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A Kuklov

Nikolai Prokof'ev
University of Massachusetts - Amherst, prokofev@physics.umass.edu

Boris Svistunov
University of Massachusetts - Amherst, svistunov@physics.umass.edu

Detecting Super-Counter-Fluidity by Ramsey Spectroscopy

Anatoly Kuklov

Department of Engineering Science and Physics, The College of Staten Island,
City University of New York, Staten Island, New York 10314

Nikolay Prokof'ev and Boris Svistunov

Department of Physics, University of Massachusetts, Amherst, MA 01003 and
Russian Research Center "Kurchatov Institute", 123182 Moscow

Spatially selective Ramsey spectroscopy is suggested as a method for detecting the super-counter-fluidity of two-component atomic mixture in optical lattice.

Recent advances in experimental studies of ultra-cold gases in optical lattices [1, 2] signal a major breakthrough in the field of strongly-correlated quantum lattice systems. Theoretical studies of ultracold atomic mixtures in optical lattices have revealed a variety of non-trivial ordered states [3, 4, 5, 6, 7, 8, 9, 10]. Developing experimental schemes able to resolve these states is of crucial importance. Recently, Altman *et al.* [11] has considered generic possibilities of revealing pairing orders through the density-density correlation properties. The proposed in Ref. [11] scheme for detecting pairing correlations relies on analyzing noise in absorptive imaging. Yet, a direct imaging of the non-single-component superflow as well as of the non-trivial topological interplay between the order parameters [9, 10] (see also below) are very desirable. Here we observe that there exists a very simple (though not a generic) method which immediately reveals the super-counter-fluidity (SCF) [6, 7, 9, 10] in the systems formed out of the two *interconvertible* species (like hyperfine states $|F = 1, m_f = -1\rangle$ and $|F = 2, m_f = 1\rangle$ of ^{87}Rb, which can be converted into each other by rf radiation).

The SCF state occurs in a two-component lattice system at integer total filling under the conditions of strong enough intra- and inter-component repulsion. Basically, the state can be considered as a condensate of pairs formed by particles of one component and holes of another component. This picture is relevant to both boson-boson, fermion-fermion and boson-fermion mixtures [6]. In general, the components can be different elements. The commensurability of the total filling factor guarantees that the number of atoms of one component coincides with the number of holes of another component, so that there is no non-paired atoms. Under these conditions the net superfluid motion is impossible, and only super-counter-flow can be realized [6]. The order parameter associated with the SCF state is

$$\Phi_{SCF} = \langle \Psi_B^\dagger \Psi_A \rangle = |\Phi_{SCF}| \, \mathrm{e}^{i\phi} \,, \tag{1}$$

where Ψ_A and Ψ_B are the field operators for the component A and B, respectively. The absence of condensate of (bosonic) atoms implies

$$\langle \Psi_A \rangle = \langle \Psi_B \rangle = 0 \,. \tag{2}$$

Another way of looking at the super-counter-fluid state—which is especially relevant for the present consideration—is mapping it onto the easy-plane ferro- (bosons) or anti-ferro- (fermions) magnet [6, 7]. In general, the operators $S_z = \frac{1}{2} \int (\Psi_A^\dagger \Psi_A - \Psi_B^\dagger \Psi_B) d\mathbf{x}$, $S_+ = \int \Psi_A^\dagger \Psi_B d\mathbf{x}$, $S_- = S_+^\dagger$ represent the $su(2)$ algebra of the angular momentum operators and thus can always be interpreted as (pseudo-)spin operators [12]. Eq. (1) then means the easy-plane spin order $\langle S_\pm \rangle \neq 0$.

In the spin terms, the *equilibrium* ordering described by the requirements (1)-(2) is exactly equivalent to the *non-equilibrium* ordering that arises in a *normal* cloud of two-component mixture of, say, ^{87}Rb atoms created by the $\pi/2$ rf pulse [13] out of one component. The spatially selective Ramsey spectroscopy (RS) techniques has been successfully applied for detecting such non-equilibrium spin order and its dynamics [13]. Hence, one immediately concludes that the same technique should be applicable for revealing the SCF phase. The only difference is that the SCF order is formed spontaneously from two components, which have no memory of each other. Thus, in contrast to the situation [13], only *one* $\pi/2$-pulse is needed.

Specifically, a short rf pulse produces the unitary transformation

$$U = \mathrm{e}^{-i(\omega' S_x + \omega'' S_y)}, \tag{3}$$

where $\omega = \omega' + i\omega'' = \int \Omega(t) \, dt$ with $\Omega(t)$ being the Rabi transition frequency, which enters the Hamiltnonian $H_P = (\hbar/2)[\Omega^*(t) S_+ + \mathrm{H.c.}]$ describing the effect of the rf-pulse—inter-conversion of the components; $S_+ = S_x + iS_y$.

Let us find the dominance of particles of the sort A in the final state: $\delta N = \langle N_A - N_b \rangle_{\text{fin}}/2 \equiv \langle S_z \rangle_{\text{fin}}$. In terms of the initial state, $\delta N = \langle S_z \rangle$ after the pulse is given by the relation $\delta N = \langle U^\dagger S_z U \rangle = \cos|\omega| \langle S_z \rangle + \sin|\omega| (\omega' \langle S_y \rangle - \omega'' \langle S_x \rangle)/|\omega|$ [12]. For the $\pi/2$-pulse with real negative ω we, thus, have

$$\delta N = \int d\mathbf{x}\, |\Phi_{SCF}| \sin\phi \approx V\, |\Phi_{SCF}| \sin\phi \qquad (\pi/2\text{-pulse}) , \qquad (4)$$

where V is the volume of the system, and spatial uniformity of the phase ϕ has been assumed. Given the fact that the phase ϕ is arbitrary, we see that repeating the experiment several times will result in the huge shot-to-shot noise $|\delta N| \sim N$, provided the SCF is strong, that is, given by the total number of particles N as $V|\Phi_{SCF}| \sim N$.

It is important to note that, similarly to the case [13], spatially resolved $\pi/2$-pulse will be able to detect *local* phase profile, that is, super-counter-fluid currents, including topological excitations [6, 9, 10]. An important ingredient is ability of controlling the circulation of the SCF vortex. Eq. (4) indicates that the local dominance $\delta n = |\Phi_{SCF}| \sin\phi$ is sensitive to how the phase winds. In general, should the winding of the SCF-phase exist, initially uniform mixture will exhibit a *phase-separation* pattern after the $\pi/2$-pulse. For example, in rotationally symmetric situation, the SCF phase can be represented as $\phi = k\theta$, where $k = 0, \pm1, \pm2, ...$ is the winding number [14] and θ is the polar angle in the plane, in which the SCF currents flow. Thus, the local dominance after the $\pi/2$-pulse will be

$$\delta n = |\Phi_{SCF}| \sin(k\theta). \qquad (5)$$

For $k \neq 0$, the domain boundaries are located along the lines given by $\theta = \pi m/k$, with $m = 0, 1, ..(|k|-1)$.

To exclude the possibility that the above-described interference effect is actually due to the off-diagonal order in each *separate* component, one needs just to make sure that there are no single-component condensates, which can be easily done by the absorption imaging technique [15].

In this report, we have concentrated on imaging the pure SCF phase. However, properties of the mixture of two superfluids (2SF phase discussed in Refs. [9, 10]), in which $\langle \Psi_A \rangle \neq 0$, $\langle \Psi_B \rangle \neq 0$ and, yet, the SCF correlations remain strong (that is $|\langle \Psi_A \rangle| \approx |\langle \Psi_B \rangle| \ll |\Phi_{SCF}|$), are quite unusual and deserve detailed experimental study. Such a state can be realized by decreasing the lattice potential, so that the SCF-2SF transition occurs [10]. One of the effects is preserving circulation of the difference of the phases of the two components, while the circulation of the sum is not preserved in the SCF state [9]. Accordingly, if, initially, the 2SF state had one vortex of, e.g., sort A, in the SCF phase it will become one SCF vortex, which can be viewed as a bound pair of *1/2-vortices* of opposite circulations of the A and B components (the circulation of the SCF vortex must be the same as of the original vortex [9]). Lowering the lattice potential, so that the system returns back to the 2SF state, will result in either reappearing of the vortex A or appearing of the anti-vortex B. The outcome depends on which superfluid stiffness (A or B) is smaller, so that the final vortex would have smaller energy. If the energy of the vortex B is lower, this effect will be seen as transferring circulation from A to B component by just cycling the system through the 2SF-SCF transition.

Above, we have analyzed the case of bosonic mixture. However, it is worth commenting on a case of fermionic two-component mixture. We note that the fermionic SCF [6] corresponds to the easy-plane anti-ferromagnetic order. Thus, imposing of the $\pi/2$ pulse will not result in a global dominance of one component. Instead, depending on the phase of the SCF order parameter, the checkerboard order will arise, with its contrast being modulated by the original SCF phase. Resolving this effect is, obviously, much more complicated issue and we will not consider it here.

Clearly, the above method cannot be applied for detecting the SCF of *non-convertible* species such as different elements. Furthermore, it will not work in the case of pairing of real atoms (rather than atoms and holes). Yet, it gives a unique opportunity to study new physical effects in selected systems, which will, then, apply to the whole class of the two-component quantum mixtures.

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