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Tight focusing properties of linearly polarized Gaussian beam with a pair of vortices

Ziyang Chen [a,b], Jixiong Pu [b], Daomu Zhao [a,*]

[a] Department of Physics, Zhejiang University, Hangzhou 310027, China
[b] College of Information Science & Engineering, Institute of Optics & Photonics, Huaqiao University, Xiamen, Fujian 361021, China

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ABSTRACT

The properties of a pair of vortices embedded in a Gaussian beam focused by a high numerical-aperture are studied on the basis of vector Debye integral. The vortices move and rotate in the vicinity of the focal plane for a pair of vortices with equal topological charges. For incident beam with a pair of vortices with opposite topological charges, the vortices move toward each other, annihilate and revive in the vicinity of focal plane.

1. Introduction

Beams with a phase term of $\exp(im\theta)$ refer to vortex beams. It is predicted by Allen and his colleagues that each photon of vortex beam with rotational symmetry carries orbital angular momentum (OAM) of $m\hbar$, of which m is the topological charge [1]. The OAM of vortex beams can be transferred to the particles and can set them into rotation, which can be used as an optical spanner [2,3]. Owing to its interesting properties and potential applications, the propagation of vortex beams has been actively studied during the last two decades [4–9]. Some investigations treated two or more individual vortices in a beam [10–14]. In 1993, Indebetouw found that a pair of vortices with same charges will rotate up to $\pi/2$ in the far field, and vortices of opposite charges tend to attract each other on propagation and pairs of vortices may interfere destructively and then vanish [10]. An optical vortex dipole (a pair of vortices with opposite topological charges) propagating in a Gaussian beam may produce a variety of possible trajectories different from that of canonical vortex beams. It is found in the study of trajectories of a vortex dipole propagating in graded-index media that the vortex will annihilate and revive periodically [11]. Expressions for the trajectories of vortices launched as a canonical dipole from arbitrary locations in a Gaussian beam are derived [12]. Furthermore, the annihilation of vortex dipoles can be accelerated by using a background phase function [13]. The self-annihilation of oppositely charged vortices provides a solution to the vortex stagnation problem [14].

Focusing laser beams through a high numerical-aperture (NA) objective has attracted great attentions because it has important applications in microscopy and particle trapping [15,16]. As vortex beams have unique advantages in optical manipulation, the focusing property of vortex beams has been the topic of many researches [17–20]. The connection between spin and orbital angular momentum with cylindrical decomposition was discussed in the study of the focusing of a circularly polarized vortex beam by a high NA objective [17]. In addition, the spin angular momentum (SAM) of the circularly polarized beam can be converted to OAM [21]. Sometimes, one may have to manipulate two particles in different locations at the same time. Can it be realized by tight focusing of a beam with two vortices? In another way, will the two vortices keep invariant when focused by a high NA objective? To our knowledge, what will happen when a pair of vortices nested in a beam focused by a high NA objective is still unknown. The purpose of this Letter is to investigate the tight focusing property of a beam with two vortices.

2. Theory of tight focusing

For a high NA objective, a vectorial theory of focusing is required. The theory known as vector Debye integral was described

* Corresponding author. Tel.: +8657188863887; fax: +8657187951328.
E-mail address: zhaodaomu@yahoo.com (D. Zhao).

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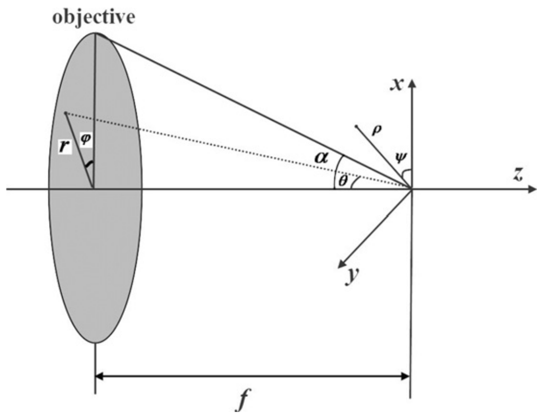

Fig. 1. Scheme of tight focusing.

originally by Richards and Wolf [22]. According to the theory, for a linearly polarized incident beam, the electric field near the focus can be expressed as [22,23]

$$E(\rho,\psi,z)=\frac{i}{\lambda}\int_0^{2\pi}\int_0^{\alpha}A(\theta,\varphi)K(\rho,\psi,z,\theta,\varphi)\,d\theta\,d\varphi, \tag{1a}$$

$$\begin{aligned}K(\rho,\psi,z,\theta,\varphi)&=\big\{\big[\cos\theta+\sin^2\varphi(1-\cos\theta)\big]\mathbf{i}+\big[\cos\varphi\sin\varphi(\cos\theta-1)\big]\mathbf{j}\\&\quad+(\cos\varphi\sin\theta)\mathbf{k}\big\}\exp\big[-ik\rho\sin\theta\cos(\varphi-\psi)\big]\\&\quad\times\exp[-ikz\cos\theta]\sin\theta\cos^{1/2}\theta,\end{aligned} \tag{1b}$$

where θ is the angle of convergence, $A(\theta)$ is the pupil apodization function at the exit pupil, λ is wavelength of the incident beam, $k=2\pi/\lambda$ is the wavenumber, and $\alpha=\sin^{-1}(\mathrm{NA})$ is the maximal angle determined by the NA of the objective. Variables ρ, ψ and z are the cylindrical coordinates of an observation point in the vicinity of focus, and $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are the unit vectors in the x, y, z directions, respectively. It is shown that a linearly polarized beam is depolarized in the focal region as focused by a high NA objective. In other words, if the incident electric field oscillates along one direction, a field component along the incident polarization direction as well as an orthogonal and a longitudinal component will produce in the focal region. A scheme of tight focusing is shown in Fig. 1.

3. A pair of vortices of equal topological charges

Gaussian beam is usually introduced as a host beam in the study of beams with a pair of vortices. For simplicity and without loss of generality, we consider that the topological charges of the two vortices have the same magnitude. First, we study a pair of vortices of equal charges. We choose two vortices of topological charge $m=1$ located at $x=\pm a$, respectively in the plane of the host beam, the electric field can be expressed as [10]

$$A(r,\varphi)=\exp\left(-\frac{r^2}{w^2}\right)\big[r\exp(i\varphi)-a\big]\big[r\exp(i\varphi)+a\big], \tag{2}$$

where r and φ represent the radial distance and angle in polar coordinates, respectively, w is a constant denoting the beam size, and the distance between the two vortices is decided by a. For $a=0$, the two vortices are overlapped, the beam is a vortex beam with topological charge $m=2$. The two vortices will separate with the increment of a, and two dark cores emerge gradually in the beam.

Since the objectives are often designed to obey sine condition, we get $r=f\sin\theta$ [23], where f is the focal length of the high NA objective, and θ is the numerical-aperture. Substituting Eq. (2) into Eq. (1) and after some simplification, the x-, y-, and z-components of the electrical field in the focal region can be simplified as

$$\begin{aligned}&E_x(\rho,\psi,z)\\&=ik\int_0^{\alpha}P(\theta)\bigg\{\frac{1}{4}f^2\sin^2\theta(\cos\theta-1)J_0(k\rho\sin\theta)\\&\quad-\frac{1}{2}(1+\cos\theta)a^2J_0(k\rho\sin\theta)\\&\quad+\frac{1}{2}(\cos\theta-1)a^2J_2(k\rho\sin\theta)\cos2\psi\\&\quad-\frac{1}{2}f^2\sin^2\theta(1+\cos\theta)J_2(k\rho\sin\theta)\exp(i2\psi)\\&\quad+\frac{1}{4}f^2\sin^2\theta(\cos\theta-1)J_4(k\rho\sin\theta)\exp(i4\psi)\bigg\}\,d\theta,\end{aligned} \tag{3a}$$

$$\begin{aligned}&E_y(\rho,\psi,z)\\&=ik\int_0^{\alpha}P(\theta)\bigg\{\frac{1}{2}a^2(\cos\theta-1)J_2(k\rho\sin\theta)\sin2\psi\\&\quad+\frac{i}{4}f^2\sin^2\theta(\cos\theta-1)\big[J_0(k\rho\sin\theta)\\&\quad-J_4(k\rho\sin\theta)\exp(i4\psi)\big]\bigg\}\,d\theta,\end{aligned} \tag{3b}$$

$$\begin{aligned}&E_z(\rho,\psi,z)\\&=ik\int_0^{\alpha}P(\theta)\bigg\{ia^2\sin\theta J_1(k\rho\sin\theta)\cos\psi\\&\quad+\frac{i}{2}f^2\sin^3\theta\big[J_3(k\rho\sin\theta)\exp(i3\psi)\\&\quad-J_1(k\rho\sin\theta)\exp(i\psi)\big]\bigg\}\,d\theta,\end{aligned} \tag{3c}$$

where

$$P(\theta)=\exp\left(-\frac{f^2\sin^2\theta}{w^2}\right)\exp[-ikz\cos\theta]\sin\theta\cos^{1/2}\theta \tag{3d}$$

and $J_0(x)$, $J_1(x)$, $J_2(x)$, $J_3(x)$, and $J_4(x)$ are the zeroth-order, the first-order, the second-order, the third-order, and the fourth-order Bessel functions of the first kind, respectively.

The influence of location of the vortices on the beam pattern in the focal plane is presented in Fig. 2. Only one dark core is observed as $a=0$ owing that the incident beam is canonical vortex beam in such case. Two dark cores emerge and separate gradually with the increment of distance between two vortices of the incident beam. It is noticed that two well-defined dark cores are preserved for certain value of a. They will disappear as the separation between the two vortices becomes sufficiently large, and a bright beam spot formed in the focal plane.

An interesting phenomenon can be found by comparing the intensity distribution of the incident beam with that in the focal beam. The two vortices of the incident beam locate in the horizontal direction, but they locate in the vertical direction in the focal plane. The evolution of the vortices can be illustrated with the help of the phase distribution in the vicinity of the focus. According to previous paper, if the incident beam is not obstructed, the magnitude of x-component of electric field in the focal plane

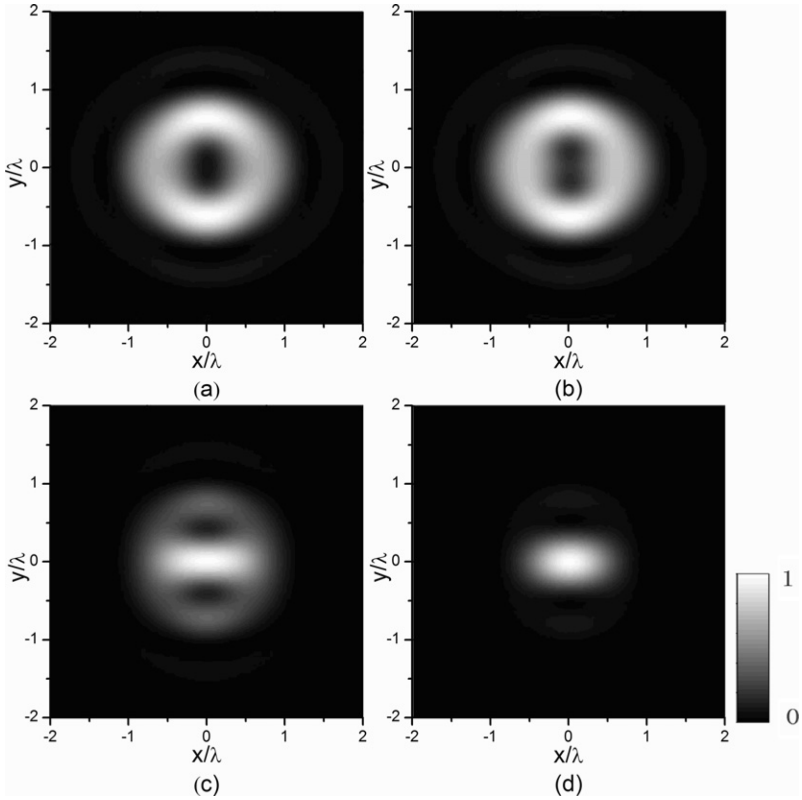

Fig. 2. Intensity distributions in the focal plane as a Gaussian beam with a pair of vortices of equal charges focused by a high NA objective. (a) $a=0$; (b) $a=0.2w$; (c) $a=0.5w$; (d) $a=0.8w$. The other parameters are chosen as $\lambda=632.8$ nm, $w=2$ mm, $f=2$ mm, NA = 0.9, $m=1$.

will be larger than that of y- and z-components [24]. The result is tenable for beams with vortices focused by a high NA objective. Therefore we only give the phase distribution of x-component, as shown in Fig. 3. The phase singularities (optical vortices) move toward each other and rotate around the center as the observation plane moves to the focus. It is found in the study of distribution of N optical vortices nested in a Gaussian beam that the whole pattern gradually rotates up to $\pi/2$ in the far field [10]. To some extent, the intensity distribution in the focal plane is equivalent to that in the far field, and such a rotation occurs as well in tight focusing of beams with a pair of vortices.

4. A pair of vortices of opposite topological charges – vortex dipole

In electromagnetic field, a dipole refers to a pair of electric charges or magnetic poles, of equal magnitude but opposite sign or polarity, separated by a small distance. A similar concept "optical vortex dipole" is introduced in the study of an optical vortex. A pair of optical vortices with topological charges of $m=+1$ and $m=-1$, respectively, form an optical vortex dipole [10,12]. It is considered that two vortices of topological charge $m=\pm 1$ locate at $x=\pm a$, embedded in a Gaussian beam, the electric field can be expressed as

$$A(r,\varphi)=\exp\left(-\frac{r^2}{w^2}\right)\left[r\exp(i\varphi)-a\right]\left[r\exp(-i\varphi)+a\right], \quad (4)$$

where the distance between the two vortices is $2a$. The beam is a non-vortex beam for $a=0$.

Substituting Eq. (4) into Eq. (1), and following the procedure in the preceding section, the x-, y-, and z-components of the electric field in the focal region of a high NA objective can be finally expressed as

$$\begin{aligned}E_x(\rho,\psi,z) &= ik\int_0^{\alpha} P(\theta)\left\{\frac{1}{2}\left(f^2\sin^2\theta-a^2\right)(1+\cos\theta)J_0(k\rho\sin\theta)\right.\\ &\quad -\frac{1}{2}(\cos\theta-1)\left(f^2\sin^2\theta-a^2\right)J_2(k\rho\sin\theta)\cos 2\psi\\ &\quad +af\sin\theta(1+\cos\theta)J_1(k\rho\sin\theta)\sin\psi\\ &\quad -\frac{1}{2}af\sin\theta(\cos\theta-1)\left[J_3(k\rho\sin\theta)\sin 3\psi\right.\\ &\quad \left.\left.+J_1(k\rho\sin\theta)\sin\psi\right]\right\}d\theta, \end{aligned} \quad (5a)$$

$$\begin{aligned}E_y(\rho,\psi,z) &= ik\int_0^{\alpha} P(\theta)\left\{-\frac{1}{2}\left(f^2\sin^2\theta-a^2\right)(\cos\theta-1)J_2(k\rho\sin\theta)\sin 2\psi\right.\\ &\quad +\frac{1}{2}af\sin\theta(\cos\theta-1)\left[J_3(k\rho\sin\theta)\cos 3\psi\right.\\ &\quad \left.\left.+J_1(k\rho\sin\theta)\cos\psi\right]\right\}d\theta, \end{aligned} \quad (5b)$$

$$\begin{aligned}E_z(\rho,\psi,z) &= ik\int_0^{\alpha} P(\theta)\left\{-i\left(f^2\sin^2\theta-a^2\right)\sin\theta J_1(k\rho\sin\theta)\cos\psi\right.\\ &\quad \left.-iaf\sin^2\theta J_2(k\rho\sin\theta)\sin 2\psi\right\}d\theta. \end{aligned} \quad (5c)$$

The electric field in the focus (i.e., $\rho=0$, $z=0$) can be further simplified as

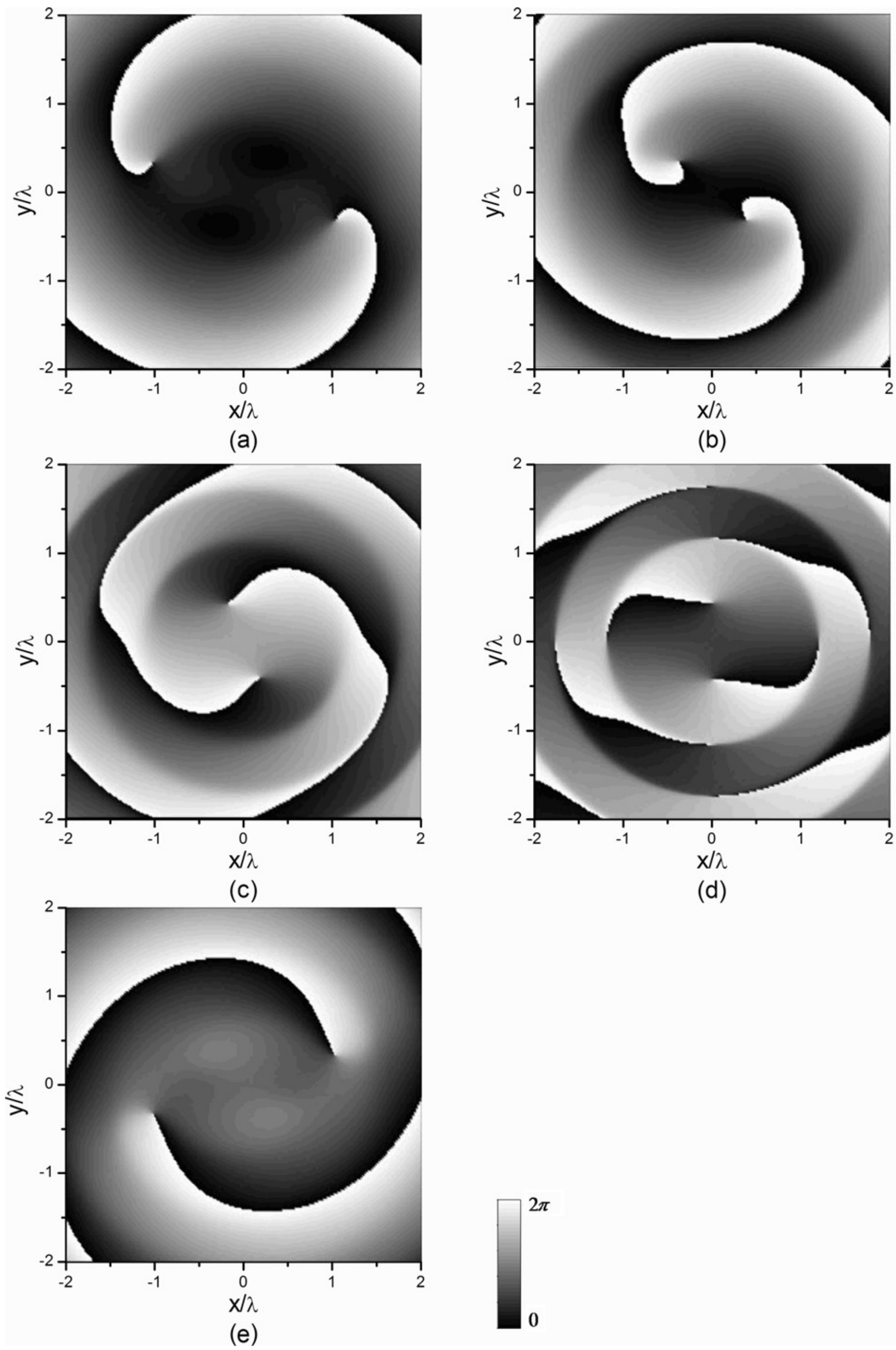

Fig. 3. Evolution of phase singularities of the x-component of the electric field as a vortex pair with equal charges focused by a high NA objective. (a) $z = -2\lambda$; (b) $z = -1\lambda$; (c) $z = -0.5\lambda$; (d) $z = 0$ (i.e., the focal plane); (e) $z = 2\lambda$. $a = 0.5w$, the other parameters are same as those in Fig. 2.

$$E_x(\rho, \psi, z) = \frac{ik}{2}\int_0^{\alpha} \exp\left(-\frac{f^2 \sin^2\theta}{w^2}\right) \times \sin\theta\sqrt{\cos\theta}\left(f^2 \sin^2\theta - a^2\right)(1+\cos\theta)\,d\theta, \quad (6a)$$

$$E_y(\rho, \psi, z) = E_z(\rho, \psi, z) = 0. \quad (6b)$$

Based on the above equations, the tight focusing property of a vortex dipole can be calculated numerically. In Fig. 4, the influence of the separation of the vortices on the beam spot in the focal plane is investigated. The incident beam is a non-vortex beam with circular symmetry for $a = 0$. A central spot surrounded by a bright annulus is produced in the focal plane. The intensity of the annulus is much weaker than the spot. With the increment of the distance between two vortices of the incident beam, i.e., the presence of two vortices, the incident beam becomes asymmetric. In the focal plane, the central spot moves away from the center and becomes weaker gradually. The original central spot will vanish, and a new spot will emerge below the center as a reaches certain value. It is clear in Eq. (6) that, the magnitude of the electric field in the focus reaches maximum when $a = 0$, and reduces gradually with the increment of a.

The evolution of vortices is an interesting phenomenon in the study of an optical vortex dipole [10–13]. The vortices embedded in the beam show a complicated dynamical behavior, such as

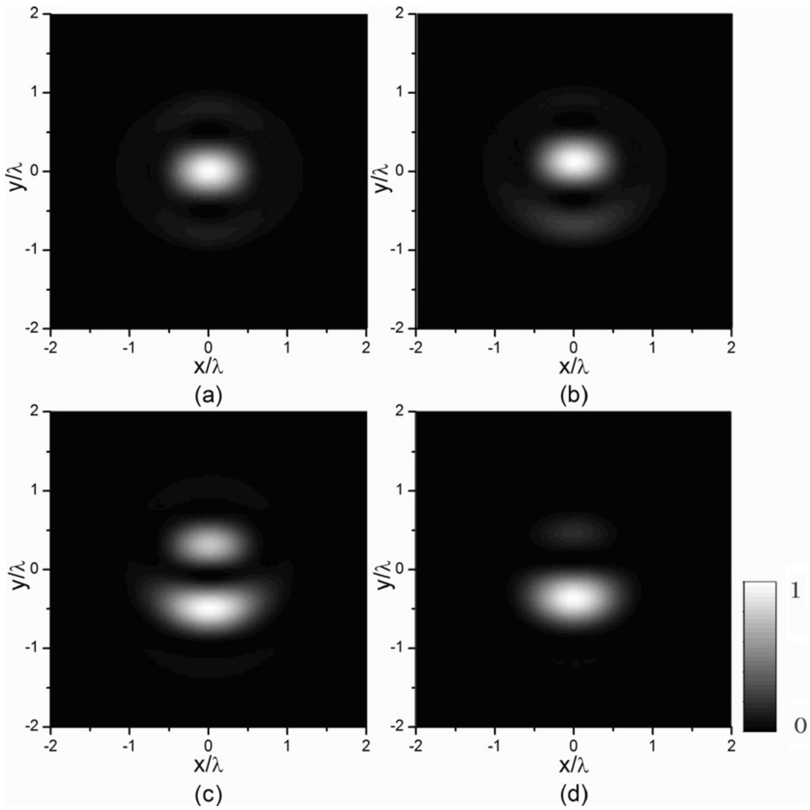

Fig. 4. Intensity distributions in the focal plane as a vortex dipole nested in Gaussian beam focused by a high NA objective. (a) $a = 0$; (b) $a = 0.2w$; (c) $a = 0.5w$; (d) $a = 0.8w$. The other parameters are same as those in Fig. 2.

moving, annihilating, reviving, etc. We plot the phase distribution in the vicinity of focus to illustrate the propagation dynamics of vortices. For the same reason as in investigation of a vortex pair with equal charges, only the phase distribution of x-component is presented, as shown in Fig. 5. The locations for Fig. 5(a)–(e) are $z = -2\lambda$, $-\lambda$, -0.5λ, 0, 0.5λ, respectively, where $z = 0$ refers to the focal plane. It is found that the separation distance between two vortices (i.e., phase singularities) decreases gradually. For $z = -2\lambda$, the distance is 1.15λ, and it reduces to 0.65λ in the plane of $z = -0.5\lambda$. Furthermore, no phase singularities are presented in the focal plane ($z = 0$), namely, the vortices annihilate. In the further study of phase distribution after the focal plane, it is noticed that the phase singularities emerge again, which means that the vortices revive.

5. Conclusions

In conclusion, we have studied the tight focusing property of a pair of vortices embedded in Gaussian beam with two different cases being discussed: one is two vortices with equal topological charge of $m = +1$, while the other one is vortex dipole.

Only one dark core is observed in the focal plane when two vortices with equal topological charges both located in the center of the incident beam, and it gradually evolves into two dark cores and then disappears as the vortices move away from the center. In addition, the position of the vortices rotates in the vicinity of the focal plane, and the angle of the rotation is $\pi/2$ in the focal plane.

When two vortices with opposite charges are overlapped in the center, a beam spot is produced in the focal plane. The focal spot will move away from the center with intensity becoming weaker, and a new spot will emerge gradually as the separation distance between two vortices of the incident beam increases. The evolution of vortices can be illustrated with the help of phase distribution. Two vortices are presented in front of the focal plane. The vortices move toward each other as the plane near to the focal plane. No vortices are found in the focal plane, which means that the vortices are annihilated, but nearly behind the focal plane, the vortices emerge again. In general, the annihilation and the revival of the vortices are observed in the tight focusing of an optical vortex dipole.

It is known that optical-trapping can be divided into two types of particles, one of which is trapping particles with refractive indices higher than that of ambient; while the other is trapping particles with refractive indices less than that of the ambient. Gaussian beam with the largest intensity at the focus are usually used to trap the first type particles [25]. In the second type, the particles can be trapped in the dark part of the doughnut-shaped beams (such as the Laguerre–Gaussian beam and dark hollow beams) [26, 27]. Moreover, a recent survey found that the red blood cells could be driven as micro-rotors by a transfer of orbital angular momentum from the LG beam (one example of vortex beams) [28]. It is shown that two dark cores are formed in the focal plane for beams with a pair of vortices of equal topological charges, which may be used to trap and rotate two particles of the second type separately at the same time. Particles can be actively trapped, manipulated and rotated by controlling the location of the vortices of the incident beam. For a beam with a vortex dipole, the first type particles can be trapped and moved by controlling the incident beam.

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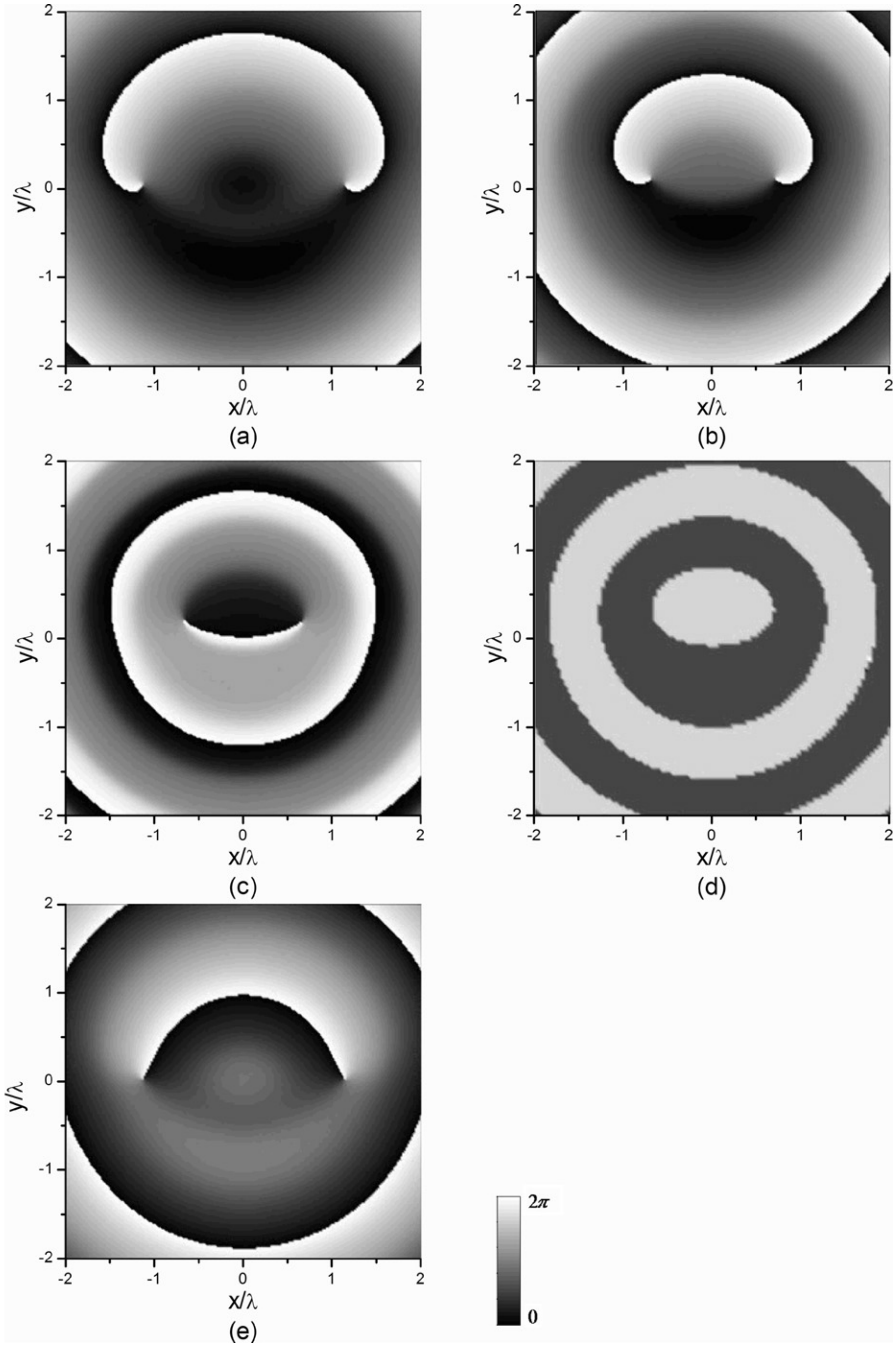

Fig. 5. Evolution of phase singularities of the x-component of the electric field as a vortex dipole nested in Gaussian beam focused by a high NA objective. (a) $z = -2\lambda$; (b) $z = -1\lambda$; (c) $z = -0.5\lambda$; (d) $z = 0$ (i.e., the focal plane); (e) $z = 2\lambda$. $a = 0.5w$, the other parameters are same as those in Fig. 2.

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