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低 Hom-Leibniz

[1] , [2] , [3] , [3]
(1 . , 137000 ; 2 . , 130052 ;
3 . , 130024)

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Classification of Low-Dimensional Hom-Leibniz Algebras

XU Li-yuan[1] , WANG Chun-yue[2] , ZHANG Ruo-lan[3] , ZHANG Qing-cheng[3]
(1 , , 137000 , , ;
2 , , 130052 , ;
3 , , 130024 ,)

Abstract : The authors determined the two dimensional and three dimensional endomorphism of Leibniz algebras on complex field using undetermined coefficients method , and then clssified the Hom-Leibniz algebras of their associated non-Lie algebra .
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≠ l′ 1) ($^{1}_{3}$) [(4 $_2$4) , ($_2$)] = $^{2}_{2}$ ő $_3$ 1; - ų l′

2) ($^{2}_{3}$) [($_2$) , ($_1$)] = $_1$ $_2$ $_3$;

3) ($^{3}_{3}$) [($_1$) , ($_1$)] = ($^{2}_{1}$ + $^{2}_{2}$) $_3$, [($_2$) , ($_2$)] = ($^{2}_{1}$ + $^{2}_{2}$) $_3$ (| $_1$ | + | $_2$ | ≠ 0) ;

4) ($^{4}_{3}$) [($_1$) , ($_1$)] = ($^{2}_{1}$ + $^{2}_{2}$ + $_1$ $_2$) $_3$, [($_2$) , ($_1$)] = ($^{2}_{1}$ + $^{2}_{2}$ + $_1$ ≠

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$$A=\begin{pmatrix} {}_1 & 0 & {}_3 \\ {}_1 & {}_2 & {}_3 \\ 0 & 0 & {}^2_2 \end{pmatrix}$$: $[\ (_2),\ (_2)] = {}^2_2\ _3$, $_2 \neq 0$

2) $[_2, _1] = \ _3$: $([_1, _1]) = 0$, $([_1, _2]) = 0$, $([_1, _3]) = 0$, $([_2, _1]) = \ (_3)$, $([_2, _2]) = 0$, $([_2, _3]) = 0$

$$\begin{cases} _1 \quad _2 \quad 0 , \\ _2\ _1 \quad 0 , \\ _2\ _1 \quad 0 , \\ _2\ _1 \quad _3 , \\ _2\ _1 \quad 0 \end{cases}$$

:

$_2 = 0$, $_3 = 0$; $_2 = 0$, $A=\begin{pmatrix} _1 & 0 & _3 \\ _1 & 0 & _3 \\ 0 & 0 & 0 \end{pmatrix}$;

$_2 \neq 0$, $A=\begin{pmatrix} 0 & _2 & _3 \\ 0 & _2 & _3 \\ 0 & 0 & 0 \end{pmatrix}$;

$_2 \neq 0$, $_1 = 0$; $_2 = 0$, $A=\begin{pmatrix} _1 & 0 & _3 \\ 0 & _2 & _3 \\ 0 & 0 & _1\ _2 \end{pmatrix}$;

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$_2 \neq 0$, $_1 = \ _3 = 0$, $A=\begin{pmatrix} 0 & _2 & _3 \\ 0 & _2 & _3 \\ 0 & 0 & 0 \end{pmatrix}$; $0 = \ _2\ _1 [_2, _1] = \ _2\ _1\ _3$, $_2\ _1 = 0$

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$[\ (_2),\ (_1)] = \ _1\ _2\ _3$, $_1\ _2 \neq 0$

3) $([_1, {}^1_1]) = \ (_3)$, $([_1, _2]) = 0$, ([

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$([_3, _2]) = 0$， $([_3, _3]) = 0$

$$\begin{cases} {}_1 \quad {}_2 \quad 0, \\ {}_1\,{}_1 \quad {}_2\,{}_2 \quad {}_2\,{}_1 \quad {}_3, \\ {}_1\,{}_1 \quad {}_2\,{}_2 \quad {}_2\,{}_1 \quad 0, \\ {}_1\,{}_1 \quad {}_2\,{}_2 \quad {}_2\,{}_1 \quad {}_3, \\ {}_1\,{}_1 \quad {}_2\,{}_2 \quad {}_2\,{}_1 \quad {}_3 \end{cases}$$

:

$$_1 \quad \frac{\quad _2 \quad \sqrt{\quad ^2\,{}_2^2 \quad 4(\,{}_2^2 \quad {}_3)}}{2},$$

$$_1 \quad \frac{\quad _2 \quad \sqrt{\quad ^2\,{}_2^2 \quad 4(\,{}_2^2 \quad {}_3)}}{2},$$

$$_3 \quad \frac{\,_1\,_1}{} \quad {}_2\,{}_1 \quad \frac{\,_2\,_2}{}$$

: $[\ (_1), \ (_1)] = (\ _1^2 + \ _2^2 + \ _1\ _2)\ _3$， $[\ (_2), \ (_1)] = \ (\ _1^2 + \ _2^2 + \ _1\ _2)\ _3$， $[\ (_2), (_2)] = (\ _1^2 + \ _2^2 + \ _1\ _2)\ _3$

5) $[_3, _2] = \ _3$ 1， : $([_1, _1]) = 0$， $([_1,$

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$$_2 = 0 \quad , A = \begin{pmatrix} _1 & _2 & _3 \\ 0 & 0 & 0 \\ 0 & 0 & ^2_1 \end{pmatrix};$$

$$_2 \neq 0 \quad , \quad _1 = \quad _3 = 0 , \quad A = \begin{pmatrix} 1 & _2 & _3 \\ 0 & _2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

:

(i) [($_1$), ($_1$)]= $^2_1$ $_3$;

(ii) [($_1$), ($_1$)]= $_3$, [($_1$), ($_2$)]= $_2$ $_2$, [($_2$), ($_1$)]= - $_2$ $_2$, $_1$ $_2$ ≠ 0

7) [$_1$, $_2$]= $_2$, [$_2$, $_1$]= - $_2$, [$_3$, $_1$]= $_3$ 1 ,

$$\begin{cases} _1 \quad _1 \quad _3 \quad _2 \quad 0 , \\ _1 \; _2 \quad _2 , \\ _3 \; _1 \quad _3 , \\ _3 \; _1 \quad 0 , \end{cases}$$

: .

$$_1 = 0 \quad , \quad _3 = \quad _2 = 0 , A = \begin{pmatrix} 0 & _2 & _3 \\ _1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix};$$

$_1 = 1$, $_3 = 0$

‖ 9) [$_1$, $_2$]= $_1$, [$_3$, $_2$]= $_3$ 1 ,

$$\begin{cases} {}_2 \quad {}_2 \quad 0, \\ {}_1 {}_2 \quad {}_1, \quad {}_3 {}_2 \quad {}_3, \\ {}_1 {}_2 \quad {}_3 {}_2 \quad 0, \\ {}_1 {}_2 \quad {}_1, \quad {}_3 {}_2 \quad {}_3 \end{cases}$$

:

$$_2 = 0 \quad , A = \begin{pmatrix} 0 & 0 & 0 \\ {}_1 & 0 & {}_3 \\ 0 & 0 & 0 \end{pmatrix};$$

$$_2 = 1 \quad , A = \begin{pmatrix} {}_1 & 0 & {}_3 \\ 0 & 1 & 0 \\ {}_1 & 0 & {}_3 \end{pmatrix};$$

$$_2 \neq 0 , \ _2 \neq 1 \quad , A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & {}_2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

: [($_1$), ($_2$)]= $_1$ $_1$ + $_3$ $_3$, [($_3$), ($_2$)]= $_1$ $_1$ + $_3$ $_3$, | $_1$ | + | $_3$ | + | $_1$ | + | $_3$ | $\neq 0$

10) [$_1$, $_2$]= $_1$, [$_2$, $_2$]= $_3$ 1 ,

$$\begin{cases} {}_2 \quad {}_3 \quad {}_2 \quad 0, \\ {}_1 {}_2 \quad {}_1, \\ {}_1 {}_2 \quad {}_1, \ {}^2_2 \quad {}_3, \\ {}_1 {}_2 \quad 0 \end{cases}$$

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$_2 = 0$, $A =$

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$_2 \neq 0$，$_1 = {}_3 = 0$，$_1 = 0$，$_2 = 1$，$_3 = {}_1 + {}_3 - {}_3$，$A = \begin{pmatrix} _1 & 0 & _3 \\ 0 & 1 & 0 \\ 0 & 0 & _1 + {}_3 - {}_3 \end{pmatrix}$；

$_2 \neq 0$，$_1 = {}_3 = 0$，$_1 = 0$，$_2 \neq 1$，$_1 = 0$，$_3 = 0$，$_3 = 0$，$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & _2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$；

$_2 \neq 0$，$_1 = {}_3 = 0$，$_1 = 0$，$_2 \neq 1$，$_1 = 0$，$_3 = 0$，$_2 = \frac{1}{}$，$A = \begin{pmatrix} 0 & 0 & _3 \\ 0 & 1/ & 0 \\ 0 & 0 & 0 \end{pmatrix}$；

$_2 = 0$，$_1 = {}_3 = 0$，$_1 = {}_3 = 0$，$A = \begin{pmatrix} 0 & 0 & 0 \\ _1 & 0 & _3 \\ 0 & 0 & 0 \end{pmatrix}$

4 ：

(i) $[(_1), (_2)] = {}_1{}_1 + {}_2 + {}_3{}_3$，$[(_3), (_2)] = (1 - {}_2)_1{}_1 + {}_1{}_3$；

(ii) $[(_1), (_2)] = {}_1{}_1 - {}_2 + {}_3{}_3$，$[(_3), (_2)] = (- 1)_1{}_1 - {}_1{}_3$；

(iii) $[(_1), (_2)] = (1 -)_3{}_1 + {}_2 + {}_3{}_3$，$[(_3), (_2)] = - (- 1)^2{}_3{}_1 + (- 1)_3$；

(iv) $[(_1), (_2)] = {}_1{}_1 + {}_2 + {}_3{}_3$，$[(_3), (_2)] = ({}_1 - {}_3 +$

$$A \quad \begin{pmatrix} ({}_3 \quad 1) {}_2 & {}_2 & 0 \\ \frac{2}{} \; {}_3 & \frac{2 \quad 2}{} \; {}_3 & {}_2 \; {}^2_3 \quad 0 \\ 0 & 0 & {}_3 \end{pmatrix}$$

:

(i) $[\ ({}_2),\ ({}_3)] = ({}_1 + {}_1)_1 + ({}_1 + {}_1 + {}_1)_2,\ [\ ({}_1),\ ({}_3)] = {}_1\ {}_1 + ({}_1 + {}_1)_2$;

(ii) $[\ ({}_2),\ ({}_3)] = ({}_1 + {}_1)_1 + ({}_1\ {}_3 + {}_1 + {}_1)_2,\ [\ ({}_1),\ ({}_3)] = {}_1\ {}_1 + ({}_1\ {}_3 + {}_1)_2$

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