boussinesq type equations in dispersion and non-linearity to extend it to intermediate water depth and to simulate the wave deformation of diffraction, refraction, reflection, shoaling and breaking. A lot of works and applications of the Boussinesq type equations have been done in coastal engineering, for example, Li Shaowu, Gu hanbin, Zhu Liangsheng, Liu Shuxue et al. have done many works in this aspect.

The numerical wave generating is important in a wave model. The often used source function wave generating method can avoid reflection by the wave generator region. In this paper, we studied the adoption of the coefficient in this method which is based on the fully nonlinear Boussinesq equations (Wei and Kirby 1995), and how to generate irregular waves using this method, then we analyzed the irregular wave spectra from autocorrelation function, and verified this method's validity. It is useful ,in fact, that a wave breaking model is introduced into the Boussinesq equations, because wave breaking is a nature phenomena near sea shore. In this paper, an improved wave breaking model is proposed and is applied to the same condition as Sato's flume tests. It is good agreement that the wave height and averaged water level in the surf zone by numerical tests are compared with experimental data.

1.1 NUMERICAL WAVE GENERATOR

There are two kinds of wave-making method, which are boundary wave-making method and inner wave-making method. In the former one, the boundary can be made absorbable for the weakly reflected outgoing waves in some degree (Li, 1998). In the present model, source function wave-making method is adopted in order to let the reflected waves outgo through the wave generator freely. In order to obtain a desired oscillation signal in the wave generating area, a source function *f(x,y,t)* is added into the mass conservation equation (Wei et al., 1999), which is expressed as

$$f(x,y,t) = g(x)s(y,t) \tag{1}$$

in which,

$$g(x) = \exp\left[-\beta(x - x_s)^2\right] \tag{2}$$

$$s(y,t) = D\sin(\lambda y - \omega t) \tag{3}$$

$$\beta = \frac{5}{(\delta L/4)^2} = \frac{80}{\delta^2 L^2} \tag{4}$$

where, L is the wave length, λ ($=k_y=k\sin\theta$) the wave number in y direction. δ is factor. x_s is the location of the center of wave-making area, D is the source function's amplitude. For monochromatic wave, D is defined as

$$D = \frac{2a_0\cos\theta\left(\omega^2 - \alpha_1 g k^4 h^3\right)}{\omega k\sqrt{\frac{\pi}{\beta}}\exp\left(-l^2/4\beta\right)\left[1 - \alpha(kh)^2\right]} \tag{5}$$

where, h is the still water depth at the wave generator region, a_0 the wave amplitude, ω the wave frequency, θ the wave incident angle and l($=k_x=k\cos\theta$) the wave number in x direction., $\alpha=-0.390$, $\alpha_1=\alpha+1/3$.

The source function consists of two parts—the shape function *g(x)*, defining the distribution of the wave generating strength, and the time function *s(y,t)*, being related to wave frequency and amplitude. The selection of the value of δ is important to implement the source function wavemaking method, so we determine the value of δ through a set of numerical tests. To study how to extend this method to irregular wave, the linear superposition of component waves is tested. According to the irregular wave concept of Longuet-Higgins (1961), the water surface elevation can be described by

$$\eta(t) = \sum_{i=1}^{\infty} a_i\cos(\omega_i t + \varepsilon_i) \tag{6}$$

where, a_i and ω_i represent the amplitude and frequency of the component wave respectively and ε_i denotes the initial phase of the component wave, which is distributed randomly in the range of 0-2π. This concept implies that each component wave has its deterministic amplitude and frequency. Supposing the nonlinear effect in the wave-generation region is negligible, we can calculate the time function *s(y,t)* by

$$s(y,t) = \sum_{i=1}^{M} D_i\sin(\lambda_i y - \omega_i t + \varepsilon_i) \tag{7}$$

where, the source function's amplitude of each component wave is defined as

$$D_i = \frac{2a_i\cos(\theta_i)\left(\omega_i^2 - \alpha_1 g k_i^4 h^3\right)}{\omega_i k_i I_i\left[1 - \alpha(k_i h)^2\right]} \tag{8}$$

$$I_i = \int_{-\infty}^{\infty}\exp(-\beta x^2)\exp(-i l_i x)dx = \sqrt{\frac{\pi}{\beta}}\exp\left(-l_i^2/4\beta\right) \tag{9}$$

$$l_i = k_i\sin(\theta_i) \tag{10}$$

Similarly to the case of regular wave, the shape function and the span of wave-making area W are also related to the wave length in irregular wave case, in which the wave length is replaced by a characteristic wave length, corresponding to the average wave period.

1.2 WAVE BREAKING MODEL

In recent two decades, many researchers made more effort to extend the Boussinesq equations into surf zone. There are two kinds of manner. In the former one, a dissipation term is introduced into the process of solving（Kobayashi et al 1989）, in the later, a modified momentum term or addition term is added into the momentum equations, in which there are many methods such as eddy viscosity mode, surface roller mode, modified momentum mode, vortex mode and spectrum method（Li Mengguo 2002）. In these methods, each one has its own characteristics and is used by some researchers.

In terms of the forms of wave breaking mode, the surface roller mode is derived from the assumption of vertical distribution of horizontal velocity, in which the breaking wave velocity is calculated by iterative. If this mode is extended to 2D, the iterative process is very complicated. The vertex term is added into the momentum equations in vertex mode. And the vertex equation is determined by Reynolds equation, which is also complicated in solving. In spectrum method the wave breaking energy is expressed by semi-experience formula. In eddy viscosity and modified momentum mode, the wave breaking terms is expressed as two order derivation of velocity or flux to space length, only the viscosity coefficient is different from one mode to another. The form of wave breaking term $\partial(\nu_t \partial U/\partial x)/\partial x$ seams to be reasonable (Li Shaowu1999), as the wave breaking transformation to turbulence energy is concerned. Zelt and Qin Chen also use this formation, moreover, Qin Chen slightly improved this formation when he extended it to 2D model. In this paper, the similar formation is adopted

$$F_{br} = \left(\nu (u_\alpha)_x\right)_x + \frac{1}{2}\left(\nu\left((u_\alpha)_y + (v_\alpha)_x\right)_y\right) \tag{11}$$

$$G_{br} = \left(\nu (v_\alpha)_y\right)_y + \frac{1}{2}\left(\nu\left((u_\alpha)_y + (v_\alpha)_x\right)_x\right) \tag{12}$$

where ν is the eddy viscosity coefficient， and is expressed as

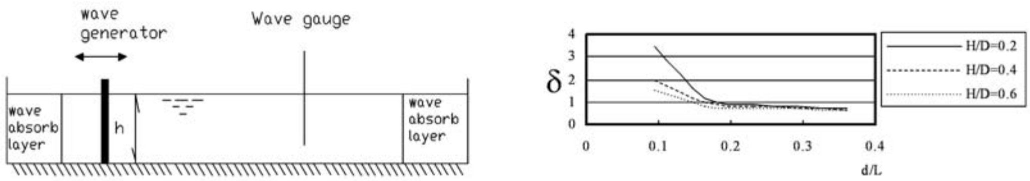
$$\nu = B|h+\eta|\sqrt{g(h+\eta)+\eta_t^2} \tag{13}$$

In this mode, we not only considered the concept of mixing length, but also considered the effect of horizontal and vertical velocity to wave breaking.

Rankine(1864) proposed that wave breaks as the water particle velocity excess the wave velocity in wave motion. Now it is a standard as wave breaking and many researchers use it. We also use it in calculation and once wave breaking occurs the breaking terms are added into momentum equations.

2. EXPERIENCE COEFFICIENT δ NUMERICAL TESTS

The shape coefficient δ is expressed as $\delta=5/(\delta L/4)^2=80/(\delta L)^2$, and the width of wave generator area is $W=\delta L/2$, in which the value of δ influences the width of wave generator area, the larger the value of δ, the wider the width of wave generator area. We perform the tests to verify the effect of δ to the calculating wave height in condition with variance of relative water depth and input wave height. Fig1(a) shows the Configuration of calculating. It is found that if the value of δ is in a certain range, the output wave height is consistent with input one, and if the value of δ is beyond the certain range, the output wave height is larger than input one. So the variance value of δ with relative water depth and input wave height is given as fig1(b) when the difference between output wave height and input one is less than 5%. It is shown that when the water depth is shallow, the value of δ should be large, but when the water depth is deep, the value of δ should be small and the value of δ has the same variance trend with relative wave height.

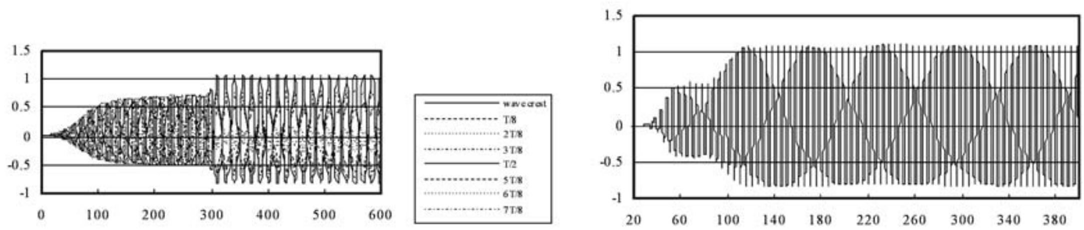


(a) (b)

Fig. 1 (a) Configuration of numerical tests (b) the value of δ

3. NON-REFLECTION WAVEMAKING CHARACTERISTICS

Non-reflection wavemaking is the most important characteristic of the source function wavemaking method. To verify it, numerical tests are conducted under constant water depth, in which the wave making region is set to the center of the calculating region, the left side boundary is set as absorbing layers and the right one as vertical wall. Fig2 shows the results of the case that the calculating region is 600m long and 20m wide, and the water depth 5.0m, the input wave height 1.0m and wave period 5.0s. In this case, the wave length is 30.27m, $\Delta x = \Delta y$= 2.0m, Δt=0.1s. Fig2 (a) shows the vertical sections of water surface, which are in the condition of fully developed waves, and begin at a wave crest, there are 8 section at the same time interval in a wave period. It is shown that the complete standing wave with clear wave node is formed at right side, and the fully absorbing is obtained at left side. Fig2(b) shows the water surface process with time marching at the location of 480m. This point is just at wave antinode. It is shown that if the wave front does not arrive at this point, the water surface is the still water level, if the wave front arrives at this point, the water surface fluctuate and the wave amplitude gradually reaches 1.0m, then after 30 second the reflection wave of the right side vertical wall return to this point, and the wave amplitude reaches the standing wave amplitude, at last the wave amplitude is almost a constant. So we say that the source function wavemking method is non-reflect wave maker.



(a) (b)

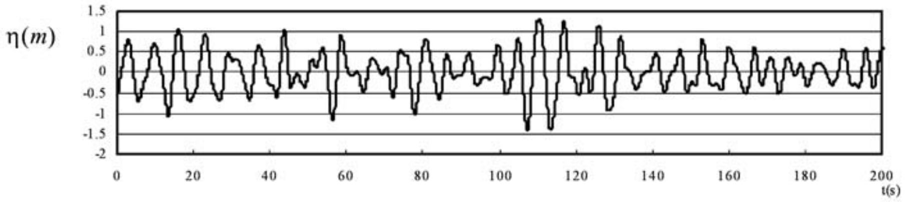
Fig. 2 (a)Horizontal coordinate is distance, vertical coordinate is the variance of water surface (unit m), wavemaking region is set at 286--314m.(b) Horizontal coordinate is time, unit s, vertical coordinate is the variance of water surface (unit: m)

3.1 IRREGULAR WAVE

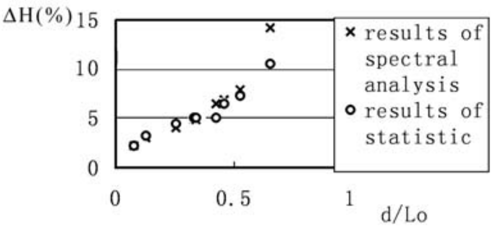
Numerical tests are performed with the above irregular wave model in condition of that wave period is 5.6s, 8.0s, 10.0s, the effective wave height is 2.0m with three different water depth. The configuration of the numerical flume is also shown in Fig 1(a). In each of the test, the number of component frequencies is set to be 400, the space interval is evaluated by $\overline{L}/15$，the time interval $T_{1/3}/50$. An example of the water surface elevation of the random wave is shown in Fig.3(a). In analysis of numerical results of the wave height, the zero-cross-up approach and the spectral evaluation method are used. The calculated results are shown in Fig.3(b). It can be seen from Fig.3(b) that the calculated results of wave height with small relative water depth are closer to the input ones than that of larger relative water depth. The output result of spectrum of the water surface elevation series, sampled at the numerical wave gauge as shown in Fig.1(a), is compared with the input one (as shown in Fig.2(c)). The output result agrees well with the input one.

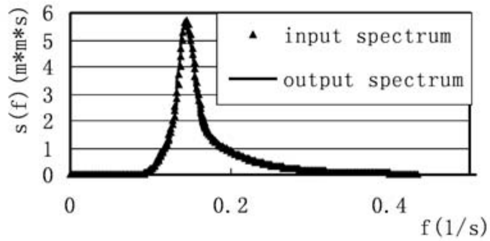
3.2 SPECTRUM ESTIMATION

In this paper, we adopt autocorrelation function method(Yu Yixiu 2000) to estimate the irregular wave spectra. In order to improve the accuracy of the autocorrelation function, Hanning window is adopted. The basic parameters Δt, m and N used in the above method of spectrum evaluation do affect interactively. Goda (1985) once suggested that the sampling interval Δt is evaluated in the range of $(1/10-1/20)T_{H_{1/3}}$ .While Yu (2000)recommended that m equals $N/(15\sim20)$ when $\Delta t=0.5$s and m equals $N/(20-40)$ when $\Delta t=1.0$s . In this paper, a series of tests are performed with different values of m and N with a fixed value of Δt being equal to $T_{1/3}/50$, for the Δt often is small (0.1−0.2s). The improved JONSWAP spectrum is used to generate a wave train and then the spectrum is evaluated by the autocorrelation function method. The results indicate that the value of m mainly affects the peak frequency and the spectral density near the peak frequency. Satisfied result of evaluated spectra can only be obtained when m ranges from $N/10$ to $N/25$, beyond that the errors between the evaluated spectrum and the target spectra will be larger than 5%.



(a)





(b) (c)

Fig. 3 (a) Configuration of numerical tests, (b) Water surface elevation of random waves, (c) Numerical results of significant wave heightin different water depth,
(d) Comparison between output spectrum and input spectrum

3.3 SIMULATION OF BREAKING WAVE

The improved wave breaking mode is tested by Sato's experimental data. Table1 shows the Sato's experimental conditions. Fig. 4 is the numerical calculation scheme. Temporal interval is $T/50$, and special interval $L/20$. Forward marching method is used in treating the moving boundary. Naturally before wave breaking, the wave front is steep. Then the spray occurs at the front top of wave profile or the water tongue is turning down. As above mentioned, we adopt Rankine's wave breaking standard, It is that when wave breaks the water particle velocity is equal or greater than phase celerity. The water particle velocity is written as

$$\boldsymbol{u} = \boldsymbol{u}_\alpha + \left(\frac{z_\alpha^2}{2} - \frac{z^2}{2}\right)\nabla(\nabla \cdot \boldsymbol{u}_\alpha) + (z_\alpha - z)\nabla[\nabla \cdot (h\boldsymbol{u}_\alpha)] \tag{14}$$

where $z = \eta$. Fig 5 shows the comparison between the results of numerical simulation of the five case and that of Sato's experiment. H/H_0 is the ratio of local wave height to that at deep water, $\bar{\eta}$ the averaged water level, d/Lo the relative water depth. It is found that the results of numerical calculation is good agreement to that of experiment.

Table 1 Sato's flume experimental condition

Date	case	Wave height at deep water (cm)	Wave period (s)	The gradient of bottom
1989	Case-1	5.42	0.98	1/40
1989	Case-2	2.86	0.98	1/40
1988	Case-3	10.80	1.18	1/20
1988	Case-4	8.76	1.18	1/20
1988	Case-5	6.68	1.18	1/20

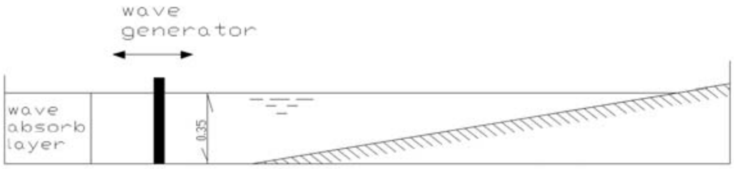


Fig. 4 Numerical test scheme of wave breaking

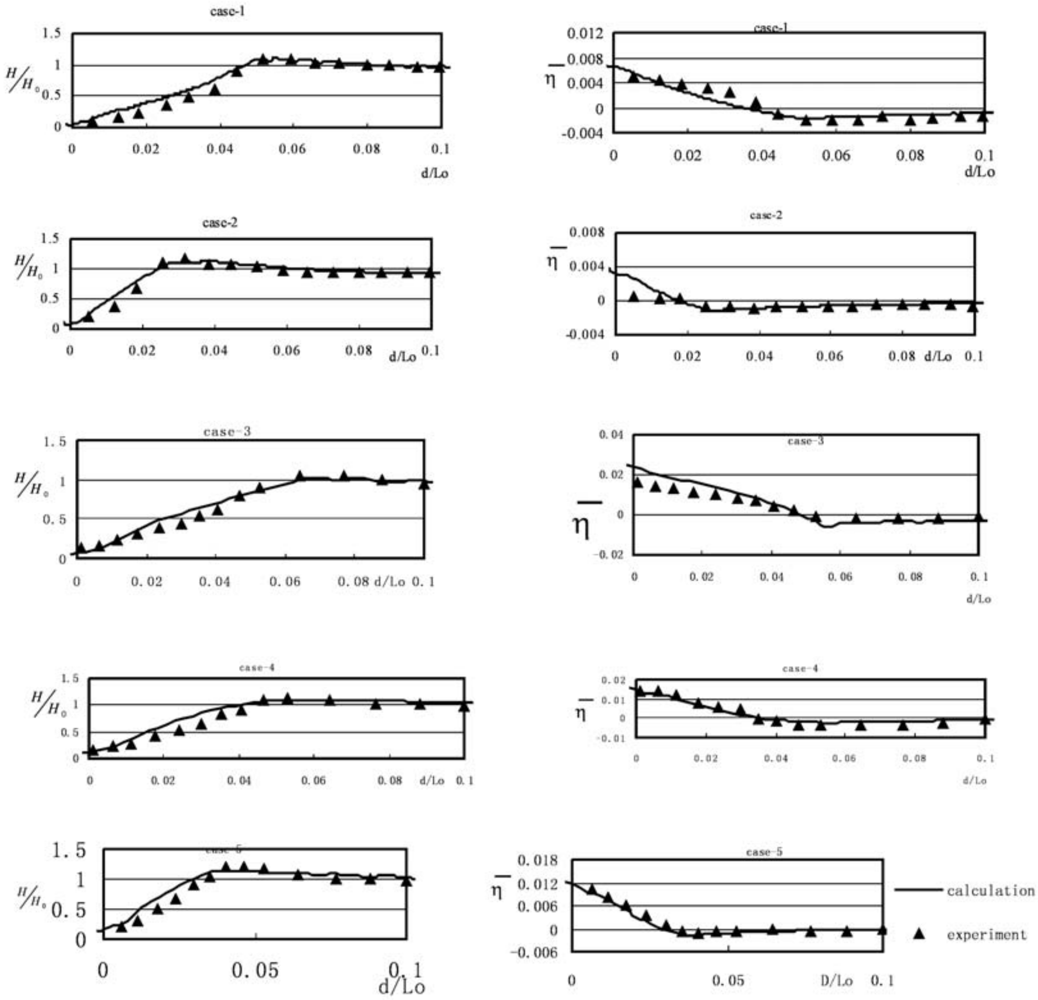


Fig. 5 Results comparison between numerical and experimental test

4. CONCLUSION

In this paper, a wave model based on fully non-linear Boussinesq equations is studied in the aspects of wave-making and wave breaking. Numerical wave generating in regular and irregular wave is discussed. The reasonable parameter δ value in the source function wave generator is found. The relation among Δt, m and N is indicated in autocorrelation function spectrum estimation. And a improve wave breaking method is validated by experimental test. In the future, we'll use the wave model to research the issues such as wave-induced current and sediment transpotation.

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