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Measuring the Topological Charge of Integer and Fraction Vortices Using Multipoint Plates

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The topological charge of integral vortex beams with a single circle of multipoints has previously been measured. In this work, we theoretically and experimental study the diffraction patterns of vortex beams of integral and fractional topological charges using a single circle and two circles of multipoint plates. It is found that the diffraction patterns are dependent not only on the multipoint plates, but also on the topological charge of the vortex beams. On the basis of this property, we can measure the topological charges of integral and fractional vortex beams.

Keywords: vortex beams, multipoint plates, interference

1. Introduction

Vortex beams have attracted much interest in recent years.[1,2] A vortex beam having a spiral phase wavefront $\exp(il\theta)$ (θ is the azimuthal angle) can have an orbital angular momentum (OAM), i.e., each photon of the vortex beam possesses $l\hbar$ OAM (l is called the topological charge). Because it has the OAM, the vortex beam can make microparticles spin. Therefore, such vortex beams have special applications in many areas, such as the trapping and rotating of micro-particles,[3,4] optical tweezers,[5] and information encoding.[6]

Owing to the high applicability of vortex beams, the measurement of the topological charge of vortex beams is important in exploiting their applications.[7,8] Up to now, several techniques for measuring the topological charge of vortex beams have been developed.[9,10] Recently, Berkhout and Beijersbergen have proposed a multipoint method for measuring the topological charge of the vortex beams.[11] They observed the diffraction patterns of the vortex beams as they passed through a single circle of a multipoint plate. It was found that different interference intensity patterns correspond to different topological charges of optical vortices. Therefore, the topological charge of the vortex beams can be determined from the interference intensity patterns.

In this work, we design a single circle and dual circle of multipoint plates, and study the diffraction patterns of integer and fraction vortices passing through the multipoint plates. We find that the diffraction patterns are dependent not only on the multipoint plates, but also on the topological charge of the vortex beams. On the basis of this property, we can measure the topological charges of integral and fractional vortex beams.

2. Theory

The major purpose of this study is to investigate the diffraction pattern of the vortex beam diffracted by two kinds of multipoint plates, in order to develop a novel method for measuring the topological charge of the vortex beams. To obtain such a method, we study the interference of vortex beams both theoretically and experimentally. We will show that topological charge can be quantitatively measured from the interference patterns.

(a) (b)

Fig. 1. Images of two kinds of multipoint plates with the number of points (a) $N = 6$ and (b) $N = 6 \times 2 = 12$.

The two kinds of multipoint plates with the number of points $N = 6$ and $N = 6 \times 2 = 12$ used in the experiment are shown in Fig. 1. In single-circle multipoints, the multipoints are arranged into a circle, however, in the dual-circle multipoint plate, the multipoints are along two circles with two different radii.

First of all, we theoretically investigate the interference of the vortex beams diffracted by single-circle multipoints. The phase is different at every point. The electric field expression of the vortex beam is $E = A\exp(il\theta)$, where A is the amplitude and l is the topological charge of the vortex beam. The distribution of intensity for the interference is[7]

$$I = EE^* = \left|\sum_{n=0}^{N-1} \exp(-il\alpha_n) \times \exp\left[i\frac{ka}{z}(x\cos\alpha_n + y\sin\alpha_n)\right]\right|^2, \quad (1)$$

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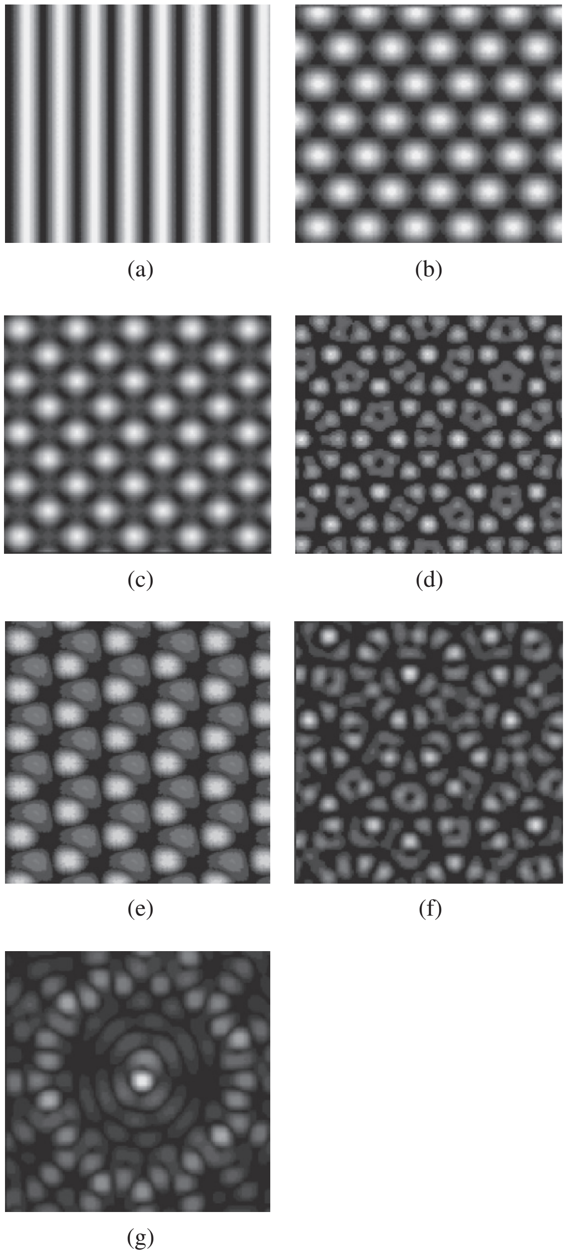

Fig. 2. Interference intensity patterns (calculated numerical results) of vortex beam through single-circle multipoints for the topological charge $l = 0.5$ and the number of apertures (a) $N = 2$; (b) $N = 3$; (c) $N = 4$; (d) $N = 5$; (e) $N = 6$; (f) $N = 7$; (g) $N = 16$.

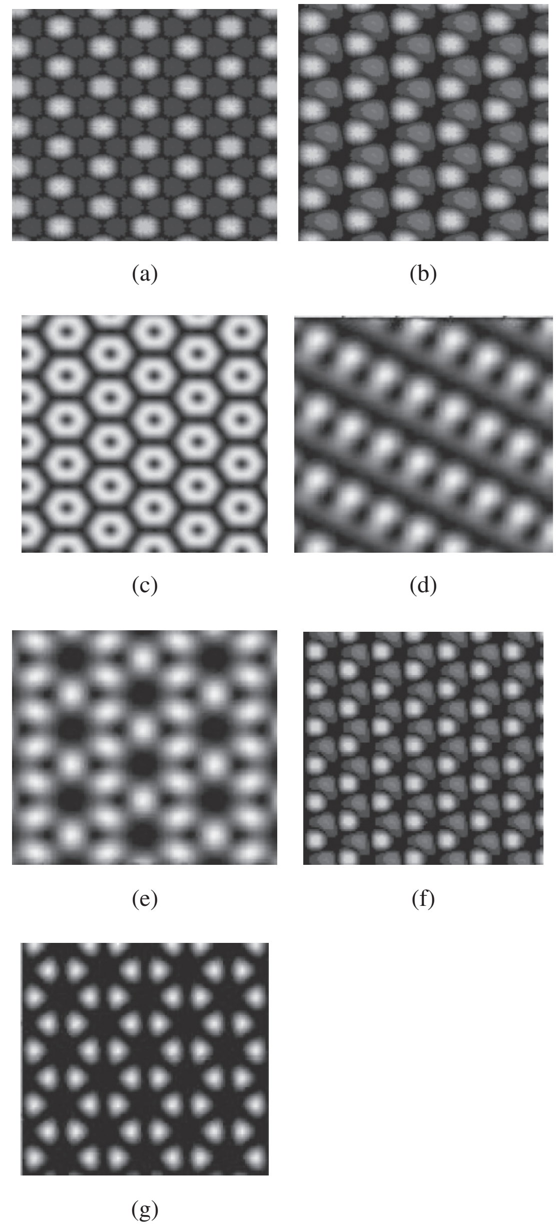

Fig. 3. Interference intensity patterns (calculated numerical results) of the vortex beam diffracted by single-circle multipoints of $N = 6$ with (a) $l = 0$; (b) $l = 0.5$; (c) $l = 1$; (d) $l = 1.5$; (e) $l = 2$; (f) $l = 2.5$; (g) $l = 3$.

where l is the topological charge of the vortex beams, $\alpha_n = 2\pi n/N$, N is the number of apertures, $k = 2\pi/\lambda$, a is the radius of the circle, and z is the distance between the multipoint plate and the observed plane.

Using eq. (1) we can simulate the vortex beam interferogram, as shown in Fig. 2. The interference image for the vortex beams through an aperture is as follows. Figures 2(a)–2(g) are the interference graphs of vortex beams of topological charge 0.5 through the multipoint plates with the number of aperutres of 2, 3, 4, 5, 6, 7, and 16, respectively. From the figure, we find that increasing the number of multipoints, changes the interference graph. Also, because different numbers of apertures limit different light intensities, the interference intensity patterns correspond to the number of apertures.

The interference intensity patterns (calculated numerical results) of vortex beams of topological charges 0, 0.5, 1, 1.5, 2, 2.5, and 3, diffracted by the multipoint plate of $N = 6$ are shown in Fig. 3. It is well known that for the vortex beam of topological charge 1, the azimuthal phase has a change of 2π in periodicity, and the change of phase corresponds to the topological charge. With a change in the topological charge, the interference between the two points introduces a phase difference.

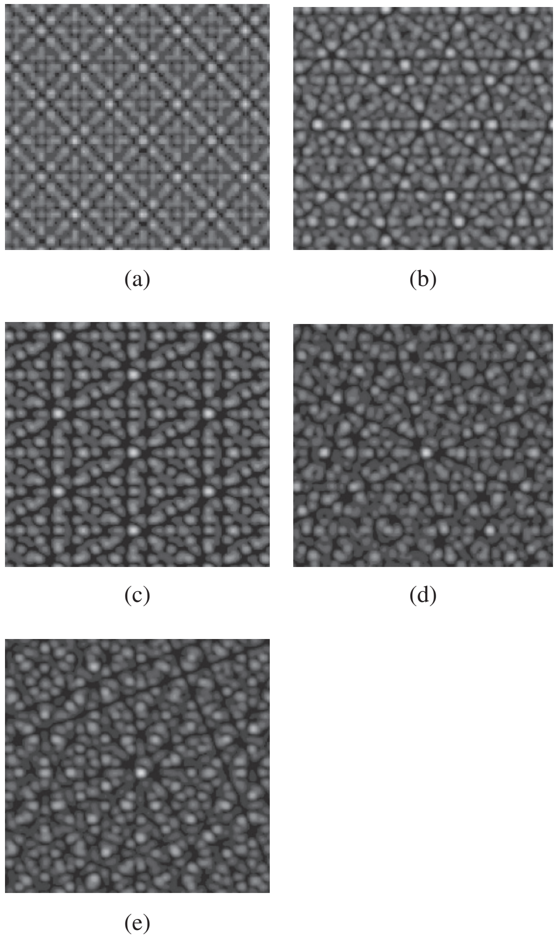

Fig. 4. Interference intensity patterns (calculated numerical results) of vortex beam diffracted by dual-circle multipoint plates for $l = 0.5$ with (a) $N = 4 \times 2$; (b) $N = 5 \times 2$; (c) $N = 6 \times 2$; (d) $N = 7 \times 2$; (e) $N = 8 \times 2$.

Now, we investigate the diffraction of the vortex beam using dual-circle multipoint plates. The distribution of the intensity of the interference can be given by

$$I = EE^* = \left| \sum_{n=0}^{N-1} \left\{ \exp(-il\alpha_n) \exp\left[\frac{ika}{z} (x \cos\alpha_n + y \sin\alpha_n) \right] + \exp(-il\alpha_n) \exp\left[\frac{ikb}{z} (x \cos\alpha_n + y \sin\alpha_n) \right] \right\} \right|^2, \quad (2)$$

where a is the radius of the first circle and b is the radius of the second circle.

The vortex beam interferogram of the vortex beams diffracted by the dual-circle multipoint plate have been theoretically simulated in Figs. 4 and 5. The results are easily understood. We can see that the interference patterns are different from the patterns obtained using the multipoint plate and shown in Fig. 2. This results from the interference generated by the multipoints along the two circles with different radii. We demonstrate that the patterns change according to the combination of topological charge l and number of apertures N, and while multiple values of l give

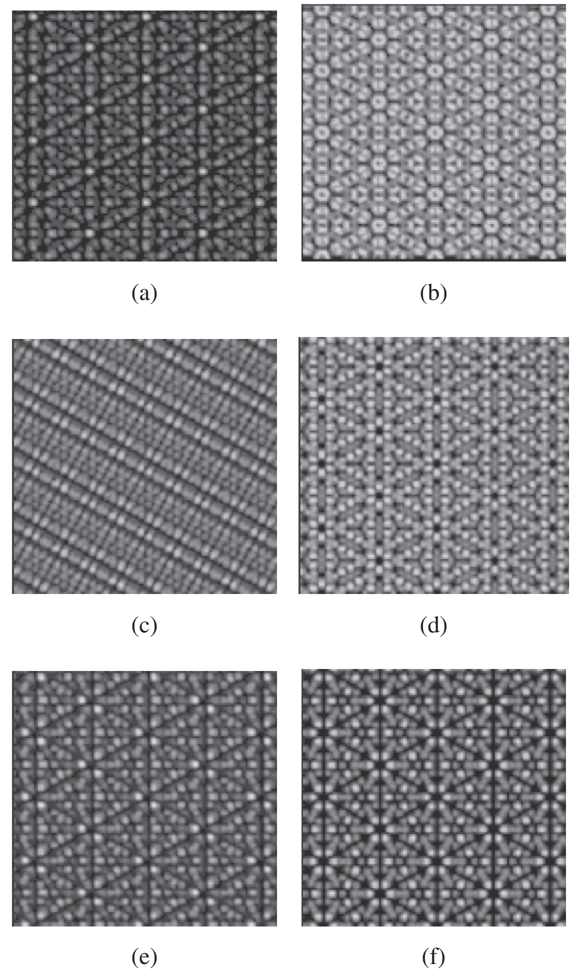

Fig. 5. Interference intensity patterns (calculated numerical results) of vortex beam diffracted by dual-circle multipoint plates with $N = 6 \times 2$ and (a) $l = 0.5$; (b) $l = 1$; (c) $l = 1.5$; (d) $l = 2$; (e) $l = 2.5$; (f) $l = 3$.

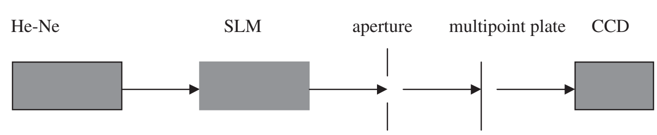

Fig. 6. Diagram of interference equipment with the vortex beam passing through the multipoint plates.

the same result for a specific value of N (for example $l = 1$ and 3 for $N = 2$[11]), they can be distinguished when applying another value of N. Thus, we can also determine the topological charge of the vortex beam by using the dual-circle multipoint plates.

3. Experimental Results

The experimental setup is shown in Fig. 6. The Gaussian beam generated by a helium–neon (He–Ne) laser is incident on the spatial light modulator (SLM). The vortex beam can be obtained by loading a computer-generated hologram (CGH) onto the SLM. The aperture is used to select a vortex beam to illuminate the multipoint plate, and the

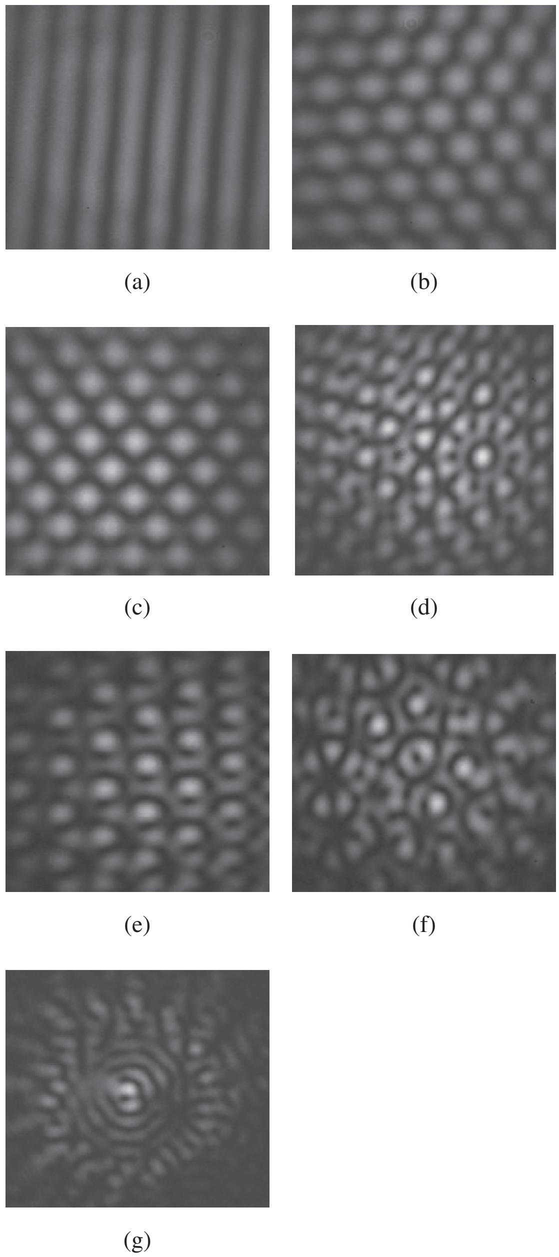

Fig. 7. Experimentally obtained interference intensity patterns of vortex beam through single-circle multipoints for $l = 0.5$ and (a) $N = 2$; (b) $N = 3$; (c) $N = 4$; (d) $N = 5$; (e) $N = 6$; (f) $N = 7$; (g) $N = 16$.

CCD camera is used to record the interference intensity patterns.

The beam width at the SLM is 2 mm and the distance from the SLM to the multipoint plate is 0.5 m. Each "point" in the multipoint plate was realized as a circular area of higher transmission with diameter 2 μm. In our experiment, the multipoint plates were made of films. The radius of the circular patterns is 1 mm, which is equal to the size of vortex beams. The distance from the multipoint plate to the CCD is 1 m. The input beam at the SLM comes from the He–Ne laser that corresponds to a plane wave, the beam curvature border on infinite. We should make sure that

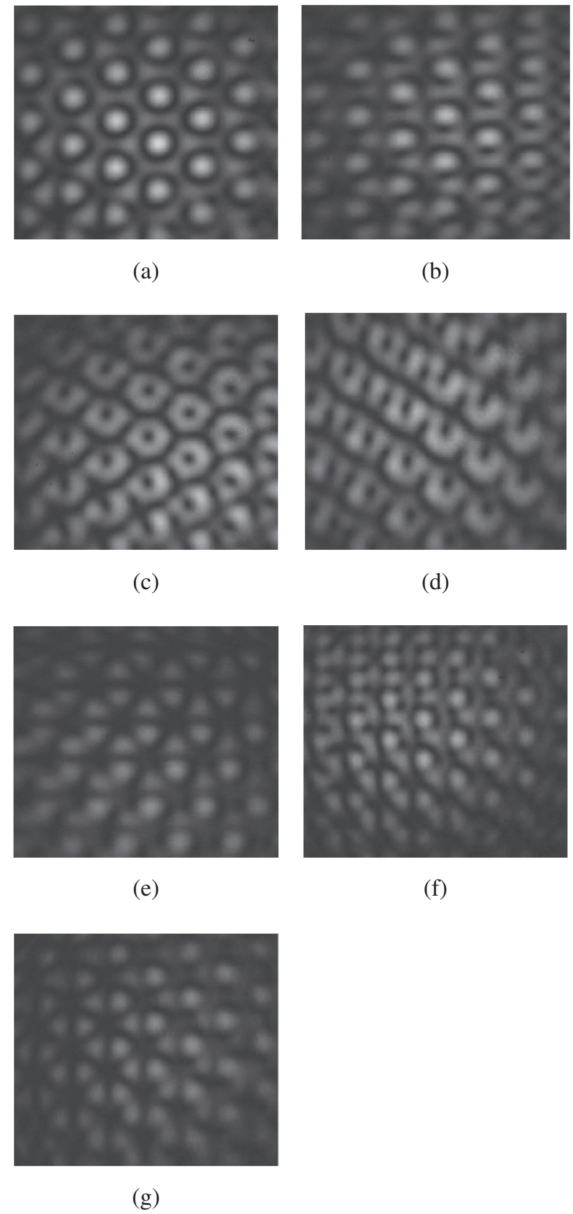

Fig. 8. Experimental interference intensity patterns of vortex beam through single-circle multipoints for $N = 6$ and (a) the topological charge $l = 0$; (b) $l = 0.5$; (c) $l = 1$; (d) $l = 1.5$; (e) $l = 2$; (f) $l = 2.5$; (g) $l = 3$.

the center of vortex beams and the circular patterns are coincident.

The interference intensity distributions recorded in our experiments are shown in Fig. 7. The intensity patterns are measured for a vortex beam with topological charge of 0.5 through two, three, four, five, six, seven, and sixteen apertures. We find that incident vortex beams with different topological charges lead to different interference intensity patterns. Because of this, we can measure the topological charge of the vortex beam.

We also experimentally examined how the change in the topological charge influences the interference intensity patterns. In Fig. 8, we show the experimentally obtained interference intensity patterns of the vortex beams diffracted

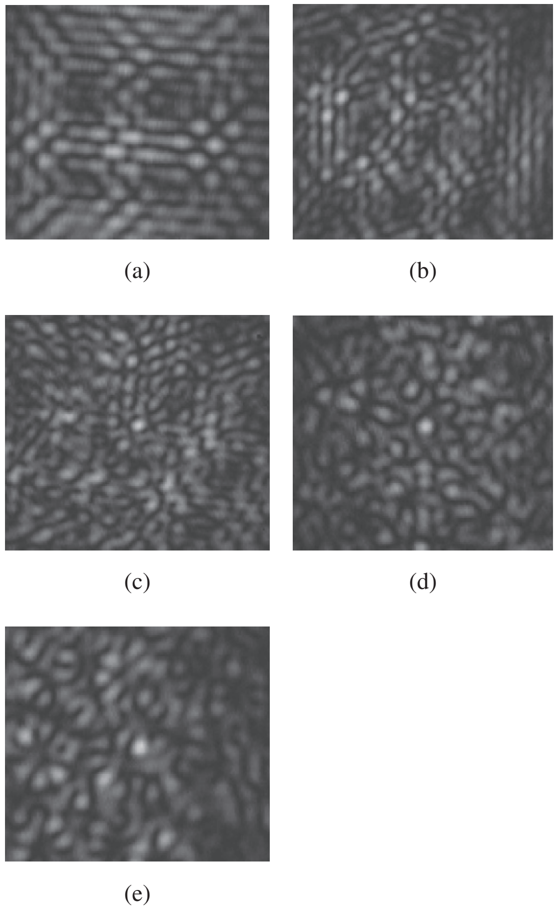

Fig. 9. Experimental interference intensity patterns of vortex beam through dual-circle multipoint plates for $l = 0.5$ and (a) $N = 4 \times 2$, (b) $N = 5 \times 2$, (c) $N = 6 \times 2$, (d) $N = 7 \times 2$, and (e) $N = 8 \times 2$.

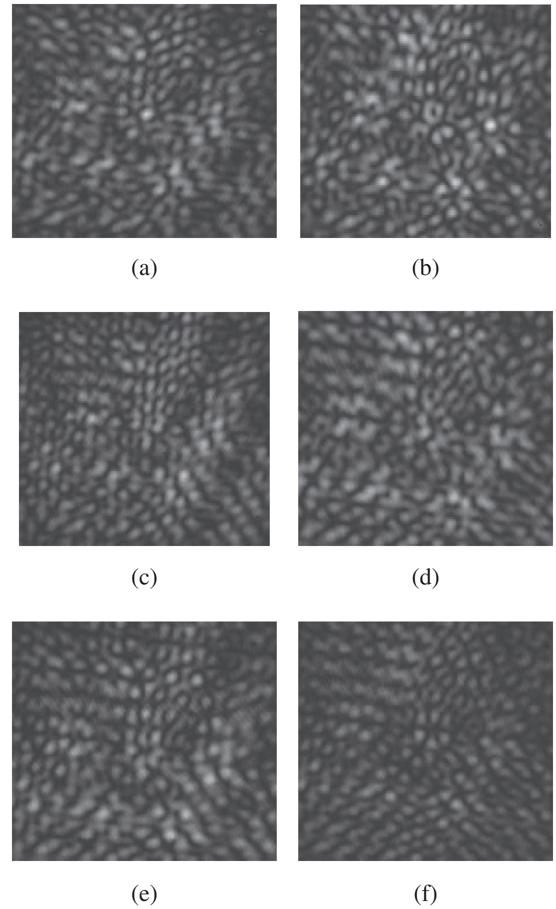

Fig. 10. Experimental interference intensity patterns of vortex beam through dual-circle multipoint plates for $N = 6 \times 2$ and (a) $l = 0.5$, (b) $l = 1$, (c) $l = 1.5$, (d) $l = 2$, (e) $l = 2.5$, and (f) $l = 3$.

by single-circle multipoints of $N = 6$. The topological charges of the vortex beams are 0, 0.5, 1, 1.5, 2, 2.5, and 3. Upon comparison with Fig. 2, it is found that the experimental results in Fig. 8 are consistent with the theoretical results.

We experimentally studied the interference intensity patterns of the vortex beams diffracted by dual-circle multipoint plates, and show them in Figs. 9 and 10. Obviously, with increasing number of apertures, the interference pattern changes. Interference intensity patterns correspond to the number of apertures. The intensity patterns shown in Figs. 9 and 10 are somewhat different from the theoretical results in Figs. 4 and 5. The reason is that the multipoint plates used in our experiments are clearly different from those assumed in calculating the ideal simulated result. Moreover, the vortex beams generated by the SLM in the experiment are not uniform compared with the ideal vortex beams. In this way, the interference intensity patterns are not in perfect accord with the theoretical results.

4. Conclusions

We investigated a method of determining the topological charge l of integer and fraction vortices. In a previous work, Berkhout and Beijersbergen *et al.* obtained results for integer vortices with a single circle of multipoints. We can see the interference patterns change upon changing the topological charge l and the number of apertures of the multipoint plate. By measuring the phases through the multipoint plate, we can achieve different patterns. The probing vortex beam produced by a spatial light modulator (SLM) is convenient in this experiment. Using the multipoint plate, we can easily measure the diversification of the phase of one vortex beam. The method of using the multipoint plate is useful for measuring the integral and fractional vortex beams, and also can be used for X-ray and electron beams.

Acknowledgments

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References

1) L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman: Phys. Rev. A **45** (1992) 8185.
2) A. T. O'Neil, I. MacVicar, L. Allen, and M. J. Padgett: Phys. Rev. Lett. **88** (2002) 053601.
3) H. He, N. R. Heckenberg, and H. Rubinsztein-Dunlop: J. Mod. Opt. **42** (1995) 217.
4) K. T. Gahagan and G. A. Jr Swartzlander: Opt. Lett. **21** (1996) 827.
5) G. Molina-Terriza, J. P. Torres, and L. Torner: Phys. Rev. Lett. **88** (2002) 013601.
6) Y. D. Liu, C. Q. Gao, M. W. Gao, and F. Li: Acta Phys. Sin. **56** (2007) 854.
7) S. G. Cheng, J. Y. Shu, and X. W. Gong: Appl. Phys. Lett. **94** (2009) 231104.
8) H. W. Yan, K. Cheng, and B. D. Lu: Acta Phys. Sin. **40** (2008) 967.
9) Q. S. Xie and D. M. Zhao: Opt. Commun. **281** (2008) 7.
10) F. A. Starikov, G. G. Kochemasov, M. O. Koltygin, S. M. Kulikov, A. N. Manachinsky, N. V. Maslov, S. A. Sukharev, V. P. Aksenov, I. V. Izmailov, F. Yu. Kanev, V. V. Atuchin, and I. S. Soldatenkov: Opt. Lett. **34** (2009) 2264.
11) G. C. G. Berkhout and M. W. Beijersbergen: Phys. Rev. Lett. **101** (2008) 100801.