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Short Distance Analysis of $D^0 - \overline{D}^0$ Mixing

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Abstract

We study the Standard Model short-distance prediction for the mass and lifetime differences between the two neutral D meson mass eigenstates. We find that, despite $\alpha_s/4\pi$ suppression, next-to-leading order (NLO) short-distance QCD corrections exceed the corresponding leading order (LO) amplitudes. For the lifetime difference, this stems from the lifting of helicity suppression of a light-quark intermediate state. We find $y_{\rm D}$ is given by $y_{\rm NLO}$ to a reasonable approximation but $x_{\rm D}$ is greatly affected by destructive interference between $x_{\rm LO}$ and $x_{\rm NLO}$. The net effect is to render $y_{\rm D} \sim x_{\rm D} \simeq 6 \cdot 10^{-7}$. Our NLO short-distance results, still smaller than most long-distance estimates, depend on the same two nonperturbative matrix elements of four-quark operators as in leading order.

I. INTRODUCTION

Experimental efforts to detect $D^0 - \overline{D}^0$ mixing are longstanding and remain an active area to this day. [1, 2, 3] The theory of $D^0 - \overline{D}^0$ mixing is relevant both in lending phenomenological guidance to ongoing experimental work and in better understanding the workings of the Standard Model and of various New Physics scenarios [4, 5]. In this paper, we present new results — the perturbative QCD NLO contributions in the framework of the $1/m_c$ expansion for $\Delta\Gamma_D$ and ΔM_D. The complex of D-meson phenomena presents a nontrivial theoretical laboratory for studying applicability of heavy quark methods. One can argue that $m_c \gg \Lambda_{\text{QCD}}$ justifies the use of heavy quark methods. However, the scale $\mu \simeq M_D$ lies in the meson resonance region, so QCD dynamics is clearly present [6]. As such, there is inherently a degree of interest in the numerical aspect of our findings. Our calculation also touches on matters of principle, such as the degree of m_q/m_c suppression in $\Delta\Gamma_D$ and ΔM_D at NLO order.

We begin by reviewing the theoretical context of $D^0 - \overline{D}^0$ mixing. The mixing arises from $\Delta C = 2$ interactions that generate off-diagonal terms in the mass matrix for D^0 and $\overline{D}^0$ mesons. The expansion of the off-diagonal terms in the neutral D mass matrix to second order in perturbation theory is

$$\left(M - \frac{i}{2}\Gamma\right)_{12} = \frac{1}{2M_D} \langle \overline{D}^0 | \mathcal{H}_w^{\Delta C=2} | D^0 \rangle + \frac{1}{2M_D} \sum_n \frac{\langle \overline{D}^0 | \mathcal{H}_w^{\Delta C=1} | n \rangle \langle n | \mathcal{H}_w^{\Delta C=1} | D^0 \rangle}{M_D - E_n + i\epsilon} \quad , \quad (1)$$

where $\mathcal{H}_w^{\Delta C=2}$ is the effective $\Delta C = 2$ hamiltonian and $\mathcal{H}_w^{\Delta C=1}$ is

$$\mathcal{H}_w^{\Delta C=1} = \frac{G_F}{\sqrt{2}} \sum_{q,q'} V_{cq}^* V_{uq'} \left[C_1(\mu) Q_1 + C_2(\mu) Q_2 \right] \quad . \quad (2)$$

In $\mathcal{H}_w^{\Delta C=1}$, the flavor sum on q, q' extends over the d, s quarks,[1] the quantities $C_{1,2}(\mu)$ are Wilson coefficients evaluated at energy scale μ, and $Q_{1,2}$ are the four-quark operators

$$Q_1 = (\overline{q}_i c_j)_{V-A} \left(\overline{u}_i q'_j\right)_{V-A} \qquad \text{and} \qquad Q_2 = (\overline{q}_i c_i)_{V-A} \left(\overline{u}_j q'_j\right)_{V-A} \quad . \quad (3)$$

The first term in Eq. (1) represents $\Delta C = 2$ contributions that are local at scale $\mu \sim M_D$, so it contributes to the M_{12} (but not to the Γ_{12}) part of the mixing matrix. For example, in the Standard Model this term is generated by the contribution of the b quark. It can also receive

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[1] In this paper, we work with $m_u = m_d = 0$.

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a potentially large enhancement from new physics. The second term in Eq. (1) comes from a double insertion of $\Delta C = 1$ operators in the SM lagrangian, and it contributes to both M_{12} and Γ_{12}. It is dominated by SM contributions even in the presence of new physics. At scale $\mu \sim M_D$, the contributions are from the strange and down quarks and these have relatively large CKM factors. By contrast the $\Delta C = 2$ term is expected to give a negligible contribution (*e.g.* in the SM there is the severe CKM suppression $|V_{ub}V_{cb}^*|^2/|V_{us}V_{cs}^*|^2 = \mathcal{O}(10^{-6})$). Thus, we omit it henceforth.

The off-diagonal mass-matrix terms induce mass eigenstates D_L and D_S which are superpositions of the flavor eigenstates D^0 and $\overline{D}^0$,

$$|D_{L,S}\rangle = p\,|D^0\rangle \pm q\,|\overline{D}^0\rangle \ , \tag{4}$$

where $|p|^2 + |q|^2 = 1$. In the Standard Model CP violation in D mixing is negligible, as is CP violation in D decays both in the Standard Model and in most scenarios of new physics. We therefore assume in the rest of this paper that CP is a good symmetry, and adopt the phase convention [7]

$$\mathcal{CP}|D^0\rangle = -\,|\bar{D}^0\rangle \ , \tag{5}$$

Then we have $p = q$, and $|D_{L,S}\rangle$ become the CP eigenstates $|D_\pm\rangle$ with $\mathcal{CP}|D_\pm\rangle = \pm|D_\pm\rangle$. We then define the mass and width differences

$$\Delta M_{\rm D} \equiv M_{D_+} - M_{D_-} \qquad \text{and} \qquad \Delta\Gamma_{\rm D} \equiv \Gamma_{D_+} - \Gamma_{D_-} \ . \tag{6}$$

It is, however, customary to work directly with the dimensionless quantities,

$$x_{\rm D} \equiv \frac{\Delta M_{\rm D}}{\Gamma_{\rm D}}, \qquad y_{\rm D} \equiv \frac{\Delta\Gamma_{\rm D}}{2\Gamma_{\rm D}}, \tag{7}$$

where $\Gamma_{\rm D}$ is the average width of the two neutral D meson mass eigenstates.

The discussion thus far covers relevant background material. We conclude this section by addressing three particularly important additional points:

1. Our calculation adopts an operator product expansion (OPE) [8, 9]. In the limit $m_c \gg \Lambda$, where Λ is some soft QCD scale, the momentum flowing through the light degrees of freedom in the intermediate state is large. As such, an OPE is implemented by expanding the second term in Eq. (1) in series of matrix elements of local operators. For example, one writes for $\Delta\Gamma_{\rm D}$,

$$\Delta\Gamma_{\rm D} = -2\Gamma_{12} = -\frac{1}{M_D}\,\text{Im}\,\langle\overline{D}^0|\,i\int {\rm d}^4x\,T\big\{\mathcal{H}_w^{\Delta C=1}(x)\,\mathcal{H}_w^{\Delta C=1}(0)\big\}|D^0\rangle \ , \tag{8}$$

and expands the time ordered product in Eq. (8) in local operators of increasing dimension (higher dimension operators being suppressed by powers of Λ/m_c).

2. We calculate $\Delta\Gamma_D$ by making direct use of work available in the literature [10], but not heretofore applied to $D^0 - \overline{D}^0$ mixing. We then compute the mass difference ΔM_D from an unsubtracted dispersion relation,[2]

$$\Delta M_D(m_c^2) = -\frac{1}{2\pi} P \int_{s_0}^{\infty} ds \, \frac{\Delta\Gamma_D(s)}{s - m_c^2} \quad , \tag{9}$$

which follows from the analyticity of $\Delta\Gamma_D$ in the complex s-plane with a unitarity branch cut along the $\mathcal{Re}\,s$ axis [11].

3. We expand all our expressions for $x_{|rmD}$ and y_D in powers of the ratio $z = m_s^2/m_c^2$.

II. ANALYSIS

In what follows we compute LO and NLO contributions to y and then x,

$$y_D = y_{LO} + y_{NLO} \qquad \text{and} \qquad x_D = x_{LO} + x_{NLO} \quad . \tag{10}$$

We depict in Fig. 1 how QCD affects the D^0-to-$\overline{D}^0$ mixing amplitude: (a) the limit of no QCD corrections, (b) the LO component in which QCD dresses the interaction vertices, and (c) an example of a NLO correction.
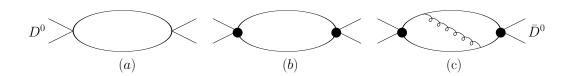

$$(a) \qquad\qquad (b) \qquad\qquad (c)$$

FIG. 1: $D^0 \to \overline{D}^0$: (a) No-QCD, (b) QCD-corrected vertices, (c) an example of NLO correction.

The leading contribution to $\Delta\Gamma_D$ in the $1/m_c$ expansion to $D^0 - \overline{D}^0$ mixing comes from the dimension-six $|\Delta C| = 2$ four-quark operators,

$$Q = \bar{u}_\alpha \gamma_\mu P_L c_\alpha \, \bar{u}_\beta \gamma_\mu P_L c_\beta \,, \qquad Q_S = \bar{u}_\alpha P_L c_\alpha \, \bar{u}_\beta P_L c_\beta \,,$$

$$Q' = \bar{u}_\alpha \gamma_\mu P_L c_\beta \, \bar{u}_\beta \gamma_\mu P_L c_\alpha \,, \qquad O'_S = \bar{u}_\alpha P_L c_\beta \, \bar{u}_\beta P_L c_\alpha \quad , \tag{11}$$

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[2] The tiny b-quark contribution, neglected here, would contribute to a subtraction constant.

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where $P_L = (1 + \gamma_5)/2$. One can use Fierz identities and equations of motion to eliminate Q' and Q'_S in favor of Q and Q_S. The resulting expression for $\Delta\Gamma_D$ is then

$$\Delta\Gamma_D = \frac{G_F^2 m_c^2}{12\pi M_D} \left[F(z) \langle \overline{D}^0 | Q(\mu') | D^0 \rangle + F_S(z) \langle \overline{D}^0 | Q_S(\mu') | D^0 \rangle \right], \qquad (12)$$

Coefficients $F(z)$ and $F_S(z)$ are defined as

$$F(z) = \sum_{qq'} \xi_q \xi_{q'} \left[F_{11}^{qq'}(z) C_1^2(\mu) + F_{12}^{qq'}(z) C_1(\mu) C_2(\mu) + F_{22}^{qq'}(z) C_2^2(\mu) \right],$$

$$F_{ij}^{qq'}(z) = F_{ij}^{(0)qq'}(z) + \frac{\alpha_s(\mu)}{4\pi} F_{ij}^{(1)qq'}(z), \qquad (13)$$

and similarly for $F_S(z)$. Here $\xi_q \equiv V_{cq}^* V_{uq}$ is a CKM factor for the intermediate s, d quarks, the $\{F_{ij}^{(0)qq'}(z)\}$ functions are given in the discussion to follow and the $\{F_{ij}^{(1)qq'}(z)\}$ are considered later in our NLO analysis. As usual, the D^0-to-$\overline{D}^0$ matrix elements of Q and Q_S are parameterized in terms of B-factors,

$$\langle \overline{D}^0 | Q | D^0 \rangle = \frac{8}{3} f_D^2 M_D^2 B_D \qquad \text{and} \qquad \langle \overline{D}^0 | Q_S | D^0 \rangle = -\frac{5}{3} f_D^2 M_D^2 \overline{B}_D^{(S)}, \qquad (14)$$

where $\overline{B}_D^{(S)} \equiv B_D^{(S)} M_D^2/m_c^2$. There are limits on the precision of B_D and $\overline{B}_D^{(S)}$ because the calculable short distance component most likely gives a negligibly small contribution. The most recent result for the quenched lattice calculation of B_D is reported in Ref. [12].

A. Leading Order (LO) Contributions

At leading order in α_s, one finds for the $s\bar{s}$ intermediate state contributions to $F(z)$ and $F_S(z)$,

$$F_{11}^{(0)ss}(z) = 3\sqrt{1-4z}\,(1-z) \qquad F_{S11}^{(0)ss}(z) = 3\sqrt{1-4z}\,(1+2z)$$

$$F_{12}^{(0)ss}(z) = 2\sqrt{1-4z}\,(1-z) \qquad F_{S12}^{(0)ss}(z) = 2\sqrt{1-4z}\,(1+2z) \qquad (15)$$

$$F_{22}^{(0)ss}(z) = \frac{1}{2}(1-4z)^{3/2} \qquad F_{S22}^{(0)ss}(z) = -\sqrt{1-4z}\,(1+2z) \quad,$$

and for the $d\bar{s}$ and $s\bar{d}$ contributions,

$$F_{11}^{(0)ds}(z) = 3(1-z)^2\left(1+\frac{z}{2}\right) \qquad F_{S11}^{(0)ds}(z) = 3(1-z)^2\,(1+2z)$$

$$F_{12}^{(0)ds}(z) = 2(1-z)^2\left(1+\frac{z}{2}\right) \qquad F_{S12}^{(0)ds}(z) = 2(1-z)^2\,(1+2z) \qquad (16)$$

$$F_{22}^{(0)ds}(z) = \frac{1}{2}(1-z)^3 \qquad F_{S22}^{(0)ds}(z) = -(1-z)^2\,(1+2z) \quad.$$

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TABLE I: **LO Values**

	Without QCD	With QCD (LO in z)	With QCD (Exact)
y_{LO}	$-(2.9 \to 4.8) \cdot 10^{-8}$	$-(5.6 \to 9.4) \cdot 10^{-8}$	$-(5.7 \to 9.5) \cdot 10^{-8}$
x_{LO}	$-(0.53 \to 1.05) \cdot 10^{-6}$	$-(1.3 \to 2.3) \cdot 10^{-6}$	$-(1.4 \to 2.4) \cdot 10^{-6}$

In addition, we have $F_{ij}^{(0)dd}(z) = F_{ij}^{(0)ss}(0)$. Insertion of Eqs. (13),(15),(16) into Eq. (12) results in the following expression for the leading $\mathcal{O}(z^3)$ contribution,

$$y_{\mathrm{LO}}^{(z^3)} = \frac{G_F^2 m_c^2 f_D^2 M_D}{3\pi \Gamma_D} \, \xi_s^2 \, z^3 \, \left(C_2^2 - 2C_1 C_2 - 3C_1^2\right) \left[B_{\mathrm{D}} - \frac{5}{2}\overline{B}_{\mathrm{D}}^{(S)}\right] \ , \qquad (17)$$

where $\Gamma_D \simeq 1.6 \cdot 10^{-12}$ GeV is the experimentally determined D^0 decay rate. The above expression for $y_{\mathrm{LO}}^{(z^3)}$ agrees in the no-QCD limit of $C_1 = 0$ and $C_2 = 1$ with that found in the literature [13]. Since we expect $5\overline{B}_{\mathrm{D}}^{(S)}/2 > B_{\mathrm{D}}$, it follows that $y_{\mathrm{LO}} < 0$.

An expression for x_{LO} is recovered by inserting $\Delta\Gamma_{\mathrm{LO}}$ into the dispersion relation of Eq. (9). One disperses in the variable m_c^2 so that $z = m_s^2/m_c^2 \to m_s^2/s$. The functions $\{F_{ij}^{(0)}(z)\}$ of Eqs. (15),(16) are employed above the threshold for each intermediate state. Although the dispersion integral diverges separately for each of the $s\bar{s}$, $d\bar{d}$, $d\bar{s}$, $s\bar{d}$ intermediate states, the flavor-summed expression for ΔM_{D} is rendered finite by GIM cancellations. All integrals are first evaluated analytically and the results are then expanded in powers of z. We find that the leading order in the z-expansion for x_{LO} occurs at $\mathcal{O}(z^2)$,

$$x_{\mathrm{LO}}^{(z^2)} = \frac{G_F^2 m_c^2 f_D^2 M_D}{3\pi^2 \Gamma_D} \, \xi_s^2 \, z^2 \, \left[C_2^2 B_{\mathrm{D}} - \frac{5}{4}(C_2^2 - 2C_1 C_2 - 3C_1^2)\overline{B}_{\mathrm{D}}^{(S)}\right] \ . \qquad (18)$$

As with y_{LO}, we again regain the standard no-QCD result [9, 13]. Terms occurring at next-to-leading order in the z-expansion are straightforward to determine, and we find

$$y_{\mathrm{LO}}^{(z^4)} = \frac{G_F^2 m_c^2 f_D^2 M_D}{3\pi \Gamma_D} \, \xi_s^2 \, z^4 \, \left[B_{\mathrm{D}}\left(C_2^2 - 4C_1 C_2 - 6C_1^2\right) - \frac{15}{4}\overline{B}_{\mathrm{D}}^{(S)}\left(C_2^2 - 2C_1 C_2 - 3C_1^2\right)\right]$$

$$x_{\mathrm{LO}}^{(z^3)} = \frac{G_F^2 m_c^2 f_D^2 M_D}{3\pi^2 \Gamma_D} \, \xi_s^2 \, z^3 \, \left[\frac{1}{2}B_{\mathrm{D}}\left(C_2^2 + 2C_1 C_2 + 3C_1^2\right)\right. \qquad (19)$$

$$\left. - \ln z \, \left(B_{\mathrm{D}} - \frac{25}{12}\overline{B}_{\mathrm{D}}^{(S)}\right)\left(C_2^2 - 2C_1 C_2 - 3C_1^2\right)\right] \ . \qquad (20)$$

Notice that at order $x_{\mathrm{LO}}^{(z^3)}$, there is now dependence also on $\ln z \simeq -5$. However, these contributions are quite small relative to those of Eqs. (17),(18).

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Numerical evaluations for y_{LO} and x_{LO} appear in Table I. The initial two columns display the leading z-dependences first with QCD turned off (*cf* Fig. 1(a)) and then with QCD included via the Wilson coefficients of Eq. (2) (*cf* Fig. 1(b)). The final column exhibits the exact LO results. The spread of values reflects uncertainties in input parameters (in particular, we have allowed for the range $B_{\text{D}}^{(S)}/B_{\text{D}} = 0.8 \to 1.2$.

The collection of LO results in Table I gives rise to several interesting questions, but the most obvious one involves the tiny magnitudes. The main suppression arises from the presence of $z^3 \sim 2 \cdot 10^{-7}$ in y_{LO} and $z^2 \sim 4 \cdot 10^{-5}$ in x_{LO}, even though the expansions for $F_{ij}(z)$ and $F_{Sij}(z)$ begin at $\mathcal{O}(1)$. Such $\mathcal{O}(1)$ contributions would be enormous, but they are in fact cancelled away as are $\mathcal{O}(z)$ terms. As a result, y_{LO} and x_{LO} are rendered tiny. A numerical by-product of the dependence $y_{\text{LO}} \sim \mathcal{O}(z^3)$ and $x_{\text{LO}} \sim \mathcal{O}(z^2)$ is that $|x_{\text{LO}}| \gg |y_{\text{LO}}|$. There is of course a corresponding physics explanation. In the diagrams of Fig. 1, the b-quark contribution is severely CKM suppressed, so only the light s, d quarks propagate on internal legs. Since the mixing amplitude will vanish in the $m_d = m_s = 0$ limit, the breaking of chiral symmetry and of SU(3) flavor symmetry play crucial roles. Thus, a factor of m_s^2 comes from an $SU(3)$ violating mass insertion on each internal quark line and another from an additional mass insertion on each line to compensate the chirality flip from the first insertion. This mechanism of chiral suppression accounts for the z^2 dependence of x_{LO}. In addition, y_{LO} requires yet another factor of $m_s^2 \propto z$ to lift the helicity suppression for the decay of a scalar meson into a massless fermion pair.

B. Next-to-Leading Order (NLO) Contributions

Any way of reducing the chiral and helicity supression in x and y should lead to an enhancement. In principle, there are both nonperturbative and perturbative ways to achieve this.

One might associate nonperturbative effects with the presence of quark condensates in the QCD vacuum [8, 9]. These contributions (suppressed by powers of $1/m_c$) lead to chirality flip the same way mass insertions do, but have an intrinsic scale of $\Lambda \sim 1$ GeV $\gg m_s$. In the realistic case of not-so-large m_c, such power suppressions are not always sufficient to ensure the smallness of higher order contributions. Therefore, Eqs. (17) and (18) cannot contribute to leading order in the dual expansion in m_s and $1/m_c$ if higher order terms in the $1/m_c$

expansion contain lower powers of z than do x_{LO} and y_{LO}. It has already been shown that this is indeed the case [8, 9].

There are also perturbative QCD corrections to x and y, but these have heretofore not been given serious consideration due to the negligibly small LO values for the $D^0 - \overline{D}^0$ mixing parameters. Even taking into account large scale dependence, the LO result gives a tiny contribution. This has stimulated a shift of attention towards the computation of the long-distance sector with varying degrees of model dependence [11, 14, 15].

But due to their milder dependence on m_s, the higher order QCD corrections might be able to give relatively large contributions [16]. This occurs, for example, in the $c \to s\gamma$ short distance amplitude, which receives a huge QCD correction [17]. In this paper, we consider a specific means by which the helicity supression in y can be lifted — a perturbative gluon correction (*e.g.* as in Fig. 1(c)). Having a perturbative gluon traversing the graph for the correlation function is the same as a well-known effect of lifting of helicity supression which follows from having three particles in the intermediate state instead of two [3]. The addition of the 'intermediate-state' gluon can lift one power of z, which characterizes the helicity suppression in y. Also, the relative lightness of m_c implies that higher order perturbative QCD corrections are suppressed by a relatively large factor of $\alpha_s(m_c) \sim 0.4$. It is therefore expected that the NLO corrections to y should dominate the LO result. Moreover, the existence of a dispersion relation implies that x might well be enhanced at NLO.

In order to systematically include the effects of intermediate-state gluons, a complete calculation of NLO corrections to $D^0 - \overline{D}^0$ mixing is needed. The NLO corrections to lifetime difference y can be readily computed. All the relevant NLO contributions to $F(z)$ and $F_S(z)$ for two massive quarks and one massive, one massless quark can be found by adopting the formulas in Refs. [10] (which considered the case of $B_s - \bar{B}_s$ mixing) to computing $F_{ij}^{(1)qq'}$ of Eq. (13). That calculation has been performed in the NDR-scheme (dimensional regularization with anti-commuting γ_5 and $\overline{MS}$ subtraction). We shall not present explicit formulas for the $\{F_{ij}^{(1)qq'}(z)\}$ and $\{F_{S,ij}^{(1)qq'}(z)\}$ functions as they are rather cumbersome.

Scale dependent quantities used in our numerical work and evaluated at $\mu = 1.3$ GeV

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[3] This mechanism leads to the prediction that the rate for the weak radiative decay $B \to \gamma e\nu$ is much larger than the rate of weak leptonic decay $B \to e\nu$

$y_{\mathrm{NLO}}^{(z^2)}$	$y_{\mathrm{NLO}}^{(z^3)}$	y_{NLO}	x_{NLO}	$y_{\mathrm{NLO}}^{(\mathrm{PENG})}$
$(2.2 \to 6.3) \cdot 10^{-7}$	$(1.7 \to 2.8) \cdot 10^{-7}$	$(3.9 \to 9.1) \cdot 10^{-7}$	$(1.7 \to 3.0) \cdot 10^{-6}$	$(0.6 \to 0.8) \cdot 10^{-9}$

were:

$$m_c = 1.3 \text{ GeV} , \quad B_{\mathrm{D}} = 0.82 , \quad C_1 = -0.411 , \quad C_2 = 1.208 , \quad \alpha_s = 0.406 . \tag{21}$$

The value for B_{D} at scale $\mu = 1.3$ GeV is obtained by referring the lattice determination at $\mu = 2$ GeV and employing the scale invariant quantity $\hat{B}_{\mathrm{D}}$,

$$\hat{B}_{\mathrm{D}} = B_{\mathrm{D}}(\mu_0)[\alpha_s(\mu_0)]^{-6/25} \left[1 + \frac{\alpha_s(\mu_0)}{4\pi} J_4 \right] , \tag{22}$$

with $J_4 \simeq 1.792$. Also we allow for a range of the ratio $\overline{B}_{\mathrm{D}}^{(S)}/B_{\mathrm{D}}$.

Using the $\{F_{ij}^{(1)qq'}(z)\}$ and $\{F_{S,ij}^{(1)qq'}(z)\}$ functions, we have calculated y_{NLO} exactly and also have expressed it in terms of a power series in z. The leading term is $\mathcal{O}(z^2)$,

$$y_{\mathrm{NLO}}^{(2)} = \frac{G_F^2 m_c^2 f_D^2 M_D}{3\pi \Gamma_D} \xi_s^2 \frac{\alpha_s}{4\pi} z^2 \left(B_{\mathrm{D}} \left[-\left(\frac{77}{6} - \frac{8\pi^2}{9} \right) C_2^2 + 14 \, C_1 C_2 + 8 \, C_1^2 \right] \right.$$
$$\left. -\frac{5}{2}\overline{B}_{\mathrm{D}}^{(S)} \left[\left(\frac{8\pi^2}{9} - \frac{25}{3} \right) C_2^2 + 20 \, C_1 C_2 + 32 \, C_1^2 \right] \right) , \tag{23}$$

and the corresponding $\mathcal{O}(z^3)$ contribution is

$$y_{\mathrm{NLO}}^{(3)} = \frac{2G_F^2 m_c^2 f_D^2 M_D}{3\pi \Gamma_D} \xi_s^2 \frac{\alpha_s}{4\pi} z^3$$
$$\times \left(B_{\mathrm{D}} \left[(15 + 7 \ln z) \, C_2^2 - \left(\frac{77}{9} + \frac{103}{3} \ln z \right) \, C_1 C_2 - (18 + 58 \ln z) \, C_1^2 \right] \right.$$
$$\left. -\frac{5}{2}\overline{B}_{\mathrm{D}}^{(S)} \left[\left(\frac{28}{3} + 6 \ln z \right) \, C_2^2 + \left(\frac{49}{9} - \frac{118}{3} \ln z \right) \, C_1 C_2 - \left(\frac{31}{3} + 58 \ln z \right) \, C_1^2 \right] \right) . \tag{24}$$

The numerical results, displayed in Table II, reveal that y_{NLO} is almost an order of magnitude larger than y_{LO} and that the subleading term $y_{\mathrm{NLO}}^{(3)}$ is smaller than $y_{\mathrm{NLO}}^{(2)}$ but not at all negligible.

The corresponding expression for x_{NLO} has, as before, been obtained by means of a dispersion relation. We evaluated the dispersion integral numerically to obtain the value presented in Table II. As regards an analytical expression for x_{NLO}, the intent was again to by first exactly perform the dispersion integrals and then expand each contribution in

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a z power series. It turned out possible to do this for the $d\bar{d}$, $d\bar{s}$ and $s\bar{d}$ intermediate states, but not for $s\bar{s}$. It is, however, nonetheless useful to have an approximate analytic representation for x_{NLO}. By exploring a variety of approximation techniques, we found the expected $\mathcal{O}(z^2, z^2 \ln z)$ leading behavior for x_{NLO} but encountered scatter in the $\mathcal{O}(z^2)$ coefficients, although less so for the $\mathcal{O}(z^2 \ln z)$ coefficients. Upon accepting the latter and fitting the $\mathcal{O}(z^2)$ coefficients to the numerical evaluations of individual dispersion integrals, we arrived at the 'effective' formula:

$$
\begin{aligned}
x_{\mathrm{NLO}} \simeq &-\frac{G_F^2 m_c^2 f_D^2 M_D}{3\pi^2 \Gamma_D} \, \xi_s^2 \, \frac{\alpha_s}{4\pi} \, z^2 \\
&\times \Bigg(B_{\mathrm{D}} \left[(11.3 - 4.1 \ln z) \, C_2^2 + (49.2 + 15.8 \ln z) \, C_1 C_2 + (37.9 + 10.7 \ln z) \, C_1^2 \right] \\
&- \frac{5}{8} \overline{B}_{\mathrm{D}}^S \left[(37.9 + 2.2 \ln z) \, C_2^2 + (-33. + 81.8 \ln z) \, C_1 C_2 + (32.0 + 125.3 \ln z) \, C_1^2 \right] \Bigg) .
\end{aligned}
\tag{25}
$$

This relation, although approximate, is nonetheless useful in understanding the magnitude of the various contributions to x_{NLO}.

Since the NLO results found for the box contributions are larger than their LO counterparts, we consider here for the sake of completeness the NLO penguin contribution $y_{\mathrm{NLO}}^{(\mathrm{P})}$ to the width difference. We have

$$
y_{\mathrm{NLO}}^{(\mathrm{P})} = -\frac{4 G_F^2 m_c^2 f_D^2 M_D}{9\pi \Gamma_D} \, \xi_s^2 \, \frac{\alpha_s}{4\pi} \, z^3 \, C_2^2 \, \left(B_{\mathrm{D}} + 5\overline{B}_{\mathrm{D}}^S \right) + \dots \ .
\tag{26}
$$

The result shown in Table II clearly shows the penguin amplitude for $y_{\mathrm{NLO}}^{(\mathrm{P})}$ is negligible compared to the box contribution. The mass splitting $x_{\mathrm{NLO}}^{(\mathrm{P})}$ is likewise $\mathcal{O}(z^3)$ and hence negligible.

III.   CONCLUDING COMMENTS

We have calculated LO and NLO contributions to the leading dimension-six component in the OPE for $D^0 - \overline{D}^0$ mixing. Numerical results appear in Table I for LO and Table II for NLO. As a partial check of our analysis, we found our results (in cases of overlap) to agree with work carried out previously. Our formulae for x and y involve not simply expansions in $1/m_c$, but rather combined expansions in m_s (m_d is negligible), α_s, and $1/m_c$. As a technical aside, we performed the calculations at scale $m_c \simeq 1.3$ GeV.

The two most noteworthy numerical features found for x and y are:

1. They are small at LO and even at NLO. This is because $z \equiv m_s^2/m_c^2$ is small and the leading dependence on z is found to be

$$y_{\text{LO}} \sim z^3 \qquad x_{\text{LO}} \sim z^2 \qquad y_{\text{NLO}} \sim z^2 \qquad x_{\text{NLO}} \sim z^2 \quad . \tag{27}$$

Although contributions from individual intermediate states are *not* small, CKM factors cancel away the $\mathcal{O}(1)$ and $\mathcal{O}(z)$ components.

2. The NLO terms are larger than the LO terms. This requires somewhat more explanation, especially since NLO amplitudes contain the small perturbative QCD factor $\alpha_s/4\pi$. As regards the dimensionless width difference y_{D}, the ratio of leading terms in the z expansion is

$$\frac{y_{\text{NLO}}^{(z^2)}}{y_{\text{LO}}^{(z^3)}} = \frac{\alpha_s}{4\pi} \times \frac{1}{z} \times \frac{W_y^{(\text{NLO})}}{W_y^{(\text{LO})}} \simeq 0.03 \times 169 \times (-0.73) \simeq -4 \quad . \tag{28}$$

In the above $W_y^{(\text{NLO})}/W_y^{(\text{LO})}$ is the ratio of terms containing the Wilson coefficients in Eqs. (17),(23) and for definiteness we have considered the case $B_{\text{D}}^{(S)} = 0.8 B_{\text{D}}$. Eq. (28) shows that $|y_{\text{NLO}}^{(z^2)}|$ exceeds $|y_{\text{LO}}^{(z^3)}|$ because the extra factor of z overwhelms the $\alpha_s/4\pi$ suppression. We have already discussed the physics of this – the helicity suppression mechanism which affects any LO $q\bar{q}$ intermediate state is removed via the presence of a virtual gluon in the NLO $q\bar{q}G$ intermediate state. Also, the difference in sign between $y_{\text{NLO}}^{(z^2)}$ and $y_{\text{LO}}^{(z^3)}$ arises from the factor $W_y^{(\text{NLO})}/W_y^{(\text{LO})}$.

Since, to leading order in z, x_{NLO} and x_{LO} both behave as z^2, something else must account for the result $|x_{\text{NLO}}| > |x_{\text{LO}}|$. From Eq. (18) and the approximate formula Eq. (25), we have

$$\frac{x_{\text{NLO}}}{x_{\text{LO}}^{(z^2)}} \simeq -\frac{\alpha_s}{4\pi} \times \frac{W_x^{(\text{NLO})}}{W_x^{(\text{LO})}} \simeq -0.03 \times (41.4) \simeq -1.3 \quad , \tag{29}$$

where $W_x^{(\text{NLO})}$ and $W_x^{(\text{LO})}$ are again the contributions from the Wilson constants and their coefficients. In this case, the suppression in $\alpha_s/4\pi$ is overcome by the large size of $W_x^{(\text{NLO})}/W_x^{(\text{LO})}$. In particular, the largest contributor to $W_x^{(\text{NLO})}$ is from the $\overline{B}_{\text{D}}^S$ term in Eq. (25), roughly equally between log and non-log terms.

3. We conclude that, citing just central values, the net effect of the short distance contributions is

$$y_{\text{D}} = y_{\text{LO}} + y_{\text{NLO}} \simeq 6 \cdot 10^{-7} \quad , \qquad x_{\text{D}} = x_{\text{LO}} + x_{\text{NLO}} \simeq 6 \cdot 10^{-7} \quad . \tag{30}$$

In brief, y_D is given by y_{NLO} to a reasonable approximation but x_D is greatly affected by destructive interference between x_{LO} and x_{NLO}. The net effect is to render y_D and x_D of similar magnitudes, at least at this order of analysis.

$D^0 - \overline{D}^0$ mixing thus provides a concrete example of a well-defined observable for which NLO perturbative QCD corrections dominate the LO result. Will it follow that the NNLO contributions are larger still? Of course, one cannot know without doing the calculation. We feel, however, it may not necessarily be the case, at least as regards the width difference y. The three-particle $q\bar{q}G$ intermediate states were able to lift the helicity suppression experienced by $q\bar{q}$ intermediate states. In passing to the NNLO sector, there is no analogous suppression factor to be lifted. Of course, there is always the possibility that large numerical coefficients can overturn the $\alpha_s/4\pi$ counting.

The question remains – just how large is $D^0 - \overline{D}^0$ mixing? Evidently, it is still not possible to provide a definitive theoretical answer and experiment will presumably decide the issue. On a relative basis, our 'short-distance' numerical results are smaller than most 'long-distance' estimates (although the model dependence and uncertainty present in even modern and improved versions of the latter is less significant here). Experimentalists might find it useful to interpret our numerical NLO values as lower bounds to y_D and x_D.

We conclude by considering our analysis in the context of operator product expansions. As we have seen above, the prediction of x and y is a result of expanding the correlation function Eq. (8) in terms of *three* 'small' quantities, z, Λ/m_c, and α_s. Since the first quantity is significantly smaller than the other two, the structure of the series is rather different from other (usual) applications of the OPE, *e.g.* $B^0 - \overline{B}^0$ mixing or b-hadron lifetimes [18].

Working with this combined expansion, we computed the leading contribution originating from matrix elements of dimension-six operators. These matrix elements are commonly parameterized in terms of the two nonperturbative parameters, B_D and $\overline{B}_D$. The applications of techniques of lattice and QCD sum rule evaluations of these operators can hopefully further improve the precision of our prediction.

At higher orders in this expansion one would need to take into account $\mathcal{O}(z^{3/2})$ corrections (multiplied by about a dozen matrix elements of dimension-nine operators) and $\mathcal{O}(z)$ corrections (with more than twenty matrix elements of dimension-twelve operators). This would introduce a veritable multitude of unknown parameters whose matrix elements cannot be computed at this time. Simple dimensional analysis [9] suggests magnitudes

$x_D \sim y_D \sim 10^{-3}$, but order-of-magnitude cancellations or enhancements are possible. However, any effect of higher orders in $1/m_c$ or $\alpha_s(m_c)$ which could render the result to be proportional to z^n in the lowest possible power $n = 1$ [15] would presumably produce a dominant contribution to the prediction of x and y.

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