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RESEARCH AND IMPLEMENT OF A 2-D UNSTEADY VISUAL MATHEMATICAL MODEL OF TIDAL ESTUARY

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Abstract: Based on triangle grids, a planar 2-D unsteady mathematical model of tidal flow is established by the finite element method in this article. Based on the model, a visual software is developed to process the data before and after model calculation. Then the visual mathematical model is made up from the two parts above. Since the methods for lumping mass and compressing memory and predictor-corrector are used to solve the equations, the questions of memory units and calculation speed are settled quite well. Simulation calculation is carried out on the south and north branches of the Yangtze River Estuary. Result shows that the model can simulate the tidal flow correctly after the model verification between calculated results and field data. From the practice implement, it shows that the visual model can not only make the calculation visual and overcome the disadvantage of traditional models, but also save much time of model debugging, and improve the efficiency of solving practical problems with mathematical model.

The model is very efficient and useful, so it is worthy of being carried out in the simulation calculation of tidal flow and sedimentation transportation at estuary and seashore.

Key words: Planar 2-D, Finite element method, Tidal flow, Mathematical model, Visualization

1. INTRODUCTION

In the study of estuary reach tidal problem, mathematical model is becoming an important research means to replace physical model. With the development of computer technology, many different numeric methods are developed in mathematical model region now. The finite element method can adopt non-frame grids to suit for various complex boundary conditions among those. Narrow river boundary, wide sea boundary and islands often coexist in estuary reach, which furthers the boundary conditions complicated. The finite element method is applicable to numerical simulation of tidal estuary very much for its flexibility. So, a planar 2-D unsteady mathematical model of tidal estuary is developed by finite element method.

The research of mathematical model should not be limited to numerical calculation. Based on the traditional mathematical model, developing multi-purposes system to process data before and after model calculation with intuitive show is urged for numerical simulation. Combining the original planar 2-D water and sediment mathematical model with software for pre- and post- process, a visual mathematical model is established which realizes the visualization and dynamic demo of whole computed course. Compared with traditional mathematical model, it has advantages of convenient debugging and live show, thus improves practicability and maneuverability of mathematical model significantly.

2. PLANAR 2-D TIDAL MATHEMATICAL MODEL

2.1 THE GOVERNING EQUATIONS AND DETERMINISTIC CONDITIONS

The Yangtze River estuary is wide and shallow bulk water, whose variation in vertical is smaller than that in other two directions of plane. So, it is viable to use depth averaged planar 2-D shallow water equations as the governing equations for tidal computation. They are as follows:

$$\frac{\partial z}{\partial t}+\frac{\partial (hu)}{\partial x}+\frac{\partial (hv)}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+g\frac{\partial z}{\partial x}+\frac{gn^2u\sqrt{u^2+v^2}}{h^{4/3}}-fv-\upsilon_t\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)=0 \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+g\frac{\partial z}{\partial y}+\frac{gn^2v\sqrt{u^2+v^2}}{h^{4/3}}+fu-\upsilon_t\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)=0 \tag{3}$$

where u, v is x, y direction components of depth averaged velocity; z 、 h is water level (or tidal level) and depth; g is acceleration due to gravity; υ_t is turbulent viscosity coefficient; n is Manning roughness coefficient; f is Coriolis force coefficient, $f=2\omega\sin\Phi$, ω is rotation angular velocity of earth, Φ is the latitude of computed reach.

The deterministic conditions involve boundary conditions and initial conditions. Boundary conditions include opening boundary and closing boundary. The former is inlet and outlet water boundary, and is governed by field tidal process for model. The latter is land boundary and the normal velocity is treated as zero for model. The initial water level is given by measured tidal level, the initial velocity is zero, deviation of initial conditions disappears quickly and does not affect the precision of computed result.

2.2 THE FINITE ELEMENT EQUATIONS

Firstly, plot the whole computed region by triangle grids. Secondly, integrate the equations (1) – (3) among each triangle with Galerkin method. Lastly you can get following finite element equations after simplification and settlement.

$$A_{ij}\frac{dz_j}{dt}=-B1_{ij}(hu)_j-B2_{ij}(hv)_j \tag{4}$$

$$A_{ij}\frac{du_j}{dt}=-C_{ij}u_j-gB1_{ij}z_j-A_{ij}\frac{gn_j^2u_j\sqrt{u_j^2+v_j^2}}{h_j^{4/3}}+A_{ij}fv_j-\upsilon_t(P_{ij}+Q_{ij})u_j \tag{5}$$

$$A_{ij}\frac{dv_j}{dt}=-C_{ij}v_j-gB2_{ij}z_j-A_{ij}\frac{gn_j^2v_j\sqrt{u_j^2+v_j^2}}{h_j^{4/3}}-A_{ij}fu_j-\upsilon_t(P_{ij}+Q_{ij})v_j \tag{6}$$

$$A_{ij}=\iint_\Omega \varphi_j\varphi_i dxdy \qquad B1_{ij}=\iint_\Omega \varphi_i\frac{\partial \varphi_j}{\partial x}dxdy$$

$$B2_{ij}=\iint_\Omega \varphi_i\frac{\partial \varphi_j}{\partial y}dxdy \qquad C_{ij}=C1_{ijk}u_k+C2_{ijk}v_k$$

$$C1_{ijk}=\iint_\Omega \varphi_k\varphi_i\frac{\partial \varphi_j}{\partial x}dxdy \qquad C2_{ijk}=\iint_\Omega \varphi_k\varphi_i\frac{\partial \varphi_j}{\partial y}dxdy$$

$$P_{ij}=\iint_\Omega\left(\frac{\partial \varphi_i}{\partial x}\frac{\partial \varphi_j}{\partial x}+\frac{\partial \varphi_i}{\partial y}\frac{\partial \varphi_j}{\partial y}\right)dxdy \qquad Q_{ij}=\oint_\Gamma\left(\frac{\partial \varphi_j}{\partial y}\varphi_i dx-\frac{\partial \varphi_j}{\partial x}\varphi_i dy\right)$$

in which φ is shape function, i, j, k is three vertex node serial number of triangle.

2.3 THE SOLUTION APPROACH TO DISCRETE EQUATIONS

For the convenience of handwriting, the universal variable P is adopted to substitute variables z、u and v of equation (4) − (6), and FP to denote the right item of equations. The above equations can be simplified as uniform format by lumping mass, which replaces A_{ij} with diagonal matrix $\overline{A}_{ij}$ and memories matrix with compressed memory method.

$$\overline{A}_{ij}\frac{dP_i}{dt} = FP_i \tag{7}$$

The time-advanced method of prediction-correction is used to solve the equations. That means using quadratic explicit *Adams* formula to predict, using implicit trapezium formula to correct. That consists of quadratic *Adams pc* formula. The discrete equations need not be joined. It has the characteristic of good stability, great speed and higher precision.

predict:
$$\overline{A}_{ij}P_i^{n+1*} = \overline{A}_{ij}P_i^n + \frac{\Delta t}{2}\left(3FP_i^n - FP_i^{n-1}\right) \tag{8}$$

correct:
$$\overline{A}_{ij}P_i^{n+1**} = \overline{A}_{ij}P_i^n + \frac{\Delta t}{2}\left(FP_i^n + FP_i^{n+1*}\right) \tag{9}$$

Iterative course is required to heighten calculation precision. Given small error δ, if $\left|P_i^{n+1**} - P_i^{n+1*}\right| < \delta$ under the condition of $1 \le i \le NP$, you can suppose $P_i^{n+1} = P_i^{n+1**}$, else suppose $P_i^{n+1*} = (P_i^{n+1**} + P_i^{n+1*})/2$; Go to second correction to iterative further until the precision is satisfied. Above, n is time interval, * means prediction value, ** means correction value, non-mark means the final result of that time interval.

2.4 CORRELATIVE PROBLEM TREATMENTS OF NUMERICAL COMPUTATION

(1) Automatic generation of grids

A new gradual change triangle grids automatic plotting method for random plane region is set up by combining popular *Delaunay* triangle method and advancing wave method. It has following advantages:

- plot and grid the computed area automatically once given little boundary information and give node serial number and correlative information of units;
- protract and show grids on the screen automatically and adjust or modify grids generation conveniently;
- realize the automatic local densification treatment and keep the uniformity transition of grid spacing by adjusting correlation parameters;
- make the random single point or several single point of given segments among calculated area become the node of triangle grid;
- obtain the topography elevation of grid nodes automatically by given topography data (section terrains or dispersed dot terrain data) and reduce the difficulty of finite element pre-treatment greatly.

(2) Related coefficients and parameters such as roughness coefficient

Roughness coefficient is a synthetic coefficient in 2-D mathematical model computation. It reflects the roughness, the comprehensive effect of river bed shape diversification and topography treatment. Debug and verify roughness by parting segments and blocks with field tidal data.

Turbulent viscosity coefficient $\upsilon_t = \alpha u_* h$, u_* is friction velocity, α is a constant.

The time spacing of model computation is selected according to *Courant* condition. It has relation with minimal edge length and maximal depth of computed area. It is commonly chosen to 10−20 s.

(3) Disposal of moving boundaries

Some nodes may submerge seeming "wet" under flood tide and appear "dry" under ebb tide during the course of computation. To reflect these variations, following simulation technique is adopted. Select a critical water depth (for example 0.005m), regard the node dry and give its velocity zero when practical water depth less than the critical water depth, interpolate tidal level from dry node nearby when practical water depth equal to the critical water depth, resume procedure computation when practical water depth great than the critical water depth.

3. MATHEMATICAL MODEL VISUALIZATION

3.1 THE PRINCIPLES OF VISUALIZATION

Dynamic demo of concentration scalar field and Euler flow field can be realized by showing a series static graphs quickly, while the dynamic demo of Lagrange flow field need the transform from the Euler flow field. Based on the Euler flow field calculated, compute the space coordinate of fluid particle after a Δt time interval, regard the linking line of two position coordinate as the path of water particle among Δt when Δt is small enough. The concrete computational steps (taking triangle grids as example) are as follows:

(1) The coordinate of given particle P is (x_0, y_0) at t and is (x_1, y_1) after Δt, u、v is velocity components of particle P at (x_0, y_0), then $x_1 = x_0 + u\Delta t$, $y_1 = y_0 + v\Delta t$, it is needed to confirm u、v for the confirmation of the coordinate (x_1, y_1).

(2) The value of u、v can be interpolated from flow field at t. The key lies in seeking triangle unit of (x_0, y_0) at t and it can be obtained through area approach. Regard the spot in the triangle when the three triangles areas sum consisting of three vertex and that spot equal to the triangle area.

(3) Compute above circulation by time and connect the position of fluid particle P at each time. Thus, particle path is gained and the transform from Euler flow field to Lagrange flow field is done.

3.2 THE PROCEDURE REALIZATION OF VISUALIZATION

The program of visual mathematical model and dynamic demo system is made of two parts. The model computation part uses traditional FORTRAN yet, thereby can make full use of existing reliable FORTRAN codes. The visualization and dynamic demo part adopts Visual Basic programming language. The combination of two parts realizes the computation visualization by making best of mighty computation function of FORTRAN and plentiful graphical interfaces of VB.

The combination can be realized by exterior Shell function of VB, it also can be realized by the mixed program language of VB and FORTRAN. Adapt existing FORTRAN program properly such as designing, editing data interface and appending illumination statements for database function output, compile the adapted program to dynamic link library (DLL file) and transfer the DLL file from VB or VC language at appropriate time to realize the real time display of computation production.

The interfaces with some common used commercial software such as Excel, Surfer, AutoCAD, Arc View are considered during the course of system programming.

3.3 MAIN ADVANTAGES

Visual mathematical model have many advantages compared with traditional mathematical model:

- Automatic grids generation according to boundary conditions and gain of grid node elevation via CAD electronic map or DEM map solve the drawback of large workload and low precision of manual work effectively.
- Checkout dynamically by timing show during the course of procedure debugging, terminate the running when error appears and modify procedure or data files immediately, thus saves much debugging time.
- Flow field visualization software can generate contour and field vector pictures of physical variables such as velocity, water level, depth, do favor to the analysis of result rationality and contrast between different projects, improve the efficiency of solving practical problems with mathematical model.
- Overcome the shortcoming of the indirect displaying of traditional mathematical model and can perform practical flow movement on screen at any moment after computation.

4. THE APPLICATION IN THE YANGTZE RIVER ESTUARY

The mathematical model is used in the influence computation and analysis of Chonghai road-bridge engineering to north and south branch reach of the Yangtze River estuary. The range computed shows as Fig.1.It is up to Xuliujing tidal station, down to Yanglinkou in south branch with 39km length; and down to Santiao harbor tidal station in north branch with 66 km length. The river reach topography of 2001−2002 is adopted.

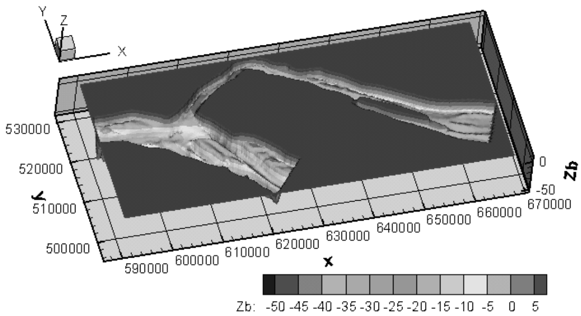

Fig. 1 The 3D morphologic map of calculated river reach

The whole region is plotted with 16355 triangle units and 8663 grid nodes by the triangle grids auto-generating method above-mentioned. The edge length of triangle grid is changed gradually with 70−100 m length and local length is compressed to 50 m.

The validation is carried out on the field's tidal level and velocity process of 2002.3.23−31 and the boundary conditions are governed by the tidal level process of Xuliujing, Yanglinkou and Santiao harbor tidal station. The flow of whole reach is smooth, the distinction between plain and channel velocity is obvious, the flood tide overflows floodplain and the ebb tide returns to channel The velocity of flood tide is larger than that of ebb tide, the conflux is between Chongtou and Qinglong harbour. The flow field and water depth during flood and ebb nearby Chongtou is as Fig.2. The analysis of flow field computed shows that it accords with facts. The statistic of results shows that the process of tidal level and velocity calculated accords well with field data, the variety of phase agrees with the measured, the error of tidal level is limited to 0.1m, the error of velocity is less than 0.2m/s generally and the relative error of velocity is less than 10 %.It can be seen that the model in the article can simulate north and south branch's tidal flow of the Yangtze River estuary well.

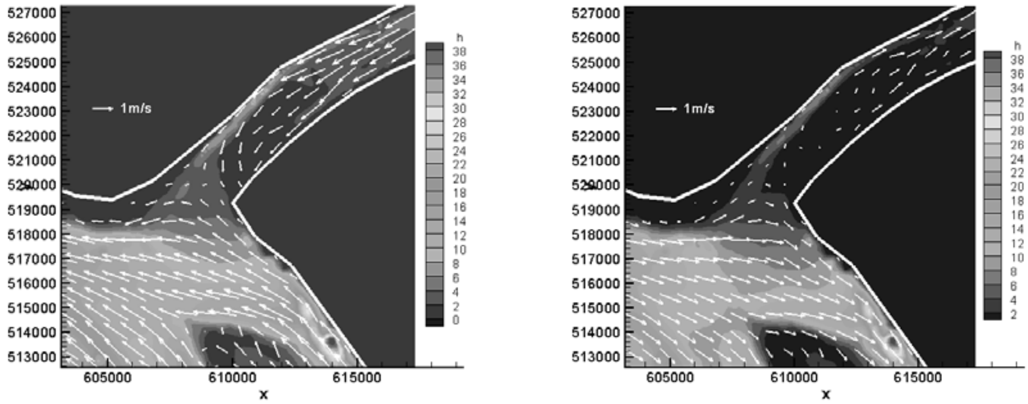

Fig. 2 Flow field and water depth during flood and ebb near ChongTou

The model can simulate the tidal flow of the north and south branch reach of Yangtze River estuary reasonably while the computation is difficult for the large region, complex river bed terrain condition, complicated boundary condition and tidal process. From the practice implement, it shows that the visual model can not only make the calculation visual and overcome the indirect showing disadvantage of traditional models, but also save much time of model debugging, and improve the efficiency of solving practical problems with mathematical model.

5. CONCLUSIONS

A planar 2-D unsteady mathematical model of tidal estuary is established by finite element method. The pre-treatment of model adopts triangle grids plotting automatically method with edge length changed gradually for random plane region. It can fit for irregular river boundary and complicated underwater terrain condition quite well, thus improves the efficiency of solving practical problems with mathematical model.

The visualization system of mathematical model is developed by VB language and combined with 2-D mathematical model organically. The system can realize the visualization of whole computation process, 2-D and 3-D show of computed data and dynamic demo of the 2-D and 3-D flow field and density field.

Simulation calculation is carried out on the south and north branch reach of the Yangtze River estuary and result shows that the model can simulate the tidal flow correctly. From the practice implement, it proves that the visual model can overcome the disadvantage of traditional mathematical model and improve the efficiency of solving practical problems with mathematical model.

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