STATISTICAL TESTINGS FOR DETERMINING SIGNIFICANT SAR-IMAGE ORIENTATION PARAMETERS

Joz WU, Chia-Jun LIU, De-Chen LIN

National Central University Chungli 320, Taiwan, ROC
Center for Space and Remote Sensing Research
jozwu@csrsr.ncu.edu.tw

(Paper number: 1400)

KEY WORDS: Airborne SAR-Image, Time Polynomials, Significance/Optimization.

ABSTRACT

For geometric analysis of a side-looking SAR image, it is frequently of interest to determine the time-varying radar antenna's position parameters along a flight path. In this paper, the time polynomials are employed to model the position (orientation) parameters, in association with the basic ranging/Doppler (radargrammetric) conditions. As usual, a least-squares adjustment yields both parameter corrections and measurement residuals. The estimated corrections and their variances can be used to statistically investigate their respective parameter significance. Equally essential are the estimated measurement residuals because the corresponding residual quadratic-form should theoretically reflect a minimum value. Based on the explicit expressions cited for both the parameter significance and the model optimization, the results of an experimental airborne SAR slant-range image indicate that the proposed methodology satisfactorily works.

1. INTRODUCTION

Side-looking synthetic aperture radar (SAR) (Moore et al., 1983) represents an active remote sensing technology. For a variety of technical issues, the current paper is concerned with the space resection of imagery resulting from the SAR sensing/data-processing. Geometrically, an SAR image is in (slant) range projection. Radargrammetric equations express both the radar cross-track range measurement and the along-track Doppler image-forming requirement. The equations are well suited for an SAR-image resection topic (Leberl, 1979). Based on some control points, the radargrammetric conditions serve to describe a functional relationship between the two-dimensional image point coordinate measurements and the three-dimensional ground point measurements.

The unknown parameters in the functional relationship are, among others, the radar antenna's time-varying positions. By employing a least-squares method, the SAR antenna's orientation parameters can be estimated (Koch, 1999). When using a quadratic form of the estimated measurement residuals, one can further perform parameter significance tests (Zhong, 1997). In this manner, the SAR-image orientation parameters may be considered statistically optimal.

2. SIDE-LOOKING SAR GEOMETRY

Unlike passive optical imagery resulting from a (central) perspective projection, SAR imaging geometry is related to a slant-range projection, leading to such an image displacement as foreshortening. Depending on an off-nadir illuminating beam at an elevated target and its terrain slope (aspect angle), layover and shadowing phenomena can also occur. Active side-looking SAR imagery has its own radiometric and geometric characteristics (Fullerton et al., 1986; Gelautz et al., 1998; Toutin, 1997).

2.1 Radargrammetric Equations

The following range and Doppler expressions are frequently utilized for radargrammetric image processing (Curlander et al., 1987; Dowman, 1992; Leberl, 1979):

$$R_{ji} = [(X_i - X_{oj})^2 + (Y_i - Y_{oj})^2 + (Z_i - Z_{oj})^2]^{\frac{1}{2}} \tag{1}$$

$$R_{ji} \sin\tau = u_x(X_i - X_{oj}) + u_y(Y_i - Y_{oj}) + u_z(Z_i - Z_{oj}) \tag{2}$$

$$R_{ji} = M_b r_i(t_j) + R_n \tag{3}$$

where R_{ji} (m) is the measured one-way slant range from radar path/station (X_{oj}, Y_{oj}, Z_{oj}) to point target (X_i, Y_i, Z_i); for the same point i, there may be multiple flight paths j over i; τ (deg) stands for the squint angle/parameter; (u_x, u_y, u_z) in m/s units is the instantaneous unit radar velocity; M_b (m/pixel) represents the pixel spacing; R_n (m) is the constant slant-range delay r_i (pixels) is the cross-track image coordinate measurement, with the along-track image coordinate t_j, acting as an argument, in pixels (or in seconds). The time polynomial expansions are used to model the exterior orientation parameters, as follows:

$$\begin{aligned} X_{oj} &= a_0 + a_1 t_j + a_2 t_j^2 + a_3 t_j^3 \\ Y_{oj} &= b_0 + b_1 t_j + b_2 t_j^2 + b_3 t_j^3 \\ Z_{oj} &= c_0 + c_1 t_j + c_2 t_j^2 + c_3 t_j^3 \end{aligned} \tag{4}$$

The higher-order coefficients ($a_2, b_2, c_2, \cdots$) are to be statistically tested for their parametric significance, on a one-by-one basis.

2.2 Parameter Estimation

By reviewing Eqs. (1-4), the line and pixel coordinate measurements ($\cdots t_j, r_i \cdots$), and the unknown parameters ($\tau, M_b, a_0, \cdots, b_0, \cdots, c_0, \cdots$) are readily identified. In general, initial approximations for the parameters can be made available. An iterative least-squares adjustment algorithm provides us with both the measurement and the parameter solutions (Koch, 1999; Leick, 1995). The relevant formulae are listed here:

$$\mathbf{Bv} + \mathbf{Ax} = \boldsymbol{l}; \qquad \sigma_0^2 \mathbf{Q} \tag{5a}$$

$$\mathbf{Q}_l = \mathbf{BQB}^T \tag{5b}$$

$$\mathbf{x} = \mathbf{Q}_x \mathbf{A}^T \mathbf{Q}_l^{-1} \boldsymbol{l} \tag{6a}$$

$$\mathbf{v} = \mathbf{Q}_v \mathbf{B}^T \mathbf{Q}_l^{-1} \boldsymbol{l} \tag{6b}$$

$$\hat{\sigma}_0^2 = \mathbf{v}^T \mathbf{Q}^{-1} \mathbf{v} / (n - u) \tag{7}$$

where the n-vector $\mathbf{v}$ contains the image control point measurement residuals; the $n \times n$ matrix $\mathbf{B}$ consists of the corresponding partial derivatives; the u-vector $\mathbf{x}$ represents the parameter corrections; the $n \times u$ design matrix $\mathbf{A}$ holds the corresponding partial derivatives (gradients); the n-vector $\boldsymbol{l}$ is a reduced observation vector; σ_0^2 represents the unit-weight reference variance, serving as a scaling factor.

The $n\times n$ matrix $\sigma_0^2\mathbf{Q}$ contains the measurement variances and covariances (=0), so $\mathbf{Q}$ is a diagonal matrix. The $n\times n$ $\mathbf{Q}_l$, $u\times u$ $\mathbf{Q}_x$ and $n\times n$ $\mathbf{Q}_v$ (not fully expressed) are the scaled covariance matrices, arising from the $\boldsymbol{l}$-, $\mathbf{x}$-, and $\mathbf{v}$-vector error propagations, respectively. $\mathbf{v}^T\mathbf{Q}^{-1}\mathbf{v}$ is a quadratic form of the estimated measurement residuals; $n-u$ denotes the degree of freedom. Eq. (7) then leads to the a posteriori reference variance estimate $\hat{\sigma}_0^2$.

3. STATISTICALPARAMETER ANALYSIS

Statistical testings are very useful algorithmic routines to evaluate the least-squares parameter and measurement estimates, see Eqs. (6a, 6b). The quadratic form (7) of the estimated measurement residuals can be used to build a chi-square test statistic, involving the a priori reference variance (Leick, 1995; Wang et al., 1998). By specifying a significance level, usually at 5%, the lower and upper critical chi-square distributed values can be derived from a statistical look-up table. If the chi-square test statistic fails to fall within the confidence interval, it can be stated that, with a 95% confidence level, the functional and stochastical modeling equation (5a) is incorrect.

3.1 Parameter-SignificanceTest

According to Zhong (1997), an F-test statistic is given as the left-hand term in the following inequality equation:

$$\frac{xq_x^{-1}x}{\hat{\sigma}_0^2} \le F_{1-\alpha;1,n-u} \qquad (8)$$

where x is a component parameter in the $\mathbf{x}$-solution equation (6a); q_x stands for the corresponding cofactor (scaled variance) of x; $\hat{\sigma}_0^2$ is the estimated reference variance, as from Eq. (7). $F_{1-\alpha;1,n-u}$ is the critical value that results from an F-distribution of (1, n–u) degrees of freedom and at a $1-\alpha$ confidence level. If the F-test (8) is passed, the parameter x is not significant, and should be deleted from the model-parameter set. Thus, a new model is formed, the appearance of which is the same as expressed in Eq. (5). According to Eq. (6), the new (u–1)-vecto $\mathbf{x}$ and n-vector $\mathbf{v}$ solutions are readily obtained.

3.2 Optimization Criteria

In order to distinguish an old model having an $u\times1$ $\mathbf{x}$-vector of parameter corrections and the new/alternate model holding the (u–1)-component $\mathbf{x}$, their respective n-component measurement residuals are employed to produce the corresponding $\mathbf{v}$-quadratic forms. The next minimum criteria will be used as the optimization indices: i.e.,

$$\hat{\sigma}_0^2 = \mathbf{v}^T\mathbf{Q}^{-1}\mathbf{v}/(n-u) \qquad \rightarrow \quad \min \qquad (9a)$$

$$V_p = (n+u)\hat{\sigma}_0^2 \qquad \rightarrow \quad \min \qquad (9b)$$

$$AIC = n\ln(\mathbf{v}^T\mathbf{Q}^{-1}\mathbf{v}) + 2u \qquad \rightarrow \quad \min \qquad (9c)$$

It is noted that the suitable/stable criteria (9a-9c) are commonly used to select an optimal regression equation (Zhong, 1997). The optimization criteria could successfully be applied to the selection of polynomial parameters involving the GPS geoid height interpolation.

After the deletion of a least significant polynomial parameter by means of Eq. (8), the resulting model is checked out as to its optimality on the basis of Eq. (9). This process is iterated until no improvements on model optimization can be made. By analogy, an optimal set of model parameters can be determined by using some other statistical methods. The discrimination test given by Wang et al. (1998) represents an interesting alternative.

4. EXPERIMENTS

The C-band HH-polarized 6-look SAR image of Yangmei town and its neighboring area was a result of the Canadian CV-580 GlobeSAR campaign in Taiwan. The 7.0-km high aerial flight, cruising at around 240 knots (120 m/s), was flown on 28 October, 1993. The nominal ground resolution was 4.0 m by 4.0 m. The experimental slant-range SAR image was of 9.0 km along-track by 12.0 km cross-track, and had 35 distributed image/ground control points, reflecting the modest terrain height variations from 75.1 m to 210.9 m.

Both the image-space and the object-space control point coordinates were carefully measured by two independent operators, leading to an averaged input dataset needed for any radargrammetric processing. In fact, the same set of image/ground coordinate measurements were once used to study the time-dependent orientation parameters. The feasibility study by Wu and Lin (2000) was focused on the linear prediction parameter-modeling. In the current paper, the existing Fortran-software program YeSir has been edited to implement the significance test (8) and the optimization checks (9) on the polynomial model parameters (4).

4.1 Parameter Estimates

The fourth-order time polynomials were initialized, creating an unknown 17-paramete **x**-vector. By utilizing a set of 30 image control points, each involving a pair of the line and pixel coordinates (t_j, r_i), there were 60 independent and identically-distributed measurements. The degree of freedom was 43, well over a generally acceptable number of 30. Table 1 summarizes the corresponding stage-by-stage results.

Table 1 Significant parameter determination with a set of 30 image/ground control points

	Stage-1	Stage-2	Stage-3	Stage-4
Parameter set	$\tau, M_b,$ $a_0,\cdots,a_4,$ $b_0,\cdots,b_4,$ c_0,c_1,c_2,c_3,c_4	$\tau, M_b,$ $a_0,\cdots,a_4,$ $b_0,\cdots,b_4,$ c_0,c_1,c_3,c_4	$\tau, M_b,$ $a_0,\cdots,a_4,$ $b_0,\cdots,b_4,$ c_0,c_1,c_3	$\tau, M_b,$ $a_0,\cdots,a_3,$ $b_0,\cdots,b_4,$ c_0,c_1,c_3
Parameter with minimum F test statistic	c_2	c_4	a_4	-
Criteria:				
$\hat{\sigma}_0^2$	0.471	0.460	0.454	0.478
V_p	121.0	117.7	115.7	121.4
AIC	665.2	663.0	662.4	671.5
Optimization	No	No	Yes	No

At the third stage, the three optimization criteria (9a -9c) all reached minima, indicating that the set of 15 significant orientation parameters has been determined. Their respective estimated values are: τ =0.0 deg, $M_b = 4.04$ m/pixel; $a_0 = 256208.1$ m, $a_1 = -0.41$ m/s, $a_2 = 2.0\times10^{-4}$ m/s^2, $a_3 = -4.2\times10^{-8}$ m/s^3, $a_4 = 3.2\times10^{-12}$ m/s^4; $b_0 = 2740272.5$ m, $b_1 = 9.2$ m/s, $b_2 = -2.3$ m/s^2, $b_3 = 4.3\times10^{-7}$ m/s^3, $b_4 = -3.0\times10^{-11}$ m/s^4; $c_0 = 7095.4$ m, $c_1 = 0.018$ m/s, $c_3 = -4.5\times10^{-10}$ m/s^3. Notably, the insignificant parameters c_2 and c_4 are the polynomial coefficients used to model airplane altitude variations. Their deletion is justified because the real flight, at 7.0 km above ground, was intended to be level.

4.2 Horizontal RMS-Errors

In the preceding subsection, 5 points were withdrawn to serve as independent horizontal ground check points. In total, 6 cases were studied, each with 5 randomly located check points (McGwire, 1996). The *X*/*Y* root-mean-square (RMS) errors were based on the 30 check point coordinate differences. To make these horizontal positionings possible, every check-point terrain elevation Z_i was treated as a known parameter, in the space intersection with Eqs. (1-3; 4). The positioning RMS results are listed in Table 2.

Table 2 Horizontal position RMS errors

	X/East (m)	*Y*/North (m)
At 5×6 check points	± 5.3	± 4.2

5. CONCLUDING REMARKS

The space resection concerning an airborne SAR slant-range image is based on the radargrammetric (range/Doppler) equations, and is enhanced through parameter-significance testings. The applied parametric *F*-test statistic and the optimization criteria are given to demonstrate how their feasibility is achieved. The adaptive orientation parameter determination is a result of statistical inferences and hardly requires any human intervention.

It is thinkable that, whenever parameter/measurement estimations are desired, the same optimality reasoning can be introduced into a data processing algorithm. Some related SAR-image processing themes are such as image matching for conjugate points, geocoding of spaceborne remote sensing imagery (Tannous and Pikeroen, 1994), and integration of ancillary navigational data.

ACKNOWLEDGMENTS

The authors would like to thank the Council of Agriculture for supporting the 1993 SAR campaign, and the NSC Satellite Remote Sensing Laboratory for providing them the experimental image.

REFERENCES

Curlander, J. C., Kwok, R., Pang, S. S., 1987. A post-processing system for automated rectification and registration of spaceborne SAR imagery. International Journal of Remote Sensing, 8(4), pp. 621-638.

Dowman, I., 1992. The geometry of SAR images for geocoding and stereo applications. International Journal of Remote Sensing, 13(9), pp. 1609-1617.

Fullerton, J. K., Leberl, F., Marque, R. E., 1986. Opposite-side SAR image processing for stereo viewing. Photogrammetric Engineering and Remote Sensing, 52(9), pp. 1487-1498.

Gelautz, M., Frick, H., Raggam, J., Burgstaller, J., Leberl, F., 1998. SAR image simulation and analysis of alpine terrain. ISPRS Journal of Photogrammetry and Remote Sensing, 53(1), pp. 17-38.

Koch, K. R., 1999. Parameter Estimation and Hypothesis Testing in Linear Models. Springer-Verlag, Berlin.

Leberl, F., 1979. Accuracy analysis of stereo side-looking radar. Photogrammetric Engineering and Remote Sensing, 45(8), pp. 1083-1096.

Leick, A., 1995. GPS Satellite Surveying. John Wiley & Sons, Inc., New York.

McGwire, K. C., 1996. Cross-validated assessment of geometric accuracy. Photogrammetric Engineering and Remote Sensing, 62(10), pp. 1179-1187.

Moore, R. K., Chastant, L. J., Porcello, L., Stevenson, J., 1983. Imaging radar systems. In: Simonett, D. S., Ulaby, F. T. (Eds.), Manual of Remote Sensing, American Society of Photogrammetry, Falls Church, Virginia, Vol. I, pp. 429-474.

Tannous, I., Pikeroen, B., 1994. Parametric modeling of spaceborne SAR image geometry. Application: SEASAT/SPOT image registration. Photogrammetric Engineering and Remote Sensing, 60(6), pp. 755-766.

Toutin, Th., 1997. Accuracy assessment of stereo-extracted data from airborne SAR images. International Journal of Remote Sensing, 18(18), pp. 3693-3707.

Wang, J., Stewart, M.P., Tsakiri, M., 1998. A discrimination test procedure for ambiguity resolution on-the-fly. Journal of Geodesy, 72(11), pp. 644-653.

Wu, J., Lin, D.-C., 2000. Radargrammetric parameter evaluation of an airborne SAR image. Photogrammetric Engineering and Remote Sensing, 66(1), pp. 41-47.

Zhong, D., 1997. Robust estimation and optimal selection of polynomial parameters for the interpolation of GPS geoid heights. Journal of Geodesy, 71(9), pp. 552-561.