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Realization of quantum symplectic group $\mathcal{O}(Sp_q(8))$ via vector representation of $U_q(sp(8))$

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Abstract: To the quantum enveloping algebra $U_q(sl_2(\mathbf{C}))$ of type A Lie algebra A_2, Jantzen gave the defining relations of $SL_q(2)$ arising from the representations of $U_q(sl_2(\mathbf{C}))$ by the R-matrix. This paper studied the similar question to $U_q(sp(8))$ of type C Lie algebra $sp(8)$ which is more difficult than type A, and gave the realization of $\mathcal{O}(Sp_q(8))$ arising from the representations of $U_q(sp(8))$ via Jantzen's approach.

Key words: quantum symplectic group; realization; vector representation

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利用$U_q(sp(8))$的向量表示实现辛型量子群$\mathcal{O}(Sp_q(8))$

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摘要: Jantzen[1] 利用 A 型李代数 A_2 的量子包络代数 $U_q(sl_2(\mathbf{C}))$, 借助于其表示论并利用 R-矩阵的方法给出了 $SL_q(2)$ 的定义关系式. 本文对结构更为复杂的 C 型李代数做了类似的研究, 通过 $U_q(sp(8))$ 的表示理论实现了 $\mathcal{O}(Sp_q(8))$ 的定义关系式.

关键词: 辛型量子群; 实现; 向量表示

0 Preliminaries

Let $\mathbf{C}\langle v_{ij}\rangle_{i,j=1,2,\cdots 8}$ denote the free algebra with generators v_{ij}, $i'=9-i, \rho_i=i-5(\rho_{i'}=-\rho_i$ if $i<i')$, $\epsilon_1=\epsilon_2=\epsilon_3=\epsilon_4=1, \epsilon=\epsilon_5=\epsilon_6=\epsilon_7=\epsilon_8=-1$; $D=(d_{ij}), d_{ij}=\epsilon_i\delta_{ij'}q^{\rho_i}$, $k^{ji}_{mn}=\epsilon d_{ji}d_{mn}, i,j,m,n=1,2,\cdots,8$; $T(k)=1, k>0$; $T(k)=0, k\leqslant 0$. $I(K)$ be the two-sided ideal of $\mathbf{C}\langle v_{ij}\rangle$generated by the elements

$$I^{ij}_{mn}:=\sum_{k,l}K^{ji}_{kl}v_{km}v_{ln}-K^{lk}_{mn}v_{ik}v_{jl},\quad i,j,m,n=1,2,\cdots,8,$$

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where

$$K_{mn}^{ij}=q^{\delta_{ij'}-\delta_{ij}}\delta_{im}\delta_{jn}+(q^{-1}-q)T(m-i)(\delta_{jm}\delta_{in}-k_{mn}^{ji}).$$

Let $A(K)$ denote the quotient algebra $\mathbf{C}\langle v_{ij}\rangle/\ I(K)$. That is $A(K)$ is the algebra generated by elements $v_{ij}, i,j=1,2,\cdots 8$. subject to the relations

$$\sum_{k,l}K_{kl}^{ji}v_{km}v_{ln}=\sum_{k,l}K_{mn}^{lk}v_{ik}v_{jl}\,.$$

Let $\mathbf{V}=(v_{ij})_{8\times 8}$, then the algebra $\mathcal{O}(Sp_q(8))$[2] be difined as the quotients of the algebra $A(K)$, by the metric condition

$$\mathbf{V}D\mathbf{V}^tD^{-1}=D\mathbf{V}^tD^{-1}\mathbf{V}=I_8.$$

I_8 is the 8×8 identity matrix. Let $\mathfrak{g}$ be a complex semisimple Lie algebra. Let Φ be the root system of $\mathfrak{g}$ and Π be the prime root system with respect to a fixed Cartan subalgebra. Let W be the Weyl group of Φ. If Φ is irreducible, then there is a unique W-invariant scalar product $(\,,)$ on the vector space generated by Φ over the reals such that $(\alpha,\alpha)=2$ for all short roots α in Φ. If Φ is of type C_n, then $(\alpha,\alpha)=2$ for the short roots and $(\alpha,\alpha)=4$ for the long roots. Set for each $\alpha\in\Phi$,

$$d_\alpha=\frac{(\alpha,\alpha)}{2}\in\{1,2\},\quad q_\alpha=q^{d_\alpha}=q^{\frac{(\alpha,\alpha)}{2}}\,.$$

The quantized enveloping algebra $U_q(\mathfrak{g})$(Let us abbreviate $U=U_q(\mathfrak{g})$) is defined as the K-algebra with generators $E_\alpha, F_\alpha, K_\alpha, K_\alpha^{-1}(\alpha\in\Pi)$, then there is on U a unique $Hopf$ algebra structrue (Δ,ϵ,S)[1-3]. Let $U^+(U^-)$ be the subalgebra of U with generators $E_\alpha(F_\alpha)$, U^0 be the subalgebra with generatora K_α, K_α^-. The maps

$$\begin{array}{cccccccc} U^-\otimes U^0\otimes U^+ & \longrightarrow & U, & & U^+\otimes U^0\otimes U^- & \longrightarrow & U,\\ u_1\otimes u_2\otimes u_3 & \longmapsto & u_1u_2u_3, & & u_1\otimes u_2\otimes u_3 & \longmapsto & u_1u_2u_3,\end{array}$$

are isomorphisms of vector spaces. We denote $U^{\leqslant 0}, U^{\geqslant 0}$ are the image of $U^-\otimes U^0$ and $U^+\otimes U^0$.

Theorem 0.1[1] There exists a unique bilinear pairing $(,):U^{\leqslant 0}\times U^{\geqslant 0}\longrightarrow K$, such that for all $x,x'\in U^{\geqslant 0}$, $y,y'\in U^{\leqslant 0}$, all $\mu,\nu\in\mathbf{Z}\Phi$, and all $\alpha,\beta\in\Pi$,

$$\begin{array}{ll}(y,xx')=(\Delta(y),x'\otimes x), & (yy',x)=(y\otimes y',\Delta(x)),\\ (K_\mu,K_\nu)=q^{-(\mu,\nu)}, & (F_\alpha,E_\beta)=-\delta_{\alpha\beta}(q_\alpha-q_\alpha^{-1})^{-1},\ ,\\ (K_\mu,E_\alpha)=0, & (F_\alpha,K_\mu)=0.\end{array}$$

Assume that char(K)=0 and that q is not a root of unity, then the restriction of $(,)$ to $U_{-\mu}^-\times U_\mu^+(\mu\in\mathbf{Z}\Phi,\mu\geqslant 0)$ is nondegenerate. Choose for each $\mu\in\mathbf{Z}\Phi,\mu\geqslant 0$ a basis $u_1^\mu,u_2^\mu,\cdots,u_{r(\mu)}^\mu$ of U_μ^+, then we can find a basis $v_1^\mu,v_2^\mu,\cdots v_{r(\mu)}^\mu$, of $U_{-\mu}^-$ such that $(v_i^\mu,u_j^\mu)=\delta_{ij}$ for all i and j. Set

$$\Theta_\mu=\sum_{i=1}^{r(\mu)}v_i^\mu\otimes u_i^\mu\ \in U\otimes U,\quad \Theta=\sum_{\mu\geqslant 0}\Theta_\mu.$$

By linear algebra Θ_μ does not depend on the choice of the basis $(u_i^\mu)_i$. We set $\Theta_0=1\otimes 1$, $\Theta_\mu=0$, if $\mu<0$. Let M and M' be finite dimensional U-modules. We have

$$\Theta_\mu(M_\lambda\otimes M'_{\lambda'})\subset M_{\lambda-\mu}\otimes M'_{\lambda'+\mu},\quad \lambda,\lambda'\in\Lambda,\mu\in\mathbf{Z}\Phi.$$

Since the set of weights of M is finite, there are only finitely many $\mu \in \mathbf{Z}\Phi$ that are different weights of M. Therefore we have $\Theta_\mu(M \otimes M') = 0$ for almost all μ and we can define a linear transformation

$$\Theta = \Theta_{M,M'}: \quad M \otimes M' \longrightarrow M \otimes M'.$$

We define for all finite dimensional U-modules M and M' bijective linear maps:

$$\begin{array}{llllllll} P: & M \otimes M' & \longrightarrow & M' \otimes M, & \widetilde{f}: & M \otimes M' & \longrightarrow & M \otimes M', \\ & m \otimes m' & \longmapsto & m' \otimes m, & & m \otimes m' & \longmapsto & f(\lambda, \mu) m \otimes m', \end{array}$$

where $f: \Lambda \times \Lambda \longrightarrow K^\times$, with

$$f(\lambda + \nu, \mu) = q^{-(\nu,\mu)} f(\lambda, \mu), \quad f(\lambda, \mu + \nu) = q^{-(\nu,\lambda)} f(\lambda, \mu).$$

for all $\lambda, \mu \in \Lambda, \nu \in \mathbf{Z}\Phi$.

Theorem 0.2[1] For all finite U-modules M, M' the map

$$R = \Theta \circ \widetilde{f} \circ P: \quad M' \otimes M \longrightarrow M \otimes M'$$

is an isomorphism of U-modules[1].

If f, $g \in U^\circ$(subspace in the dual space of U), $u \in U$, and $\Delta(u) = \sum_i u_i \otimes u_i'$, then we can define the product $fg(u) = \Sigma_i f(u_i) g(u_i')$. Let $\mathfrak{g} = sp(2n)$, M is the $2n$-dimensional verctor space, as the module of the representation of $U = U_q(\mathfrak{g})$, We can define matrix coefficients $c_{f,m} \in U^\circ$ for all $m \in M, f \in M^\star$, such that $c_{f,m}(v) = f(vm)$, $v \in U$, then the space of U° spanned by all $c_{f,m}$(for all finite dimensional U-modules M and all m, f) is closed under multiplication. It contains the ring identity ϵ of U°, so this space is a subalgebra of U°.

Let M be a finite dimensional U-module, Choose a basis $e_1, e_2, \cdots, e_n$, and denote by c_{ij} the matrix coefficients with $ue_j = \sum_i c_{ij}(u) e_i, j = 1, 2, \cdots, n, u \in U$. Let M' be another finite dimensional U-module, with basis $e_1', e_2', \cdots, e_m'$, and correspongding matrix coefficients c_{ij}', R: $M' \otimes M \longrightarrow M \otimes M'$ is an isomorphism of U-modules, satisfying

$$R(e_i' \otimes e_j) = \sum_{h,l} R_{ij}^{hl} e_h \otimes e_l'.$$

Theorem 0.3[1] We have for all i, j, r, s

$$\sum_{h,l} R_{hl}^{rs} c_{hi}' c_{lj} = \sum_{h,l} R_{ij}^{hl} c_{rh} c_{sl}'.$$

1 Main result

For $M = \mathbf{C}^8$, as the module of the vector representation of $U_q(sp(8))$, choose a basis of M:

$$e_i = (0, 0, \cdots, 0, \overset{i}{1}, 0, \cdots, 0)', \quad 1 \leqslant i \leqslant 4,$$

$$e_i = (0, 0, \cdots, 0, \overset{13-i}{1}, 0, \cdots, 0)', \quad 5 \leqslant i \leqslant 8.$$

and let

$$
\begin{aligned}
&E_{\alpha_1}=E_{4,8}, \quad E_{\alpha_i}=E_{5-i,6-i}-E_{10-i,9-i}, \quad (2\leqslant i\leqslant 4),\\
&F_{\alpha_1}=E_{8,4}, \quad F_{\alpha_i}=E_{6-i,5-i}-E_{9-i,10-i}, \quad (2\leqslant i\leqslant 4),\\
&K_{\alpha_i}=D_{5-i}D_{6-i}^{-1}D_{4+i}^{-1}D_{3+i}, \quad 2\leqslant i\leqslant 4,\\
&K_{\alpha_1}=D_4^2D_5^{-2}.
\end{aligned}
$$

Where D_j is the diagonal matrix $\Sigma_i q^{\delta_{ij}}E_{ii}$, E_{ij} is the 8×8 matrix with 1 in the (i,j)-position and 0 elsewhere,α_1 is the longest root. Then we have

$$
\begin{aligned}
&F_{\alpha_{5-i}}e_i=e_{i+1}, \quad E_{\alpha_{5-i}}e_{i+1}=e_i, \qquad (1\leqslant i\leqslant 4),\\
&F_{\alpha_i}e_{3+i}=-e_{4+i}, \quad E_{\alpha_i}e_{4+i}=-e_{3+i}, \quad (2\leqslant i\leqslant 4).
\end{aligned},
$$

$$
\begin{aligned}
M_{\epsilon_{5-i}}&=Ce_i, \quad (1\leqslant i\leqslant 4),\\
M_{-\epsilon_{i-4}}&=Ce_i, \quad (5\leqslant i\leqslant 8).
\end{aligned}
$$

c_{ij} are the matrix coefficient of M, to any $u\in U_q(sp(8))$, we have $ue_j=\sum_i c_{ij}(u)e_i$,

$$
\begin{aligned}
c_{i+1,i}(F_{\alpha_{5-i}})&=1, \quad c_{i,i+1}(E_{\alpha_{5-i}})=1, \quad 1\leq i\leqslant 4,\\
c_{4+i,3+i}(F_{\alpha_i})&=-1, \quad c_{3+i,4+i}(E_{\alpha_i})=-1, \quad 2\leqslant i\leqslant 4,\\
c_{ii}(K_{\alpha_j})&=q^{\delta_{j,5-i}-\delta_{j-1,5-i}}, \quad 1\leqslant i\leqslant 4, 2\leqslant j\leqslant 4,\\
c_{ii}(K_{\alpha_j})&=q^{\delta_{j-1,i-4}-\delta_{j,i-4}}, \quad 5\leqslant i\leqslant 8, 2\leqslant j\leqslant 4,\\
c_{ii}(K_{\alpha_1})&=q^{2(\delta_{1,5-i}-\delta_{1,i-4})}, \quad 1\leqslant i\leqslant 8.
\end{aligned}
$$

For $\mathfrak{g}=sp(8)$, $\omega_0=s_4s_3s_2s_1s_2s_3s_4s_3s_2s_1s_2s_3s_2s_1s_2s_1$[4] as a reduced expression of the longest element of the Weyl group of $\mathfrak{g}$, then $\beta_1=\alpha_4, \beta_2=\alpha_3+\alpha_4, \beta_3=\alpha_2+\alpha_3+\alpha_4, \beta_4=\alpha_1+2\alpha_2+2\alpha_3+2\alpha_4, \beta_5=\alpha_1+\alpha_2+\alpha_3+\alpha_4, \beta_6=\alpha_1+2\alpha_2+\alpha_3+\alpha_4, \beta_7=\alpha_1+2\alpha_2+2\alpha_3+\alpha_4$, $\beta_8=\alpha_3, \beta_9=\alpha_2+\alpha_3, \beta_{10}=2\alpha_2+2\alpha_3+\alpha_1, \beta_{11}=\alpha_2+\alpha_3+\alpha_1, \beta_{12}=2\alpha_2+\alpha_3+\alpha_1$, $\beta_{13}=\alpha_2, \beta_{14}=2\alpha_2+\alpha_1, \beta_{15}=\alpha_2+\alpha_1, \beta_{16}=\alpha_1$ are all the positive roots of $\mathfrak{g}$. If we only reserve the positive(negative) root vectors of $\mathfrak{g}$ that have effect on the R-matrix, we have

$$
\begin{aligned}
&X_1=E_{\alpha_4}, \quad X_2=E_{\alpha_4}E_{\alpha_3}-q^{-1}E_{\alpha_3}E_{\alpha_4}, \quad X_3=E_{\alpha_4}E_{\alpha_3}E_{\alpha_2}+q^{-2}E_{\alpha_2}E_{\alpha_3}E_{\alpha_4},\\
&X_4=-q^{-3}E_{\alpha_4}E_{\alpha_3}E_{\alpha_2}E_{\alpha_1}E_{\alpha_2}E_{\alpha_3}E_{\alpha_4}, \quad X_5=E_{\alpha_4}E_{\alpha_3}E_{\alpha_2}E_{\alpha_1}-q^{-4}E_{\alpha_1}E_{\alpha_2}E_{\alpha_3}E_{\alpha_4},\\
&X_6=E_{\alpha_4}E_{\alpha_3}E_{\alpha_2}E_{\alpha_1}E_{\alpha_2}+q^{-5}E_{\alpha_2}E_{\alpha_1}E_{\alpha_2}E_{\alpha_3}E_{\alpha_4},\\
&X_7=E_{\alpha_4}E_{\alpha_3}E_{\alpha_2}E_{\alpha_1}E_{\alpha_2}E_{\alpha_3}-q^{-6}E_{\alpha_3}E_{\alpha_2}E_{\alpha_1}E_{\alpha_2}E_{\alpha_3}E_{\alpha_4},\\
&X_8=E_{\alpha_3}, \quad X_9=E_{\alpha_3}E_{\alpha_2}-q^{-1}E_{\alpha_2}E_{\alpha_3}, \quad X_{10}=q^{-2}E_{\alpha_3}E_{\alpha_2}E_{\alpha_1}E_{\alpha_2}E_{\alpha_3},\\
&X_{11}=E_{\alpha_3}E_{\alpha_2}E_{\alpha_1}+q^{-3}E_{\alpha_1}E_{\alpha_2}E_{\alpha_3}, \quad X_{12}=E_{\alpha_3}E_{\alpha_2}E_{\alpha_1}E_{\alpha_2}-q^{-4}E_{\alpha_2}E_{\alpha_1}E_{\alpha_2}E_{\alpha_3},\\
&X_{13}=E_{\alpha_2}, \quad X_{14}=-q^{-1}E_{\alpha_2}E_{\alpha_1}E_{\alpha_2}, \quad X_{15}=E_{\alpha_2}E_{\alpha_1}-q^{-2}E_{\alpha_1}E_{\alpha_2} \quad X_{16}=E_{\alpha_1}.\\
&Y_1=F_{\alpha_4}, \quad Y_2=F_{\alpha_3}F_{\alpha_4}-qF_{\alpha_4}F_{\alpha_3}, \quad Y_3=F_{\alpha_2}F_{\alpha_3}F_{\alpha_4}+q^2F_{\alpha_4}F_{\alpha_3}F_{\alpha_2},\\
&Y_4=-q^3F_{\alpha_4}F_{\alpha_3}F_{\alpha_2}F_{\alpha_1}F_{\alpha_2}F_{\alpha_3}F_{\alpha_4}, \quad Y_5=F_{\alpha_1}F_{\alpha_2}F_{\alpha_3}F_{\alpha_4}-q^4F_{\alpha_4}F_{\alpha_3}F_{\alpha_2}F_{\alpha_1},
\end{aligned}
$$

$$Y_6 = F_{\alpha_2}F_{\alpha_1}F_{\alpha_2}F_{\alpha_3}F_{\alpha_4} + q^5F_{\alpha_4}F_{\alpha_3}F_{\alpha_2}F_{\alpha_1}F_{\alpha_2},$$
$$Y_7 = F_{\alpha_3}F_{\alpha_2}F_{\alpha_1}F_{\alpha_2}F_{\alpha_3}F_{\alpha_4} - q^6F_{\alpha_4}F_{\alpha_3}F_{\alpha_2}F_{\alpha_1}F_{\alpha_2}F_{\alpha_3},$$
$$Y_8 = F_{\alpha_3}, \quad Y_9 = F_{\alpha_2}F_{\alpha_3} - qF_{\alpha_3}F_{\alpha_2}, \quad Y_{10} = q^2F_{\alpha_3}F_{\alpha_2}F_{\alpha_1}F_{\alpha_2}F_{\alpha_3},$$
$$Y_{11} = F_{\alpha_1}F_{\alpha_2}F_{\alpha_3} + q^3F_{\alpha_3}F_{\alpha_2}F_{\alpha_1}, \quad Y_{12} = F_{\alpha_2}F_{\alpha_1}F_{\alpha_2}F_{\alpha_3} - q^4F_{\alpha_3}F_{\alpha_2}F_{\alpha_1}F_{\alpha_2},$$
$$Y_{13} = F_{\alpha_2}, \quad Y_{14} = -qF_{\alpha_2}F_{\alpha_1}F_{\alpha_2}, \quad Y_{15} = F_{\alpha_1}F_{\alpha_2} - q^2F_{\alpha_2}F_{\alpha_1} \quad Y_{16} = F_{\alpha_1}.$$

Note For convinence, we take $F_{i_1i_2\cdots i_k}$ as $F_{\alpha_{i_1}}F_{\alpha_{i_2}}\cdots F_{\alpha_{i_k}}$, take $E_{i_1i_2\cdots i_l}$ as $E_{\alpha_{i_1}}E_{\alpha_{i_2}}\cdots E_{\alpha_{i_l}}$, $q_{i_1i_2i_3\cdots i_l} = q_{\alpha_{i_1}}q_{\alpha_{i_2}}q_{\alpha_{i_3}}\cdots q_{\alpha_{i_l}}$, $q_{\alpha_i} = q^{\frac{(\alpha_i,\alpha_i)}{2}}$.
Let

$\Theta' = 1\otimes 1+(q^{-2}-1)(1+q^2)F_1\otimes E_1+(q^{-1}-q)F_2\otimes E_2+(q^{-1}-q)F_3\otimes E_3+(q^{-1}-q)F_4\otimes E_4+(q^{-1}-q)F_{12}\otimes E_{21}+(q^{-1}-q)F_{21}\otimes E_{12}+q_{12}(1-q^{-2})F_{12}\otimes E_{12}+q_{21}(1-q^{-2})F_{21}\otimes E_{21}+(q^{-1}-q)F_{32}\otimes E_{23}+(q^{-1}-q)F_{23}\otimes E_{32}+q_{32}(1-q^{-2})F_{32}\otimes E_{32}+q_{23}(1-q^{-2})F_{23}\otimes E_{23}+(q^{-1}-q)F_{34}\otimes E_{43}+(q^{-1}-q)F_{43}\otimes E_{34}+q_{34}(1-q^{-2})F_{34}\otimes E_{34}+q_{43}(1-q^{-2})F_{43}\otimes E_{43}+(q^{-1}-q)F_{321}\otimes E_{123}+(q^{-1}-q)F_{123}\otimes E_{321}+q_{321}(-1)^3(1-q^{-2})F_{321}\otimes E_{321}+q_{123}(-1)^3(1-q^{-2})F_{123}\otimes E_{123}+(q^{-1}-q)F_{432}\otimes E_{234}+(q^{-1}-q)F_{234}\otimes E_{432}+q_{432}(-1)^3(1-q^{-2})F_{432}\otimes E_{432}+q_{234}(-1)^3(1-q^{-2})F_{234}\otimes E_{234}+(q^{-2}-1)(1+q_{212})F_{212}\otimes E_{212}+(q^{-1}-q)F_{2123}\otimes E_{3212}+(q^{-1}-q)F_{3212}\otimes E_{2123}+q_{2123}(1-q^{-2})F_{2123}\otimes E_{2123}+q_{3212}(1-q^{-2})F_{3212}\otimes E_{3212}+(q^{-1}-q)F_{4321}\otimes E_{1234}+(q^{-1}-q)F_{1234}\otimes E_{4321}+q_{4321}(1-q^{-2})F_{4321}\otimes E_{4321}+q_{1234}(1-q^{-2})F_{1234}\otimes E_{1234}+(q^{-2}-1)(1+q_{32123})F_{32123}\otimes E_{32123}+(q^{-1}-q)F_{21234}\otimes E_{43212}+(q^{-1}-q)F_{43212}\otimes E_{21234}+q_{21234}(-1)^5(1-q^{-2})F_{21234}\otimes E_{21234}+q_{43212}(-1)^5(1-q^{-2})F_{43212}\otimes E_{43212}+(q^{-1}-q)F_{321234}\otimes E_{432123}+(q^{-1}-q)F_{432123}\otimes E_{321234}+q_{321234}(1-q^{-2})F_{321234}\otimes E_{321234}+q_{432123}(1-q^{-2})F_{432123}\otimes E_{432123}+(q^{-2}-1)(1+q_{4321234})F_{4321234}\otimes E_{4321234}$.

It is easy to prove

$$R = \Theta\circ\widetilde{f}\circ P = \Theta'\circ\widetilde{f}\circ P: \quad M\otimes M \longrightarrow M\otimes M$$

is an isomorphism of U-modules.Let $\mathsf{C} = (c_{ij})$, according to therom 0.3, we have

$$\sum_{h,l} R^{rs}_{hl}c_{hi}c_{lj} = \sum_{h,l} R^{hl}_{ij}c_{rh}c_{sl}, \quad r,s,i,j = 1,2,\cdots,8.$$

We can get $\mathsf{C}D\mathsf{C}^tD^{-1} = D\mathsf{C}^tD^{-1}\mathsf{C} = I_8$ and $K^{kl}_{mn} = R^{kl}_{mn}$,then we get the defining relations of $\mathcal{O}(Sp_q(8))$.

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