A NEW APPROACH FOR MATCHING SURFACES FROM LASER SCANNERS AND OPTICAL SCANNERS

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ABSTRACT

Surfaces play an important role in diverse applications, such as orthophoto production, city modeling, ice sheet monitoring, and object recognition. Surfaces are usually obtained by a sampling process. The raw sampled data must be processed further. A frequently occurring task is the comparison of two surfaces. In the most general case, the two surfaces are described by discrete sets of points, whereby the point density may be different as well as the reference systems. We propose to compare two surfaces by computing the shortest distance between points in one surface and locally interpolated surface patches of the second surface. This entails a correspondence between points and surface patches. We describe a solution to this matching problem that is based on a parameter space representation. After a brief problem statement we explain the proposed matching scheme by way of an example. We then apply the method to determine the transformation parameters between the two surfaces. To arrive at an operational solution, we reduce the n-parameter space to one dimension by an iterative solution. The feasibility of our matching scheme is demonstrated with simulated data sets as well as real data. We show how a surface determined by laser scanning can be compared with the same physical surface but established by photogrammetry. As a natural extension, one can use the method for change detection.

1 Introduction

There is an increasing demand for the rapid generation of digital surface models (DSM). The production of digital orthophotos as backdrops for GIS requires DSMs, for example. More recently, city modeling is an application that poses a challenge for generating DSMs. Surfaces play also an important role in such diverse applications as ice sheet monitoring and recognizing objects in aerial and satellite scenes.

Surfaces are typically determined by a sampling process. This is certainly true for airborne laser scanning and stereo photogrammetry. The net result of data acquisition is a set of points that constitutes a discrete surface description. In case of laser scanning, the point distribution is irregular and the surface characteristics, for example breaklines, are not explicitly encoded.

The set of points obtained during data acquisition is hardly a useful end product. There are a number of basic operations that must be performed on surfaces. One of the first steps usually involves what we may consider a resampling process. The classical example is interpolating the original set into a regular grid (gridding) because most every subsequent process assumes regularly spaced data.

In one way or another, many processes involve the comparison between surfaces. Examples are abundant; calibrating data acquisition systems involves the comparison between the observed surface and the known surface (e.g. test field); change detection compares two surfaces sampled at different times; merging two or more data sets for a combined surface (fusion) requires quality control; a data set acquired in a local reference system must be transformed into a differently registered set.

Surface comparison is usually performed by interpolating both data sets into a regular grid. Then, the comparison is reduced to analyzing the elevations at the grid posts. Not all applications allow this simple procedure, however. Take the example of two irregularly spaced data sets that are acquired in different reference systems with unknown transformation parameters. We describe in this paper a new approach for solving this general problem.

Ebner and Strunz (1988) and *Ebner and Ohlhof* (1994) describe a solution that is based on interpolating the data to a grid, subject to a transformation with unknown parameters, which are determined in an adjustment procedure whereby the elevation differences at the grid posts are minimized. We propose to minimize the distance between the points of one set along surface normals to locally interpolated surface patches of the other surface. As shown in *Schenk* (1999a) this makes a weak-posed problem well-posed.

The next section provides a more detailed problem statement and discusses solutions. We then concentrate on the solution of the matching problem and illustrate the proposed approach of using a voting scheme to analyze the parameter space by an example. Finally, we present experimental results obtained from synthetic and real data sets.

2 Problem Statements and Solutions

2.1 Simple Case

Given are two sets of points that describe the same surface. Let $S_1 = \{\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n\}$ be the first set and $S_2 = \{\mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_m\}$ the second set, $n \neq m$. Suppose the points are randomly distributed (no point to point correspondence). The problem is to determine how well the two data sets agree describing the same surface.

The traditional approach to this problem is to interpolate both data sets to a regular grid, followed by determining the $z-$differences at the grid posts. There are problems with this simple approach, however. First, the $z-$differences that serve as a comparison criteria are affected by interpolation errors. More critical is the restriction to compare differences only along the $z-$axis. Take the extreme example of a vertical surface; here $z-$differences would be meaningless to capture surface differences.

An improved solution is to compute the difference between the two sets along surface normals and at the original point location, to avoid interpolation. Suppose now that local surface patches for S_1 are generated. The simplest approach would be to create a TIN model—quite adequate for laser surfaces. Let surface patch SP in S_1 be defined by the 3 points $\mathbf{p}_a, \mathbf{p}_b, \mathbf{p}_c$ and let $\mathbf{q}_i$ be a point in the second set. Then, Eq. 2 is the shortest distance between a point in the second set to the corresponding surface patch, as illustrated in Fig. 1. If we want to impose the condition that $\mathbf{q}_i$ lies on the surface patch (coplanarity condition), then we have $D = 0$ in Eq. 1.

$$D = \begin{vmatrix} xq_i & yq_i & zq_i & 1 \\ xp_a & yp_a & zp_a & 1 \\ xp_b & yp_b & zp_b & 1 \\ xp_c & yp_c & zp_c & 1 \end{vmatrix} \tag{1}$$

$$d_i = \frac{D}{\sqrt{D_1^2 + D_2^2 + D_3^2}} \tag{2}$$

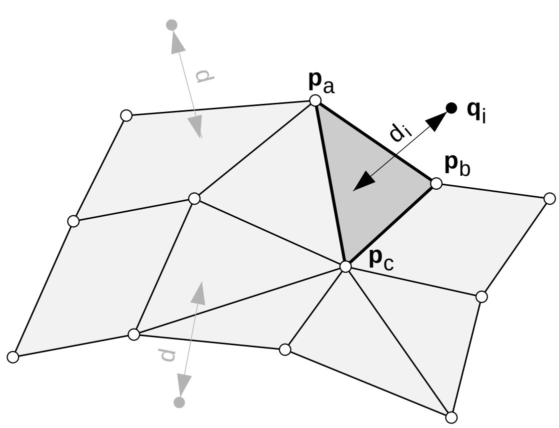

Figure 1: Illustration of comparing two data sets that describe the same surface. The points of one set are shown by circles. Solid circles represent a few points of the second data set. The comparison is achieved by computing the shortest distances d_i from points in one data set to the corresponding surface patches of the other data set, here shown as triangles.

We call the association of point $\mathbf{q}_i$ with the proper triangle $\mathbf{p}_a, \mathbf{p}_b, \mathbf{p}_c$ the matching problem in this paper. In the simple case of both data sets being registered in the same reference system, the matching can be solved in the $x-, y-$coordinate plane by selecting that surface patch which contains point $\mathbf{q}_i$ within its perimeter.

2.2 General Case

We generalize now the surface comparison problem by allowing that the two data sets S_1 and S_2 are in different reference systems. We assume that there is a known functional relationship between the two sets but with unknown parameters. An example would be the knowledge that the two sets are related by a 3-D similarity transformation; and the seven parameters should be determined without identical points. This situation exists when merging two data sets that may be affected by uncompensated systematic errors. Calibrating laser systems is a classical case; here the surface defined by laser points from an uncalibrated system is compared with a known surface (control surface). The expected errors must be modeled, e.g. by an affine transformation, and the unknown parameters of that model—the calibration parameters—can be determined with the method described here (see, e.g. *Filin and Csathó* (1999)).

It is important to realize that any functional relationship between the two sets can be used in our proposed matching scheme. We use a 3-D similarity transformation as an example to aid the following discussions. The solution sketched above must be extended by subjecting S_2 to the relevant function. With the example of a similarity transformation, we have

$$\mathbf{q}'_i = s \cdot \mathbf{R} \cdot \mathbf{q}_i + \mathbf{t} \tag{3}$$

where s is the scale factor, $\mathbf{t}$ the translation vector, and $\mathbf{R}$ a 3-D orthogonal rotation matrix. We can solve the parameters in an adjustment procedure using Eq. 2 as the target function (see *Schenk* (1999a) for details). Such a procedure would determine transformation parameters that minimize the distances d_i according to the least-squares principle. This implies that the differences between the two surfaces are assumed to be random; hence, the remaining distances after establishing the transformation parameters are residuals.

We are faced with a new problem, however. To compute the distances d_i, a correspondence between the points $\mathbf{q}_i$ and the surface patches must be established. This matching problem is no longer trivial because the two sets are in different reference systems. The proposed solution to this intricate problem is based on searching the solution in the parameter space by a voting scheme. Before delving into details we first introduce the notion of parameter space and voting scheme by way of an example.

3 The Notion of Parameter Space and Voting Scheme

The method of determining parameters by a voting scheme was first proposed by *Hough* (1962). Variants of this approach are known as Hough technique or Hough transform. Let us introduce the notion of parameter space and voting scheme by way of an example. Suppose we are interested in detecting points that happen to lie on a circle of known radius. Fig. 2(top) depicts a cloud of points. It would be quite cumbersome to solve this problem in the spatial domain. Instead, we represent it in the parameter space, motivated by the following considerations.

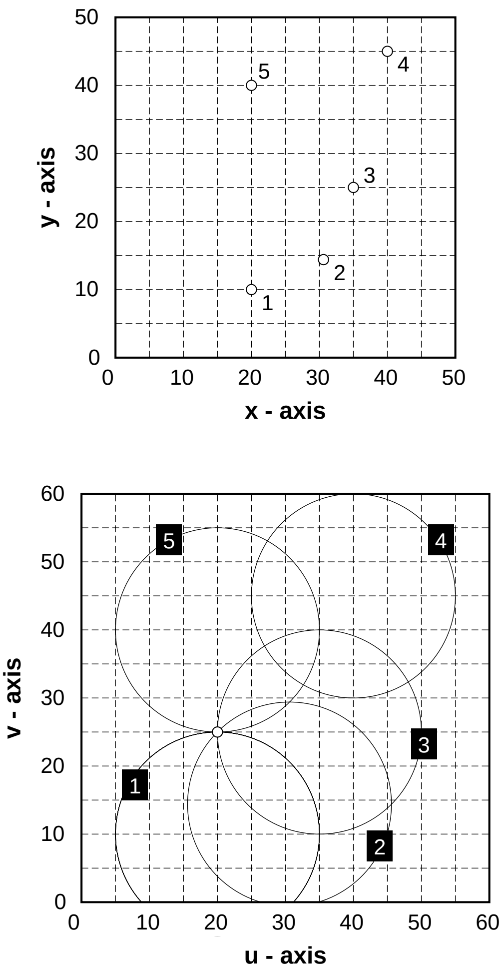

Figure 2: Illustration of finding circles through data points. A point in the spatial domain (top) corresponds to a circle in the parameter space (bottom) and vice versa. Here, the intersection of circles determines the center of the sought circles in the spatial domain. The intersection of four circles at $u = 20, v = 25$ identifies points 1,2,3 and 5 as belonging to circle whose center $\mathbf{c}$ in the spatial domain is $\mathbf{c} = [20, 25]^T$.

A circle of radius r can be defined by

$$(x - u)^2 + (y - v)^2 - r^2 = 0 \tag{4}$$

with x, y the spatial variables and u, v the parameters of the circle (center) in the spatial domain. Let us now introduce the parameter space, represented by the coordinate system u, v. After a moment's thought we realize that in this representation, variables and parameters switch roles; u, v are now variables and x, y become parameters. A point x_i, y_i in the spatial domain corresponds to a circle in the parameter space, centered at x_i, y_i. What do we gain? For every point in the spatial domain there exists a circle in the parameter space and vice versa. The intersection of circles define centers of circles in the spatial domain—a simple solution of our original problem. The number of intersecting circles in the parameter space is directly related to the number of points that lie on this circle.

The Hough method is usually implemented by a so called accumulator array which is an n-dimensional, discrete space where n is identical to the number of parameters. In our example with circles of known radii, the accumulator array is two-dimensional. Each circle is discretely represented in the parameter space. To keep track of all the circles, we simply increment the cells that are turned on by every circle. After having processed all points in this fashion, we analyze the accumulator array and determine the number of hits per cell. Every hit casts one vote for a point lying on that particular circle. The cell with the maximum number of hits, located at $u_{\max}, v_{\max}$, yields the center of a circle in the spatial domain that passes through *max* points. Similarly, other peaks in the accumulator identify additional circle centers.

In order to identify the points belonging to the circles found by analyzing the accumulator array, the procedure is repeated. This time we already know which accumulator cells yielded a circle. Whenever a point happens to turn a peak cell on, it is immediately labeled.

4 Surface Matching in Parameter Space

In this section we apply the concept of determining the parameters by a voting scheme to solve the surface matching problem as stated in Sec. 2.2. To determine the seven parameters of the similarity transformation, seven equations of the type of Eq. 2 are required. Since there is no redundancy, we introduce the condition $d_i = 0$. That is, Eq. 2 becomes the coplanarity condition. Theoretically, we can select seven points $\mathbf{q}$ in set S_2 and match them with all possible surface patches of S_1. For every such combination, a set of seven equations is found and solved. The discretized solution yields those cell addresses of the 7-D accumulator array that need to be incremented. Once all possible combinations are explored, we select again seven points $\mathbf{q}$ and repeat the procedure. The correct solution will emerge as a peak in the accumulator array.

Of course, this trial and error approach is not practical at all. To explore all combinations leads quickly to combinatorial explosion. The maximum number of combinations of points $\mathbf{q}$ with surface patches is roughly $n \cdot m$. We need seven independent combinations with repetitions allowed. Thus the total number of solutions s is

$$s = \frac{n!m!}{7!(n-7)!(m-7)!} \tag{5}$$

With a modest number of $n = m = 100$ we get $\approx 10^{11}$ solutions, a vivid impression of the combinatorial problem indeed!

The memory request of the 7-D accumulator array creates another problem. Even restricting the solution space to plausible solutions, the size of the parameter space may get astronomical, depending on the discretization size. Take ten arc second for the three rotation angles and a range of $\pm 10^o$, for example. Each of the three parameter axes would require $7,200$ units.

Similar considerations for the translational parameters lead a request of $7.2 \cdot 10^{10}$ cells capable to store the number of solutions. Again, it appears that the approach is highly impractical.

The problems just identified are caused by the attempt to determine all seven transformation simultaneously. Let us pursue the other extreme and calculate the parameters sequentially, in an iterative fashion. Consequently, the accumulator array will be one dimensional and the memory problem disappears. The total number of point to surface patch combinations reduces to $m \times n_s$ with m the number of points in set S_2 and n_s the number of surface patches, e.g. triangles, in S_1. Since this is now an iterative process that has to be repeated for every parameter, the computational complexity is proportional to $m \times n_s \times 7 \times$ maximum number of iterations.

The method proceeds along the following steps:

1. Select one of the parameters, e.g. t_k. The current values of the other parameters are considered constants. Initialize the 1-D accumulator array for parameter t_k.
2. Pick point $\mathbf{q}_i$ in set S_2.
3. Select surface patch SP_j in S_1, e.g. defined by points $\mathbf{p}_a, \mathbf{p}_b, \mathbf{p}_c$ and compute parameter t_k by solving the coplanarity condition.
4. Update accumulator array.
5. Repeat steps 3 to 4 until all plausible point to surface patch correspondences have been explored.
6. Repeat steps 2 to 5 until all points $\mathbf{q}$ have been evaluated.
7. Analyze accumulator array for a distinct peak. Update parameter t_k with the peak value.
8. Repeat steps 1 to 7 until all parameters have been updated.
9. Repeat the entire procedure if the parameters changed more than a predefined threshold.

This procedure can be executed under a coarse-to-fine strategy that controls the precision of the solution (discrete interval) and the permissible range. As one proceeds from coarse to fine, the range becomes smaller as well as the discrete solution steps. The dimension of the accumulator array may remain constant.

So far we have determined the transformation parameters iteratively, one by one; we have yet to solve the surface matching problem. For explicitly labeling the correct point to surface patch correspondence we simply repeat the procedure described above. This time we already know the correct transformation parameters, however. Hence, whenever a correspondence is found with the correct solution (correct accumulator cell), the point is labeled accordingly. Now, as a mandatory final step we could determine the transformation parameters simultaneously, for example by the adjustment procedure described in *Schenk* (1999a). Like every non-linear adjustment problem, reasonable approximations are required. Of course, the iteratively determined transformation parameters are excellent approximations.

An important aspect in comparing surfaces is concerned with detecting blunders in the data. It is well known that undetected blunders that participate in a least-squares adjustment may greatly influence the solution. How robust is our proposed approach in this respect? Step 3 of the procedure computes values for parameter t_k with point $\mathbf{q}_i$ and all surface patches. The values are entered into the accumulator array. Suppose now point $\mathbf{q}_i$ is wrong (blunder). As a result, wrong parameter values are computed and cells in the accumulator array are incremented which are separated from the peak. It follows that blunders have no impact on the solution—an important property of our approach that can be applied to detect blunders.

Let us again analyze the final step, involving the explicit labeling of matches. Points that remain unlabeled have never contributed to the correct solution of a transformation parameter. Such points are obviously not part of a consistent surface description; they can be labeled as blunders. This allows for change detection. Here, we would analyze the spatial distribution of blunders and signal a significant difference between the two surfaces whenever blunders are locally concentrated.

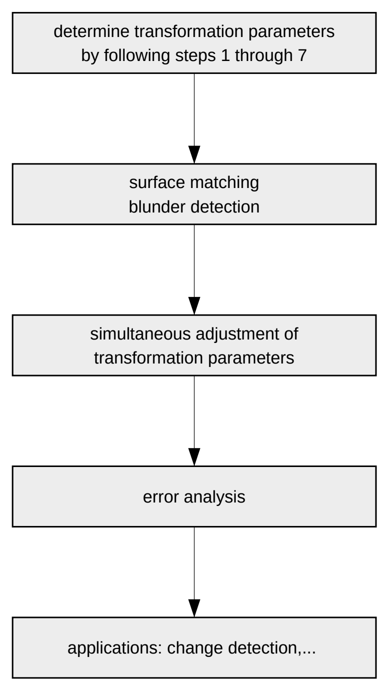

Figure 3: Schematic diagram of surface matching, blunder and change detection. The iterative determination of the transformation parameters is accomplished by a voting scheme in the parameter space, described above by steps 1–9. Surface matching is obtained by repeating the procedure, but now with known parameters. At the same time, blunders are detected and labeled accordingly. A mandatory step is the simultaneous adjustment of the transformation parameters, using the previous results as approximations. Other steps may follow, for example error analysis and applications such as change detection.

5 Experiments

In order to test the feasibility and performance of the proposed surface matching method, we have performed several experiments with synthetic and real data. This section briefly summarizes the most pertinent results.

5.1 Tests with Synthetic Data

Fig. 4 depicts the synthetic data set. Following the notation used in the previous sections, data set S_2 consists of the points $\mathbf{q}_i, i = 1, 2, \dots, 30$. Surface S_1 on the other hand is given in form of five surface patches $SP_1, \dots, SP_5$. The true correspondence of points $\mathbf{q}$ to the surface patches is known in this simulation, as well as the transformation parameters. The tests served the purpose of recovering the parameters and the correspondences. Moreover, the convergence rate was examined as a function of surface topography.

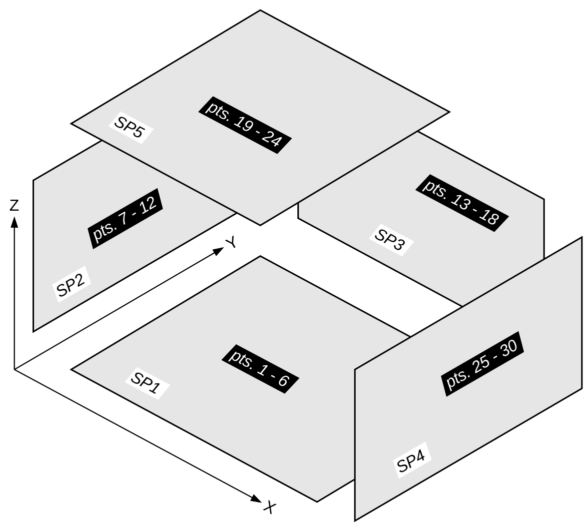

Figure 4: Synthetic data sets for simulation studies with the proposed matching method. Data set S_1 is given by the five surface patches $SP_1, \dots, SP_5$ and data set S_2 is represented by 30 points. The figure also shows the correct correspondence of points to surface patches.

The initial values of the parameters were set off by 3^o for the angles, 2 meters for the translation parameters, and 40% for the scale factor. All parameters were determined correctly. Fig. 5 shows the accumulator array for the scale factor. The number of non-zero elements in the accumulator array corresponds to the number of correspondences evaluated—in our example $30 \times 5 = 150$ (every point $\mathbf{q}$ with every surface patch SP). The distinct peak with a value of 30 indicates that for all points one correct correspondence was found.

Not all parameters exhibit the same behavior as a closer examination of Fig. 4 reveals. Take the shift parameter along the $Y-$axis, for example. It can only be determined from a correspondence to SP_3; all other surface normals have no $Y-$component. Thus, the accumulator array has a peak value of six, referring to the correct correspondence $\mathbf{q}_{13}, \dots, \mathbf{q}_{18}$ to SP_3.

Finally, Fig. 6 shows the change of parameters as a function of number of iterations. As expected, the convergence rate depends on how separable a parameter is. The translation parameters are linear hence fewer iterations are required. By the same token, the angular elements need more iterations because of the highly non-linear rotation matrix.

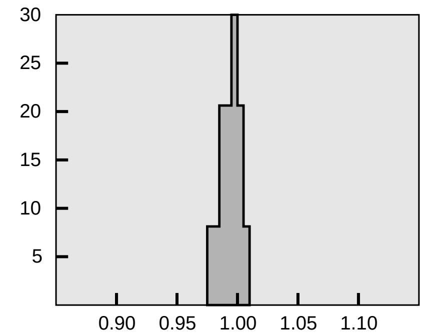

Figure 5: One-dimensional accumulator array (histogram) for the scale parameter. The peak value of 30 indicates that all 30 points of data set S_2 contributed in one correspondence to the correct scale factor.

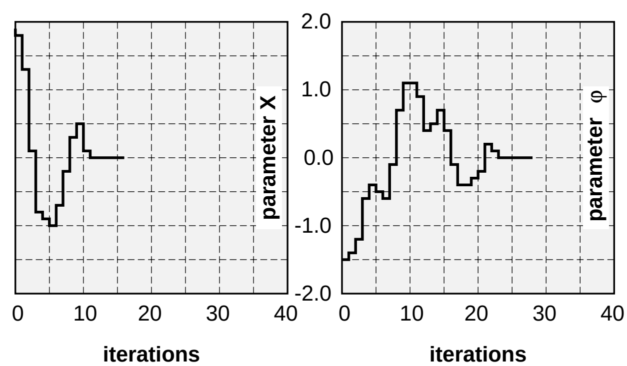

Figure 6: Change of parameters as a function of number of iterations. After initial fluctuations, the parameter stabilizes after a few iterations. The iterations are terminated once the changes become marginal.

There is another factor that greatly influences the convergence rate, however. Remember that we impose the coplanarity condition to compute the transformation parameters. In essence, the distance d_i in Eq. 2 is set to zero. The distance is parallel to the normal of the surface patches. To obtain a good solution for our transformation problem surface patches with normals oriented in all directions are necessary. The topography of S_1 is important. As shown in *Schenk* (1999a), the surface slopes should point in different directions. The slope angle directly influences the goodness of the solution.

5.2 Experiments with Real Data

As reported by *Csathó et al.* (1998), ISPRS Technical Commission III has acquired a multisensor/multispectral data set over Ocean City, with several laser data sets provided by NASA Wallops, and aerial imagery flown by NGS (National Geodetic Survey). The data provide an excellent opportunity to test the proposed procedure on a real world problem; how well does a laser surface agree with a photogrammetrically derived surface?

Fig. 7 shows a portion of an aerial image covering three apartment buildings (top) and a wire frame diagram of the laser data of the same area (bottom). We used the laser data set in the sense of S_2, that is, as a sequence of unrelated points. The necessary surface patches of S_1 were obtained from measuring the stereo model on a softcopy workstation.

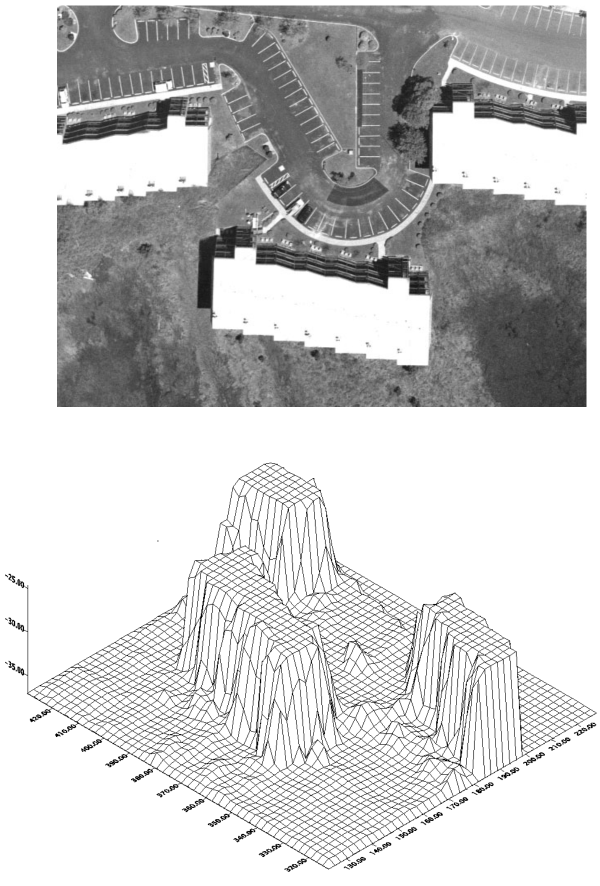

Figure 7: Aerial image patch showing an apartment complex of the test site Ocean City (top). The laser data set of the same area is represented by a wire frame diagram (bottom).

We skip the details here but present a short summary of the results. Fig. 8 (left) shows the laser surface represented as a gray level image. Superimposed are the photogrammetrically measured points (crosses) and a few triangles that were formed when generating a TIN model. The triangles served as surface patches SP. The parameters found by our approach indicate very good agreement between the two data sets.

A more meaningful check is to perform the transformation with the parameters found, followed by computing the distance of the transformed points to the surface patches S_1. The average distance of 0.03 m between the laser and stereo surface confirms the accuracy potential of both methods. Fig. 8 (right) is a graphical illustration of the matching. The white crosses show all the laser points that were found as correct matches.

Finally we show the result of detecting blunders. In the area examined, one laser point did not correspond to any surface patch. As discussed in the previous section, such points are labeled as blunders. Fig. 9 depicts the laser point and the triangle to which it should correspond. A closer analysis reveals that the laser point is on top of a tree. The planar surface patch, determined by photogrammetry, is on the ground. Hence, the distance from the laser point to the surface patch exceeded the tolerance.

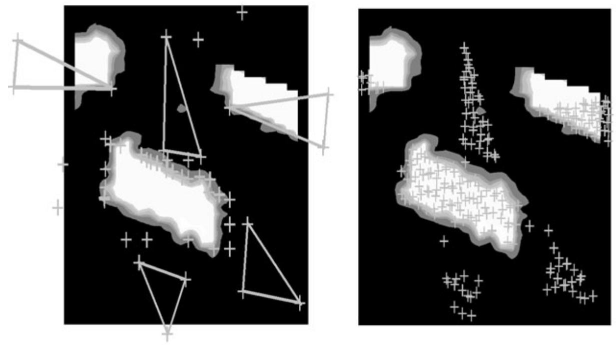

Figure 8: The left part shows the laser surface represented as a gray level image. Superimposed are the points measured photogrammetrically. Also shown are a few triangles formed by generating a TIN model. The result of the establishing the correspondence between the two surfaces, the laser points that were matched with the triangles are shown in the right part of the figure.

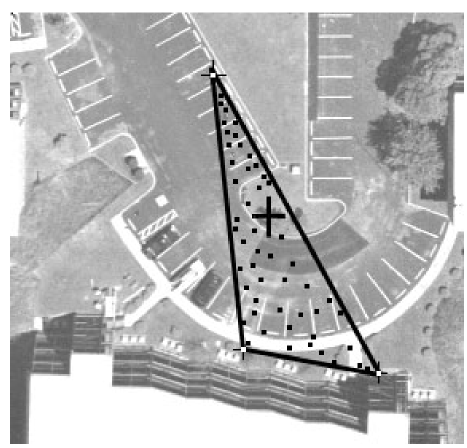

Figure 9: Small squares identify correct matches of laser points within one triangle, established by photogrammetry. The cross identifies a point that should lie on the triangle. However, the distance exceeded the tolerance and the point is considered a blunder. The laser footprint is on the top of a tree while the surface patch was measured on the ground.

6 Conclusions

Comparing surfaces is a frequently occurring task and a prerequisite for merging data sets that describe the same physical surface but with different sets of discrete points. If the two data sets are in different reference systems then the comparison is quite challenging because neither can we count on identical points nor are the transformation parameters known. Our proposed matching scheme solves this problem in a general, effective, and robust fashion.

The reliability of the transformation parameters between the two sets depends on the surface geometry. Using distances along surface normals requires reasonably sloped surfaces, with different slope directions. Of course, this is not specific to our method; rather, it is a general requirement.

The proposed approach of reducing the $n-$parameter space to one or a few dimensions hinges on the separability of the parameters. Highly non-linear transformations have a slower convergence rate. However, more objective criteria must be established to assess the convergence, for example as a function of the surface normal distribution. Moreover, the correlation among parameters depends on where it is measured on surface.

The proposed voting scheme is essentially a statistical method. As such, many votes are necessary for a reliable analysis. One distance between a point in one surface to a surface patch allows the computation of one transformation parameter and its discretized value casts one vote. Hence, the more independent point to surface patch relations exist the more reliable becomes the solution. Generally, laser altimetry determines many points. Thus, the method described here is particularly suited for dealing with laser surfaces. Moreover, laser data sets are irregularly distributed. Another advantage of our approach is that no interpolation to a regular grid is necessary. Finally, the identification of blunders is of great practical importance. Contrary to adjustment methods, undetected blunders do not affect the solution. This makes the proposed surface matching method robust.

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