



Propagation of the degree of cross-polarization of a stochastic electromagnetic beam through the turbulent atmosphere

Jixiong Pu^a, Olga Korotkova^{b,*}

^a Department of Electronic Science and Technology, Huaqiao University, Quanzhou, Fujian 362021, China

^b Department of Physics, University of Miami, Coral Gables, FL 33146, USA

ARTICLE INFO

Article history:

Received 11 July 2008

Received in revised form 29 December 2008

Accepted 5 January 2009

ABSTRACT

The behavior of the recently introduced spectral degree of cross-polarization of a stochastic electromagnetic beam-like field on propagation through the turbulent atmosphere is analyzed. Unlike the spectral degree of coherence, the degree of cross-polarization is generally unbounded and does not decrease with propagation distance. We support the analysis by numerical examples relating to two model sources: with uncorrelated and partially correlated mutually orthogonal transverse components of the electric field.

© 2009 Published by Elsevier B.V.

1. Introduction

The concept of cross-polarization of stochastic electromagnetic beam-like fields was first introduced by Ellis and Dogariu [1] where the parameter called the mutual degree of cross-polarization [MDCP] was defined in a somewhat heuristic manner. Namely, a normalization factor required for the MDCP not to exceed unity was chosen to be just the same as that for the spectral degree of coherence [2]. This new quantity is a measure of correlation of the degrees of polarization at any two points in the field and reduces to the ordinary degree of polarization when the points coincide.

Recently, the alternative definition for the degree of cross-polarization was introduced by Shirai and Wolf [3] (see also [4]) where the analysis of the intensity fluctuations of stochastic electromagnetic beams was carried out. The quantity introduced in [3] was called the spectral degree of cross-polarization [SDCP], it agrees in concept with the MDCP and also reduces to the ordinary degree of polarization for coinciding spatial arguments but it has a different normalization factor. We believe that the later definition is a more adequate measure of correlation in polarization properties and we will use it as the basis for our study.

The interest in propagation of the degree of cross-polarization was expressed recently in Refs. [5,6] but the analysis was restricted to free-space propagation alone. In this paper, we will consider propagation of cross-polarization of beams in the turbulent atmosphere using the propagation law based on the extended Huygens–Fresnel integral. We will restrict our study to the class of stochastic electromagnetic beams called the Gaussian Schell-model [EGSM]

beams. These beams can efficiently illustrate all the interesting phenomena pertaining to stochastic beams while providing with analytically tractable results [7–9]. While other classes of stochastic beams have been recently introduced the majority of papers still use the EGSM beams as the basic model. Moreover EGSM beams form the only class of beams that have been produced in the laboratory so far [10].

We will first introduce the formulas relating to propagation of the SDCP of an EGSM beam in the turbulent atmosphere (Section 2) then consider separately the cases when the electric field components are uncorrelated in the source plane (Section 3) and are correlated (Section 4). Finally we will summarize the obtained results.

2. Propagation of the degree of cross-polarization in the turbulent atmosphere

In Ref. [3] the SDCP of an electromagnetic beam-like field at two spatial positions (\mathbf{r}_1, z) and (\mathbf{r}_2, z), oscillating at frequency ω , was introduced by means of the expression

$$P(\mathbf{r}_1, \mathbf{r}_2, z, \omega) = \sqrt{1 - \frac{4\text{Det} \vec{W}(\mathbf{r}_1, \mathbf{r}_2, z, \omega)}{[\text{Tr} \vec{W}(\mathbf{r}_1, \mathbf{r}_2, z, \omega)]^2}}, \quad (1)$$

where Det and Tr stand for the trace and the determinant, and $\vec{W}(\mathbf{r}_1, \mathbf{r}_2, z, \omega)$ is the 2×2 cross-spectral density matrix of the field, whose elements are defined by the formulas [2]

$$\vec{W}(\mathbf{r}_1, \mathbf{r}_2, z, \omega) = [E_i^*(\mathbf{r}_1, z, \omega)E_j(\mathbf{r}_2, z, \omega)], \quad (i, j = x, y). \quad (2)$$

Here, E_x and E_y are the two mutually orthogonal components of the electric field, the star stands for complex conjugate and the angular

* Corresponding author.

E-mail address: korotkova@physics.miami.edu (O. Korotkova).

brackets denote the average over ensemble of monochromatic realizations, according to the coherence theory in space-frequency domain (see Ref. [2]). We assume that the beam is generated in the plane $(x, y, 0)$, $\mathbf{r} = (r_x, r_y, 0)$ denoting a transverse position vector, and propagates close to direction z , into the positive half-space, filled with a random medium, e.g. turbulent atmosphere. Generally, for propagation in a medium with linear frequency response the elements of the cross-spectral density matrix propagate according to the laws

$$W_{ij}(\mathbf{r}_1, \mathbf{r}_2, z, \omega) = \left(\frac{k}{2\pi z}\right) d^2 \rho_1 d^2 \rho_2 W_{ij}^{(0)}(\rho_1, \rho_2, \omega) K(\rho_1, \rho_2; \mathbf{r}_1, \mathbf{r}_2, z, \omega),$$

$$(i, j = x, y), \tag{3}$$

where ρ_1, ρ_2 are dummy variables denoting two source points, the integration is performed twice over the source plane, K is the propagator depending on a Green's function for a given medium and k is the wave number. For propagation in the homogeneous and isotropic atmospheric turbulence the propagator K was shown to be given by the expression

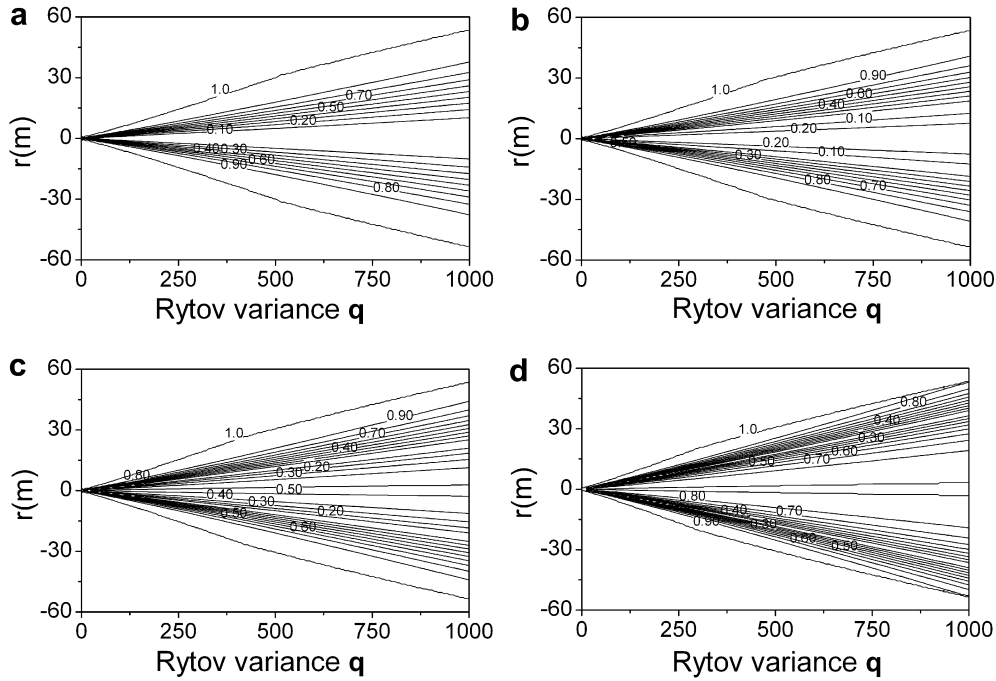


Fig. 1. The contour of the spectral degree of polarization $P(r_1 = r_2 = r; q)$. (a) $I_y = I_x$; (b) $I_y = (5/3)I_x$; (c) $I_y = 3I_x$ and (d) $I_y = 19I_x$; The values of the source parameters are: $\delta_{xx} = 5$ mm, $\delta_{yy} = 7.5$ mm.

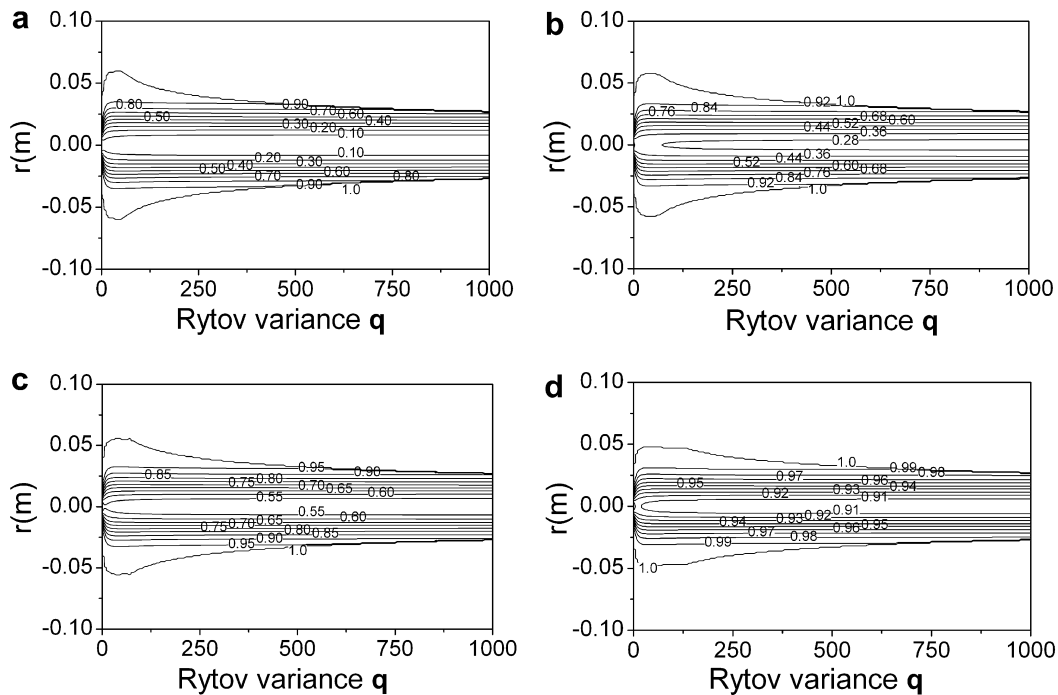


Fig. 2. The same as Fig. 1 but the contour of the spectral degree of cross-polarization $P(r_1 = 0, r_2 = r; q)$.

$$K(\rho_1, \rho_2; \mathbf{r}_1, \mathbf{r}_2, \omega) = \exp \left[-ik \frac{(\mathbf{r}_1 - \rho_1)^2 - (\mathbf{r}_2 - \rho_2)^2}{2z} \right] \times \exp \left[-\frac{\pi^2 k^2 z}{3} \left[(\mathbf{r}_1 - \mathbf{r}_2)^2 + (\mathbf{r}_1 - \rho_2) \cdot (\rho_1 - \rho_2) + (\rho_1 - \rho_2)^2 \right] \int_0^\infty \kappa^3 \Phi_n(\kappa) d\kappa \right], \quad (4)$$

where $\Phi_n(\kappa)$ is the one-dimensional power spectrum of atmospheric fluctuations [11].

As we mentioned earlier, as a model for the source of a stochastic electromagnetic beam we will use the EGSM [7] beam with uniform polarization for all our numerical examples. The electric cross-spectral density matrix of such a source can be expressed in the form

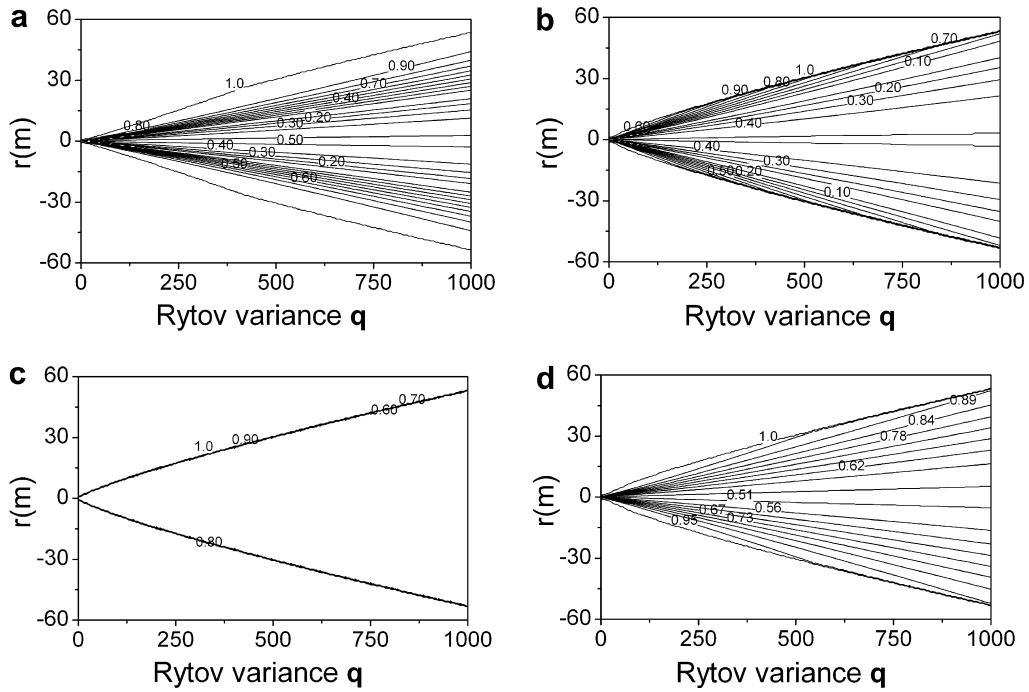


Fig. 3. The contour of the spectral degree of polarization $P(r_1 = r_2 = r; q)$. (a) $\delta_{xx} = 5$ mm; (b) $\delta_{xx} = 6.5$ mm; (c) $\delta_{xx} = 7.5$ mm and (d) $\delta_{xx} = 9.5$ mm. The values of the source parameters are: $\delta_{yy} = 7.5$ mm, $l_y = 3l_x$.

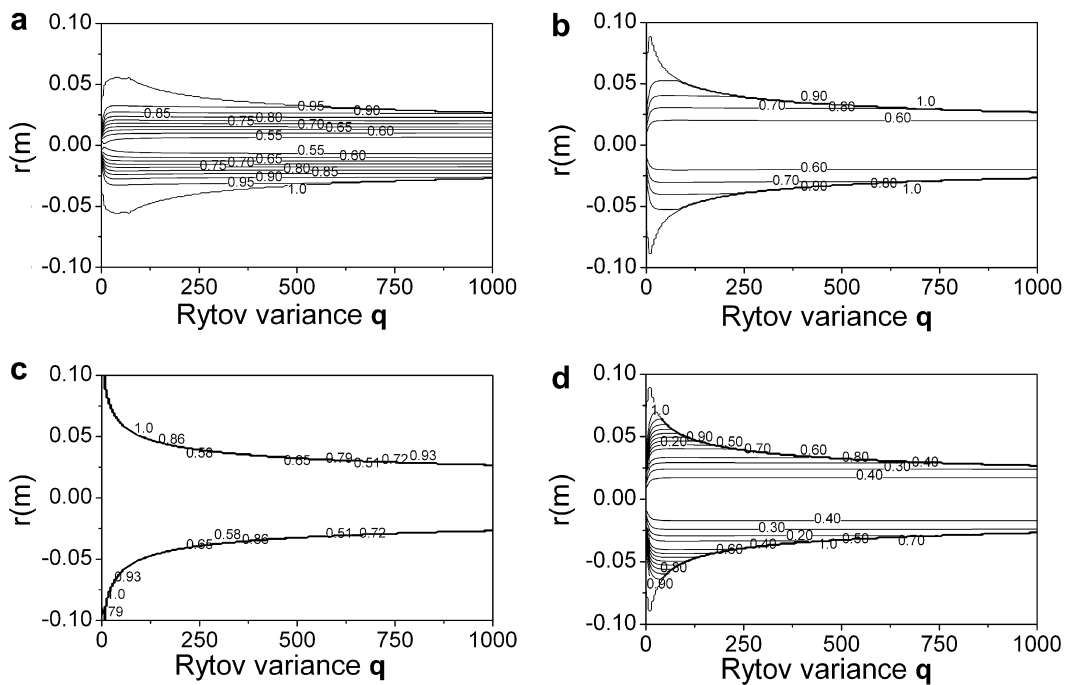


Fig. 4. The same as Fig. 1 but the contour of the spectral degree of cross-polarization $P(r_1 = 0, r_2 = r; q)$.

$$\vec{W}^{(0)}(\rho_1, \rho_2, \omega) = \exp\left(-\frac{\rho_1^2 + \rho_2^2}{4\sigma^2}\right) \times \begin{bmatrix} I_x B_{xx} \exp\left[-\frac{(\rho_2 - \rho_1)^2}{2\delta_{xx}^2}\right] & \sqrt{I_x} \sqrt{I_y} B_{yx} \exp\left[-\frac{(\rho_2 - \rho_1)^2}{2\delta_{xy}^2}\right] \\ \sqrt{I_x} \sqrt{I_y} B_{yx} \exp\left[-\frac{(\rho_2 - \rho_1)^2}{2\delta_{xy}^2}\right] & I_y B_{yy} \exp\left[-\frac{(\rho_2 - \rho_1)^2}{2\delta_{yy}^2}\right] \end{bmatrix}. \quad (5)$$

Here I_x and I_y are the spectral densities of x and y components of the electric field in the source plane, which may depend on frequency, and the parameters relating to spatial variation should meet the following set of conditions ([7], see also [2]):

$$B_{xx} = B_{yy} = 1, \quad |B_{xy}| = |B_{yx}| = B, \quad \delta_{xy} = \delta_{yx},$$

$$\max\{\delta_{xx}, \delta_{yy}\} \leq \delta_{xy} \leq \min\left\{\frac{\delta_{xx}}{\sqrt{B}}, \frac{\delta_{yy}}{\sqrt{B}}\right\}. \quad (6)$$

The elements of the cross-spectral density matrix of the EGSM beam propagating in the atmosphere in the plane located at dis-

tance z from the source plane were derived in Ref. [12] and are given by the expressions

$$W_{ij}(\mathbf{r}_1, \mathbf{r}_2, z; \omega) = \frac{\sqrt{I_i} \sqrt{I_j} B_{ij}}{\Delta_{ij}^2(z)} \exp\left(-\frac{(\mathbf{r}_1 + \mathbf{r}_2)^2}{8\sigma^2 \Delta_{ij}^2(z)}\right) \times \exp\left(-\left[\frac{1}{2\Delta_{ij}^2(z)}\left(\frac{1}{4\sigma^2} + \frac{1}{\delta_{ij}^2}\right) + M(1 + \sigma^2) - \frac{M^2 z^2}{2k^2 \sigma^2 \Delta_{ij}^2(z)}\right](\mathbf{r}_1 - \mathbf{r}_2)^2\right) \times \exp\left(\frac{ik(\mathbf{r}_2^2 - \mathbf{r}_1^2)}{2R_{ij}(z)}\right) \quad (7)$$

with

$$\Delta_{ij}^2(z) = 1 + \frac{z^2}{(k\sigma)^2} \left(\frac{1}{4\sigma^2} + \frac{1}{\delta_{ij}^2}\right) + \frac{2Mz^2}{k^2 \sigma^2},$$

$$R_{ij}(z) = \frac{k^2 \sigma^2 \Delta_{ij}^2(z) z}{k^2 \sigma^2 \Delta_{ij}^2(z) + Mz^2 - k^2 \sigma^2}, \quad (8)$$

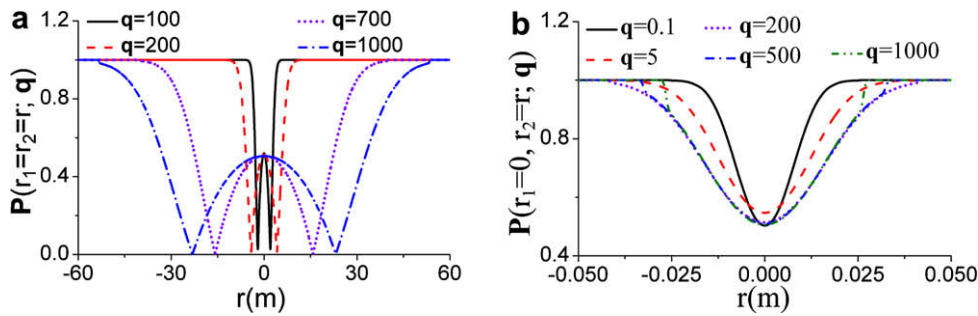


Fig. 5. (a) The spectral degree of polarization $P(r_1 = r_2 = r; q)$ and (b) the spectral degree of cross-polarization $P(r_1 = 0, r_2 = r; q)$ for four different values of q . The source parameters are $\delta_{xx} = 5$ mm, $\delta_{yy} = 7.5$ mm, $I_y = 3I_x$.

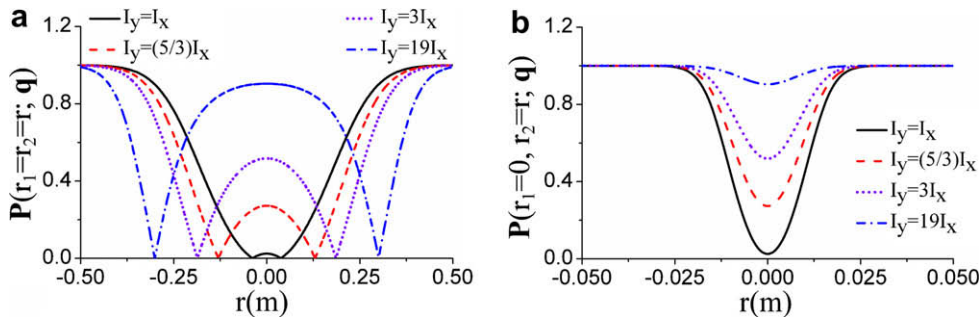


Fig. 6. (a) The spectral degree of polarization $P(r_1 = r_2 = r; q)$ and (b) the spectral degree of cross-polarization $P(r_1 = 0, r_2 = r; q)$ for four different values of the initial degree of polarization at the $q = 1$ plane. The source parameters are $\delta_{xx} = 5$ mm, $\delta_{yy} = 7.5$ mm.

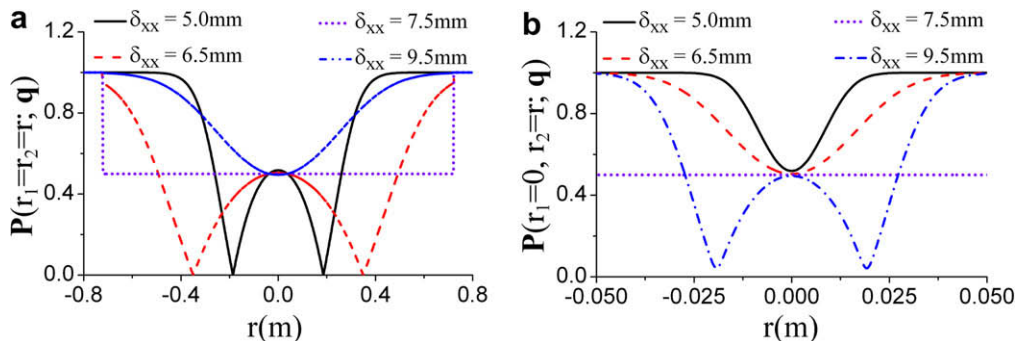


Fig. 7. (a) The spectral degree of polarization $P(r_1 = r_2 = r; q)$ and (b) The spectral degree of cross-polarization $P(r_1 = 0, r_2 = r; q)$ for four different values of δ_{xx} at the $q = 1$ plane. The source parameters are $\delta_{yy} = 7.5$ mm, $I_y = 3I_x$.

where parameter M entering expressions (7) and (8) depends on the power spectrum of turbulence as

$$M = \frac{1}{3} \pi^2 k^2 z \int_0^\infty \kappa^3 \Phi_n(\kappa) d\kappa. \quad (9)$$

In particular, it was also shown in Ref. [13] that for the Tatarskii model of the spectrum (see [11]) this parameter reduces to the form

$$M = 0.5465 C_n^2 I_0^{-1/3} k^2 z, \quad (10)$$

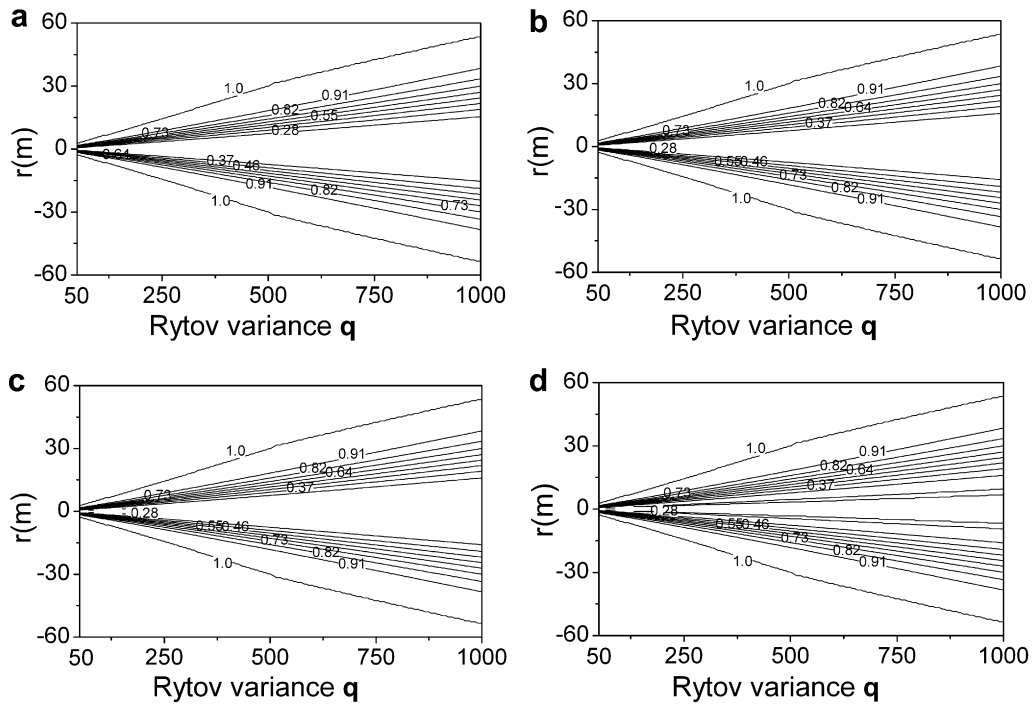


Fig. 8. The contour of the usual spectral degree of polarization $P(r_1 = r_2 = r; q)$. (a) $\delta_{xy} = 7.5$ mm; (b) $\delta_{xy} = 9$ mm; (c) $\delta_{xy} = 10$ mm; (d) $\delta_{xy} = 11$ mm. The values of the source parameters are: $\delta_{xx} = 5$ mm, $\delta_{yy} = 7.5$ mm, $B = 0.2$, $I_x = I_y = 1$.

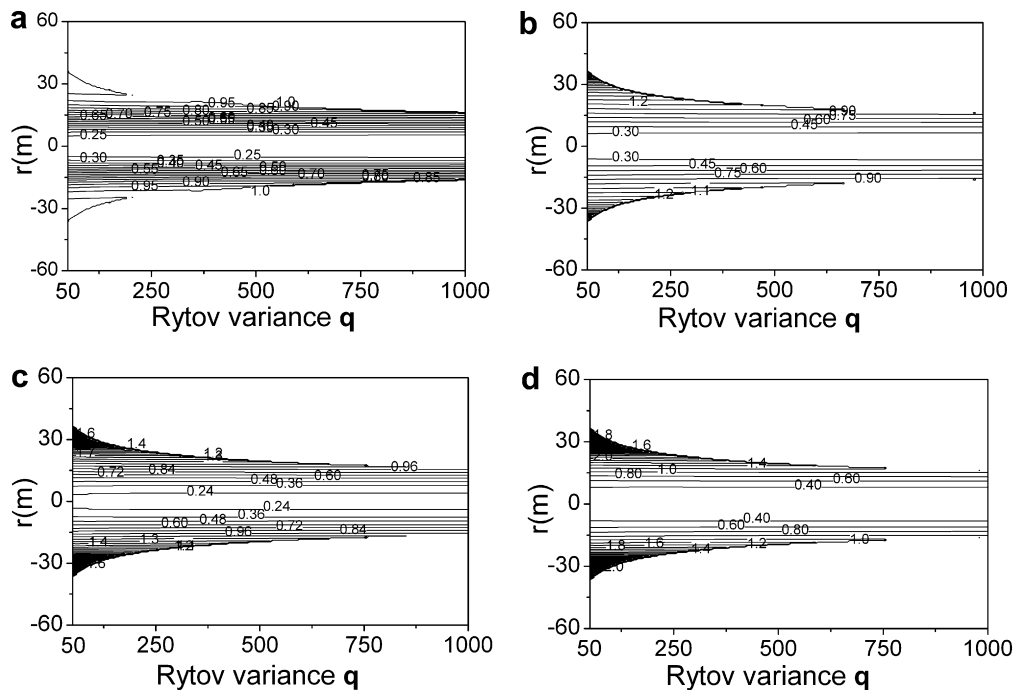


Fig. 9. The contour of the spectral degree of cross-polarization $P(r_1 = 0, r_2 = r; q)$. (a) $\delta_{xy} = 7.5$ mm; (b) $\delta_{xy} = 9$ mm; (c) $\delta_{xy} = 10$ mm; (d) $\delta_{xy} = 11$ mm. The values of the source parameters are: $\delta_{xx} = 5$ mm, $\delta_{yy} = 7.5$ mm, $B = 0.2$, $I_x = I_y = 1$.

where C_n^2 is the refractive index structure parameter and l_0 is the inner scale of turbulence. On substituting from expressions (7), (8), and (10) into formula (1) the SDCP of the propagating beam can be determined.

3. Uncorrelated electric field components

We begin by considering maybe the simplest possible EGSM sources, namely the sources with $B_{xy} = 0$, i.e. sources whose cross-spectral density matrix has diagonal form, viz.,

$$\overline{W}^{(0)}(\rho_1, \rho_2, \omega) = \exp\left(-\frac{\rho_1^2 + \rho_2^2}{4\sigma^2}\right) \times \begin{bmatrix} I_x \exp\left[-\frac{(\rho_2 - \rho_1)^2}{2\delta_{xx}^2}\right] & 0 \\ 0 & I_y \exp\left[-\frac{(\rho_2 - \rho_1)^2}{2\delta_{yy}^2}\right] \end{bmatrix}. \quad (11)$$

For all numerical calculations we assume that $\lambda = 632.8$ nm, $\sigma = 2.5$ cm, $l_0 = 2.5$ mm, $C_n^2 = 10^{-13}$ m^{-2/3} unless different values are specified.

We will now illustrate by numerous numerical examples how in case of uncorrelated electric field components in the source plane the SDCP of the propagating beam depends on all source parameters and on the parameters of the turbulence.

In Figs. 1–7 we compare the behavior of the ordinary degree of polarization and of the SDCP $P(r_1 = 0, r_2 = r; q)$. In particular in Fig. 1 the contours of the ordinary degree of polarization are shown as functions of Rytov variance $q = 1.23C_n^2 k^{7/6} z^{11/6}$ and radial range $r = |r|$ for several combinations of initial intensities I_x and I_y . Rytov variance q is a commonly used measure of the strength of the atmospheric turbulence ($q \ll 1$ for weak and $q \gg 1$ for strong fluctuations) [11]. Fig. 2 illustrates the dependence of the SDCP on q and r for the same combinations of initial intensities as in Fig. 1. From the comparison of Figs. 1 and 2 it becomes evident that there is a qualitative difference in the changes of the ordinary degree of polarization and of the degree of cross-polarization: while the radial range of the former quantity grows with q that of the later

quantity does not, and, in fact, even slightly narrows down for large values of q . Both quantities vary mostly in the central part of the beam and for more polarized sources (larger ratios of intensities I_x and I_y) the transverse cross-sections of both quantities become more uniform for large values of q .

Figs. 3 and 4 illustrate the behavior of the ordinary degree of polarization and the SDCP $P(r_1 = 0, r_2 = r; q)$ of the beam generated by the source with $I_y = 3I_x$ for several different values of one of the source correlation coefficients while the other one is fixed. In the set of four figures the “degenerate” case (c), for which the correlation coefficients of x and y components of the electric field are the same, should be distinguished. In this case neither quantity vary with q as a result of source correlations. We note that the only contourplot curve in Figs. 3c and 4c appear on the boundary of the beam and should be regarded simply as an edge effect.

In Fig. 5a and b we show the typical behavior of the degree of polarization and the SDCP $P(r_1 = 0, r_2 = r; q)$, respectively, in planes transverse to the direction of propagation, for several fixed values of Rytov variance q . From this different perspective on the dependence of the two quantities on q we find that, typically, the SDCP has more monotonic behavior in the central part of the beam, compared with the ordinary degree of polarization, however, both quantities tend to value 1 toward the edge of the beam.

Fig. 6a illustrates the changes in the spectral degree of polarization and Fig. 6b the SDCP $P(r_1 = 0, r_2 = r; q)$ for four different values of the initial degree of polarization in the fixed transverse plane where $q = 1$. Finally, in Fig. 7a the spectral degree of polarization and the SDCP $P(r_1 = 0, r_2 = r; q)$ Fig. 7b for four different values of δ_{xx} is shown at the plane where the Rytov variance $q = 1$. It can now be seen much better than from contourplots that in the degenerate case mentioned earlier (dotted curves) both quantities indeed remain constants over all the transverse cross-section of the beam.

At this point we would like to make one more remark about new features of the SDCP: unlike the spectral degree of coherence of the beam propagating in the atmosphere, which at any two fixed points in the transverse cross-sections tends to zero with growing

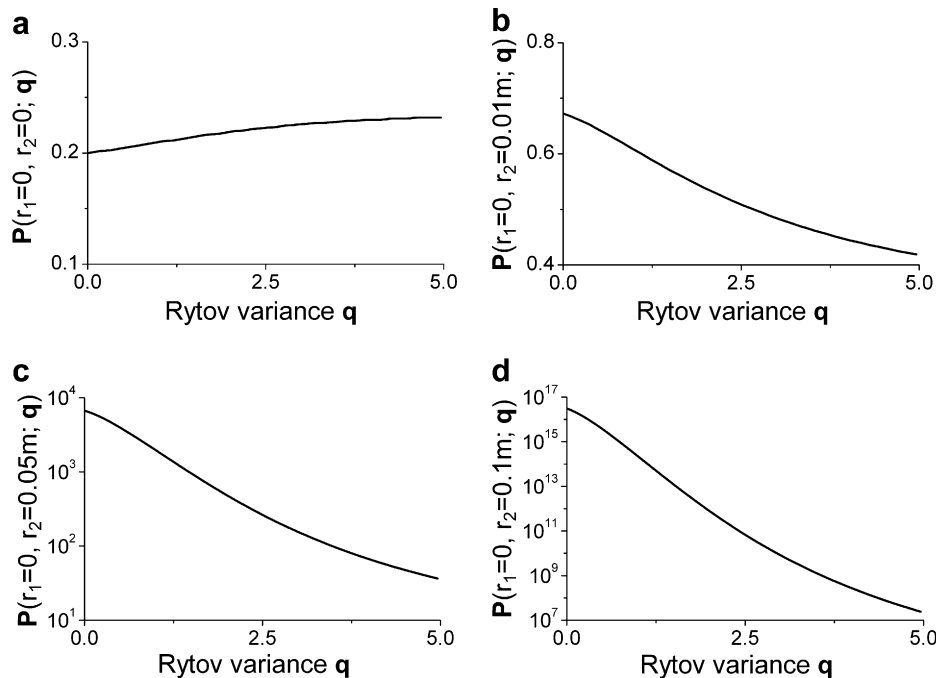


Fig. 10. The spectral degree of cross-polarization $P(r_1 = 0, r_2 = r; q)$ for different values of r . The values of the source parameters are: $\delta_{xx} = 5$ mm, $\delta_{xy} = 10$ mm, $\delta_{yy} = 7.5$ mm, $B = 0.2$.

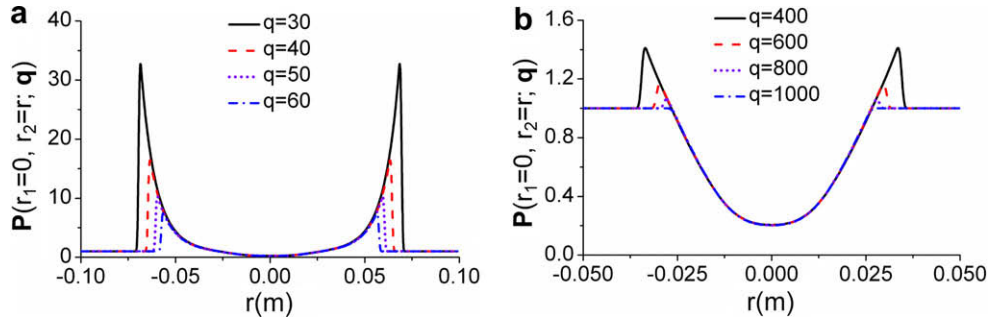


Fig. 11. The spectral degree of cross-polarization $P(r_1 = 0, r_2 = r; q)$ for different values of q . The values of the source parameters are: $\delta_{xx} = 5$ mm, $\delta_{yy} = 10$ mm, $\delta_{xy} = 7.5$ mm, $B = 0.2$.

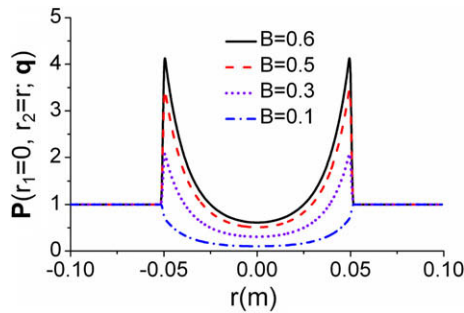


Fig. 12. The spectral degree of cross-polarization $P(r_1 = 0, r_2 = r; q)$ for four different values of the initial degree of polarization at the $q = 100$ plane. The source parameters are $\delta_{xx} = 7.5$ mm, $\delta_{yy} = 10$ mm, $\delta_{xy} = 7.5$ mm.

distance, the SDCP can approach non-zero values depending on the parameters of the source. This is seen at best from Fig. 5b.

4. Partially correlated electric field components

We will now discuss propagation of the degree of cross-polarization of beams generated by the most general (uniformly polarized) EGSM sources, namely the sources with non-zero B_{xy} coefficient, whose cross-spectral density matrix has form (5). We will primarily concentrate here on the effect of parameters δ_{xy} and B_{xy} on the evolution of the SDCP, since the effect of all other source parameters was discussed in Section 3 for uncorrelated sources and is similar for partially correlated ones.

In Figs. 8 and 9 the degree of polarization (Fig. 8) and the degree of cross-polarization (Fig. 9) are plotted as functions of Rytov variance and radial distance r from the center of the beam for several

values of the correlation coefficient δ_{xy} entering the off-diagonal components of the matrix while the other parameters are being fixed. The main difference between these figures and the corresponding figures for the uncorrelated case is that the degree of cross-polarization might attain values larger than 1. This result coincides with the one for free-space propagation [6].

Fig. 10 represents the SDCP $P(r_1 = 0, r_2 = r; q)$ for four different values of r . Horizontal scale shows the Rytov variance q with values corresponding to weak and moderate regimes of atmospheric fluctuations. One can see that as radial separation between points increases the SDCP can attain very large values close to the source plane.

In Fig. 11 the SDCP of a typical EGSM beam is shown as a function of separation r between points 0 and \mathbf{r} , for several different values of q , corresponding to strong regime of atmospheric fluctuations. From comparison of Fig. 11 with the corresponding Fig. 5b we see that there is no significant difference in the asymptotic behavior of the SDCP whether the x and y field components are initially correlated or not.

Figs. 12 and 13 illustrate the behavior of the spectral degree of cross-polarization $P(r_1 = 0, r_2 = r; q)$ as a function of separation distance r , in the transverse plane $q = 100$, i.e. for the case of very strong atmospheric fluctuations. In particular, Fig. 12 shows the dependence for four different values of the correlation parameter B_{xy} (in other words, for four different values of the initial degree of polarization). One can see that for larger values of this parameter the SDCP is typically larger in the central part of the beam. Fig. 13 shows the same type of plot as Fig. 12 but for different values of source correlation parameter δ_{xy} . We note here that only when all three correlation coefficients coincide, i.e. $\delta_{xy} = \delta_{xx} = \delta_{yy}$ (see Fig. 13a, solid curve), the SDCP remains constant across the transverse plane.

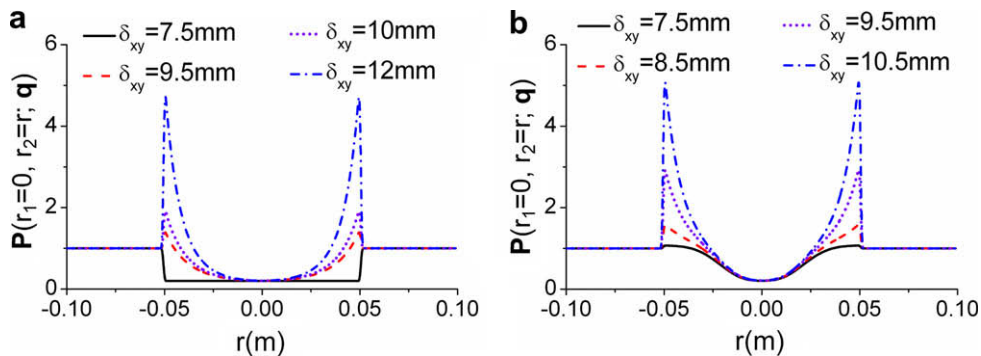


Fig. 13. (a) The spectral degree of cross-polarization $P(r_1 = 0, r_2 = r; q)$ for different values of δ_{xy} at the $q = 100$ plane, with $\delta_{xx} = 7.5$ mm, $\delta_{yy} = 7.5$ mm, $B = 0.2$. (b) The same as in (a) but $\delta_{xx} = 5$ mm, $\delta_{yy} = 7.5$ mm.

5. Summary

Polarization correlation properties in stochastic electromagnetic beams at a pair of points in space may be characterized with the help of one of the quantities the MDCP or the SDCP. This may provide some structural or correlation information about the random medium in which the beam travels. In fact, the MDCP has been successfully employed for sensing of biological tissues [14,15]. In this paper, we have studied how the SDCP evolves when an EGSM beam travels in a turbulent atmosphere, and showed far from trivial behavior of this quantity, not being similar to behavior of either spectral degree of coherence or the ordinary degree of polarization studied earlier [12,13]. Considering separately the cases when the transverse components of the electric field are uncorrelated and partially correlated we have found that all the parameters of the source affect the evolution of the SDCP. We believe that our results will find applications in imaging and sensing through the turbulent atmosphere.

Acknowledgements

O.K. acknowledges the AFOSR support via Grant FA 9550-08-1-0102.

P.J. acknowledges the Key Project of Science and Technology of Fujian Province support via Grant No. 2007H0027

References

- [1] J. Ellis, A. Dogariu, *Opt. Lett.* 29 (2004) 536.
- [2] E. Wolf, *Introduction to the Theories of Coherence and Polarization of Light*, Cambridge University Press, Cambridge, 2007.
- [3] T. Shirai, E. Wolf, *Opt. Commun.* 272 (2007) 289.
- [4] S.N. Volkov, D.F.V. James, T. Shirai, E. Wolf, *J. Opt. A: Pure Appl. Opt.* 10 (2008) 055001.
- [5] Y. Xin, Y. Chen, Q. Zhao, M. Zhou, *Opt. Commun.* 281 (2008) 1954.
- [6] S. Sahin, O. Korotkova, G. Zhang, J. Pu, *Opt. Commun.*, submitted for publication.
- [7] H. Roychowdhury, O. Korotkova, *Opt. Commun.* 249 (2005) 379.
- [8] F. Gori, M. Santarsiero, G. Piquero, R. Borghi, A. Mondello, R. Simon, *J. Opt. A: Pure Appl. Opt.* 3 (2001) 1.
- [9] O. Korotkova, B. Hoover, V. Gamiz, E. Wolf, *J. Opt. Soc. Am. A* 22 (11) (2005) 2547.
- [10] G. Piquero, F. Gori, P. Romanini, M. Santarsiero, R. Borghi, A. Mondello, *Opt. Commun.* 208 (2002) 9.
- [11] L.C. Andrews, R.L. Phillips, *Laser Beam Propagation in the Turbulent Atmosphere*, second ed., SPIE Press, Bellington, 2005.
- [12] Wei Lu, Liren Liu, Jianfeng Sun, Qingguo Yang, Yongjian Zhu, *Opt. Commun.* 271 (2007) 1.
- [13] M. Salem, O. Korotkova, A. Dogariu, E. Wolf, *Wave Random Media* 14 (2004) 513.
- [14] O.V. Angelsky, A.G. Ushenko, Y.G. Ushenko, et al., *J. Biomed. Opt.* 10 (6) (2005) 060502.
- [15] O.V. Angelsky, A.G. Ushenko, Y.G. Ushenko, et al., *J. Biomed. Opt.* 10 (6) (2005) 064025.