The generation of Entangled Qudits and their Application in Probabilistic Superdense Coding *

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A scheme of the generation of entangled qutrits is presented, and then is generalized to entangled ququads and entangled qudits. With the entangled qutrits, an experimental scheme of probability superdense coding with only linear optical elements is proposed. It is shown that this scheme will be suitable for the entangled ququads, even for the entangled qudits if some nonlinearity is used. This scheme is feasible in the laboratory with the current experimental technology.

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In quantum communication and quantum computation, entanglement is the crucial resource. Much more classical information can be transmitted by some entanglement than that of classical communication, which is called the entanglement assisted quantum communication.[1] A typical proposal is the well-known superdense coding.[2] Recently, entangled qudits have arrested much more attention than entangled qubits for their stronger nonlocality and capacity of information transmission. Various proposals are provided to realize the qudits and entangled qudits in optics, for example, orbital angular momentum entangled qutrits,[3] pixel entanglement,[4] energy-time entangled qutrits and time-bin entanglement,[5] polarization degree of freedom of two-photon entangled qudits,[6] etc. In this Letter, we codify the states in the propagation paths of the photons, likely to Ref. [7], and present a scheme to generate entangled qutrits with only linear optical elements. Also, we show that this scheme is very convenient to be generalized to prepare entangled qudits which will be applied to the proposal of probabilistic superdense coding.

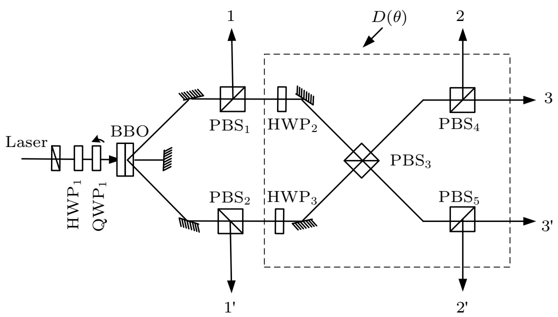

Fig. 1. Schematic diagram of the generation of the entangled qutrits. The SPDC process is used to generate a nonmaximal entangled qubit, and then an entangled qutrit encoded by paths is generated by the interference of two photons. For details, see text.

The generation scheme is shown in Fig. 1. Using a half wave plate (HWP_1, set at α), a quarter wave plate (QWP_1) and two type-I BBO ($\beta-$ BaB_2O_4) crystals, photons pairs in the non-maximally polarization-entangled (NME) state $|\Psi_0\rangle = \sin\alpha|H\rangle|H\rangle + \cos\alpha|V\rangle|V\rangle$ can be generated through the SPDC (spontaneous parametric down-conversion) process.[12] After injected into two polarizing beam splitters (PBS_1, PBS_2) and two HWP (HWP_2, HWP_3, both set at θ), the two-photon state can be described as follows:

$$|\Psi_0\rangle \rightarrow \sin\alpha(\cos\theta|H\rangle + \sin\theta|V\rangle)\cdot(\cos\theta|H\rangle + \sin\theta|V\rangle) + \cos\alpha|1\rangle|1\rangle, \quad (1)$$

where $|1\rangle$ denotes the path state of the photon. When the two photons whose states are described by the first item in Eq. (1) are interfered at PBS_3, by the coincidence measurement of the paths 2(3) and $2'(3')$ (exactly the coincidence measurement of the two sides 1,2,3 and $1', 2', 3'$ is enough), one can throw away the cross items in Eq. (1) to achieve the following two-photon state,

$$\sin\alpha\cos^2\theta|H\rangle|H\rangle + \sin\alpha\sin^2\theta|V\rangle|V\rangle + \cos\alpha|1\rangle|1\rangle. \quad (2)$$

After transmitted through two PBS (PBS_4, PBS_5), the output state is the follows:

$$|\Psi^3\rangle = \sin\alpha\cos^2\theta|3\rangle|3\rangle + \sin\alpha\sin^2\theta|2\rangle|2\rangle + \cos\alpha|1\rangle|1\rangle, \quad (3)$$

which is a entangled qutrit with the normalized coefficient, i.e., the success probability $\sin^2\alpha(\cos^4\theta + \sin^4\theta) + \cos^2\alpha$. If we set $\theta = \pi/4$ and $\tan\alpha = 2$, the two-photon state is the maximally entangled qutrit $\frac{1}{\sqrt{3}}(|3\rangle|3\rangle + |2\rangle|2\rangle + |1\rangle|1\rangle)$ with the success probability 3/5. In addition, if the same device denoted by $D(\theta)$ (where θ denotes the parameter of the two

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HWP) is applied to the two photons whose states are $\cos\alpha|V\rangle|V\rangle$, the entangled ququads

$$|\Psi^4\rangle = \sin\alpha(\cos^2\theta|4\rangle|4\rangle + \sin^2\theta|3\rangle|3\rangle) + \cos\alpha(\cos^2\theta|2\rangle|2\rangle + \sin^2\theta|1\rangle|1\rangle), \quad (4)$$

would be generated.

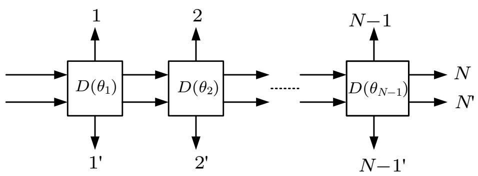

Fig. 2. Schematic diagram of the generation of the entangled qudits. $D(\theta)$ is the device shown in the dashed line of Fig. 1. Injecting a photon pair in the state $|H\rangle|H\rangle$, then associated with the coincidence measurement of the paths $1(2\cdots N)$ and $1'(2'\cdots N')$ respectively, the entangled qudits would be achieved. For details, see text.

Generalizing this method to entangled qudits is straightforward, and this scheme is shown in Fig. 2. Here we change to use a photon pair in the state $|H\rangle|H\rangle$ as the initial state which is injected into a set of devices $D(\theta)$ shown by the dashed line of Fig. 1. Associated with the coincidence measurement of the paths $1(2\cdots N)$ and $1'(2'\cdots N')$ respectively, in each devices $D(\theta_i)$, the following transformation is implemented,

$$|H\rangle|H\rangle \to \cos^2\theta_i|H\rangle|H\rangle + \sin^2\theta_i|V\rangle|V\rangle \to \cos^2\theta_i|H\rangle|H\rangle + \sin^2\theta_i|i\rangle|i\rangle, \quad (5)$$

where $|i\rangle$ denotes the path state of the photon. Then the $N-1$ devices $D(\theta)$ will yield the following state in the output,

$$|\Psi^N\rangle = \sin^2\theta_1|1\rangle|1\rangle + \sum_{k=2}^{N-1}\Big(\prod_{i=1}^{k-1}\cos^2\theta_i\Big)\sin^2\theta_k|k\rangle|k\rangle + \Big(\prod_{i=1}^{N-1}\cos^2\theta_i\Big)|N\rangle|N\rangle, \quad (6)$$

which is an unnormalized nonmaximal entangled qudit. To make all the coefficients equal, we set $\sin^2\theta_k = \frac{1}{N-k+1}$ $(k=1,\ldots,N-1)$, then the following maximal entangled qudit (unnormalized) can be obtained,

$$|\Psi^N\rangle = \sum_{k=1}^{N}\frac{1}{N}|k\rangle|k\rangle. \quad (7)$$

The corresponding success probability is $1/N$. This scheme is carried out in the condition of the coincidence measurement of the paths $1(2\cdots N)$ and $1'(2'\cdots N')$ respectively, while the condition of the case of entangled qutrits (ququads) may be the coincidence measurement of the two sides $1,2,\cdots,N$ and $1',2',\ldots,N'$. As a result, the limit of the application of the entangled qudits is stronger than the entangled qutrits (ququads) under this condition, which is discussed in the following. In these two schemes, besides the SPDC process in the BBO crystals, only the linear optical elements are used, which can be easily realized in the realistic experiment.

In the standard superdense coding proposal, two bits classical information can be transmitted only by one qubit sent, if assisted by a pair of maximal entangled qubits. It shows the advantage of the quantum communication and this proposal has been demonstrated by Mattle *et al.* in an experiment with a pair of polarization-entangled photons.[8] Also, in 2004, Mizuno *et al.* demonstrated it using the entanglement of a two-mode squeezed vacuum state.[9] However, if the entangled state is nonmaximal, much more classical information can be transmitted only with some probability, which has been discussed in the proposal of probability superdense coding (PSC).[10] In our previous work, we have presented an experimental scheme of PSC with nonmaximal entangled qubits,[11] now in this study we will consider the case of PSC with nonmaximal entangled qudits $|\Psi^N\rangle$, for which the classical information of $\log N$ bits would be transmitted by one photon sent with some probability.

In the proposal of PSC, the entangled qudit $|\Psi^N\rangle$ is shared by the sender Alice and the receiver Bob prior. First, Alice encodes the classical information to a local operation which is performed on her particle. The local operation is selected from a set of unitary operations $\mathcal{U}_{mn}$ $(m,n=1,\cdots,N)$, which can be described as follows:

$$\mathcal{U}_{mn} = (U)^m(V)^n, \quad (8)$$

where U is the shift operator and V is the rotation operator, whose action on the basis states are defined as follows:

$$U|k\rangle = |k\oplus 1\rangle, \quad V|k\rangle = e^{2\pi ik/N}|k\rangle, \quad (9)$$

and $\oplus$ is the addition modulo N. After the local operation, the entangled qudit $|\Psi^N\rangle$ is transformed to the following state,

$$(\mathcal{U}_{mn}\otimes I)|\Psi^N\rangle = \sum_{k=1}^{N}p_k e^{\frac{2\pi ink}{N}}|k\oplus m\rangle|k\rangle = |\Psi^N_{mn}\rangle. \quad (10)$$

Second, Alice sends her particle to Bob, and Bob performs a measurement to distinguish these states. In principle, Bob can distinguish all the states with some probability, i.e., he can achieve the classical information of $2\log N$ bits with some probability.

Now we first discuss the realization of PSC with entangled qutrits $(m,n=1,2,3)$, and then generalize it to the case of entangled qudits. Here we use paths as quantum states. For these states, using a multiport beam splitters, any discrete unitary operator can be realized,[7,13] which is shown in the dashed panel of Fig. 3. Since the unitary operators U^m are the exchange of the optical beams and the unitary operators V^n are the phase shifters implemented to the optical beams, all the beam splitters in the original setup can be removed or replaced by some mirrors. In detail, the unitary operator U is corresponding to the transformation $U|k\rangle = |k \oplus 1\rangle$, where $k = 1, 2, 3$. It is easy to find that only one mirror (M_1) is enough for the unitary operator U, and the other two mirrors (M_2, M_3) can be removed. Similarly, only one mirror (M_2) is required for the unitary operator U^2 ($U^2|k\rangle = |k \oplus 2\rangle$), while all the three mirrors (M_1, M_2, M_3) are required for the unitary operator U^3 ($U^3|k\rangle = |k \oplus 3\rangle$). In summary, the unitary operators U, U^2, U^3 correspond to $(1, 0, 0)$, $(0, 1, 0)$, $(1, 1, 1)$ respectively, where 1 denotes that a mirror exits in the corresponding position (M_1, M_2, M_3), and 0 denotes that no elements exit in that position. On the other hand, the unitary operators V, V^2, V^3 correspond to the three phase shifters $(\frac{2\pi}{3}, \frac{4\pi}{3}, 0)$, $(\frac{4\pi}{3}, \frac{2\pi}{3}, 0)$, $(0, 0, 0)$ respectively.

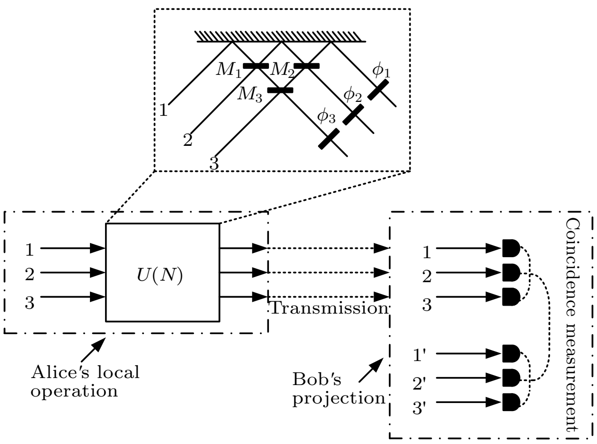

Fig. 3. Schematic diagram of probability superdense coding with entangled qutrits. The multiprot beam splitters are used to realize any discrete unitary operator, which are implemented to the photon hold by the sender Alice. In the end, Bob uses the Von Neumann measurement and the following coincidence measurement of the two sides to discriminate the states. For details, see text.

After Alice's local operation has been implemented, the rest job the ob is Bob's discrimination after the transmission. In principle, this set of linear independent states $|\Psi_{mn}^N\rangle$ can be discriminated with some probability.[14] Unfortunately, only by linear optical elements, even four Bell states can not be discriminated perfectly, while three of them can be[15]. Thus, it is impossible to distinguish all the states $|\Psi_{mn}^N\rangle$ determinately in linear optics even if they are maximal entangled states. For simplification, we only divide these states into three parts which correspond to the combination of rotation operators U, U^2, U^3 just by the Von Neumann measurement and the following coincidence measurement of the two sides, which is shown in the Fig. 3. Of course, through some discrete unitary operator which has been described above, we can discriminate the other combination of the unitary operators, for example U, U^2, V, etc. For the reason that these different measurements can not be performed at the same time, all the states $|\Psi_{mn}^N\rangle$ can not be discriminated determinately. As a result, the classical information achieved by Bob is only $\log 3$ if the entangled qutrits are maximal.

In the process of superdense coding, Alice's operation is performed on one photon, and Bob's discrimination is based on the coincidence measurement of the two sides. These operations will not lead to the confusion of the two photons, i.e., the photon on one side will not appear on the other side after these operations. As we discussed above, the generations of the entangled qutrits and ququads are based on the coincidence measurements of the two sides. Thus the entangled qutrits and ququads can be directly applied to the scheme of superdense coding.

In addition, if the entangled qudits shown in Eq. (7) can be generated perfectly, and a multiport beam splitters is designed appropriately (also remove or replace the beam splitters by mirrors), associated with the Von Neumann measurement and the following coincidence measurement of the two sides, the realization of PSC is similar to the case of entangled qutrits. Now the classical information can be achieved increasing to $\log N$, which is only half of the amount in theory. However, designing different setups to discriminate the different combinations of the unitary operators respectively, this scheme can demonstrate the proposal of PSC with entangled qudits, in principle. Unfortunately, in our proposal, the coincidence measurement of the paths $1(2 \cdots N)$ and $1'(2' \cdots N')$ respectively is required in the generation of entangled qudits, because under this condition the cross items can be throw away. While in Bob's discrimination, the coincidence measurement of the two sides is required, then the cross items would bring some errors. Thus the entangled qudits can not be applied directly. However, the entangled qutrits and ququads can be applied directly for which only the coincidence measurement of the two sides is required.

Now we discuss briefly the feasibility of our schemes. In the first, pairs of photons are required in the generation. Though we do not have an on-demand single-photon source available, we can use the SPDC process to generate simultaneous pairs of photons. The typical coincidence count rates are 10^5–$10^6\,\mathrm{s}^{-1}$ reported in Ref. [12], which is enough for the demonstration of PSC. In addition, in the generation of entangled qudits, the interference of two photons and the coincidence measurements between the same paths are required. Because the coherent time of the photons is about 0.5 ps (if a 4-nm FWHM interference filter is inserted before each detector[16]) and the time resolution of the single-photon detectors is about 100 ns for APDs,[17] the length difference between the two paths must be less than the coherent length of the photons by about 0.15 mm, which is feasible in the laboratory with the current experimental technology. After generation, no special techniques are required in the transmission and the measurement, so this proposal of PSC with entangled qutrits and ququads can be even used in the realistic communication to increase the efficiency of the information transmission.

In conclusion, we first present a scheme to generate entangled qutrits, and then generalize this scheme to entangled ququads, even entangled qudits. For their uses in the proposal of PSC, the entangled qutrits and ququads can be applied directly, even in realistic communication, while the entangled qudits can not be. However, if some nonlinearities, such as cross-Kerr nonlinearities,[18] are used to assist the coincidence measurement, the entangled qudits can also be applied to the proposal of PSC. Moreover, the cross-Kerr nonlinearities may enable us to discriminate all the state $|\Psi_{mn}^N\rangle$ more perfectly, which will be considered in the future.

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