TR-13
1968

Texas Water Resources Institute
make every drop count

A Hydrodynamic Study of Flow in Irrigation Furrows

O.C. Wilke

Texas Water Resources Institute

Texas A&M University

A HYDRODYNAMIC STUDY OF FLOW
IN
IRRIGATION FURROWS

By

Otto Charles Wilke

Technical Report No. 13

July 1968

This report was prepared originally as a dissertation for
the Department of Agricultural Engineering and submitted
to the Graduate College of Texas A&M University in
partial fulfillment of the requirements
for the degree of Doctor of Philosophy

ABSTRACT

A Hydrodynamic Study of Flow
in Irrigation Furrows

by

Otto Charles Wilke

Equations of motion describing flow in irrigation furrows are derived and presented in characteristic form. Predicted flow profiles obtained from approximate numerical solutions of the equations of motion did not compare well with measured flow profiles. An estimate of furrow hydraulic roughness was obtained from field data. A procedure for determining infiltration rates from measurements of surface flow volume and irrigation stream advance is proposed for the case for which the cumulative infiltration is described by the Kostiakov-Lewis equation. Numerical solutions of the steady-state form of the flow equations were used to prepare design curves providing estimates of cutback flow rates for preventing tailwater losses. Sample problems illustrate how these reduced rates of application can be utilized to design furrow irrigation distribution systems to obtain improved irrigation efficiencies and subsurface water distribution patterns.

Descriptors: Furrow irrigation, rates of application, infiltration, roughness (hydraulic), irrigation efficiency, distribution patterns, irrigation design, distribution systems

Identifiers: Irrigation stream advance, cutback furrow streams, tailwater losses

ACKNOWLEDGMENTS

My wife, Virginia, sustained my efforts by her faith and encouragement.

Dr. Ernest T. Smerdon directed this research, providing specific ideas and reference material as well as general guidance.

Mr. B. C. Moore, Dr. R. A. Clark, Dr. W. B. Davis, Dr. O. R. Kunze, Dr. R. R. Hocking, and Dr. E. A. Hiler reviewed this manuscript and offered many helpful suggestions and corrections.

Mr. Fred Schmer aided in the preparation of the manuscript for publication.

Mrs. Florace Kling patiently typed the text and each of the many equations.

Mr. Barney Ballard assisted in the drawing of the figures.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

Competition among industrial, municipal, and agricultural users of water for available supplies is increasing. In some areas, the High Plains of Texas for example, ground water supplies are being depleted. Pollution also decreases the supply of water suitable for irrigation purposes.

The annual income from irrigated crops in Texas alone is in excess of 600 million dollars (36). An increase in production with a given supply of water or decrease in the amount of water necessary to maintain a given production, by virtue of improvements in the design of irrigation systems, would be of considerable value.

Ideally the amount of water infiltrating into the soil should be equal at all points along the length of the irrigation run and thus no water would be wasted. Since the water advances from the supply at the head of the run to the lower end of the run, the upper end generally receives the most water. The amount of water infiltrating into the soil at any point along an irrigation run is affected most by the length of time the soil surface has been wetted. Thus, a system should be designed so that water is standing over each location along the run an approximately equal amount of time.

The citations on the following pages follow the style of the Transactions of the American Society of Agricultural Engineers.

It is desirable to be able to describe mathematically the advance of water along the run and the recession of water after the flow to the furrow or border is stopped. Continuity equations which adequately describe only the rate of advance for the case of constant inflow and variable infiltration are available, but have restrictive assumptions. Finite difference solutions of the equations of motion, i.e., the continuity equation and the momentum equation, have been obtained for the case of one-dimensional flow in an infinitely wide channel with constant or variable infiltration. Apparently no solution has been available for the more general case with variable inflow into a shallow channel of finite cross-section and having an infiltration rate that is a known function of time. Furthermore, a solution for this general case is necessary for determining both the advance and recession of a furrow irrigation stream and consequently necessary for determining the distribution of infiltration amounts along the furrow length.

Hansen (13) listed stream size, depth of flow, infiltration rate, length of run, slope of the surface, surface roughness, channel shape, erosion hazard, and depth of water to be applied as some of the variables influencing the design of an irrigation system.

In 1938, Lewis and Milne (20) applied the principle of continuity of mass to determine the rate of advance of a border irrigation stream. Their derivation involved the assumption of constant inflow to the border and constant flow depth on the border. Utilizing the Faltung theorem of Laplace transforms, Philip and Farrell (26), in 1964, presented a general solution of Lewis and Milne's equation. One particular form of their solution has been compared with field data by the writer and has been found to be adequate (40). Fok and Bishop (11) have also published an equation describing the rate of advance of an irrigation stream with constant inflow and variable infiltration. Neither method is useful for predicting the recession of an irrigation stream.

Spatially varied unsteady surface flow can be described by two simultaneous differential equations normally termed the equations of motion. These equations can be found in numerous texts, e.g., Chow (6) and Rouse (27). Lamb (19) notes that the rate of change of momentum of a fluid element in motion past a fixed region is equal to the sum of the change in momentum within the fixed region and the flux of momentum outwards across the boundaries.

Numerical solution of the equations of motion was suggested by Massau (23) and by Thomas (37) but convergence and stability must be checked for each solution. Isaacson *et al.* (15), and Stoker (35) solved the equations for gradually varied unsteady flow without lateral inflow using an explicit finite difference scheme. They also used the theory of characteristics to prevent their solution from becoming unstable. Liggett (21), Chen (3), and Kruger and Bassett (18) have used various methods in solving the equations of motion for the specific problem of overland flow. Liggett applied the theory of characteristics to a modified form of the equations. Liggett's modification of the momentum equation seems to be applicable only when the lateral inflow is very small.

Schreiber (30) used an implicit central difference technique to predict the recession of flow over an infinitely wide porous bed having constant infiltration. He showed that this technique is unconditionally stable. However, in a recent paper, Liggett and Woolhiser (22) argued that under certain conditions some finite difference techniques may yield unstable results even though they are theoretically stable. Wei (39) used an explicit central difference scheme to compute overland flow hydrographs resulting from a measured storm. Wei's analysis is based on the assumption of an infinitely wide flow plane. Chen and Hansen (5) suggest that the key to the solution of the equation may lie in the characteristics of the equations and the dimensionless parameters involved.

In any case some estimate of the friction slope must be made. The general practice is to assume that the roughness acts the same for unsteady gradually varied flow as it does for uniform flow and Manning's n is calculated from measurements of uniform flow.

The uniformity of distribution of infiltrated water has been termed the water distribution efficiency and an equation for this efficiency is given by Israelsen and Hansen (14). The effect of the variability of infiltration rate with time on water distribution efficiency has been studied by Bishop (2), Smerdon (32), and Smerdon and Glass (33).

The purpose of this study was to formulate, investigate the nature of, and seek solutions of the equations of motion describing unsteady gradually varied flow along irrigation furrows for the case in which the lateral outflow due to infiltration is a function of time. Any solution or approximate solution will be analyzed to determine the effect of time variations in the inflow into a furrow on the uniformity of distribution of infiltrated water along the irrigated runs.

CHAPTER II

EQUATIONS OF MOTION

The flow generally occurring in irrigation furrows is a spatially varied unsteady flow with lateral outflow. The equations of motion describing such flow are based on the principles of conservation of mass and momentum. The derivations will be presented in detail. The assumptions involved in the derivations are:

1. Forces normal to the direction of flow are negligible. This requires that the channel be prismatic with small, constant bottom slope. The vertical depth of flow is approximately equal to the flow depth normal to the channel bottom.

2. The momentum or Boussinesq coefficient is unity.

3. The head loss at a section is the same as for a uniform flow having the same velocity and hydraulic radius of the section.

A representative fluid element is depicted in Fig. 2.1.

The Continuity Equation

At time, t, the velocity and area of flow at section 1 are V and A. The mass per unit time entering the fluid element at section 1 at time t is

$$\rho AV. \qquad 2.1$$

The fluid density ρ is assumed to be constant. At time, t + dt, the mass per unit time entering the fluid element is given by the expression,

$$\rho(A + A_t\, dt)(V + V_t\, dt). \qquad 2.2$$

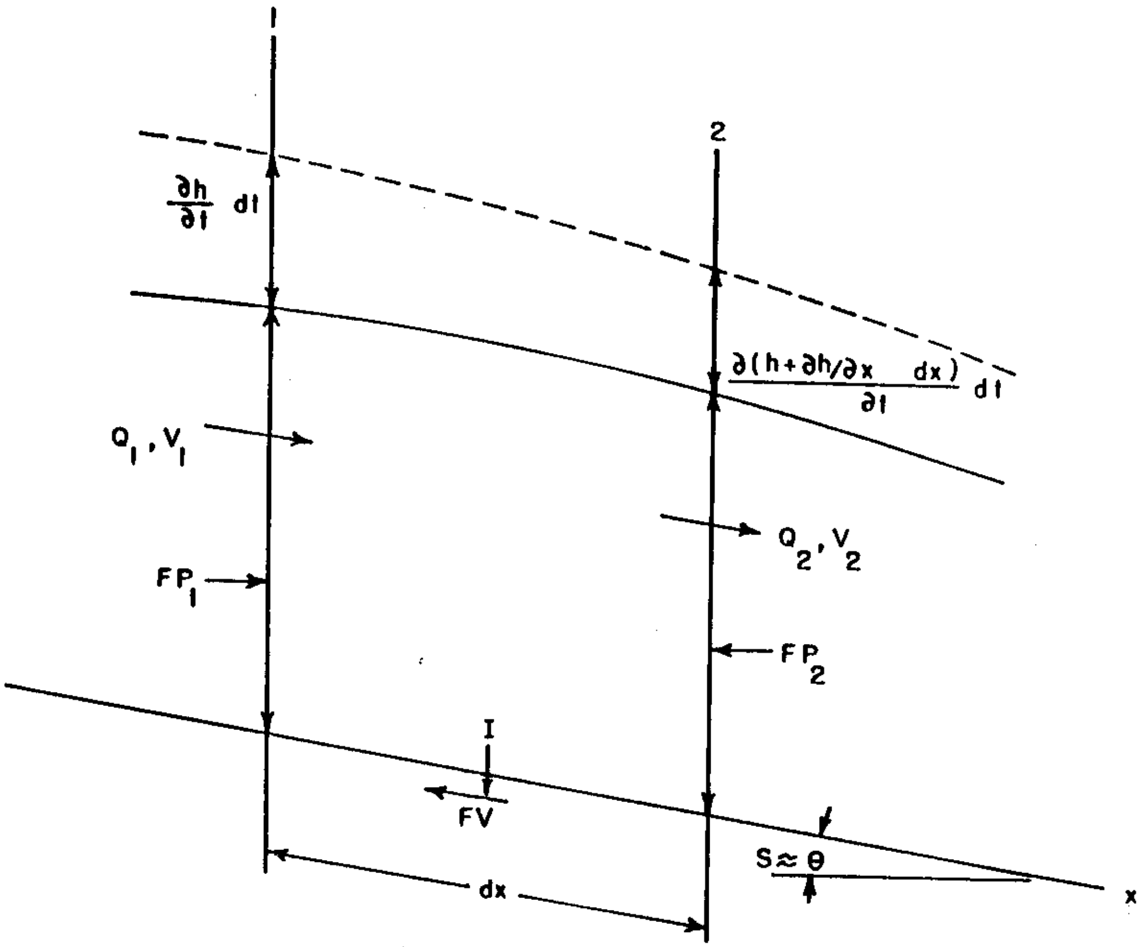

FIG. 2.1. A REPRESENTATIVE ELEMENT OF SPATIALLY VARIED UNSTEADY OPEN CHANNEL FLOW.

The symbolism f_t represents $\partial f/\partial t$. The average rate at which mass is entering the fluid element at section 1 is assumed to be the numerical average of expressions 2.1 and 2.2. Neglecting products of differentials, the average mass entering the system in time dt is

$$\tfrac{1}{2}\,\rho(AV + AV + A\,V_t\,dt + V\,A_t\,dt)dt. \qquad 2.3$$

The mass leaving the fluid element at section 2 per unit of time t by means of surface flow is

$$\rho(V + V_x\,dx)(A + A_x\,dx). \qquad 2.4$$

Because the furrow slope is assumed small, the distance, x, is measured in the direction of flow. The distance increment, dx, is the distance between section 1 and section 2.

The rate at which mass is leaving the system by means of surface flow at time t + dt is

$$\rho[V + V_x\,dx + (V + V_x\,dx)_t\,dt]$$
$$[A + A_x\,dx + (A + A_x\,dx)_t\,dt]. \qquad 2.5$$

The average rate at which mass is leaving the system is assumed to be the numerical average of expressions 2.4 and 2.5. Thus the mass leaving the system by means of surface flow in time dt, neglecting higher order products of differentials, is

$$\rho[AV + A\,V_x\,dx + V\,A_x\,dx + \tfrac{1}{2}(A\,V_t\,dt + V\,A_t\,dt)]dt. \qquad 2.6$$

Let I be the volume of water infiltrating into the soil per unit of channel length per unit of time (dimensionally, $I \doteq L^2/T$). I denotes the infiltration rate at section 1 at time t. The average rate at which mass is leaving the fluid element by means of infiltration at time t is given by the expression,

$$\tfrac{1}{2}\rho(I + I + I_x\ dx)dx. \qquad 2.7$$

At time t + dt, the average loss of mass by infiltration per unit time is

$$\tfrac{1}{2}\rho\ [I + I_t\ dt + I + I_x\ dx + (I + I_x\ dx)_t\ dt]\ dx. \qquad 2.8$$

Neglecting higher order differentials, the approximate mass leaving the system by infiltration in time dt is

$$\rho I\ dx\ dt. \qquad 2.9$$

The mass present in the fluid element at time t is approximately

$$\tfrac{1}{2}\rho(A + A + A_x\ dx)dx. \qquad 2.10$$

The mass present in the fluid element at time t + dt is

$$\tfrac{1}{2}\rho\ [A + A_t\ dt + A + A_x\ dx + (A + A_x\ dx)_t\ dt]\ dx. \qquad 2.11$$

Neglecting higher order differentials, the increase in mass present in the fluid element during time increment dt is

$$\rho A_t\ dx\ dt. \qquad 2.12$$

The continuity equation, obtained by equating the net mass entering the system to the increase in storage, is

$$A\ V_x + V\ A_x + A_t + I = 0 \qquad 2.13$$

The Momentum Equation

Newton's second law states that the unbalanced force is equal to the time rate of change of momentum,

$$F = d(mV)/dt, \tag{2.14}$$

in which F is the net force acting on the fluid element and m is the mass of the fluid contained in the element. Lamb (19) noted that the rate of change of momentum of a fluid element in motion past a fixed region is equal to the sum of the change in momentum within the fixed region and the flux of momentum outwards across the boundaries.

The average momentum entering the fluid element defined by Fig. 2.1 per unit of time is

$$\rho QV + \tfrac{1}{2}\rho(QV)_t\ dt = \rho[QV + \tfrac{1}{2}\ Q\ V_t\ dt + \tfrac{1}{2}\ V\ Q_t\ dt], \tag{2.15}$$

in which Q is the volume of fluid per unit of time entering the system at section 1. The average momentum leaving the system per unit time is

$$\tfrac{1}{2}\rho\ \{(Q + Q_x\ dx)(V + V_x\ dx) + [Q + Q_x\ dx + (Q + Q_x\ dx)_t\ dt]$$
$$[V + V_x\ dx + (V + V_x\ dx)_t\ dt]\ \}. \tag{2.16}$$

The net momentum flux outwards across the boundaries per unit time, neglecting higher order differentials, is

$$\rho[Q\ V_x\ dx + V\ Q_x\ dx]. \tag{2.17}$$

Since the velocity of that part of the fluid which infiltrates into the soil is quite small, it will be assumed to remove no momentum from the surface flow.

The average momentum within the fluid element at time t is

$$[\tfrac{1}{2}\rho(A + A + A_x\ dx)\ dx][\tfrac{1}{2}(V + V + V_x\ dx)]. \qquad 2.18$$

At time, t + dt, the approximate momentum within the fluid element becomes

$$\{\tfrac{1}{2}\ \rho[A + A_t\ dt + A + A_x\ dx + (A + A_x\ dx)_t\ dt]dx\ \}$$

$$\{\tfrac{1}{2}[V + V_t\ dt + V + V_x\ dx + (V + V_x\ dx)_t\ dt]\ \}. \qquad 2.19$$

Neglecting higher order differentials, the change in momentum within the fluid element per unit time is

$$(\rho V\ A_t\ dx\ dt + \rho A\ V_t\ dx\ dt)/dt. \qquad 2.20$$

The time rate of change of momentum is given by the sum of expressions 2.17 and 2.20;

$$d(mV)/dt = \rho[Q\ V_x\ dx + V\ Q_x\ dx + V\ A_t\ dx + A\ V_t\ dx]. \qquad 2.21$$

Three forces acting on the fluid element will be considered. These are (1) that component of the gravity force acting in the direction of flow, (2) the net pressure force acting in the direction of flow, and (3) the resisting viscous or shear force which is considered to act parallel to the direction of flow.

The component of gravity force acting in the direction of flow is

$$FG = \rho g\ (\text{average volume})\ \sin\theta\ , \qquad 2.22$$

in which θ is the angle between the furrow bed and a horizontal datum and $\sin\theta \approx S$, where S is the furrow slope. Neglecting higher order differentials, the gravity force component is given by the equation,

$$FG = \rho AgS\ dx. \tag{2.23}$$

The approximate net pressure force in the direction of flow is

$$FP = -\rho gA\ dx[(\bar{h}/A)A_x + \bar{h}_x], \tag{2.24}$$

in which $\bar{h}$ is the distance from the free surface to the center of gravity. The expression in parentheses on the right hand side of Equation 2.24 may be written

$$(\bar{h}/A)\ A_x + \bar{h}_x = [(h - d)/A]\ A_x + (h - d)_x, \tag{2.25}$$

in which d is the distance from the channel bed to the center of gravity of flow. Since the channel is prismatic, the distance to the center of gravity from the bed can be defined as

$$d = \left[\int_0^A h'\ dA'\right] /A, \tag{2.26}$$

in which h' and A' are variables of integration. Thus Equation 2.25 may be written:

$$(\bar{h}/A)\ A_x + \bar{h}_x = (h/A)A_x - \left(\int_0^A h'\ dA'\right) A_x/A^2 + h_x - \left(\int_0^A h'\ dA'/A\right)_x. \tag{2.27}$$

By expanding the derivative of the quotient contained in the final term in Equation 2.27 and applying the Leibnitz rule for differentiation under an integral sign, Equation 2.24 can be written as:

$$FP = -\rho gAh_x\ dx. \tag{2.28}$$

The final force which will be considered is the viscous force in the direction of flow,

$$FV = -\rho gA(SF)\ dx, \tag{2.29}$$

in which SF is the estimated friction slope.

The momentum equation, Equation 2.14, for spatially varied unsteady open channel flow with lateral outflow can now be written in the form:

$$\rho[gAS\ dx - gAh_x\ dx - gA(SF)\ dx] = \rho[Q\ V_x\ dx + V\ Q_x\ dx + V\ A_t\ dx + A\ V_t\ dx]. \tag{2.30}$$

Replacing Q by AV and applying the continuity equation leads to the conventional form of the momentum equation,

$$S - SF = h_x + (V/g)\ V_x - VI/Ag + V_t/g. \tag{2.31}$$

Equations 2.13 and 2.31 contain derivatives of A, y, and V. Since $\partial A = W\partial h$, where W is the top width of flow, Equation 2.31 can be written:

$$S - SF - A_x/W - V\ V_x/g + VI/Ag - V_t/g = 0. \tag{2.32}$$

The Theory of Characteristics

The following analysis is taken from Courant and Friedrichs (7). Equations 2.13 and 2.32 form a pair of non-linear partial differential equations, L_1 and L_2, of the first order. Since the equations are to be solved simultaneously there must be a linear combination,

$$L_o = \lambda_1 L_1 + \lambda_2 L_2 = 0, \qquad 2.33$$

so that the derivatives of V and A combine to derivatives in the same direction. The coefficients, λ_1 and λ_2, are functions of the variables, A, V, x, and t.

The variable, $\sigma(x,t)$, can be chosen so that the direction of these derivatives is given by the ratio x_σ/t_σ. In order that A and V be differentiated in the proper direction the ratio of the coefficients must be

$$\frac{\lambda_1 V + \lambda_2/W}{\lambda_1 + 0\lambda_2} = \frac{\lambda_1 A + V\lambda_2/g}{0\lambda_1 + \lambda_2/g} = x_\sigma/t_\sigma. \qquad 2.34$$

Multiplying Equation 2.33 by x_σ yields

$$(\lambda_1 V + \lambda_2/W)\, A_\sigma + (A\lambda_1 + V\lambda_2/g)V_\sigma$$
$$+ [-\lambda_2(VI/Ag + S - SF) + \lambda_1 I]x_\sigma = 0. \qquad 2.35$$

Multiplying Equation 2.33 by t_σ gives

$$(\lambda_1 + 0\lambda_2)\, A_\sigma + (0\lambda_1 + \lambda_2/g)\, V_\sigma$$
$$+ [-\lambda_2(VI/Ag + S - SF) + \lambda_1 I]t_\sigma = 0. \qquad 2.36$$

Equations 2.34, 2.35, and 2.36 form the following four linear homogeneous equations for λ_1 and λ_2:

$$\lambda_1(V\, t_\sigma - x_\sigma) + \lambda_2(t_\sigma/W) = 0, \qquad 2.37$$

$$\lambda_1(A\, t_\sigma) + \lambda_2(V\, t_\sigma/g - x_\sigma/g) = 0, \qquad 2.38$$

$$\lambda_1(V\, A_\sigma + A\, V_\sigma + I\, x_\sigma) + \lambda_2\, [A_\sigma/W + V\, V_\sigma/g$$
$$-x_\sigma(VI/Ag + S - SF)] = 0, \qquad 2.39$$

and

$$\lambda_1 (A_\sigma + I\ t_\sigma) + \lambda_2 [V_\sigma/g - (VI/Ag + S - SF)t_\sigma] = 0. \qquad 2.40$$

In order that Equations 2.37 through 2.40 have non-trivial solutions for λ_1 and λ_2 the coefficient determinants must vanish. From Equations 2.37 and 2.38,

$$(1/g)(x_\sigma)^2 - (2V/g)t_\sigma x_\sigma + (V^2/g - A/W)(t_\sigma)^2 = 0, \qquad 2.41$$

and

$$x_\sigma/t_\sigma = V \pm (g/2)\sqrt{4A/gW} = V \pm \sqrt{gD}\ . \qquad 2.42$$

D denotes the hydraulic depth, A/W. Since 4A/gW is real and positive, two characteristic directions, x_σ/t_σ, exist, and Equations 2.13 and 2.32 form a set of hyperbolic partial differential equations (7).

The value of the coefficient determinant of Equations 2.38 and 2.40 is

$$(A/g)\ V_\sigma t_\sigma - A(VI/Ag + S - SF)(t_\sigma)^2$$
$$- (A_\sigma + I\ t_\sigma)(V\ t_\sigma/g - x_\sigma/g) = 0. \qquad 2.43$$

Division by $(t_\sigma)^2$ and substitution of $V + \sqrt{gD}$ and $V - \sqrt{gD}$, respectively, for x_σ/t_σ yields

$$V_\sigma/t_\sigma \pm (\sqrt{gD}/A)(A_\sigma/t_\sigma) = g(S - SF)$$
$$+ IV/A \mp I\ \sqrt{gD}/A\ . \qquad 2.44$$

Data, which are presented in Chapter III, indicate that the shape of an irrigation furrow can be described by an equation of the form,

$$A = b_1 h^b, \qquad 2.45$$

in which A is the cross-sectional area, in square feet, h is the depth of flow, in feet, and b_1 and b are constants. For this case the ratio of depth to hydraulic depth, h/D, is the constant b. Thus the quantity, $(\sqrt{gD}/A)\ A_\sigma$, is equivalent to $(2b\sqrt{gD})_\sigma$, and Equations 2.42 and 2.44 can be written:

$$dx/dt = V \pm \sqrt{gD}\ , \qquad 2.46$$

and

$$d(V \pm 2b\ \sqrt{gD})/dt = g(S - SF) + IV/A \mp I\sqrt{gD/A}. \qquad 2.47$$

Other characteristic relations could be derived from Equations 2.37 through 2.40 but would provide no further information.

Equations 2.13 and 2.32 or 2.46 and 2.47 form the desired mathematical model of flow in irrigation furrows. However, before useful solutions can be obtained, an adequate determination of the friction slope, SF, and the infiltration rate, I, must be made.

CHAPTER III

FRICTION SLOPE EVALUATION

The frictional resistance to flow in open channels has historically been characterized by three parameters: Chezy's C, Manning's n, and the Darcy-Weisbach friction factor, f (1). The values of these parameters are generally considered constant for a channel having a particular surface roughness, but are known to vary with changes in the ratio of depth of flow to the height of the surface roughness elements and with changes in other flow parameters.

Derivations of relationships between these three resistance parameters and other variables of flow depend on the assumption that steady, uniform flow exists. The flow which typically occurs in irrigation furrows is a gradually varied, unsteady flow. For such flow the head loss at a section is generally assumed to be the same as for a uniform flow having the same velocity and hydraulic radius (6). This assumption has been almost universally applied in the study of the hydraulics of furrow irrigation (18, 24).

Thus the friction slope in the momentum equation can be evaluated by one of the three uniform flow formulae:

$$SF = fV^2/8Rg, \qquad 3.1$$

$$SF = (Vn/1.486R^{2/3})^2 \quad \text{[English units]}, \qquad 3.2$$

and

$$SF = V^2/C^2R. \qquad 3.3$$

V is the mean velocity obtained from Q/A, Q is the flow rate and A is the cross-sectional area of flow, R is the hydraulic radius, and g is the acceleration of gravity.

The following relationships (1) hold between the resistance parameters:

$$C = (8g/f)^{1/2}, \qquad 3.4$$

and

$$n = 1.486\ R^{1/6}\ (f/8g)^{1/2} \qquad \text{[English units].} \qquad 3.5$$

Theoretical relationships for the resistance parameters, f, n, and C (6), are based on the assumption of a logarithmic velocity profile in the turbulent region and on various other assumptions. For hydraulically smooth surfaces having roughness protuberances which do not extend through the laminar sublayer into the turbulent region of flow, the friction factor is a function of the Reynolds' number, Re. For rough surfaces, the friction factor is a function of the hydraulic radius. The following relations are given:

$$1/(f)^{1/2} \sim \log\ (Re(f)^{1/2}/c_1) \quad \text{(smooth surfaces),} \qquad 3.6$$

and

$$1/(f)^{1/2} \sim \log\ (c_2R/k_s) \quad \text{(rough surfaces),} \qquad 3.7$$

in which k_s is the height of uniformly spaced roughness elements, Re is Reynolds' number, Re = $4RV/\nu$, and c_1 and c_2 are constants.

However, as noted in 1963 by the ASCE Task Force on Friction Factors in Open Channels (1), the shape and longitudinal spacing of the roughness elements also affect the resistance to flow. In addition, none of the above relations strictly apply when erosion or silting is occurring in the channel. Finally, for Froude numbers near unity, such as might be expected near the wetting front, free-surface flow becomes unstable and the frictional resistance increases (6).

Resistance to flow in irrigation furrows is caused by soil clods, grass, and dead leaves, in addition to grain roughness. The clods may remain stable when wetted or may disintegrate. Erosion and silting may change the channel shape and roughness during the flow period. Local variations in channel shape are common and collection of debris may cause damming. If the equations of motion are to be used to predict the advance and recession of a furrow irrigation stream, then the resistance to flow that actually occurs in the field must be estimated.

Olsen (24) successfully used a chosen constant value of Chezy's C to predict the advance of water over an irrigation border. Davis (9) suggested that, at high flow velocities, "the flow tends to smooth the furrow and dissipate the cloddiness, which reduces the roughness of the furrow." He presented data relating Manning's n and the mean velocity of flow. Thornton (38) noted a similar relationship.

In the following, data from 36 furrows at four separate locations in Texas are presented. These data were collected by Smerdon and Hohn (34), and by several investigators, including the writer, over a period of years.

The flow rate into each furrow was measured either by volumetric catchment or by use of a V-notch weir. Depths of flow were measured at uniformly spaced positions along the furrow length. After the water was turned off, measurements of furrow shape were made. Average relationships between flow depth and top width are presented in Fig. 3.1. A power equation (34),

$$W = b_1 b h^{b-1}, \tag{3.8}$$

in which W is the top width, h is the depth of flow, and b_1 and b are constants, describes the relationships quite well.

Integration of Equation 3.8 results in equations for the area of flow, A,

$$A = b_1 h^b. \tag{3.9}$$

The wetted perimeter, P, is given by the equation,

$$P = 2\int_0^h \sqrt{1 + \frac{d(W/2)}{dh'}}\ dh', \tag{3.10}$$

in which h is the particular depth of the flow in question, and h' is the variable of integration. Generally when $d(W/2)/dh' > 1$,

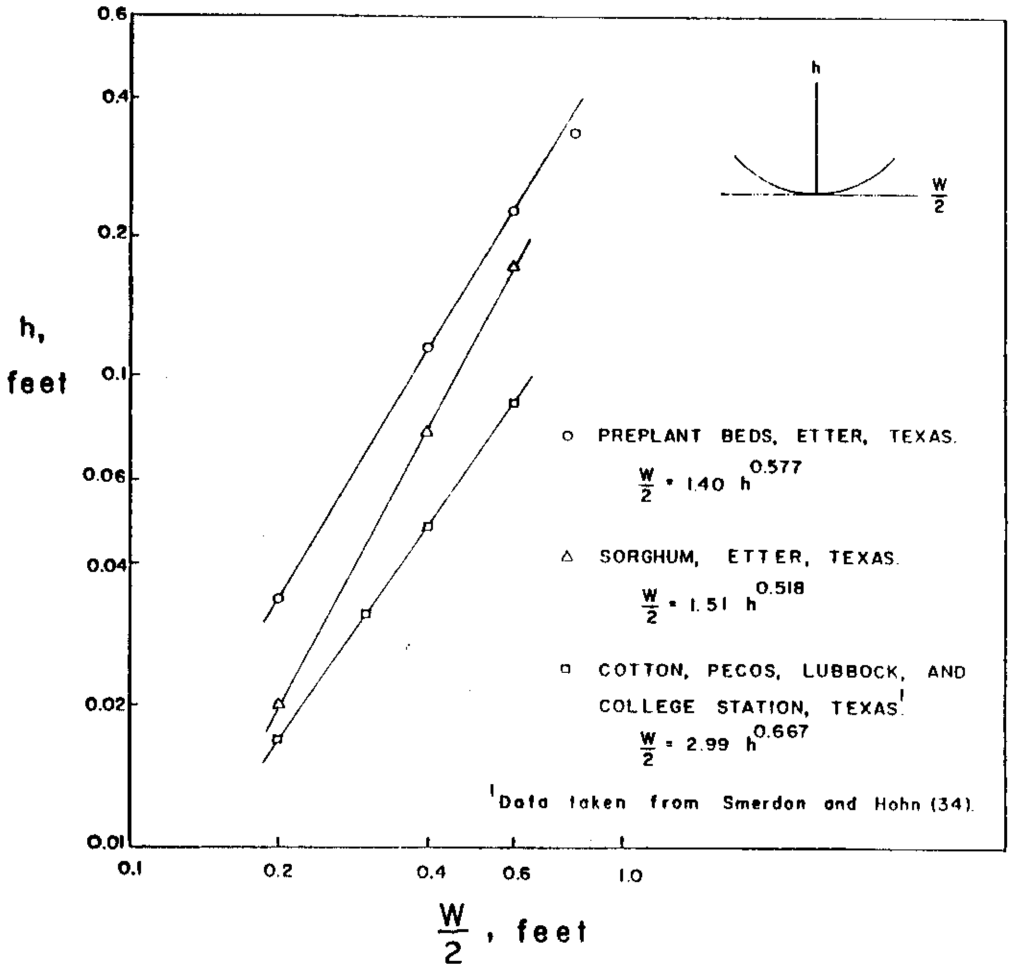

FIG. 3.1. RELATIONSHIP BETWEEN FLOW DEPTH AND TOP WIDTH FOR TYPICAL TEXAS IRRIGATION FURROWS.

the value of the integral in Equation 3.10 can be estimated by expanding the integrand into a binomial series and integrating term by term to obtain

$$P = b_1 bh^{b-1} + \frac{2h^{3-b}}{b_1 b(b-1)(3-b)} - \frac{2h^{7-3b}}{b_1^3\ b^3(b-1)^3\ (7-3b)} + \cdots \quad 3.11$$

Usually only the first two or three terms of Equation 3.11 need to be evaluated. Relationships between flow depth and wetted perimeter for the furrows studied are presented in Fig. 3.2. Due to irregularities in the channel surface, actual wetted perimeters are slightly greater than those obtained from Fig. 3.2

At this point the assumption is made that uniform flow exists except at the zone immediately behind the wetting front. Consequently, at the upstream end, (x = 0), the depth should approach the uniform flow depth corresponding to the inflow rate for that furrow.

In general, independent measurements of mean flow velocity were not made. The values of velocity were deduced from measurements of flow rate and cross-sectional area. The results of calculations leading finally to the determination of Manning's n and the Darcy-Weisbach f for each of the 36 furrows studied are presented in Table 3.1

Fig. 3.3 shows the relationship between Manning's n and velocity. Included on Fig. 3.3 are data taken from Davis (9) and Thornton (38). For a particular velocity, Manning's n appears to increase with increasing furrow slope. Illustrated in Fig. 3.4

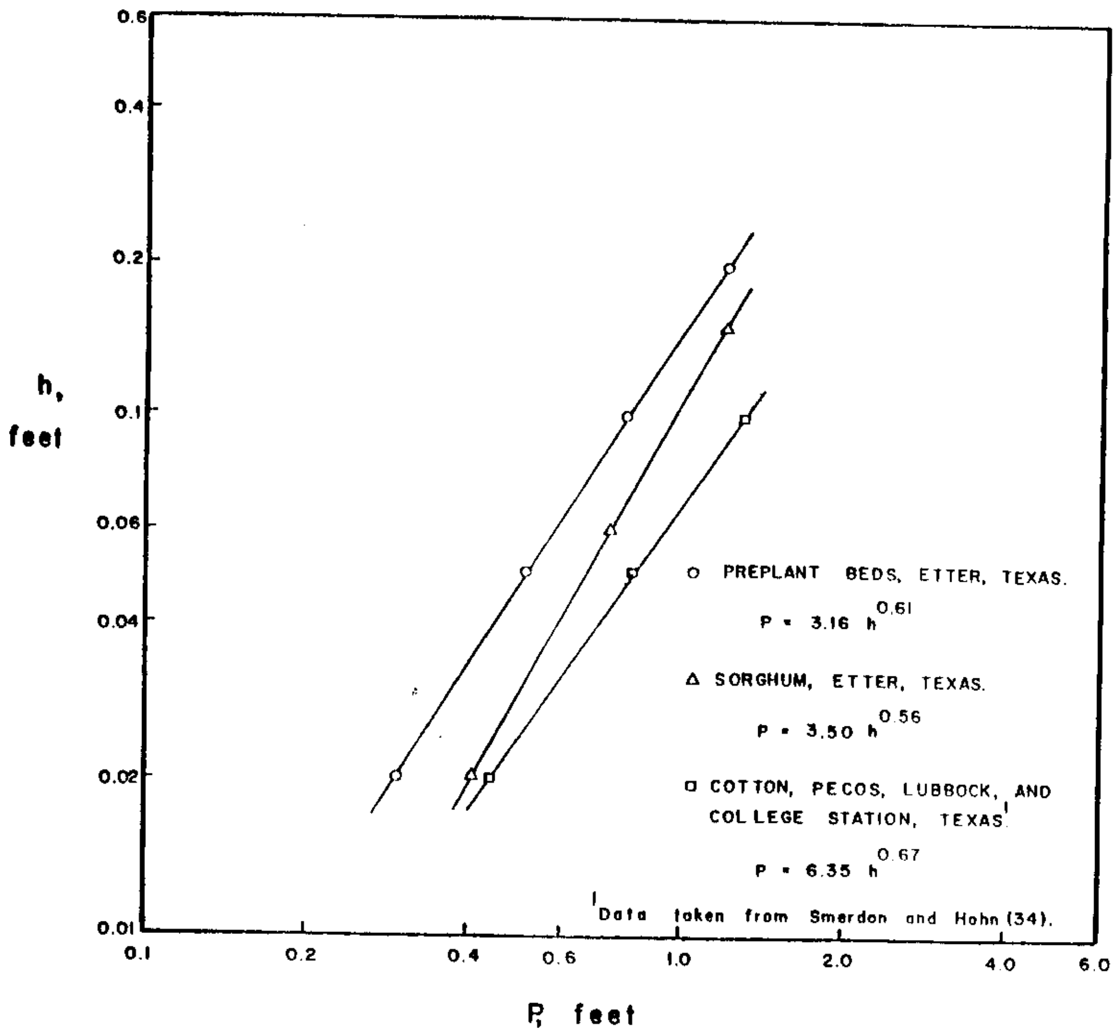

FIG. 3.2. RELATIONSHIP BETWEEN WETTED PERIMETER AND FLOW DEPTH FOR TYPICAL TEXAS IRRIGATION FURROWS.

TABLE 3.1. DATA FOR CALCULATION OF FRICTION COEFFICIENTS.

Location	Q cfs	S %	h_o ft.	A $ft.^2$	R ft.	f	n	C
Pecos Summer Irrigation of Cotton[1]	.0254	.307	.110	.090	.064	.657	.047	20.5
	.0452	.281	.135	.127	.079	.467	.041	25.6
	.0518	.285	.149	.148	.086	.551	.046	22.7
	.0445	.384	.138	.132	.081	.729	.053	19.7
	.0654	.369	.163	.170	.093	.651	.051	20.4
	.0720	.120	.200	.245	.117	.415	.042	25.0
	.0430	.126	.150	.150	.087	.361	.037	28.1
College Station Summer Irrigation of Cotton[1]	.0445	.060	.214	.268	.123	.745	.057	19.4
	.0335	.062	.179	.201	.101	.630	.051	18.6
	.0224	.035	.174	.190	.100	.683	.052	22.3
Lubbock Summer Irrigation of Cotton[2]	.0758	.10	.23	.305	.133	.584	.051	21.8
	.105	.10	.23	.305	.133	.305	.036	30.2
	.0961	.10	.20	.245	.117	.198	.029	36.9
	.0741	.10	.20	.245	.117	.331	.037	25.6
	.0730	.10	.27	.400	.155	1.20	0.74	14.8

Table 3.1. Continued.

Location	Q cfs	S %	h_o ft	A ft^2	R ft	f	n	C
	.123	.10	.23	.305	.133	.217	.031	35.3
	.106	.10	.19	.220	.110	.131	.023	46.1
	.0862	.10	.19	.220	.110	.197	.028	37.7
Etter	.0839	.53	.15	.111	.092	.22	.029	34.2
Summer Irrigation of Sorghum[2]	.0905	.53	.17	.135	.104	.28	.034	28.4
	.0405	.57	.16	.123	.098	1.24	.070	13.9
	.0750	.60	.145	.106	.089	.24	.030	30.7
	.0745	.61	.15	.111	.092	.32	.035	28.3
Etter	.120	.53	.20	.140	.118	.197	.039	34.6
Irrigation of Preplant Beds	.0853	.53	.22	.163	.130	.55	.048	19.9
	.0550	.53	.20	.140	.118	.93	.062	15.7
	.0814	.53	.22	.163	.130	.82	.059	19.1
	.0733	.53	.24	.187	.141	1.13	.087	14.4
	.0958	.53	.19	.130	.113	.25	.032	30.2
	.0850	.53	.17	.109	.101	.21	.029	33.8
	.132	.53	.25	.199	.147	.47	.046	22.1

TABLE 3.1. Continued

Location	Q cfs	S %	h_o ft	A ft^2	R ft	f	n	C
	.0916	.53	.24	.187	.141	.79	.081	18.0
	.0916	.53	.23	.176	.135	.68	.055	19.5
	.0450	.53	.17	.109	.101	.83	.058	17.9
	.0716	.53	.20	.140	.118	.55	.048	20.4
	.0416	.53	.155	.094	.093	.57	.047	19.9

[1] Data taken from Smerdon and Hohn (34).

[2] Data collected by L. J. Glass, Texas A&M University, 1966.

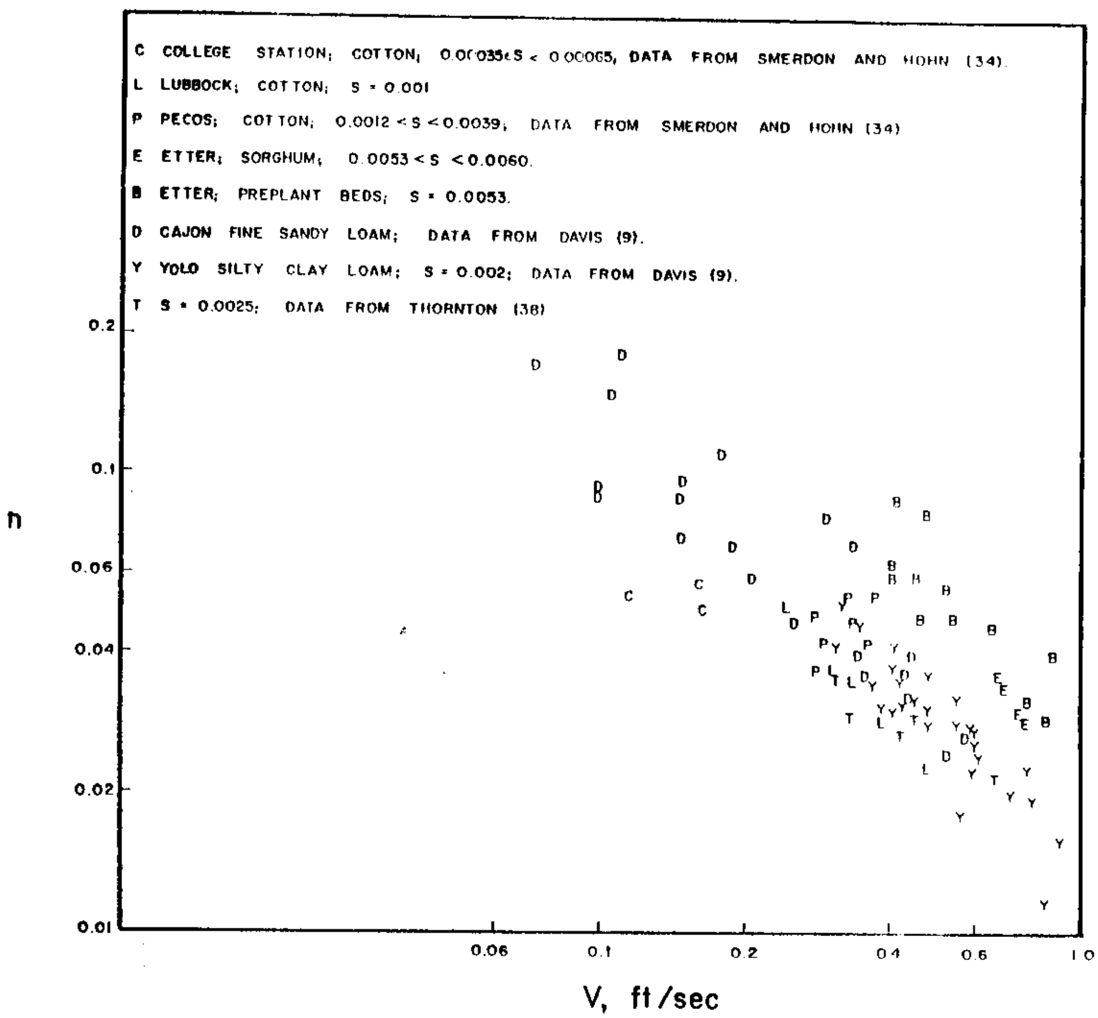

FIG. 3.3. MANNING'S n AS A FUNCTION OF FLOW VELOCITY.

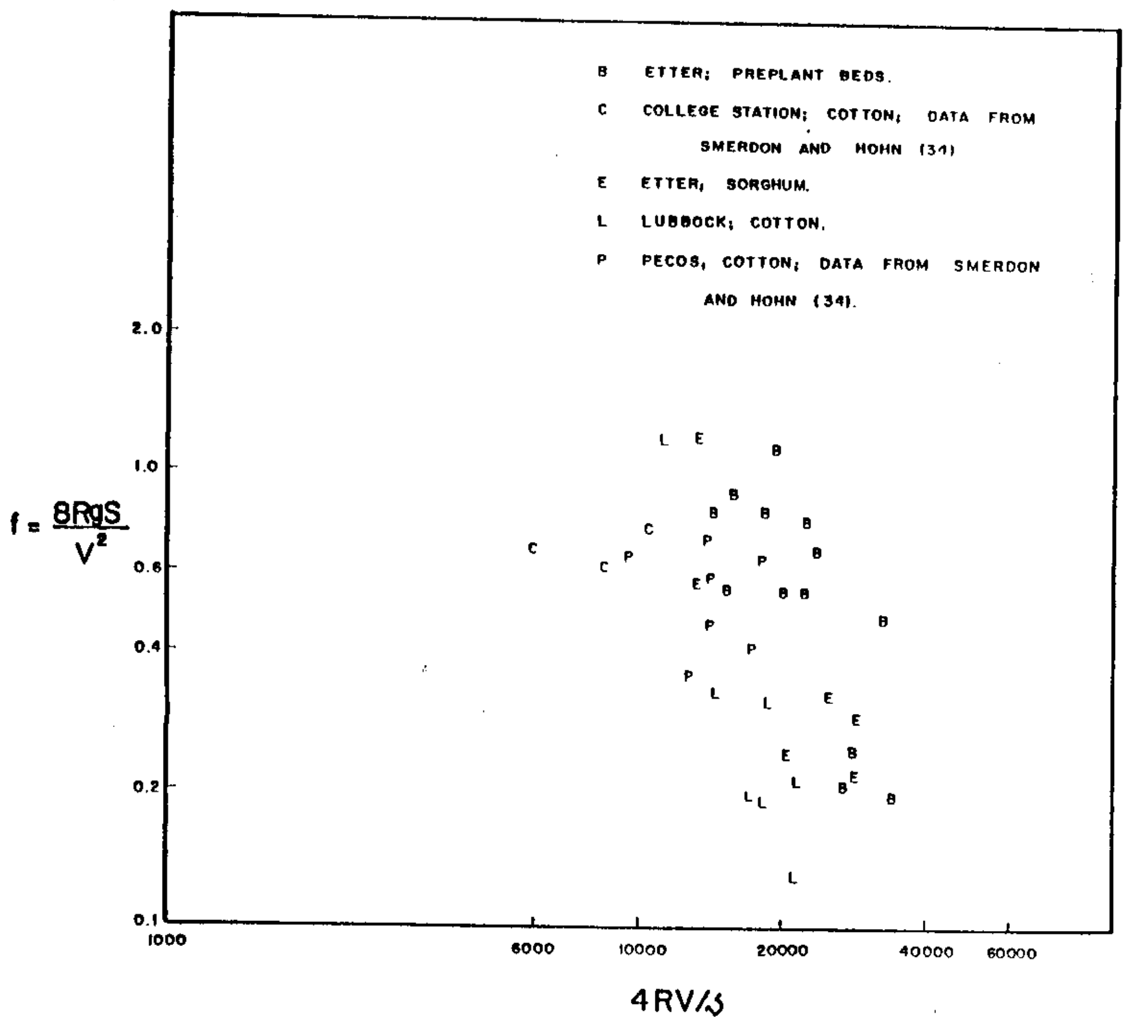

FIG. 3.4. THE DARCY-WIESBACH FRICTION FACTOR, f, AS A FUNCTION OF REYNOLDS' NUMBER.

is a logarithmic plot of friction factor versus Reynolds' number.

The accuracy of the relationships shown in Fig. 3.3 and Fig. 3.4 is questionable for several reasons. First, the velocity was not measured independently (for the 36 furrows in Texas). Errors in the calculation of velocity are due primarily to errors in the measurement of uniform flow depth and are magnified in the calculation of Manning's n and the Darcy-Weisbach f. The flow generally occurring in irrigation furrows appears turbulent in that dye is rapidly diffused and values of Reynolds' number are in the generally accepted turbulent range. Although the value of f is expected to vary some with variations in the Reynolds' number, the degree of variation illustrated in Fig. 3.4 is much greater than similar plots of flume data would lead one to expect (6). The fact that the logarithmic plot of Manning's n versus velocity has a slope approximately equal to minus one suggests that the relationship actually being presented is the relationship between an estimated velocity and its reciprocal. Again the variation of n with velocity seems excessive.

Due to local variations in furrow shape and slope, direct measurements of mean flow velocity are difficult to obtain. Independent measurements were made of flow depth, flow rate, furrow slope, and furrow shape. A logarithmic plot of $1.486AR^{2/3}$ *versus* $Q/S^{1/2}$ is shown in Fig. 3.5. The ordinate and abscissa values

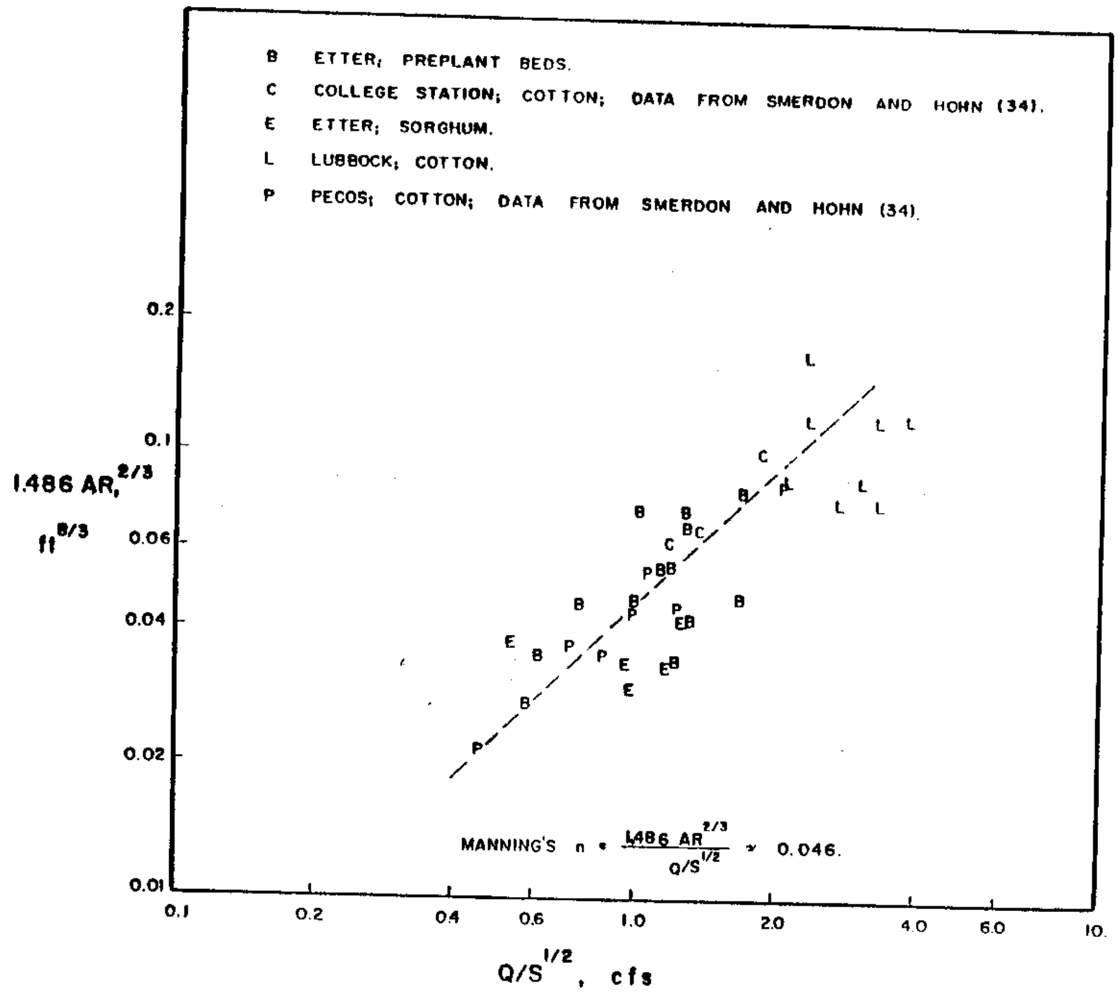

FIG. 3.5. RELATIONSHIP BETWEEN 1.486 $AR^{2/3}$ AND $Q/S^{1/2}$ FOR TEXAS IRRIGATION FURROWS.

are measured independently. The fact that the slope of this plot is approximately one supports the use of a constant value of Manning's n. An estimate of Chezy's C is similarly obtained from the logarithmic plot of $AR^{1/2}$ *versus* $Q/S^{1/2}$ shown in Fig. 3.6.

Although the value of the resistance parameters is expected to vary with variations in other flow parameters, this variation cannot be well described without an extended study which ought to include independent measurements of mean flow velocity. At the present time the assumption of a constant value of Manning's n for a particular furrow appears to be as practical and reasonable an assumption as one can make. A reasonable value of Manning's n for 40-inch irrigation furrows in Texas is 0.046.

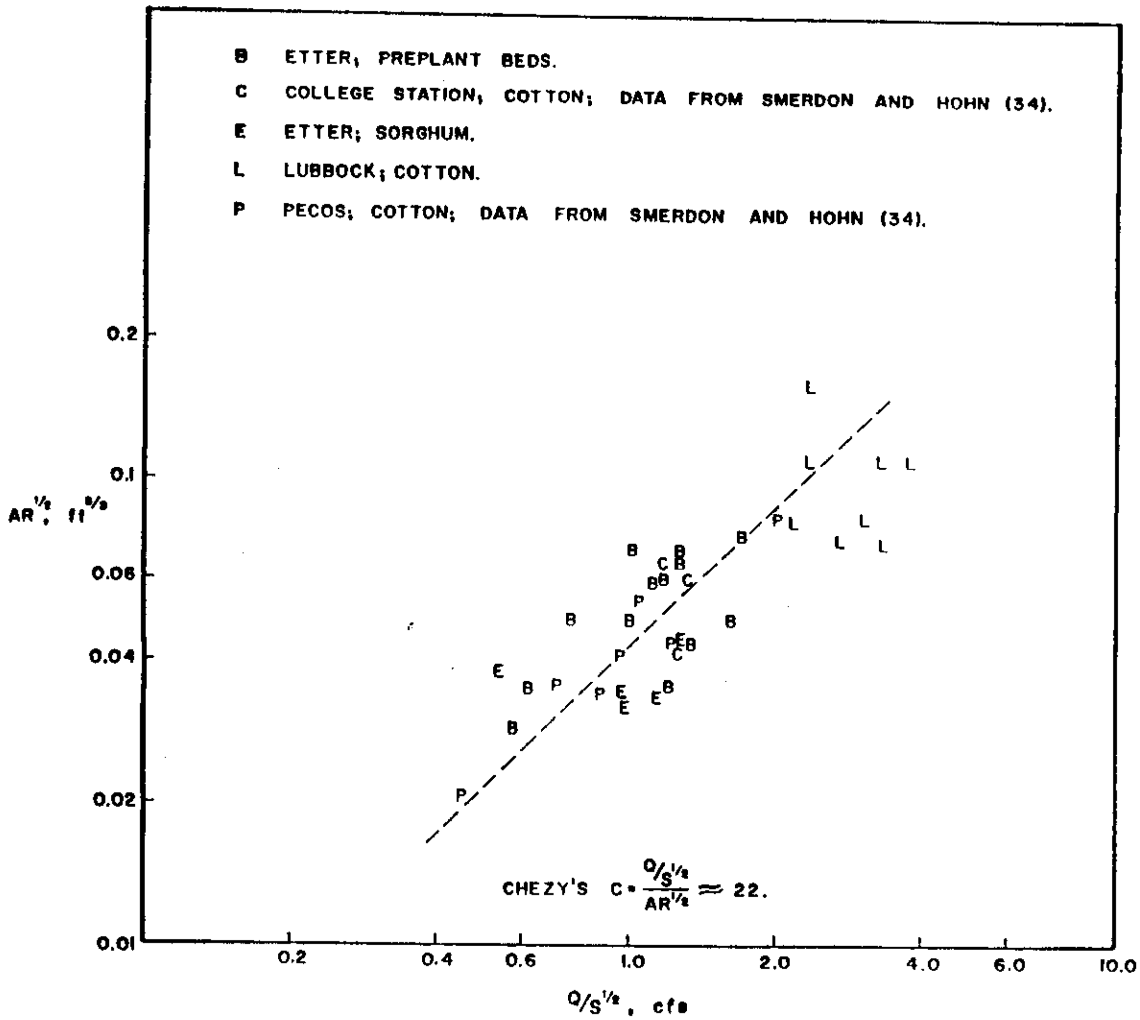

FIG. 3.6. RELATIONSHIP BETWEEN $AR^{1/2}$ AND $Q/S^{1/2}$ FOR TEXAS IRRIGATION FURROWS.

CHAPTER IV

DETERMINING THE INFILTRATION FUNCTION

One mathematical model for describing infiltration is based on the assumption that the soil water movement is primarily an isothermal liquid-phase diffusion phenomenon which obeys Darcy's law (12, 25). An intensive study of this model has been conducted by Philip (25). The potential causing flow includes gravitational and capillary components. Generally both the capillary potential and the unsaturated hydraulic conductivity are considered to be single-valued functions of the water content of a soil. Klute (17) applied the principle of continuity of mass to Darcy's law and explicitly derived a diffusion-type partial differential equation describing unsteady, unsaturated flow through porous media. Philip (25) developed a procedure for obtaining an approximate solution of Klute's equation for the case of one-dimensional infiltration into a homogeneous soil initially at a constant moisture content. A more recent approximate solution is given by Singh (31).

A related but simplified mathematical model of one-dimensional infiltration into a homogeneous soil has been developed by Fok and Hansen (12).

Although the Klute model applies to water movement in three dimensions, to the writer's knowledge no solution is yet available for two-dimensional infiltration subject to the boundary conditions present in irrigation furrows. The problem of describing infiltration into an agricultural soil is complicated by the fact that the soil typically consists of layers having different properties. Consequently, for engineering purposes, infiltration data are often described by empirical equations of various forms (20, 26).

One particular equation form which describes the intake behavior of irrigation furrows under a wide range of conditions is the Kostiakov-Lewis equation,

$$y = kt_0^{\alpha} , \qquad 4.1$$

in which y is the cumulative intake in cubic feet per foot of furrow length, t_0 is the time in minutes since the soil was wetted, and α and k are constants. For some soils the Horton equation as reported by Philip and Farrell (26) may describe infiltration data more accurately when the time range during which infiltration occurs is large.

During furrow irrigation water infiltrates into the sides of the furrow as well as vertically. In addition, the flow itself disturbs the soil surface and this may affect infiltration. Therefore, some doubt exists concerning the adequacy of ring or blocked-furrow infiltrometer data (34).

Finkel and Nir (10) have proposed a graphical procedure for deducing the infiltration rate from rate of advance data. The Philip and Farrell solution (26) of the Lewis-Milne volume-balance equation provides analytical tools for relating the irrigation stream advance and infiltration.

In the following a simple procedure is outlined for estimating the constants in Equation 4.1 when advance data are available. The Philip and Farrell solution (26) for the advance of an irrigation stream is

$$\frac{cx}{Qt} = \sum_{j=0}^{\infty} \frac{[-(kt^{\alpha}/c)\Gamma(1+\alpha)]^{j}}{\Gamma(2+j\alpha)} \qquad 4.2$$

In this discussion c is the constant average area of surface storage, in square feet; Q is the constant inflow into the furrow, in cubic feet per minute; x is the distance the wetting front has advanced, in feet; t is the time since the water was turned on, in minutes; Γ is the Gamma function; and j is an integer. The remaining variables are defined the same as for Equation 4.1. The asymtotic expansion for Equation 4.2, for large values of kt^{α}/c, also obtained by Philip and Farrell, is

$$\frac{cx}{Qt} = -\sum_{j=1}^{\infty} \frac{1}{[(-kt^{\alpha}/c)\Gamma(1+\alpha)]^{j}\Gamma(2-j\alpha)} \qquad 4.3$$

A dimensionless graphical representation of Equations 4.2 and 4.3 (40) for $0 \le kt^{\alpha}/c \le 10$ is shown in Fig. 4.1. Inspection of Equation 4.3 reveals that as the value of kt^{α}/c increases, a logarithmic plot of x *versus* t should approach a straight line having slope $1-\alpha$, as illustrated by the data shown in Fig. 4.2

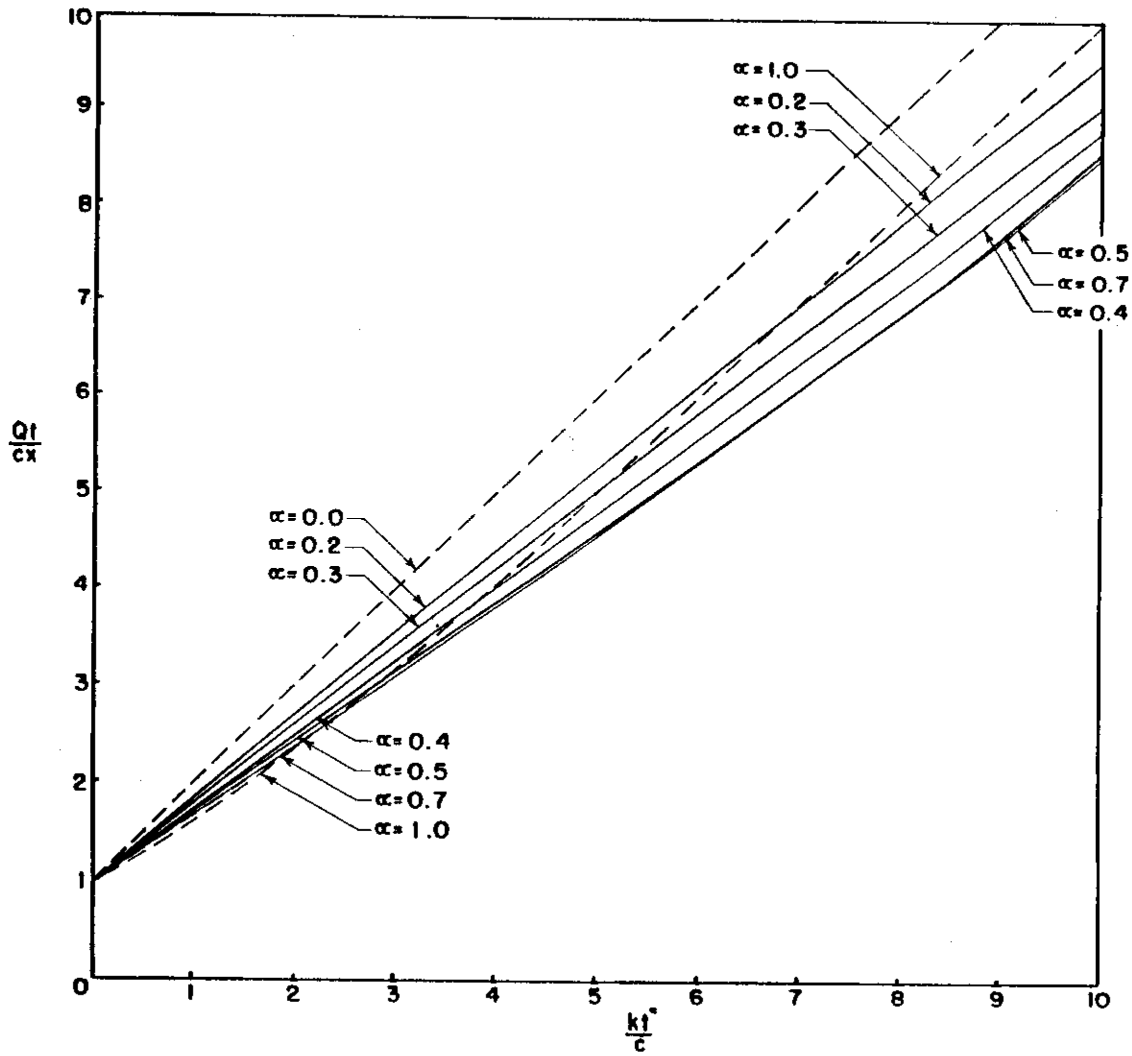

FIG. 4.1. DIMENSIONLESS ADVANCE CURVES

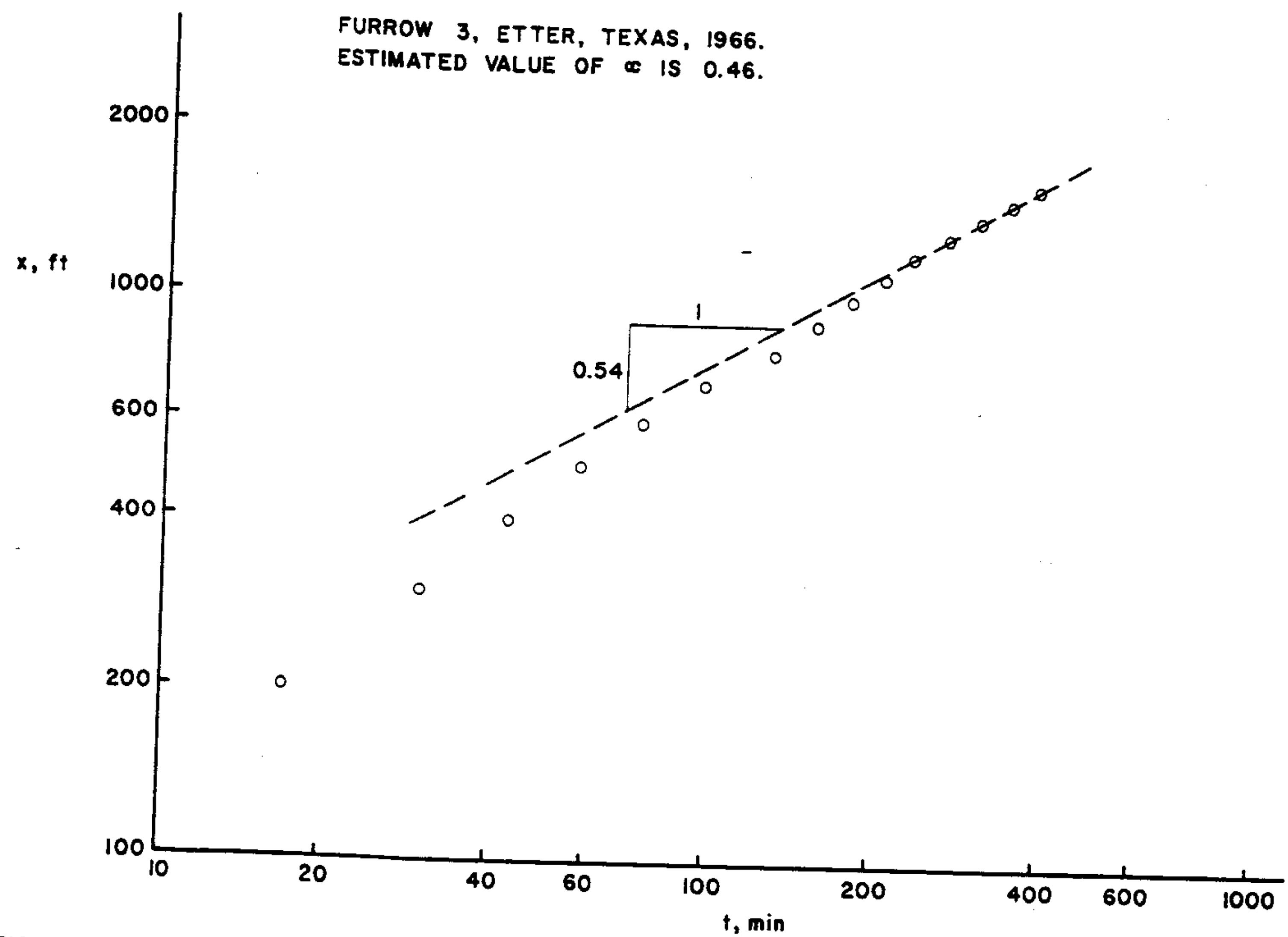

FIG. 4.2. REPRESENTATIVE ADVANCE RATE DATA.

The value of k can then be estimated from Equation 4.3 when all but the first one or two terms of the infinite series are disregarded. If advance data are not available for large enough values of kt^{α}/c, a first estimate of α can still be obtained from a logarithmic plot of x versus t. Then values of k are computed using Fig. 4.1. for several values of x and t. If the values of k are not approximately equal, a slightly different value of α is assumed and new values of k determined until the values of k do become nearly equivalent. This process is illustrated in the following example. The data for the example are the same as shown in Fig. 4.2.

Example: Furrow 3, Border Field, Etter 1966.

$Q = 2.42$ cfm, $c = 0.093$ ft ^{2}, $\alpha = 0.46$ (estimated from Fig. 4.2.)

t	x	Qt/cx	kt^{α}/c (Fig. 4.1.)	k
30	300	2.60	2.13	.0414
75	600	3.25	3.05	.0390
155	900	4.48	4.60	.0419
345	1500	5.97	6.50	.0412

$$y \approx 0.041\ t_o^{0.46}. \qquad 4.4$$

If the time range required for the desired amount of water to infiltrate into the soil during irrigation is considerably greater than time range for which advance data can be obtained, then data from blocked-furrow infiltrometers may provide a better estimate of the values of α and k. The average surface storage, c, is not always constant but generally increases slightly with time and approaches a constant value. The value of c can be determined by the method proposed by Smerdon and Hohn (34), who found that the value of c is approximately 77 per cent of the cross-sectional area of flow at the upstream end.

CHAPTER V

FINITE DIFFERENCE TECHNIQUES

A Dimensionless Form of the Equations of Motion

The number of variables which must be considered can be reduced by writing the equations of motion in dimensionless form.

Because flow depths occurring in irrigation furrows are usually less than 0.2 feet, the wetted perimeter can be approximated using only the first term in Equation 3.11. Thus, it follows that:

$$R = A/P \approx b_1 h^b / b b_1 h^{b-1} = h/b = D. \qquad 5.1$$

With the substitution of D for R and the assumption of a constant value of Chezy's C, the equations for furrow slope and friction slope are

$$S = V_o^2/C^2 D_o \qquad 5.2$$

and

$$SF = V^2/C^2 D. \qquad 5.3$$

V_o and D_o are the normal flow velocity and hydraulic depth corresponding to an initial flow rate, QI.

The following dimensionless variables can be defined:

$$x_* = x/D_o;\ t_* = tV_o/D_o;\ V_* = V/V_o;\ D_* = D/D_o;\ \text{and}$$

$$t_{o*} = t_o V_o/D_o. \qquad 5.4$$

Written in dimensionless form, Equations 2.46 and 2.47 become:

$$dx_*/dt_* = V_* \pm \sqrt{D_*}/Fr \tag{5.5}$$

and

$$d(V_* \pm 2b\sqrt{D_*}/Fr)/dt_* = r(1 - V_*^2/D_*) + z(t_{o*}^{\alpha-1}/D_*^b)(V_* \mp \sqrt{D_*}/Fr), \tag{5.6}$$

in which

$$Fr = V_o/\sqrt{gD_o}$$

$$r = g/C^2 ,$$

and

$$z = (D_o I_{t_*=1})/V_o A_{D=D_o}).$$

Equations 5.5 describe C^+ - and C^- - characteristic lines in the x_*-t_* plane (7). Equations 5.6 are valid along these characteristic lines.

Initial and Boundary Conditions

Initially no flow exists along the furrow bed. Saint-Venant (28) and Schoklitsch (29) have studied the initial propagation of a surge on a dry bed when water stored behind a vertical wall is suddenly released onto a horizontal bed. Their results indicate that critical flow exists at the point of release. Chen (4) has suggested critical flow as an initial upstream condition. Kruger and Bassett (18) found that the depth of flow at the upstream end rapidly approaches normal depth. They utilized normal flow as the initial upstream condition, as did Olsen (24).

The selection of boundary conditions for $t_*>0$ depends on the description of the flow at the wetting front. Chen (4) has discussed this problem. If one assumes that the depth at the leading edge of the wetting front is single-valued and equal to zero, then the C^+ - and C^- -characteristics at the wetting front are both defined by the equation,

$$dx_*/dt_* = V_*. \qquad 5.7$$

For this case the curve of the wetting front advance apparently represents an envelope of the C^+ - and C^- -characteristics. The pattern of characteristics for this case is shown in Fig. 5.1. Equations 5.6 are not defined, however, if $D_*=0$.

The leading edge of the wetting front is an approximately vertical wall of water. One might logically assume, therefore, that critical flow exists at the wetting front, with depth greater than zero, and, as suggested by Keulegan (16), that the front is propagated with speed $2\sqrt{gD}$. Thus, at the wetting front

$$dx_*/dt_* = 2V_* = V_* + \sqrt{D_*}/Fr. \qquad 5.8$$

For this case, the position of the "leading" C^+ -characteristic describes the advance of the wetting front. Equations 5.6 are defined since $D_*>0$. The infiltration dependent term disappears from the the first of Equations 5.6, eliminating the necessity

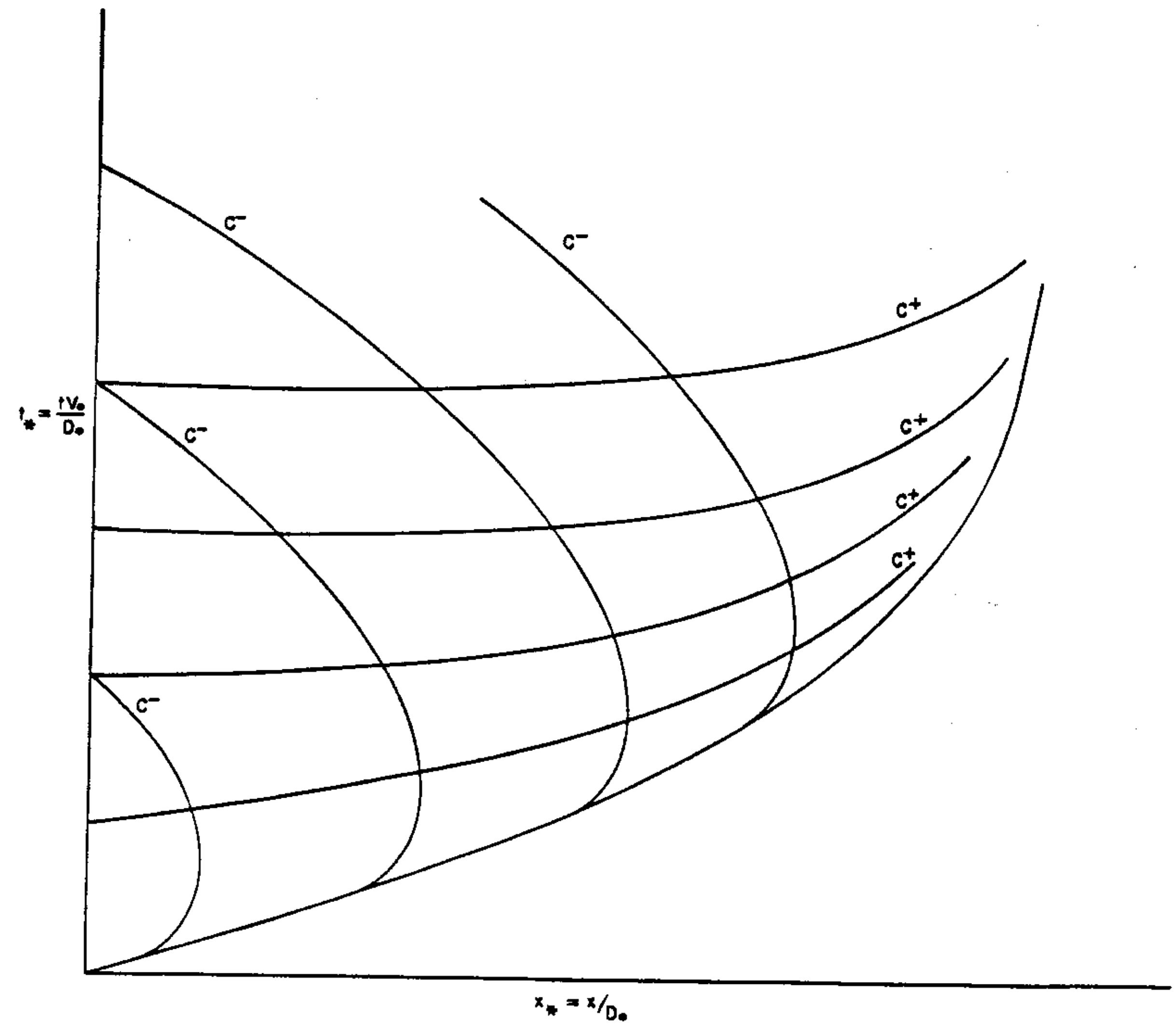

FIG. 5.1. PATTERN OF CHARACTERISTICS FOR $D_* = 0$ AT THE WETTING FRONT.

of defining the infiltration rate when the opportunity time is zero.

The rapidity with which flow conditions change in the immediate vicinity of the wetting front adds to the computational difficulties. The flow occurring only a short distance behind the front usually more nearly resembles normal flow than critical flow.

Kruger and Bassett (18) and Olsen (24) assumed that the rate of advance of an irrigation stream is equal to the velocity of flow in a segment of flow immediately behind the wetting front. If this assumption is chosen, then the curve in the x_*-t_* plane describing the advance of the wetting front is neither a characteristic curve nor an envelope of the two sets of characteristic curves. In order to advance a solution in time either the flow depth or some relation between depth and velocity must be assumed at the wetting front.

The Solution at Interior Grid Points

The procedure used in advancing the solution to a typical grid point H, illustrated in Fig. 5.2, is the same for all cases. In finite difference form Equations 5.5 become:

$$\frac{x_{*_H} - x_{*_G}}{t_{*_H} - t_{*_G}} = V_{*_G} + \sqrt{D_{*_G}}/Fr \qquad 5.9$$

and

$$\frac{x_{*_H} - x_{*_M}}{t_{*_H} - t_{*_M}} = V_{*_M} - \sqrt{D_{*_M}}/Fr. \qquad 5.10$$

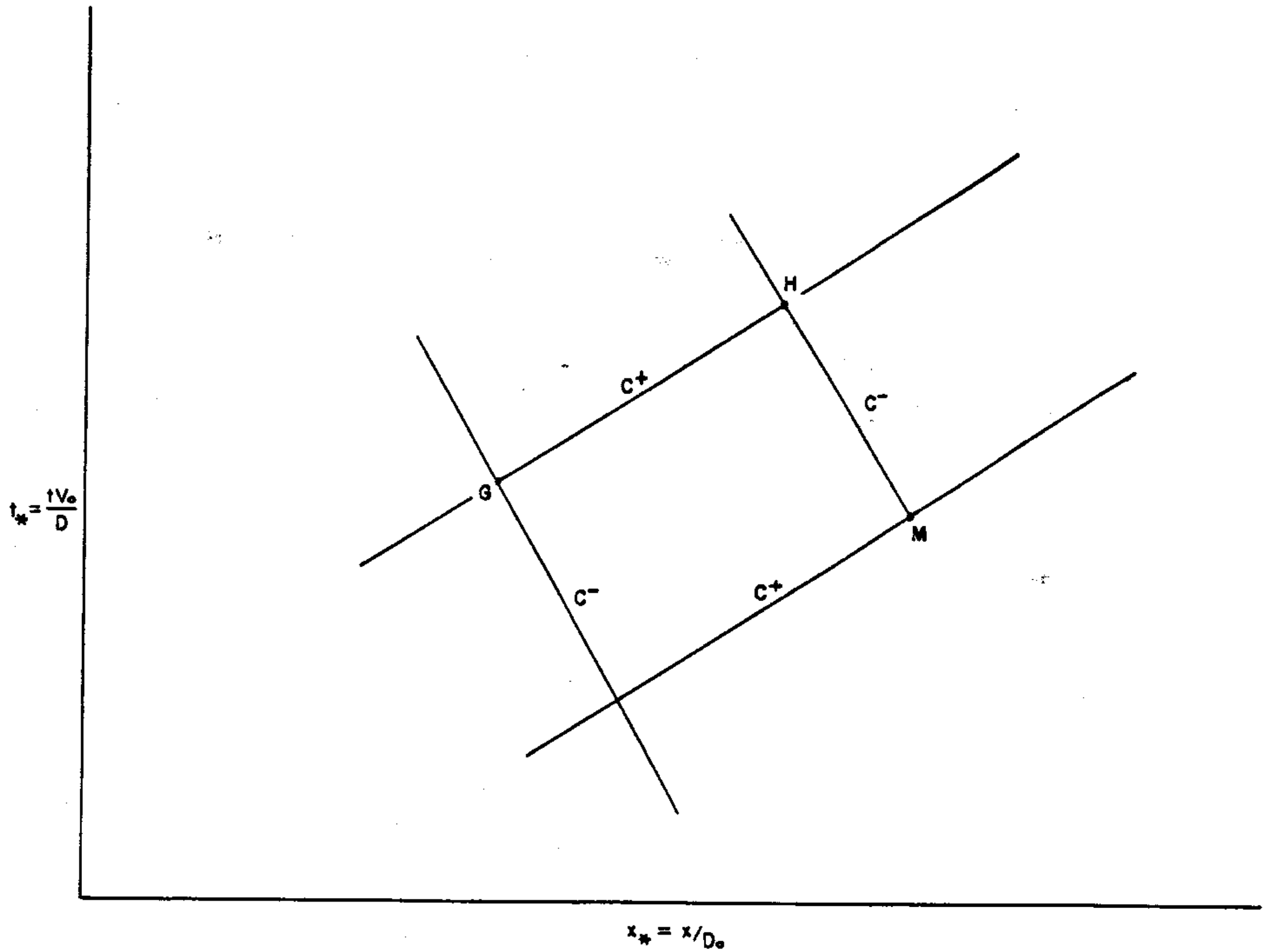

FIG. 5.2. REPRESENTATION OF A TYPICAL GRID POINT.

Equations 5.6 become:

$$(V_* + 2b\ \sqrt{D_*}/Fr)_H = (V_* + 2b\ \sqrt{D_*}/Fr)_G + \int_{t_{*_G}}^{t_{*_H}} [r(1 - V_*^2/D_*) + z(t_{o*}^{\alpha-1}/D_*^b)(V_* - \sqrt{D_*}/Fr)]dt_*, \quad 5.11$$

and

$$(V_* - 2b\ \sqrt{D_*}/Fr)_H = (V_* - 2b\ \sqrt{D_*}/Fr)_M + \int_{t_{*_M}}^{t_{*_H}} [r(1 - V_*^2/D_*) + z(t_{o*}^{\alpha-1}/D_*^b)(V_* + \sqrt{D_*}/Fr)]dt_*. \quad 5.12$$

The values of x_{*_H} and t_{*_H} are obtained from Equations 5.9 and 5.10. Addition and subtraction of Equations 5.11 and 5.12 result in two equations for V_{*_H} and D_{*_H}, respectively.

The values of the integrals in Equations 5.11 and 5.12 must be evaluated numerically, either explicitly from the values of the variables at points G and M alone or, as proposed by Liggett and Woolhiser (22), using the solution at G and M in conjunction with increasingly improved estimates of V_{*_H} and D_{*_H}. Following Liggett and Woolhiser, let the integral in Equation 5.11, for example, be approximated by the expression,

$$\frac{t_{*_H} - t_{*_G}}{2}\left[r(1 - V_{*_G}^2/D_{*_G}) + z(t_{o_{*_G}}^{\alpha-1}/D_{*_G}^b)(V_{*_G} - \sqrt{D_{*_G}}/Fr) + r(1 - V_{*_H}^2/D_{*_H}) + z(t_{o_{*_H}}^{\alpha-1}/D_{*_H}^b)(V_{*_H} - \sqrt{D_{*_H}}/Fr)\right]. \qquad 5.13$$

If the integrals in Equations 5.11 and 5.12 are evaluated from expressions of this form, increasingly improved estimates of the values of V_{*_H} and D_{*_H} can be obtained by the technique proposed by Liggett and Woolhiser (22) or by the generalized Newton-Raphson technique.

Computational schemes for numerically integrating the equations of motion by the method of characteristics were devised for two cases.

Computational Techniques

The first case is that for which critical flow exists at the wetting front and the position of the "leading" C^+-characteristic describes the advance of the wetting front. Assuming that critical flow exists initially at two upstream points, the solution may then be advanced according to the point numbering system illustrated in Fig. 5.3. The first C^+ -characteristic is extended using Equations 5.9 and 5.11 until $D_*=0$. If the initial infiltration rate is great, the grid points must be closely spaced in order to initiate a solution. Succeeding C^+ -characteristics are originated at $x_*=0$. Values of x_*, t_*, V_*, and D_* at interior grid points are computed from Equations 5.9 through 5.12. A particular C^+ -characteristic, after it overtakes the preceding one, is extended using Equations 5.9 and 5.11 and the assumption $V_* = \sqrt{D_*}/Fr$, until $D_*=0$. Points of intersection of C^- -characteristics and the line $x_*=0$ are located using Equation 5.10. Values of V_* and D_* at points on the line $x_*=0$ are obtained from Equation 5.10 and the assumption of a constant inflow, Q=DVW.

The convergence of characteristic lines in the neighborhood of the wetting front creates computational problems. Due to the high infiltration rate and small flow depths occurring near the wetting front, calculations may generate negative values of D_* at interior

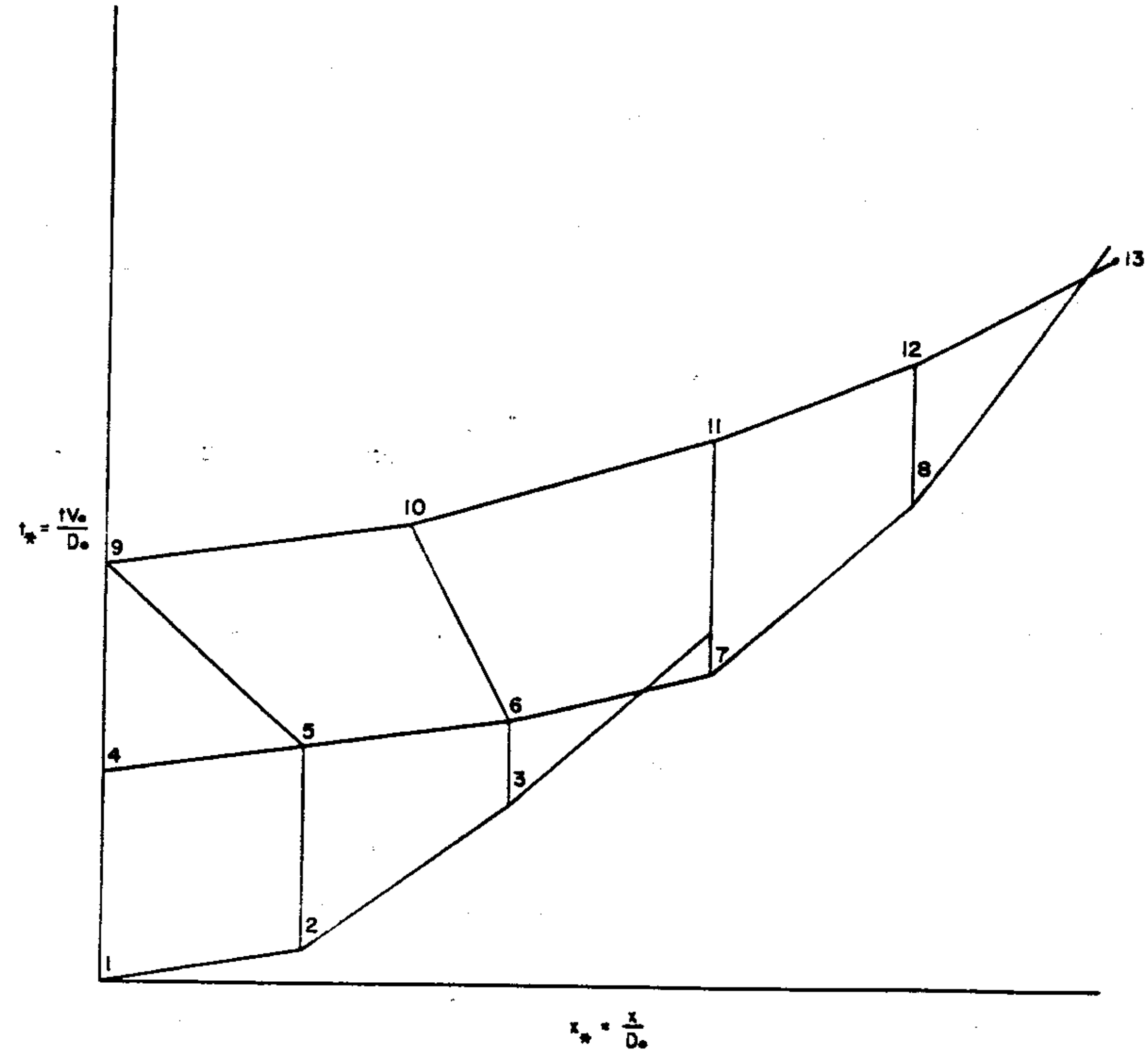

FIG. 5.3. POINT NUMBERING SYSTEM UTILIZED WHEN $dx_*/dt_* = 2V_*$ AT THE WETTING FRONT.

grid points behind the front. Improper crossing of the C^+ - or C^- -characteristics may also occur.

The second case is that for which the rate of advance is equal to the velocity of flow at the wetting front and the curve describing the advance of the wetting front is not a characteristic line. Two conditions of flow must be specified at the upstream boundary. Normal flow is assumed. Thus, $V_*=1$ and $D_*=1$. One flow condition must be specified at the wetting front. If critical flow is assumed, many of the computational difficulties associated with the previous case reappear. An alternative is to assume that the depth of flow at the wetting front is constant. The sequence of calculations is illustrated by the point numbering system in Fig. 5.4.

Predictions of wetting front positions obtained using the computational procedures just described did not compare well with measured positions. A reliable method for coping with the computational difficulties which occur due to the rapid change of flow conditions near the wetting front needs to be developed.

The region influenced by flow entering the furrow at $x_*=0$ and $t_*=t_{*_1}$ is bounded by the C^+ -characteristic originating at that point. Consequently, if irrigation is to be designed to prevent loss of water past the downstream end of the furrow, the rate of inflow at $x_*=0$ might have to be reduced before the wetting front reaches the end of the furrow. A solution of the equations of

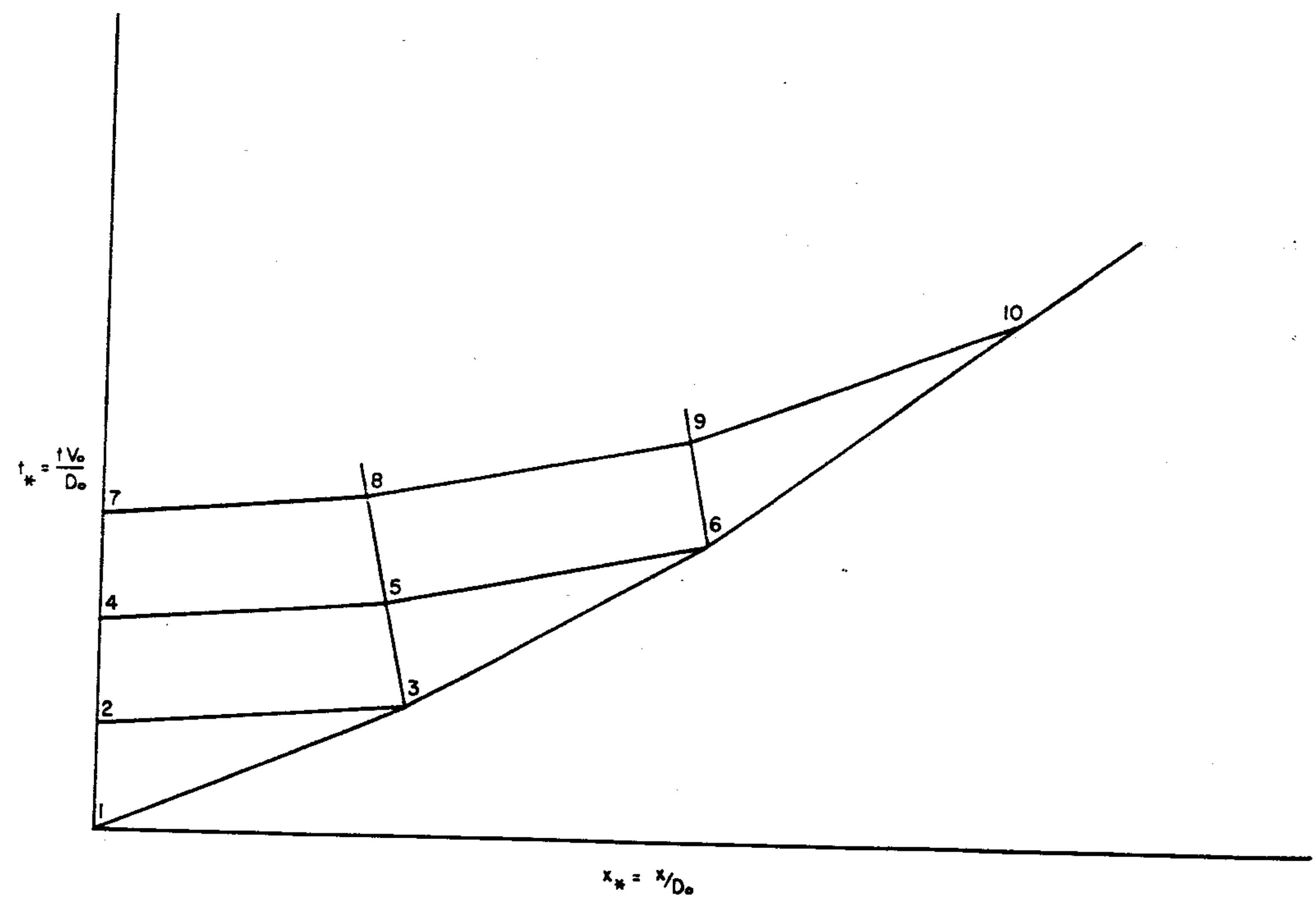

FIG. 5.4. SEQUENCE OF CALCULATIONS UTILIZED WHEN THE VELOCITY OF ADVANCE IS EQUAL TO THE FLOW VELOCITY.

motion obtained using the method of characteristics would, therefore, be much more useful than one obtained using a rectangular finite difference grid.

Two procedures for estimating the manner in which furrow inflow must be reduced in order to prevent tailwater losses are presented in the next chapter.

CHAPTER VI

ESTIMATING CUTBACK FLOW RATES

In order to achieve a uniform distribution of the applied water, the opportunity time, *i.e.*, the time the soil has been wetted, must be the same at all points along the furrow length. In other words, the curve describing the recession or disappearance of an irrigation stream must be parallel to the curve describing the advance of the wetting front. A perfectly uniform distribution can often be achieved by shortening the furrows to an uneconomical length or by applying a high rate of inflow at the upstream end. As the rate of inflow is increased, the amount of runoff, which must be recirculated or lost, increases correspondingly. Runoff losses can be avoided by reducing the inflow after the wetting front nears or reaches the downstream end of the furrow. Ideally the inflow rate should be "cut back" continuously or in increments in such a manner as to prevent flow past the downstream end.

Determination of Initial Inflow

Because the time of recession is generally less than the time of advance, the most uniform distribution of the applied water is obtained when the advancing wetting front traverses the furrow length in the minimum possible time. Thus, the proper choice of an initial upstream inflow rate is the maximum nonerosive flow rate.

One frequently used criterion for choosing a maximum nonerosive flow rate is the empirical relationship proposed by Criddle *et al.* (8),

$$Q = 0.1/S, \qquad 6.1$$

in which Q is the flow rate in gallons per minute and S is the furrow slope in feet per foot. However, the erosion resistance of a soil varies with the soil's texture and structure. Soil properties do influence the choice of a maximum nonerosive flow rate if the flow rate is chosen so that the shear exerted on the soil does not exceed the critical tractive shear, T_c, which will cause erosion, *i.e.*,

$$T_c = \gamma R_o S \qquad 6.2$$

where T_c is the frictional force exerted on a unit area of the furrow bed by the moving water, γ is the unit weight of water, and R_o is the normal hydraulic radius corresponding to the upstream flow rate. Given the furrow slope and shape and a permissible value of T_c, the maximum nonerosive flow rate can be computed from Equation 6.2 and either Manning's or Chezy's uniform flow equation.

The Volume Balance Procedure for the Case, c = 0

Often the advance of a furrow irrigation stream can be approximated by an empirical equation of the form

$$x = a_1 t^a, \qquad 6.3$$

in which x is the distance, in feet, that the wetting front has advanced from the upstream end of the furrow, t is the time, in minutes, since the water was turned on, and a_1 and a are constants. A plot of t *versus* x on logarithmic paper will approximate a straight line having slope a.

The volume of water, in cubic feet, in the furrow above the soil surface divided by the length of the wetted portion of the furrow, in feet, can be called the average area of surface storage, c. Soon after the water is turned on, the value of c generally approaches a constant value. In many cases after irrigation is essentially complete the volume of water remaining on the soil surface is small compared to the volume of water which has infiltrated into the soil. Suppose that the ratio of surface storage to infiltrated water is sufficiently small that the value of c can be assumed to be zero. For this case, $c = 0$ and $x = a_1 t^a$, Philip and Farrell (26) have shown that the amount of water which has infiltrated at any point along the run can be estimated by an equation having the form of Equation 4.1,

$$y = kt_o^{\alpha} , \qquad 4.1$$

in which y is the accumulative infiltration at a point, in cubic feet per foot of furrow length; t_o is the time, in minutes, that the surface at that point has been wet; and k and α are constants which are related to the constants a_1 and a by the equalities,

$$a = 1 - \alpha. \qquad 6.4$$

and

$$a_1 = QI/[k\Gamma(1 + \alpha)\Gamma(2 - \alpha)]. \qquad 6.5$$

QI is the initial inflow rate, which is maintained until the water reaches the end of the furrow, in cubic feet per minute, and Γ is the symbol for the Gamma function.

Calculation of cutback flow rate. Fig. 6.1 shows an arithmetic plot of a typical irrigation stream advance as a function of time. Based on the previous assumptions, the opportunity time at any point along the run is

$$t_o = t - (x/a_1)^{1/a}. \qquad 6.6$$

The volume of infiltrated water per foot of furrow length at any point, x, is

$$y(x, t) = k[t - (x/a_1)^{1/a}]^{\alpha}. \qquad 6.7$$

The rate of infiltration at that point is

$$I(x, t) = dy/dt_o = \alpha k t_o^{\alpha-1} = \alpha k[t - (x/a_1)^{1/a}]^{\alpha-1}. \qquad 6.8$$

The rate of total water infiltration per unit of time along the entire furrow length, I_L, is

$$I_L = \int_0^L I(x, t)\, dx, \qquad 6.9$$

in which L is the length of furrow, in feet, and I_L is measured in cubic feet per minute.

Substitution of Equation 6.8 into 6.9 yields

$$I_L(t) = \alpha k t^{\alpha-1} \int_0^L [1 - (x/a_1)^{1/a}/t]^{\alpha-1} dx. \qquad 6.10$$

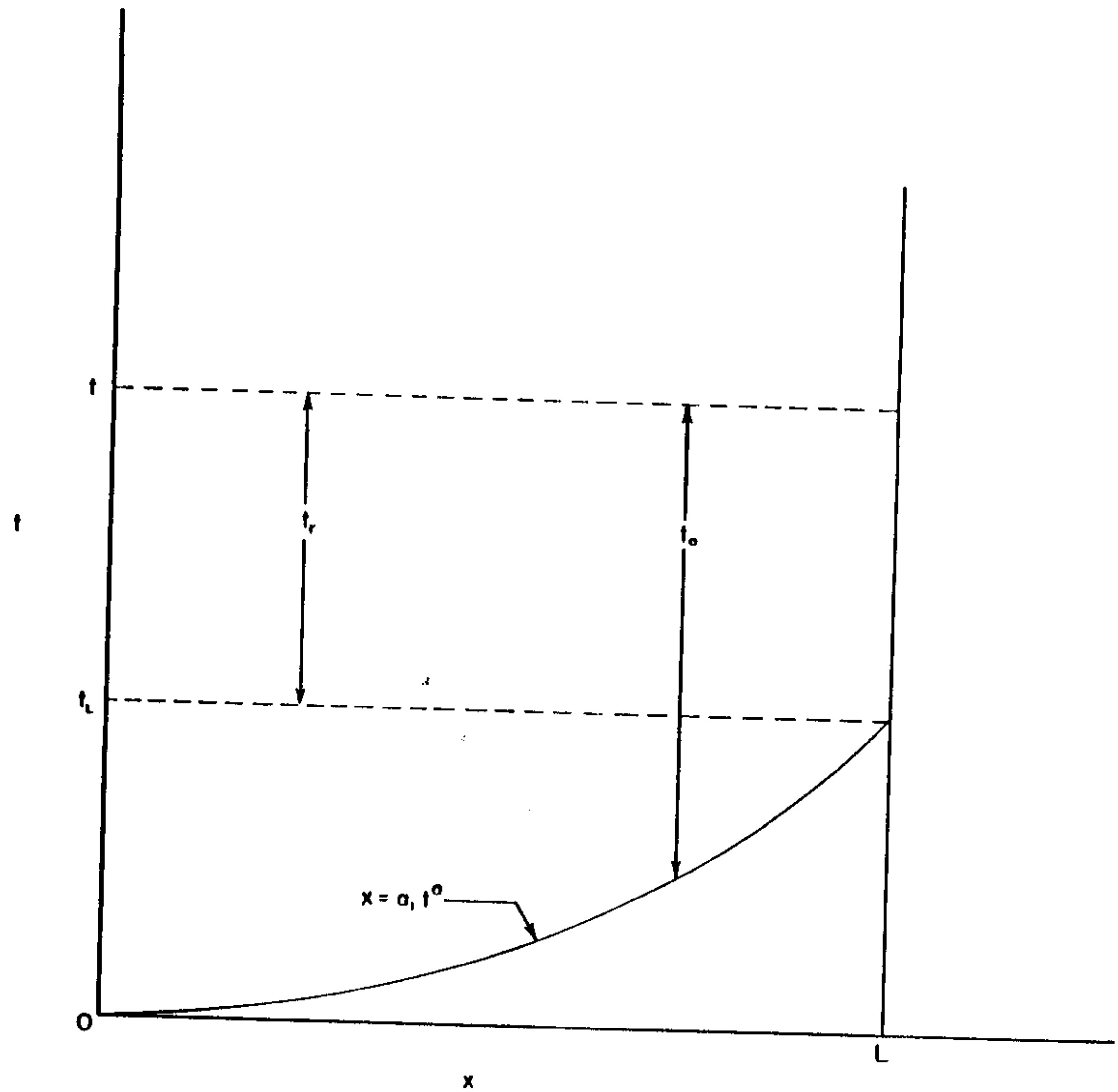

FIG. 6.1. ARITHMETIC PLOT OF A TYPICAL ADVANCE FUNCTION.

In order to avoid any runoff, the inflow, Q, should be continuously cut back to match the total infiltration rate, I_L. For the special case, $t = t_L$, where t_L is the time required for the wetting front to reach the end of the furrow, in minutes, the cutback stream size can be determined by integrating Equation 6.10 to obtain

$$Q(t_L) = \frac{QI\alpha(1 - \alpha)\Gamma(\alpha)\Gamma(1 - \alpha)}{\Gamma(1 + \alpha)\Gamma(2 - \alpha)} = QI. \qquad 6.11$$

When c = 0, the above result is expected because at each instant the infiltration must equal the inflow.

Equation 6.10 was integrated numerically for various values of the dimensionless terms, Q/QI, t_L/t, and a. These results are shown in Fig. 6.2, and provide a ready estimate of cutback stream size as a function of time for many irrigation situations. The assumptions, c = 0 and $x = a_1 t^a$, generally are not valid for small time increments, for soils having very slow infiltration rates, or for unusually large furrows or stream sizes.

Determination of water distribution efficiency. Additional approximate information about the performance of an irrigation system can be obtained when the above assumptions apply.

Define a dimensionless application ratio

$$Y = y/y_a, \qquad 6.12$$

in which y is the amount of infiltration at any point and y_a is the desired average application along the run, and is given by

$$y_a = kt_a^{\alpha}. \qquad 6.13$$

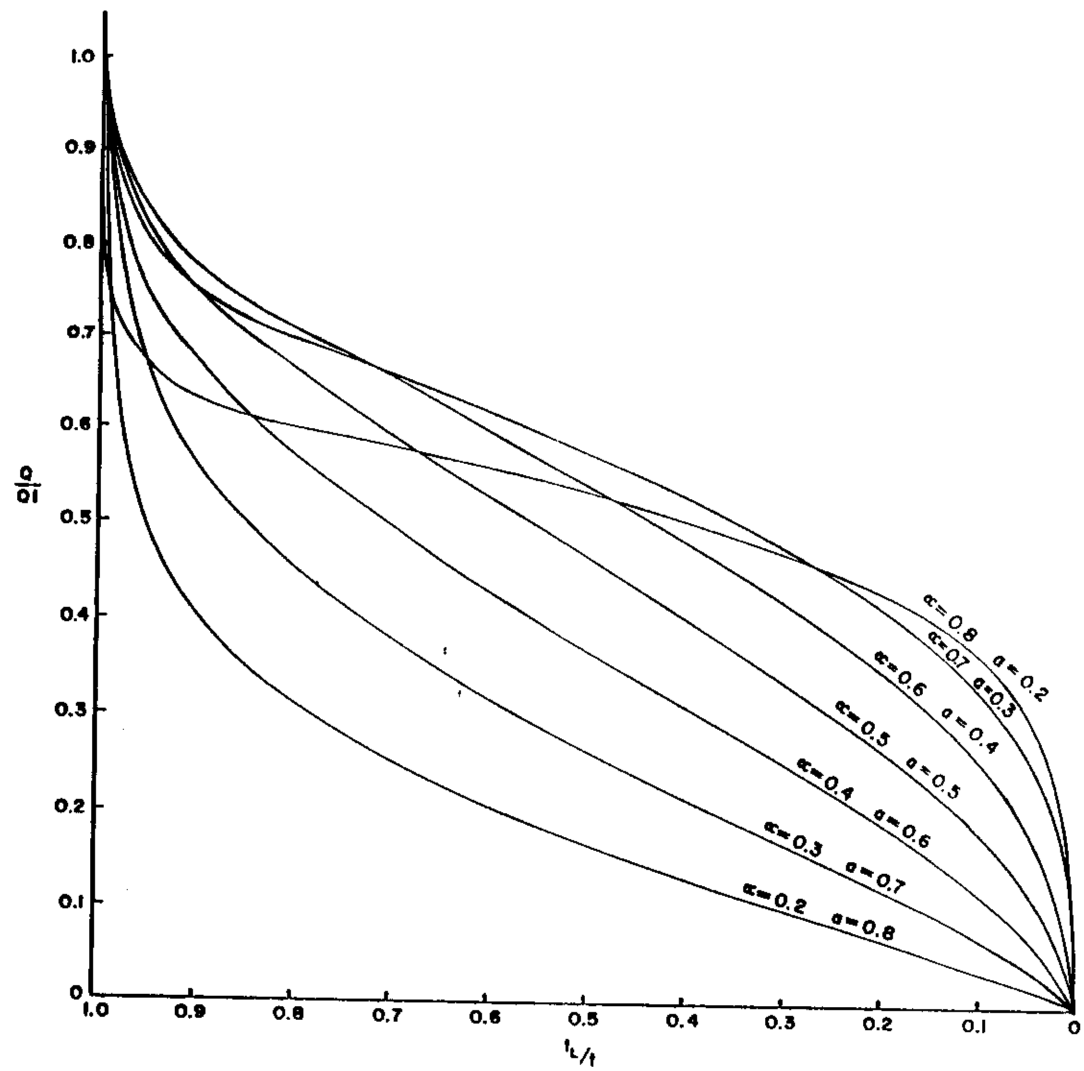

FIG. 6.2. CUTBACK FLOW RATES DETERMINED BY THE VOLUME BALANCE PROCEDURE FOR c = 0.

t_a is the time required for the desired average application to infiltrate. Substitution of Equations 6.7 and 6.13 into Equation 6.12 yields

$$Y = \frac{k[t - (x/a_1)^{1/a}]^\alpha}{kt_a^\alpha} = [t/t_a - (x/a_1)^{1/a}/t_a]^\alpha. \qquad 6.14$$

Let t_r be the time of recession after the advance has reached the end of the field so that

$$t_r = t - t_L. \qquad 6.15$$

Then

$$Y = [t_r/t_a + t_L/t_a - (x/a_1)^{1/a}/t_a]^\alpha$$

or

$$Y = [T(1 - X^{1/a}) + t_r/t_a]^{1-a} , \qquad 6.16$$

in which T is a dimensionless time ratio, $T = t_L/t_a$, and X is the dimensionless distance ratio, $X = x/L$.

By definition (32)

$$\int_0^1 Y \, dX = 1. \qquad 6.17$$

If a horizontal recession is assumed, where t_r is constant at all values of x, then

$$\int_0^1 [T(1 - X^{1/a}) + t_r/t_a]^{1-a} \, dX = 1. \qquad 6.18$$

When a, t_L, and t_a are known, t_r can be estimated by a trial and error process from successive numerical integrations of Equation

6.18 using various values of t_r. It should be noted that $t_r/t_a < 1$ for any real situation. The results of this procedure are presented in Fig. 6.3.

Water distribution efficiency has been defined by the equation (14)

$$E_d = 100(1 - \bar{d}/y_a), \tag{6.19}$$

where $\bar{d}$ is the average numerical deviation from y_a and obtained from

$$\bar{d} = \int_0^1 | y - y_a | \, dX . \tag{6.20}$$

Thus

$$E_d = 100(1 - \int_0^1 | y - y_a | \, dX/y_a)$$

or

$$E_d = 100(1 - \int_0^1 | Y - 1 | \, dX). \tag{6.21}$$

The results of a numerical evaluation of Equation 6.21 are presented in Fig. 6.4. If the desired application, y_a, is known and the values of a and t_L are determined from a plot of the advance, t_a can be estimated using Fig. 6.3. The approximate water distribution efficiency can then be obtained from Fig. 6.4.

If a cutback flow rate, as given in Fig. 6.2, is applied after the water reaches the end of the furrow, little runoff should occur. If a constant flow rate, QI, is maintained for the entire time t, then the approximate volume of runoff, in cubic feet, leaving each furrow is

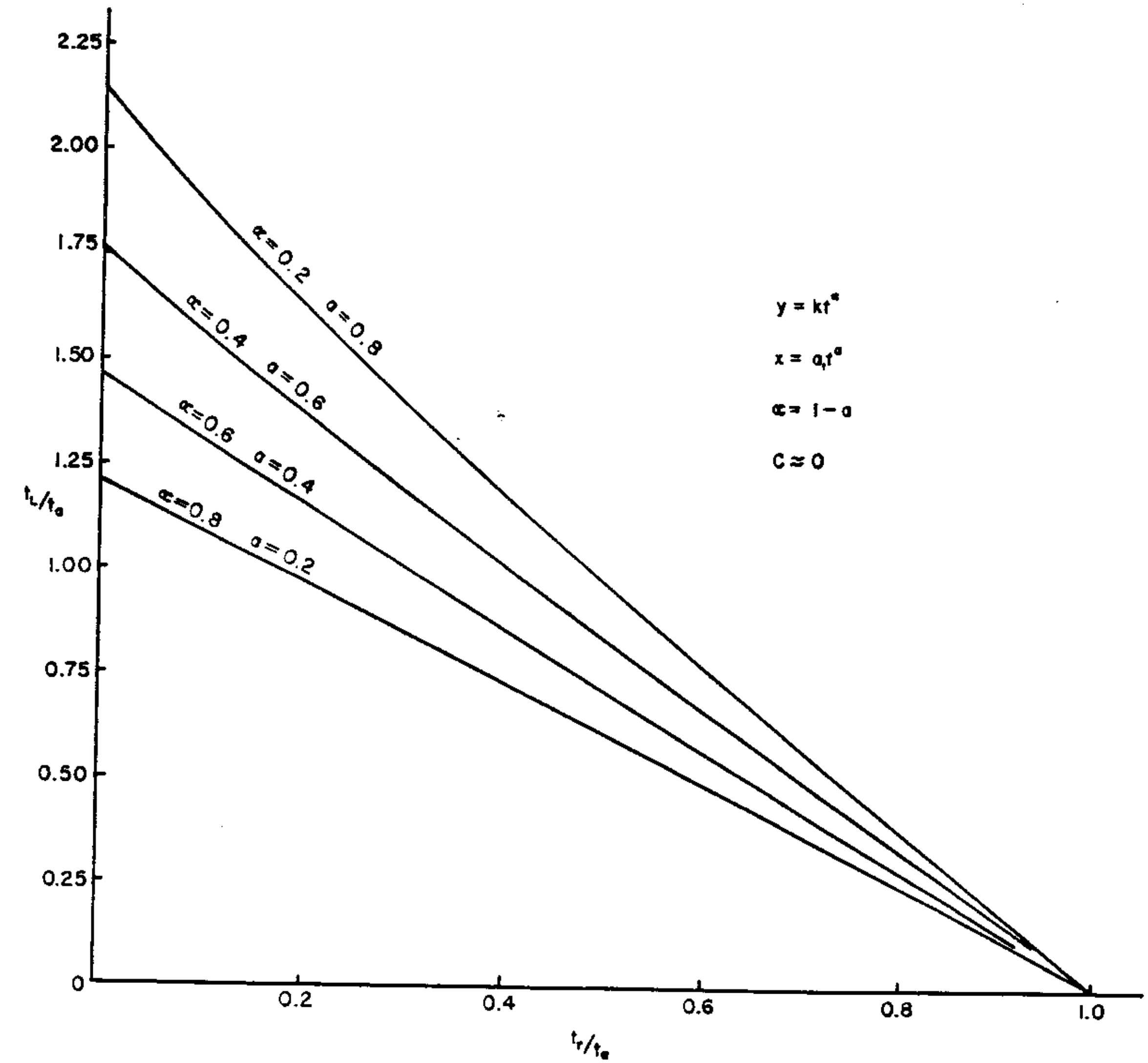

FIG. 6.3. RELATIONSHIP BETWEEN TIME OF ADVANCE AND TIME OF RECESSION.

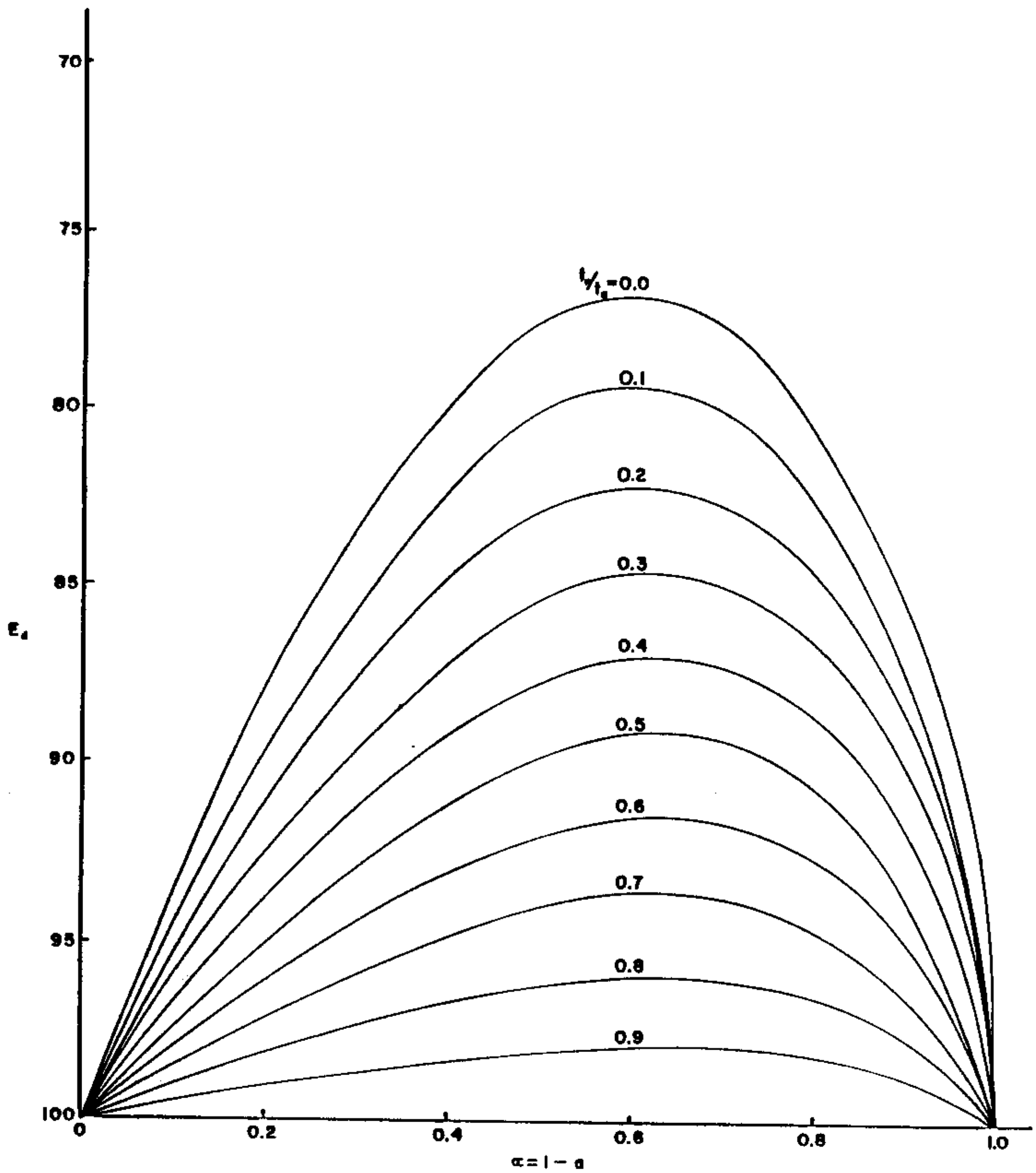

FIG. 6.4. WATER DISTRIBUTION EFFICIENCY AS A FUNCTION OF THE INFILTRATION PARAMETER, α.

$$QI\ t - y_a L. \tag{6.22}$$

The analysis presented above is based on several restrictive assumptions which are never exactly found in nature. The procedure usually does not work well for border irrigation because the amount of surface storage is generally not small compared to the amount of infiltrated water. This procedure does, however, provide a rational estimate of cutback stream size for preventing excessive runoff from irrigation furrows.

Several methods for estimating cutback flow rates for the case, $c > 0$, were investigated.

The Hydrologic Routing Procedure

If the surface storage is greater than zero but still small compared to the amount of infiltrated water, estimates of cutback flow rates can be obtained by a hydrologic routing procedure. Assume that a constant inflow rate, QI, is maintained until the wetting front reaches the end of the furrow, and further assume that the cumulative infiltration is described by an equation of the form, $y = kt_o^{\alpha}$. Divide the furrow length into p equal increments. After the wetting front reaches the end of the furrow at $t - t_L$, a constant reduced flow rate, Q, is applied during a time increment Δt. The value of Δt is chosen large enough so that at the end of the time increment normal flow exists at $x = 0$. The assumption can be made that the average surface storage, c, is approximately three-fourths of the area of flow at $x = 0$. This assumption is verified by Smerdon and Hohn (34) who found that the ratio of the average area of surface storage to the upstream cross-sectional area is about 0.77. The volume of water entering the furrow during time Δt is equated to the volume of water which infiltrates into the soil minus the reduction in surface storage, *i.e.*,

$$Q\ \Delta t = \Delta y\ \Delta x + L\Delta c. \qquad 6.23$$

More specifically,

$$Q_i\ (t_{i+1} - t_i) = \sum_{j=1}^{p} (y_{x_{j,t_{i+1}}} - y_{x_{j,t_i}})(L/p) - L(c_{t_i} - c_{t_{i+1}}). \qquad 6.24$$

where

$$c_{t_{i+1}} = 0.75\ (A_{0,t_{i+1}}), \qquad 6.25$$

$$(AR^{2/3})_{0,t_{i+1}} = Q_i n/(1.486 S^{1/2}), \qquad 6.26$$

and

$$A = b_1 h^b;\ R \simeq h/b. \qquad 6.27$$

Rearrangement of Equations 6.24 through 6.27 results in an implicit equation for Q_i from which the value of Q_i can be computed using the Newton-Raphson technique.

Sample results of this hydrologic routing procedure are presented in Fig. 6.5. If the value of kt_L^{α}/c is small, the value of Δt must be large so that the infiltration occurring during Δt

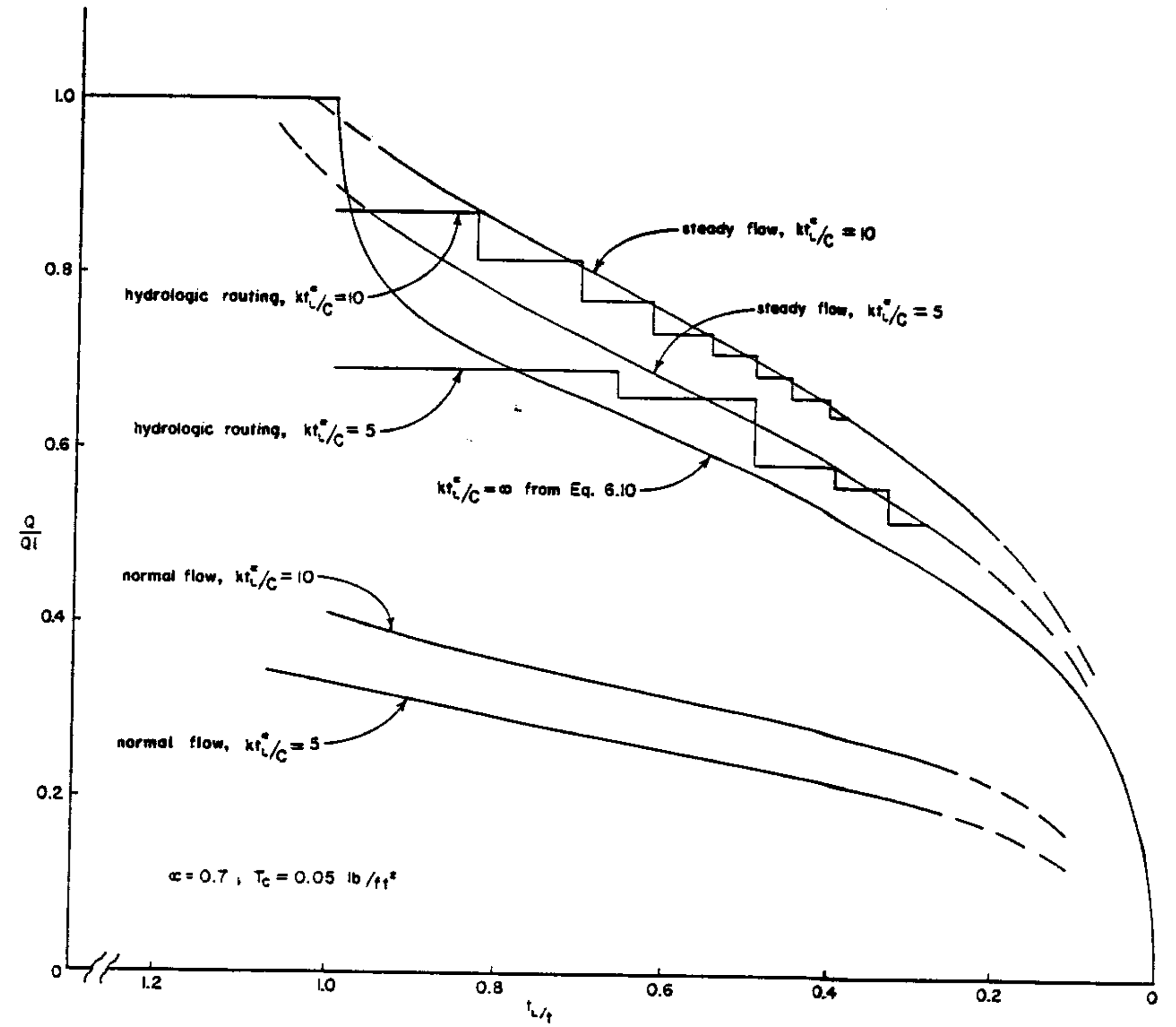

FIG. 6.5 COMPARISON OF CUTBACK FLOW RATES OBTAINED BY THREE METHODS.

is greater than the surface storage at the start of the time increment. If this condition is not met, the inflow during that time increment must be negative. No knowledge of how to reduce the inflow before water reaches the end of the furrow is obtained by this hydrologic routing method.

The Normal Flow Procedure

If the position of the particular C^+ -characteristic which terminates at $x = L$ and $t = t_L$ were specified, then the exact time at which reduction in inflow should begin would be known also. If it is assumed that normal flow exists at points along C^+ -characteristics which intersect the line $x = L$ after time $t = t_L$, then along each C^+ -characteristic

$$dx/dt = V + \sqrt{gD} \qquad 6.28$$

and

$$d(V + 2b\sqrt{gD})/dt = (I/A)(V - \sqrt{gD}). \qquad 6.29$$

Also

$$V = \frac{1.486}{n} R^{2/3} S^{1/2}. \qquad 3.2$$

If a particular flow rate is chosen at $x = L$ such that $Q_{x=L} << Q_{x=0}$, the runoff will be negligible and the values of V and D at preceding points along a C^+ -characteristic can be computed. The calculated flow rate at $x = 0$ is an estimate of the desired cutback flow rate. Sample results of the procedure are shown in Fig. 6.5. Due to the assumption of normal flow and/or due to computational errors, estimates of cutback flow rates obtained using this method do not compare well with estimates obtained using other methods.

The Steady Flow Procedure

After the flow reaches the end of the furrow, the flow profile changes slowly with time. If the time derivatives are eliminated from Equations 2.13 and 2.32, two ordinary differential equations are obtained

$$V \frac{dA}{dx} + A \frac{dV}{dx} = -I \qquad 6.30$$

and

$$(1/W)\frac{dA}{dx} + (V/g)\frac{dV}{dx} = S - SF + VI/Ag \qquad 6.31$$

Given the assumption of zero velocity and a particular depth at the downstream end of the furrow, values of V and A can be computed explicitly at a series of upstream points until x = 0. Cutback flow rates for two steady flow cases are presented in Fig. 6.5 and compare well with the results of the hydrologic routing procedure.

Suggested design curves were prepared using the steady flow procedure and are presented in Figs. 6.6, 6.7, and 6.8. The values of E_d and t_a/t_L shown were taken from Figs. 6.3 and 6.4 and are based on the assumption of a horizontal recession occurring when the furrow inflow is stopped. The relationships presented in Figs. 6.6, 6.7, and 6.8 were obtained using an assumed permissible tractive shear value of 0.05 pounds per square foot. Values of

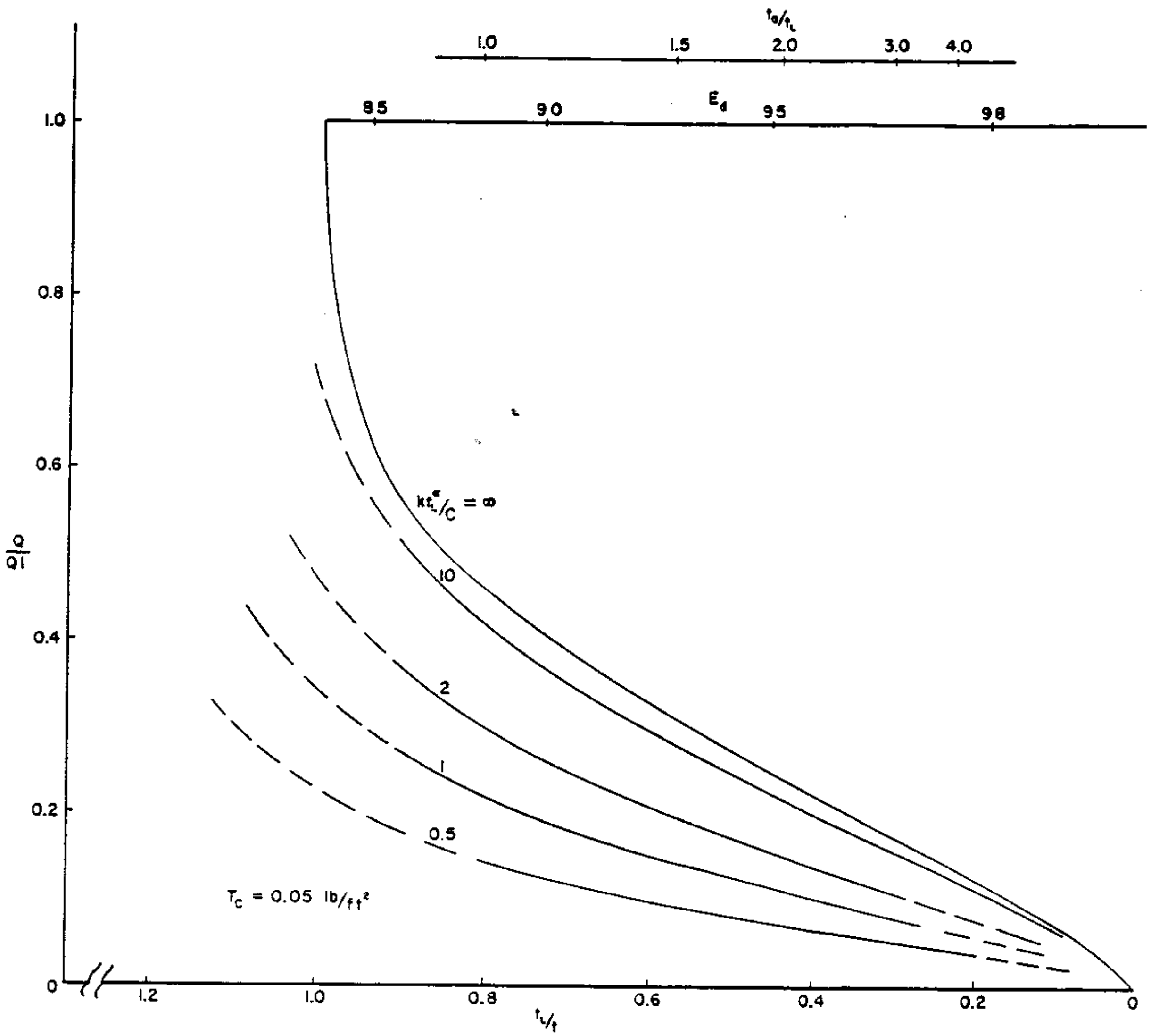

FIG. 6.6. DESIGN CURVES TO DETERMINE CUTBACK FLOW RATES DETERMINED BY THE STEADY FLOW METHOD ($\alpha = 0.3$).

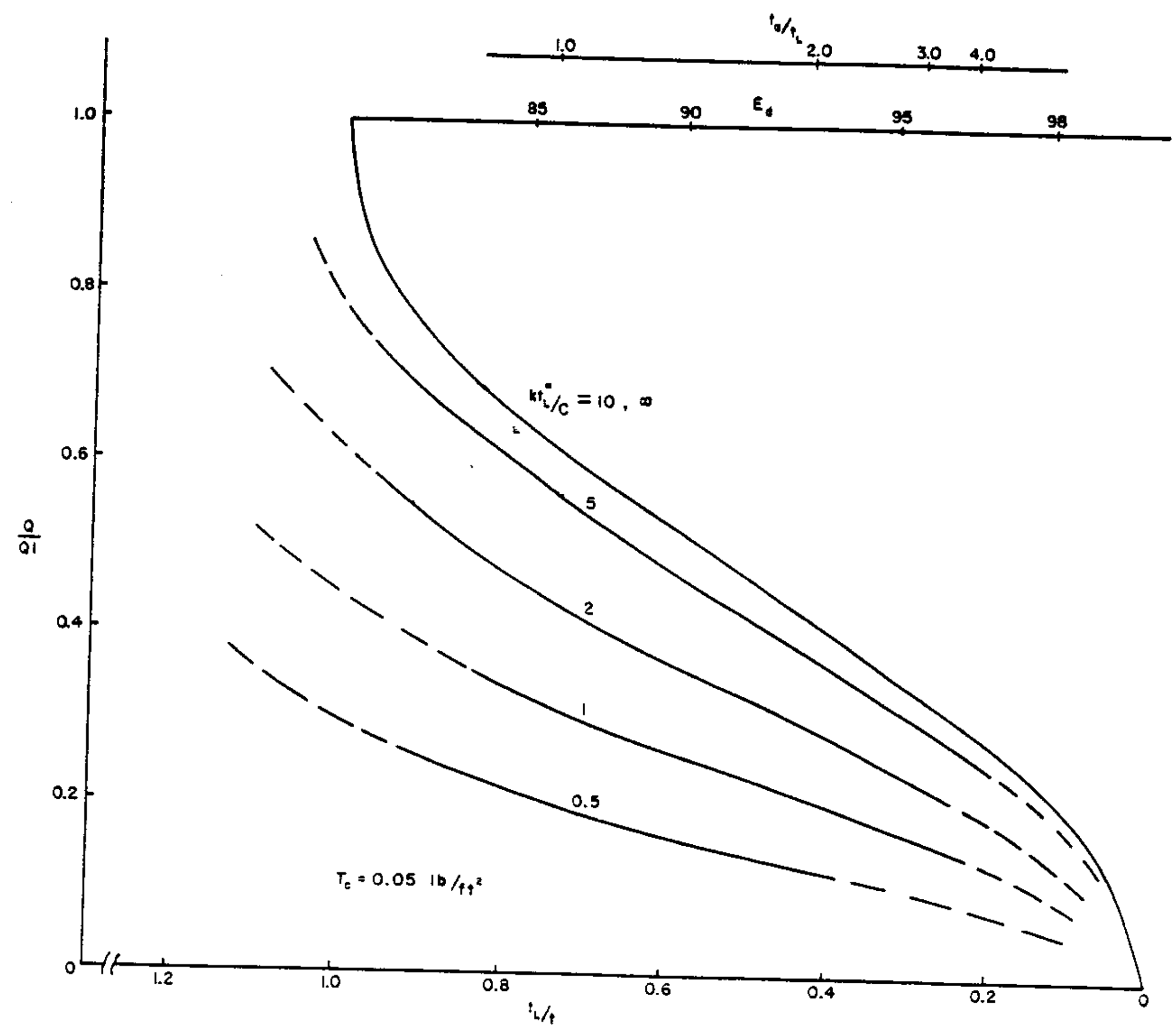

FIG. 6.7. DESIGN CURVES TO DETERMINE CUTBACK FLOW RATES DETERMINED BY THE STEADY FLOW METHOD ($\alpha = 0.5$).

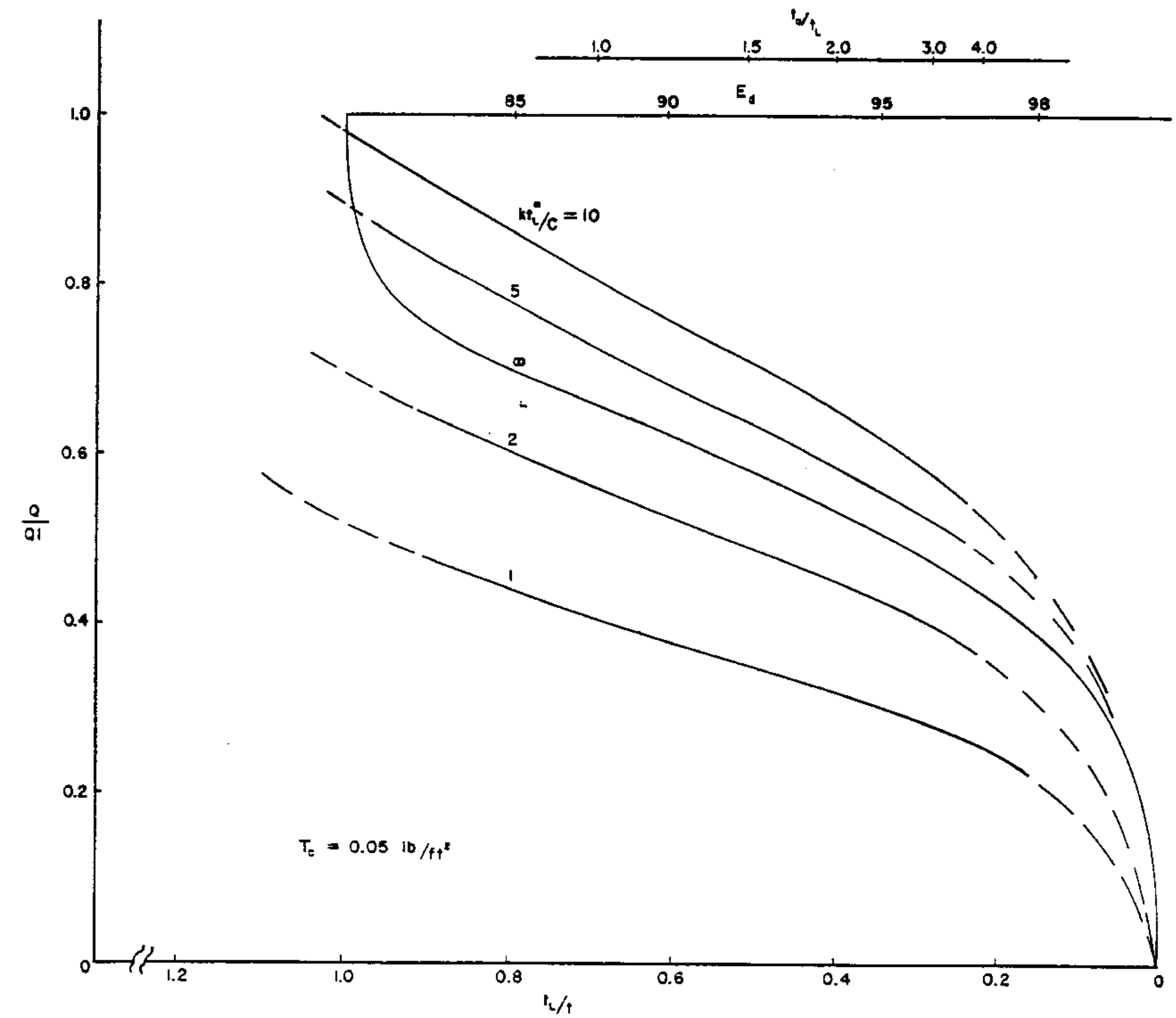

FIG. 6.8 DESIGN CURVES TO DETERMINE CUTBACK FLOW RATES DETERMINED BY THE STEADY FLOW METHOD (α = 0.7).

maximum nonerosive initial inflow rates determined using this value of T_c are generally less than flow rates obtained from Equation 6.1 for slopes greater than three per cent and greater than flow rates obtained from Equation 6.1 for slopes less than three per cent.

The time at which the reduction in inflow should begin can be obtained from an estimate of the time interval required for a flow disturbance to traverse the furrow length, *i.e.,* $\Delta t \approx L/(V_o + \sqrt{gD_o})$.

The use of Figs. 6.6, 6.7, and 6.8 in the design of a "cut-back" irrigation system and the accuracy of the suggested design curves are illustrated in sample problems appearing in Appendix B.

CHAPTER VII

SUMMARY AND RECOMMENDATIONS

A derivation of the characteristic form of the equations of motion describing flow in irrigation furrows is presented. Techniques for numerically integrating the equations are discussed.

Computational difficulties were encountered in attempting to determine the flow depth and velocity at points near the wetting front. The region near the wetting front is characterized by a high infiltration rate and rapid changes in flow depth and velocity; therefore, predictions of wetting front advance positions obtained using these techniques were not accurate.

The region in the x-t plane influenced by flow originating at time t and at x = 0 is bounded by the C^+ -characteristic originating at that point. Therefore, a solution of the equations of motion by the method of characteristics would be very useful in determining how to vary the inflow in order to prevent tailwater losses.

A reasonable estimate of Manning's n for 40-in. irrigation furrows in Texas is 0.046. However, variation in the roughness parameter does occur when other flow parameters vary. An accurate field study of furrow roughness is needed. Such a study should include independent measurements of flow velocity.

A volume balance procedure is proposed for determining furrow infiltration rates from measurements of surface storage and wetting front advance positions for the case, $y = kt_o^{\alpha}$. Repetitive values of α and k determined by this procedure, although approximate, generally exhibit less variability than values of α and k obtained using a blocked-furrow infiltrometer.

Design curves providing estimates of cutback flow rates for preventing tailwater losses are presented. Sample problems illustrate how these relationships can be utilized to obtain an efficient use of irrigation water. In order to reduce the number of independent variables so that these design relationships could more easily be presented in graphical form, a relationship between furrow slope and initial furrow stream size is assumed, *i.e.*, $T_c = 0.05$ pounds per square foot. Similar curves could be developed for other erosion resistance criteria.

Field experiments for the verification and/or adjustment of these theoretical design relationships should be conducted. Design of an automated irrigation distribution system which can apply a desired amount of water uniformly along the length of an irrigation furrow without causing tailwater losses will then be possible.

LIST OF REFERENCES

LIST OF REFERENCES

1. ASCE Task Force on Friction Factors in Open Channels. Friction factors in open channels. Journal of the Hydraulics Division, ASCE, 89:97-143. March 1963.

2. Bishop, A. A. Relation of intake rate to length of run in surface irrigation. Transactions, ASCE, 127:(III)282-288, 1962.

3. Chen, Cheng-lung. An analysis of overland flow. Ph.D. dissertation. Michigan State University, 1962.

4. Chen, Cheng-lung. Mathematical hydraulics of surface irrigation. Report PR-WR11-2, Utah Water Research Laboratory, Utah State University, June 1966.

5. Chen, Cheng-lung and Hansen, V. E. Theory and characteristics of overland flow. Transactions, ASAE, 9:(1)20-26, 1966.

6. Chow, Ven Te. Open Channel Hydraulics. McGraw-Hill Book Co., Inc., New York, 1959.

7. Courant, R. and Friedrichs, K. O. Supersonic Flow and Shock Waves. Interscience Publishers, Inc., New York, 1948.

8. Criddle, W. D., Davis, S., Pair, C. H., and Shockley, D. G. Methods for evaluating irrigation systems. Agricultural Handbook No. 82, USDA Soil Conservation Service, 1956.

9. Davis, J. R. Estimating rate of advance for irrigation furrows. Proceedings of the ARS-SCS Workshop on Hydraulics of Surface Irrigation, ARS 41-43, October 1960.

10. Finkel, H. J. and Nir, D. Determining infiltration rates in an irrigation border. Journal of Geophysical Research, 65:2125-2131, July 1960.

11. Fok, Yu-Si and Bishop, A. A. Analysis of water advance in surface irrigation. Journal of the Irrigation and Drainage Division, ASCE, 91:99-116, 1965.

12. Fok, Yu-Si and Hansen, V. E. One-dimensional infiltration into homogeneous soil. Journal of the Irrigation and Drainage Division, ASCE, 92:34-48, September 1966.

13. Hansen, V. E. Mathematical relationships expressing the hydraulics of surface irrigation. Proceedings of the ARS-SCS Workshop on Hydraulics of Surface Irrigation, ARS 41-43, October 1960.

14. Israelsen, O. W. and Hansen, V. E. Irrigation Principles and Practices. John Wiley and Sons, Inc., New York, 1962.

15. Isaacson, E., Stoker, J. J., and Troesch, A. Numerical solution of flood prediction and river regulation problems. Reports 11 and 111, New York University, January 1954 and October 1956.

16. Keulegan, G. H. Wave Motion. Engineering Hydraulics edited by Hunter Rouse. John Wiley and Sons, Inc., New York, 1950.

17. Klute, A. A numerical method for solving the flow equation for water in unsaturated materials. Soil Science, 73:105-116, 1952.

18. Kruger, W. E. and Bassett, D. L. Unsteady flow of water over a porous bed having constant infiltration. Transactions, ASAE, 8:(1)60-62, 1965.

19. Lamb, H. Hydrodynamics, 6th edition. Cambridge University Press, 1932.

20. Lewis, M. R. and Milne, W. E. Analysis of border irrigation. Agricultural Engineering, 19:267-272, June 1938.

21. Liggett, James A. Unsteady open channel flow with lateral inflow. Technical Report No. 2, Stanford University, 1959.

22. Liggett, James A. and Woolhiser, David A. Difference solutions of shallow water equation. Journal of the Engineering Mechanics Division, ASCE, 93:39-71, April 1967.

23. Massau, Junius, Jr. Appendiceau memorire sur l'integration graphique. Ann. Assoc. Ingenieurs Sortis Ecoles Speciales Gand, 12:185-444, Ghent, Belgium, 1889.

24. Olsen, Edwin C. Hydrodynamics of unsteady open channel fluid flow over a porous bed having a variable infiltration rate. Ph.D. dissertation. Utah State University, 1965.

25. Philip, J. R. The theory of infiltration: 1. the infiltration equation and its solution. Soil Science, 83:(5)345-357, May 1957.

26. Philip, J. R. and Farrell, D. A. General solution of the infiltration advance problem in irrigation hydraulics. Journal of Geophysical Research, 69:621-631, February 1964.

27. Rouse, Hunter. Engineering Hydraulics. John Wiley and Sons, Inc., New York, 1950.

28. Saint-Venant, B. de. Memoire sur un mode d'interpolation applicable a des questions relatives au mouvement des eaus, et suppleant a l'integration souvent impossible des equations aux derivees partielles. Comples rendus, 17, 1843.

29. Schoklitsch, A. Uber Dambruchwellen. Sitzungsberichte der k. Akademie der Wissenschafter, 126:1489, 1917.

30. Schreiber, D. L. Hydraulic description of recession of shallow flow over a porous bed. M.S. thesis. Washington State University, 1965.

31. Singh, R. Solution of a diffusion equation. Journal of the Hydraulics Division, ASCE, 93:43-50, September 1967.

32. Smerdon, E. T. Subsurface water distribution in surface irrigation. Journal of the Irrigation and Drainage Division, ASCE, 89:1-15, March 1963.

33. Smerdon, E. T. and Glass, L. J. Surface irrigation water distribution efficiency related to soil infiltration. Transactions, ASAE, 8:(1)76-78, 1965.

34. Smerdon, E. T. and Hohn, C. M. Relationships between the rate of advance and intake rate in furrow irrigation. Miscellaneous Publication No. 509, Texas Agricultural Experiment Station, College Station, Texas, April 1961.

35. Stoker, J. J. Numerical solution of flood prediction and river regulation problems. Report 1, New York University, October 1953.

36. Texas Almanac. A. H. Belo Corporation, Dallas, Texas, 1965.

37. Thomas, H. A. The hydraulics of flood prediction in rivers. Engineering Bulletin, Carnegie Institute of Technology, Pittsburgh, Pennsylvania, 1937.

38. Thornton, John F. Summary of hydraulics of furrow irrigation studies in Missouri. Proceedings of the ARS-SCS Workshop on Hydraulics of Surface Irrigation, ARS 41-43, October 1960.

39. Wei, T. C. Overland flow hydrograph synthesis by digital computer. M.S. thesis. Texas A&M University, 1966.

40. Wilke, O. C. and Smerdon, E. T. A solution of the irrigation advance problem. Journal of the Irrigation and Drainage Division, ASCE, 91:23-34, September 1965.

APPENDICES

APPENDIX A

NOTATION

A	cross-sectional area of flow
a, a_1	empirical constants in a power equation describing wetting front advance, $x = a_1 t^a$
b, b_1	empirical constants in a power equation describing furrow shape, $A = b_1 h^b$
C	Chezy's roughness parameter
C^+, C^-	characteristic directions in the x-t plane
c	average area of surface flow during the time of wetting front advance
c_1, c_2	constants of proportionality
D	hydraulic depth, $D = A/W$
D_o	normal hydraulic depth for $Q = QI$
D_*	dimensionless length ratio, $D_* = D/D_o$
d	distance from channel bed to the center of gravity of flow
E_d	water distribution efficiency
F	unbalanced force acting on a fluid element
FG	gravity force acting on a fluid element
FP	pressure force acting on a fluid element
FV	viscous force acting on a fluid element

Fr	Froude number, $Fr = V_o/(gD_o)^{\frac{1}{2}}$
f	Darcy-Weisbach friction factor
g	acceleration of gravity
h	depth of flow
h_o	normal depth of flow for Q = QI
$\bar{h}$	distance from the free surface to the center of gravity of flow
I	infiltration rate
I_L	total infiltration rate occurring along the entire furrow length
j	integer of summation
k	empirical constant in a cumulative infiltration equation, $y = kt_o^\alpha$
k_s	height of surface roughness elements
L	furrow length
L_1, L_2	differential equations of motion
m	mass of a fluid element
n	Manning's roughness parameter
P	wetted perimeter of flow
Q	flow rate, volume per unit of time
QI	initial flow rate entering furrow
R	hydraulic radius, R = A/P

Re	Reynold's number, $Re = 4RV/\nu$
R_o	normal hydraulic radius for Q = QI
r	dimensionless roughness parameter, $r = g/C^2$
S	furrow slope
SF	friction slope
T	dimensionless time ratio, $T = t_L/t_a$
T_c	critical tractive shear, force per unit area, $T_c = \gamma R_o S$
t	time
t_a	time required for an average application of water to infiltrate
t_L	time required for the wetting front to reach the end of the furrow
t_o	opportunity time; time the soil at a point has been wetted
t_r	time of recession after the advance has reached the end of the furrow
t_*	dimensionless time ratio, $t_* = tV_o/D_o$
t_{o*}	dimensionless time ratio, $t_{o*} = t_o V_o/D_o$
V	velocity of flow
V_o	normal flow velocity for Q = QI
V_*	dimensionless velocity parameter, $V_* = V/V_o$

W	top width of flow
X	dimensionless distance ratio, $X = x/L$
x	distance measured from the upstream end of the furrow
x_*	dimensionless distance ratio, $x_* = x/D_o$
Y	dimensionless application ratio, $Y = y/y_a$
y	cumulative infiltration, volume per unit length of furrow
y_a	average application, volume per unit length of furrow
z	dimensionless ratio, $z = (D_o I_{t_*=1})/(V_o A_{D=D_o})$
α	constant in the cumulative infiltration equation, $y = kt_o^{\alpha}$
Γ	symbol for the Gamma function
γ	unit weight of water
θ	angle between the furrow bed and a horizontal plane, in radians
λ_1, λ_2	functional coefficients of L_1 and L_2, respectively
ν	kinematic viscosity of water
ρ	fluid density
σ	a variable dependent on x and t: $\sigma = \sigma(x, t)$

APPENDIX B

ILLUSTRATIVE DESIGN PROBLEMS

Problem 1

An irrigator wishes to apply an average of three inches of water to cotton planted in 40-inch rows. The cumulative infiltration as a function of time is given by the equation, $y = 0.132t_o^{0.3}$, in which y is the application in cubic feet per linear foot of furrow length and t_o is the opportunity time in minutes. The furrow slope is one per cent and the furrows are 2,600 feet long. How should the flow rate vary with time to prevent excessive runoff? What will be the approximate water distribution efficiency? Assume that the maximum permissible tractive shear of the soil which will not cause excessive erosion in the furrows, T_c, is 0.05 pounds per square foot.

Step 1: Determination of the average application, y_a, and of the time, t_a, required for the average application to infiltrate into the soil, *i.e.*,

$$y_a = \frac{3\ \text{in.}}{12\ \text{in./ft}} \, \frac{(40/12)\ \text{ft}^2}{1\ \text{linear ft of furrow}} = 0.833\ \text{ft}^3/\text{ft}.$$

Obtain t_a from the infiltration equation,

$$t_a = (y_a/k)^{1/\alpha} = (0.833/0.132)^{3.33} = 460\ \text{min}.$$

Step 2: Determination of the maximum allowable upstream hydraulic radius (based on the size of the furrow stream which will not cause excessive erosion). From Equation 6.2,

$$R_o = T_c/\gamma S = \frac{0.05 \text{ lb/ft}^2}{(62.4 \text{ lb/ft}^3)(0.01)} = 0.08 \text{ ft.}$$

Step 3: Determination of the upstream cross-sectional area of flow and of the upstream depth of flow. Furrow shape relationships for typical cotton irrigation furrows in Texas are presented in Figs. 3.1 and 3.2. From Fig. 3.2

$$P = 6.35\ h^{0.675} \text{ft.}$$

From Fig. 3.1 and Equation 3.9

$$A = 3.6h^{1.67} \text{ft}^2$$

$$R_o = A/P = (3.6/6.35)\ h^{0.995} \text{ft} = 0.08 \text{ ft.}$$

$$h = 0.14 \text{ ft.}$$

$$A = 0.135 \text{ ft}^2.$$

Step 4: Estimation of the average area of surface storage, c. Estimate the average area of surface storage to be three-fourths of the upstream cross-sectional area of flow (Smerdon and Hohn (34) determined the average area of surface storage to be 0.77 times the upstream cross-sectional area of flow).

$$c = 0.75\ A = 0.101 \text{ ft}^2.$$

Step 5: Determination of the upstream flow velocity. Assume the value of Manning's n to be 0.046 as obtained from Fig. 3.5. Then, using Manning's equation, Equation 3.2,

$$V = \frac{1.486}{n} R^{2/3} S^{1/2} = 0.60 \text{ ft/sec.}$$

Step 6: Determination of the initial furrow stream size.

$$QI = AV = 0.081 \text{ ft}^3/\text{sec} = 4.85 \text{ ft}^3/\text{min.}$$

Step 7: Determination of the time required for the wetting front to reach the end of the furrow, t_L. Try various values of t_L until the value of kt_L^α/c obtained from Fig. 4.1 is equal to the computed value of kt_L^α/c. Values of k and α are: $k = 0.132$ and $\alpha = 0.3$. Values of c and QI were determined in Steps 4 and 6, respectively.

t_L min	$\frac{QI\ t_L}{cL}$	kt_L^α/c computed	kt_L^α/c Fig. 4.1
300.	5.54	7.24	5.62
400.	7.38	7.88	7.90
390.	7.20	7.84	7.68
398.	7.35	7.86	7.87

Use t_L = 398 min.

Step 8: Estimation of the time at which the cutback flow rate should begin. In order to reduce the inflow soon enough to prevent flow past the downstream end of the furrow, the irrigator can estimate the path of the particular C^+ -characteristic which reaches the point, $x = L$, at time, $t = t_L$. From Equation 2.46

$$dx/dt = V + \sqrt{gD}.$$

The maximum value of dx/dt occurs at the upstream end of the furrow where $V = 0.60$ ft/sec and $h = 0.14$ ft ($\sqrt{gD} = \sqrt{gh/b}$ =

1.64 ft/sec). Thus the minimum time required for the C^+-characteristic in question to traverse the furrow length of 2,600 feet is

$$\frac{2{,}600 \text{ ft}}{(V + \sqrt{gD})_{x=0}} = \frac{2{,}600 \text{ ft}}{2.24 \text{ ft/sec}} = 1{,}160 \text{ sec} \approx 20 \text{ min.}$$

If the reduction in inflow is initiated not more than twenty minutes before the wetting front reaches the point, $x = L$, then the wetting front should definitely reach the end of the furrow and insure that the entire furrow length is wetted. Therefore, begin to reduce the inflow at

$$t = t_L - 20 \text{ min} = 378 \text{ min}$$

or

$$t_L/t = \frac{398}{378} = 1.05.$$

Step 9: Determination of the time at which inflow should be terminated. Using the results of Steps 1 and 7

$$t_a/t_L = \frac{460}{398} = 1.155.$$

From Fig. 6.6 or from Fig. 6.3 and Equation 6.15, the time at which inflow is terminated is

$$t_L/t = 0.718$$

or

$$t = 554 \text{ min.}$$

The relationship utilized in determining this value is based on the assumption of a horizontal recession occurring at the time inflow is terminated. Because the recession is not instantaneous it may be desirable to terminate the inflow sooner.

Step 10: Estimation of the water distribution efficiency, E_d.

From Fig. 6.6

$$E_d \simeq 90\%.$$

Step 11: Determination of the variation in cutback flow rate with time.

From Step 7

$$kt_L^\alpha/c = 7.86.$$

Using Fig. 6.6, interpolate between the design curves for $kt_L^\alpha/c = 2$ and $kt_L^\alpha/c = 10$ to obtain the flow rate, Q, as a function of time as shown in Fig. B.1.

Total water applied $= 2{,}230\ ft^3$.

Desired application $= y_aL = (0.833\ ft^2)(2{,}600\ ft)$

$= 2{,}160\ ft^3$.

Estimated surface storage at time of inflow termination (See Step 4.): $c \simeq 180\ ft^3$.

Step 13: Determination of the application amounts at the ends of the furrow. At the upstream end the final opportunity time is approximately equal to the time the water is turned on.

$$t_{o_{x=0}} = 554\ min.$$

$$y_{x=0} = kt_o^\alpha = 0.132(554)^{0.3} = 0.88\ ft^3/ft$$

$= 3.17$ in. of applied water.

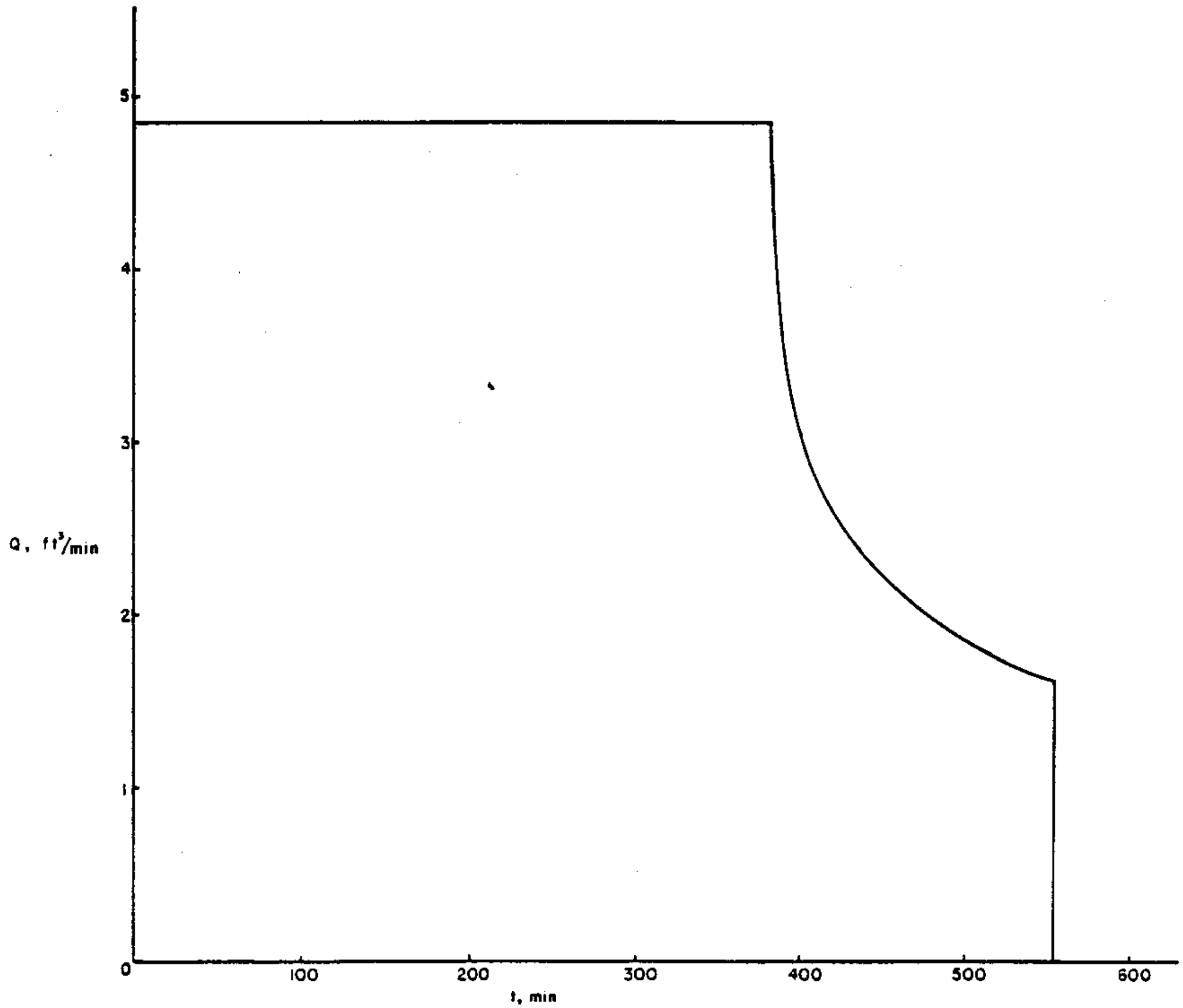

FIG. B.1. DESIGN CUTBACK FLOW RATE FOR PROBLEM 1.

The minimum possible final opportunity time at the downstream end is the time water is turned on minus the time required for the wetting front to reach the end of the furrow.

$$y_{x=2,600\ ft} > 0.132(554 - 398)^{0.3} = 0.60\ ft^3/ft$$
$$= 2.16 \text{ in. of applied water.}$$

<u>Problem 2</u>

What distribution efficiency could be obtained in Problem 1 if the furrow length were reduced to 1,700 feet? How should the flow rate be reduced to prevent excessive runoff?

Step 1: $y_a = 0.833\ ft^3/ft$; $t_a = 460$ min.

Step 2: $R_o = 0.08$ ft.

Step 3: $h_{x-0} = 0.14$ ft; $A_{x=0} = 0.135\ ft^2$.

Step 4: $c = 0.101\ ft^2$.

Step 5: $V_{x-0} = 0.60$ ft/sec.

Step 6: $QI = 4.85\ ft^3/min$.

Step 7: $t_L = 225$ min; $kt_L^\alpha/c = 6.65$.

Step 8: Begin inflow rate reduction at $t = 212$ min or $t_L/t = 1.06$.

Step 9: Terminate inflow at $t_L/t = 0.45$.

Step 10: $E_d \simeq 95\%$.

Step 11: Obtain Q as a function of time from Fig. 6.6 for $kt_L^\alpha/c = 6.65$.

Step 12: Volume of water applied equals 1,535 ft^3. Desired application, y_aL, equals 1,416 ft^3. Surface storage at time of inflow termination equals 44 ft^3.

Step 13: $y_{x=0}$ = 0.85 ft^3/ft = 3.06 in. of applied water;

$y_{x=L}$ > 0.71 ft^3/ft = 2.56 in. of applied water.

Problem 3

An irrigator wishes to apply three inches of water to cotton planted in 40-inch rows. The cumulative infiltration as a function of time is given by the equation, $y = 0.0268t_o^{0.585}$, in which y is the application in cubic feet per foot of furrow length and t_o is the opportunity time in minutes. The furrow slope is 0.5 per cent and the furrows are 2,600 feet long. How should the flow rate vary with time and what will be the approximate water distribution efficiency? The permissible tractive shear, T_c, is 0.05 pounds per square foot.

Step 1: y_a = 0.833 ft^3/ft; t_a = 353 min.

Step 2: R_o = 0.16 ft.

Step 3: $h_{x=0}$ = 0.28 ft; $A_{x=0}$ = 0.432 ft^2.

Step 4: c = 0.324 ft^2.

Step 5: $V_{x=0}$ = 0.677 ft/sec.

Step 6: QI = 17.5 ft^3/min.

Step 7: t_L = 83 min; kt_L^{α}/c = 1.1.

Step 8: Begin inflow reduction at t_L/t = 1.22.

Step 9: Terminate inflow at t_L/t = 0.236.

Step 10: $E_d \approx 97\%$.

Step 11: Determine flow rate as a function of time for α = 0.585 and kt^{α}/c = 1.1 by interpolation from Figs. 6.7 and 6.8.

Step 12: Applied water equals 2,540 ft^3. Desired application, y_aL, equals 2,160 ft^3. Estimated surface storage at time of inflow termination equals 200 ft^3.

Step 13: $y_{x=0}$ = 0.83 ft^3/ft = 3.0 in. of applied water; $y_{x=L}$ > 0.71 ft^3/ft = 2.6 in. of applied water.

Problem 4

An irrigator wishes to apply an average depth of five inches of water to cotton planted in 40-inch rows. The cumulative infiltration as a function of time is given by the equation, $y = 0.015t_o^{0.7}$, in which y is the application in cubic feet per foot of furrow length and t_o is the opportunity time in minutes. The furrow slope is one per cent and the furrows are 1,000 feet long. The maximum permissible tractive shear is 0.05 pounds per square foot. What distribution efficiency can be obtained if a cutback flow is applied?

Step 1: y_a = 1.39 ft^3/ft; t_a = 650 min.

Step 2: $R_o = 0.08$ ft.

Step 3: $h_{x=0} = 0.14$ ft; $A_{x=0} = 0.135$ ft^2.

Step 4: $c = 0.101$ ft^2.

Step 5: $V_{x=0} = 0.60$ ft/sec.

Step 6: QI = 4.85 ft^3/min.

Step 7: $t_L = 57$ min; $kt_L^{\alpha}/c = 2.5$.

Step 8: Begin inflow reduction at $t_L/t = 1.14$.

Step 9: Terminate inflow at $t_L/t \simeq t_L/t_a = 0.088$.

Step 10: $E_d \simeq 99\%$.

Step 11: Determine flow rate as a function of time for $\alpha = 0.7$ and $kt_L^{\alpha}/c = 2.5$ by interpolation from Fig. 6.8.

Step 12: Applied water equals 1,430 ft^3. Desired application equals 1,390 ft^3. Estimated surface storage at time of inflow termination equals 35 ft^3.

Step 13: $y_{x=0} = 1.4$ ft^3/ft = 5.0 in. of applied water; $y_{x=L} > 1.3$ ft^3/ft = 4.7 in. of applied water.

VITA

Otto Charles Wilke was born on April 16, 1941, to Esther and Gottlieb Wilke. The fourth of seven children, he attended Texas public elementary schools in Fort Bend and Washington counties. Wilke attended Brenham High School, Brenham, Texas, graduating in May 1959. He received a B.S. degree in Agricultural Engineering from Texas A&M University, College Station, Texas, in January 1964 and an M.S. degree in Agricultural Engineering from Texas A&M in January 1965.

Otto Wilke married Virginia Hensley of Channelview, Texas, on August 14, 1965.

In addition to working on his parents' dairy, Wilke worked four summers for the U.S. Forest Service as a surveyor and fire fighter.

The typist for this dissertation was Mrs. Florace Kling.