HIERARCHICAL RECOVERY OF EXTERIOR ORIENTATION FROM PARAMETRIC AND NATURAL 3-D CURVES

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ABSTRACT

Space resection is a fundamental task in many photogrammetric, remote sensing and computer vision applications. While in traditional photogrammetric techniques this task is exclusively solved from 3-D to 2-D point correspondences, recent advances in computer technology require us to adopt more general methods, accommodating features other than points, especially linear features. In the last decade, a substantial research on the use of higher level geometric features has been carried out in the photogrammetry and computer vision communities. However, only few types of linear features have been thoroughly addressed. In this paper we present a method that solves for the exterior orientation parameters analytically from a set of general 3-D curves in object space and their 2-D images. First, a non-linear least-squares method utilizing 3-D curves represented parametrically is developed. The proposed method is then extended to accommodate free-form curves represented as an ordered list of 3-D points. Finally, a coarse-to-fine strategy is introduced to hierarchically recover the orientation parameters. Although the correspondence between 3-D curves and their 2-D partial images is not solved in this work, one of its major contributions is the fact that given this correspondence, the association between their individual points is established as a by product from the resection solution. In that sense, the matching and the orientation tasks are carried out simultaneously. Furthermore, the beauty of the proposed method is that the underlying principal is not tied exclusively with the perspective transformation governing the image formation process. In fact, it is general enough to be employed for any other functional form relating measurable quantities in two arbitrary domains.

1 INTRODUCTION

In the last decade, the challenge of geometric modeling based on higher level features has attracted a substantial body of research activities in the photogrammetry and computer vision communities. Not surprisingly, the earliest attempts have primarily been devoted to the simplest subclass of linear features, namely straight lines. Over the years, various linear (e.g., [6, 7]) and iterative (e.g., [2, 8, 9]) algorithms have been introduced. What is common to all these estimation methods is that they use the same key idea realized by imposing the following constraint – a 3-D line as well as any vector connecting the perspective center with an arbitrary point on the 3-D line must lie in the plane defined by the perspective center and the corresponding 2-D line. The class of conic sections is another example of algebraic curves that has been effectively used to solve the resection problem. Employing different solutions Ji et al [6], Haralick and Shapiro [5] show that given a circle in 3-D space along with its corresponding ellipse in the image, the plane equation where the 3-D circle lies relative to the camera coordinate frame can be solved for analytically. Mulawa and Mikhail [9] describe a non-linear method that determines the six exterior orientation parameters (EOPs) by constraining the object space coordinates of a measured image point to obey the appropriate conic equation in 3-D space. In this method each member of the conic family is treated separately resulting in a different pair of constraints. For example, a circular feature in 3-D space can be described as the intersection of a sphere with a plane containing the sphere's center. Thus, every object point lying on that circular feature is constrained to satisfy both the sphere as well as the plane equations. In the approach presented by Haralick and Chu [4] the viewing parameters are determined from conic sections represented parametrically. Their solution is based on writing down a set of equations involving the parametrized object conic, the 2-D coordinates of an image point known to be lying on its projection, as well as the six parameters describing the position and the orientation of the camera. Next, the conic parameter as well as the three translation parameters X_0, Y_0, and Z_0 are eliminated (by introducing several image points associated with the conic) to end up with a single homogeneous equation involving the remaining three angular parameters. However, regardless of how many such equations are written the entire set of three angles can not be estimated if only one conic section is involved. That is essentially due to the inherent deficiency of a single conic (being a five-parameter feature) in recovering the entire set of six orientation parameters. An attempt to generalize the resection problem to more complex linear features has been made by Habib and Novak [3]. In their approach a relation is established between a 2-D curve in image space and a fixed object space point, the projection of which lies on the 2-D curve. Assuming that the 2-D curve can be represented implicitly by $g(x, y) = 0$, the traditional collinearity equations can be substituted into the curve equation to yield a condition that constrains the projection of the object space point to correspond to the 2-D curve. While this method is theoretically correct, its implementation in the case where an explicit representation of the feature in object space is available reveals a practical problem. The difficulty stems from the fact that image space fitting must be employed to obtain the implicit equation of the 2-D curve. Recently, Habib et. al [2] have developed a clustering technique to carry out a single photo resection from free-form curves. In this method a point in the image space is matched with every point in the object space. This approach, apart from being computationally expensive, also assumes that the pixels delineating a feature in the image space correspond to 3-D points describing the control feature in the object space.

In this paper a hierarchical least-squares method that solves for the EOPs analytically from a set of general 3-D curves in object space and their 2-D images is derived. Specifically, the remainder of this paper is organized as follows. In the next section a non-linear least-squares method utilizing 3-D curves that are represented in parametric form as "control lines", is developed. In section 3 this method is extended to accommodate general (free-form) space curves represented as an ordered list of 3-D points. A method that approximates a projectively invariant scale-space of planar curves is then presented for hierarchically recovering the EOPs. Section 5 describes the experiments that were carried out to test the feasibility and the performance of the proposed algorithms for image orientation. Finally, concluding remarks are given in section 6.

2 EXTERIOR ORIENTATION FROM PARAMETRIC CURVES

This section describes a non-linear mathematical model developed to recover the exterior orientation parameters of a single image from 3-D curves represented in parametric form. The EOPs are recovered after establishing a relationship between measured quantities in the image space and their counterparts in the object space. Two such relationships are utilized in the proposed model. In the first, presented in subsection 2.1, the collinearity equations relating object space coordinates of a point to their projected image space counterparts are extended to support the constraint that a ray from the perspective center passing through an image point must intersect the underlying 3-D space curve. In the second, described in subsection 2.2, a local tangent direction in 3-D is related to its image space 2-D projection through the six EOPs. In subsection 2.3 an integrated model is presented. Further, it is also shown there how the proposed model can be extended to accommodate higher level local characteristics, such as curvature, for recovering the orientation parameters.

In what follows we will assume that our control information is given in the form of a class of 3-D curves, called *regular curves*. A regular curve is defined as the locus of points traced out by the end point of a vector $\Gamma(t) = [X(t) \quad Y(t) \quad Z(t)]^T$ as the curve parameter $t \in \Re$ ranges from a to b. Further, $\Gamma(t)$ must have continuous second derivatives and its first derivatives must not vanish simultaneously anywhere in the interval $a \le t \le b$.

2.1 The Extended Collinearity Equation Model

With object space points being constrained to lie on $\Gamma(t)$ the traditional set of the collinearity equations takes the form

$$\begin{aligned} x_p &= -f\frac{u}{w} \\ y_p &= -f\frac{v}{w} \end{aligned} \quad \text{with} \quad \begin{bmatrix} u \\ v \\ w \end{bmatrix} = R(\omega, \varphi, \kappa)\begin{bmatrix} X(t) - X_0 \\ Y(t) - Y_0 \\ Z(t) - Z_0 \end{bmatrix} \tag{1}$$

Here, R is a rotation matrix, formed from the angular EOPs, transforming the vector from the perspective center to the ground point from the object into the camera coordinate frame. From (1) we observe that as the vector $[u, v, w]^T$ depends on the seven parameters $< X_0, Y_0, Z_0, \omega, \varphi, \kappa, t >$ so do the corresponding image space coordinates (x_p, y_p).

Linearization of (1) requires initial values of the parameter vector, which in our case also include the curve parameter t apart from the initial estimates for the six EOPs. The most natural way to obtain a reasonable estimate for t is to compute the value t_0 of the curve parameter that corresponds to the point on $\Gamma(t_0)$ that is the closest to the ray defined by the perspective center and the point (x, y) in image space. With the initial estimates of the seven parameters at hand the system (1) is linearized to result in

$$x_p + f\frac{u^0}{w^0} = A_1^x\,dX_0 + A_2^x\,dY_0 + A_3^x\,dZ_0 + A_4^x\,d\omega + A_5^x\,d\varphi + A_6^x\,d\kappa + A_7^x\,dt + e_x$$
$$y_p + f\frac{v^0}{w^0} = A_1^y\,dX_0 + A_2^y\,dY_0 + A_3^y\,dZ_0 + A_4^y\,d\omega + A_5^y\,d\varphi + A_6^y\,d\kappa + A_7^y\,dt + e_y \qquad (2)$$

with u^0, v^0, and w^0 obtained by substituting the approximate parameters $< X_o{}^0, Y_o{}^0, Z_o{}^0, \omega^0, \varphi^0, \kappa^0, t^0 >$ in (1), A_1^x through A_7^x, and A_1^y through A_7^y denoting respectively the partial derivatives of x_p and y_p with respect to $X_o, Y_o, Z_o, \omega, \varphi, \kappa$, and t ,and e_x, e_y representing the stochastic errors with zero expectation associated with the observed image coordinates x_p and y_p.

For every measured image point known to correspond to the projection of the parametric curve, two observation equations (2) are written. However, such measurement contributes only one degree of freedom to the overall redundancy budget. That is because in addition to the EOPs an individual value of the curve parameter, different from one measurement to another must also be estimated. In that sense, our model only generalizes the traditional approaches for the resection problem introduced in the literature. From the last argument the following question arises. Are there any additional local properties associated with analytical curves that could be exploited for estimating the camera parameters? Of course, an analytical curve does not just consist of a sequence of unrelated points – there is more independent information we can utilize for solving the pose problem. In the next section an example for such additional, and, in general, independent source of information will be given.

2.2 Incorporating the relation between image and object tangents

So far, all the photogrammetric methods dealing with linear features use only a relation between a point's location in object space and its counterpart in the image. Obviously, in point-based approaches the coordinates of a point are the only piece of information that could be utilized to solve the resection problem. Linear features however, carry additional independent characteristics (e.g. tangents, curvatures, etc.) that could be used to recover the EOPs if a relation between their values in both domains is properly established. In this section a relation between a local tangent vector in object space and its projected counterpart in the image space is formulated. As this relation primarily depends on the EOPs, their recovery from a correspondence between these tangents seems appealing.

Referring to the extended collinearity equations (1), and assuming for a moment that the EOPs are known and fixed, the image space coordinates of a point in the vicinity of p (the projection of P) are a function of only the curve parameter t as follows:

$$x(t) = -f\frac{u(t)}{w(t)} \quad y(t) = -f\frac{v(t)}{w(t)} \quad \text{with} \quad \begin{bmatrix} u(t) \\ v(t) \\ w(t) \end{bmatrix} = R\begin{bmatrix} X(t)-X_0 \\ Y(t)-Y_0 \\ Z(t)-Z_0 \end{bmatrix} \qquad (3)$$

An infinitesimal change in the parameter value at t_0 will give rise to an infinitesimal change along the space curve in the direction parallel to the vector $[X'(t), Y'(t), Z'(t)]^T$ and this in turn will cause the projected point in the image space to move in the 2-D direction given by $[x'(t), y'(t)]^T$. This direction is readily obtained by differentiating the equations in (3) with respect to the parameter t. The sought derivatives are hence

$$x'(t) = -f\frac{u'(t)w(t) - w'(t)u(t)}{w^2(t)} \quad y'(t) = -f\frac{v'(t)w(t) - w'(t)v(t)}{w^2(t)} \quad \text{with} \quad \begin{bmatrix} u'(t) \\ v'(t) \\ w'(t) \end{bmatrix} = R\begin{bmatrix} X'(t) \\ Y'(t) \\ Z'(t) \end{bmatrix} \qquad (4)$$

By diving these two equations we obtain the tangent direction $\tan(\Theta)$ in image space, expressed as a function of the tangent direction in object space $[X'(t), Y'(t), Z'(t)]^T$

$$\tan(\Theta) = \frac{y'(t)}{x'(t)} = \frac{v'(t)w(t) - v'(t)u(t)}{u'(t)w(t) - w'(t)u(t)} \qquad (5)$$

While each individual component of (4) is not invariant with respect to the curve parameter t, their ratio, given in (5) in fact is. The invariance of that ratio is not surprising as it corresponds to the 2-D tangent direction in image space, which is essentially an intrinsic (geometric) property of a 2-D curve. Moreover, this invariant ratio can also be measured directly from the image by estimating the tangent direction in the image from the distribution of the image points in the vicinity of (x, y). The linearization of (5) gives

$$\tan(\Theta) - \frac{v'^0 w^0 - w'^0 v^0}{u'^0 w^0 - w'^0 u^0} = A_1^{\Theta} dX_0 + A_2^{\Theta} dY_0 + A_3^{\Theta} dZ_0 + A_4^{\Theta} d\omega + A_5^{\Theta} d\varphi + A_6^{\Theta} d\kappa + A_7^{\Theta} dt + e_{\Theta} \tag{6}$$

where $\tan(\Theta)$ is a pseudo-observation of the tangent direction in image space, u_0, v_0, w_0 and u'_0, v'_0, w'_0 are the quantities evaluated at the approximated parameter values $< X_o{}^0, Y_o{}^0, Z_o{}^0, \omega^0, \varphi^0, \kappa^0, t^0 >$. $A_1^{\Theta}, \cdots, A_7^{\Theta}$ denote the derivatives of $\tan(\Theta)$ with respect to the seven parameters, and finally e_{Θ} is a stochastic error with zero expectation, the dispersion of which is essentially a function of how accurately the tangent direction is estimated in the image space.

2.3 The Integrated Model

Putting (2) and (6) together results in three observation equations written for each image space point k at which the image coordinates along with the tangent direction are introduced as observations. In vector form we get

$$y^k = \left[A_{EOP}^k \quad A_t^k \right] \xi^k + e^k \tag{7}$$

with

$$y^k = \left[x_k + f\frac{u^0}{w^0} \qquad y_k + f\frac{v^0}{w^0} \qquad \tan(\Theta_k) - \frac{v'^0 w^0 - w'^0 v^0}{u'^0 w^0 - w'^0 u^0} \right]^T$$

$$A_{EOP}^k = \begin{bmatrix} A_1^x & A_2^x & A_3^x & A_4^x & A_5^x & A_6^x \\ A_1^y & A_2^y & A_3^y & A_4^y & A_5^y & A_6^y \\ A_1^{\Theta} & A_2^{\Theta} & A_3^{\Theta} & A_4^{\Theta} & A_5^{\Theta} & A_6^{\Theta} \end{bmatrix}, \quad A_t^k = \begin{bmatrix} A_7^x \\ A_7^y \\ A_7^{\Theta} \end{bmatrix}, \text{ and}$$

$\xi^k = \left[d_{EOP}{}^T \; d\,t_k \right]^T$ as partitioned parameter vector, where the first part is associated with the six orientation parameters common to all image locations, and the second contains a single correction (dt_k) to the curve parameter at image point k. Finally, $e^k = \left[e_x{}^k \quad e_y{}^k \quad e_{\Theta}{}^k \right]^T$ is the error vector with zero expectation and usually non-diagonal dispersion matrix denoted hereafter by Σ_{e^k}. The reason why Σ_{e^k} is in general non-diagonal stems from the fact that in the proposed approach the tangent direction estimate is a function of the image coordinates of the point k. More precisely, let the tangent direction at image point k be estimated from the image coordinates of 2P+1 points around k such that $\tan(\Theta_k) = F(x_{k-P}, y_{k-P}, x_{k-P+1}, y_{k-P+1}, \cdots, x_k, y_k, \cdots x_{k+P-1}, y_{k+P-1}, x_{k+P}, y_{k+P})$, for some function F in 2*(2P+1) parameters. Then, the dispersion Σ_{e^k} is readily given by

$$\Sigma_{e^k} = H \Sigma_{4P+2} H^T \tag{8}$$

with

$$H_{3\times(4P+2)} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \cdots & 0 & 0 \\ \partial F/\partial x_{k-P} & \partial F/\partial y_{k-P} & \cdots & \partial F/\partial x_k & \partial F/\partial y_k & \cdots & \partial F/\partial x_{k+P} & \partial F/\partial y_{k+P} \end{bmatrix}$$

and with Σ_{4P+2} being the dispersion matrix associated with 4P+2 x-y coordinates of the 2P+1 points used to evaluate the tangent direction.

In system (7) three observation equations are written for each image space location. Once again, as such a 3-tuple requires the estimation of the associated space curve parameter, an overall contribution of two degrees of freedom results for any non-redundant triple of "observations". Hence, as far as the associated redundancy is concerned, we have succeeded, in increasing the number of degrees of freedom associated with a single image location from one to two. However, that has not been done without paying the price of having to estimate the tangent direction in the image space.

Finally, a set of N (≥ 3) image locations with uncorrelated triplets is formulated as a Gauss-Markov Model:

$$y = A\,\xi + e \tag{9}$$

where:

$$A_{(3N)\times(6+N)} = \begin{bmatrix} A^1_{EOP} & A^1_t & 0 & 0 & 0 \\ A^2_{EOP} & 0 & A^2_t & 0 & 0 \\ \vdots & 0 & 0 & \ddots & 0 \\ A^N_{EOP} & 0 & 0 & 0 & A^N_t \end{bmatrix},\; y_{(3N)} = \begin{bmatrix} y^1 \\ y^2 \\ \vdots \\ y^N \end{bmatrix},\; \xi_{(6+N)} = \begin{bmatrix} d_{EOP} \\ dt_1 \\ \vdots \\ dt_N \end{bmatrix}$$

and the error vector

$$e_{(3N)} = \begin{bmatrix} e^1 \\ e^2 \\ \vdots \\ e^N \end{bmatrix} \text{ with } e \sim \left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \Sigma = \begin{bmatrix} \Sigma_{e^1} & & & \\ & \Sigma_{e^2} & & \\ & & \ddots & \\ & & & \Sigma_{e^N} \end{bmatrix} \right) \tag{10}$$

In the model just stated we assume that the estimated tangent direction at location j is independent of the one estimated at k resulting in a block diagonal dispersion matrix Σ in (10). That assumption is logical as the tangent direction is estimated only from the image points in the vicinity of the examined location. The least-squares estimate for the parameter vector ξ in (9) is given by

$$\hat{\xi} = (A^T \Sigma^{-1} A)^{-1} A^T \Sigma^{-1} y \tag{11}$$

with the associated dispersion matrix

$$D(\hat{\xi}) = (A^T \Sigma^{-1} A)^{-1} \tag{12}$$

As the model (9) is linearized the estimation of the EOPs proceeds in an iterative fashion.

Before concluding, let us describe how the idea used to form the tangent equation can be further extended. Recall that the 2-D curvature is an intrinsic property of a 2-D parametric curve and is given by

$$k(t) = \frac{x'(t)y''(t) - y'(t)x''(t)}{(x'(t)^2 + y'(t)^2)^{3/2}} \tag{13}$$

As it contains up to second order derivatives, they can be obtained by a differentiation of the extended collinearity equations (3). Further, like the tangent vector the curvature can also be estimated independently from the image space. Employing the same approach used to derive the tangent equation an appropriate *curvature equation* can be devised.

3 EXTERIOR ORIENTATION FROM FREE-FORM CURVES

This section is concerned with the development of a space resection algorithm utilizing free-form curves as control information. More formally, let $P = \{\vec{P}_i\}$ with $1 \leq i \leq N_P$ be a sequence of N_P 3-D points forming a control curve. The point set P induces an ordered set $S = \{\vec{s}_i\}$ of N_{P-1} line segments with $\vec{s}_i$ connecting points $\vec{p}_i$ and $\vec{p}_{i+1}$. The proposed resection algorithm from free-form curves, being essentially an adaptation of the model presented in the last section to a sequence of line segments, is now stated. Given the data set P along with the initial values of the EOPs the following steps are carried out iteratively until the ultimate convergence. For each image space location (x_i, y_i), the direction vector ρ of the ray from the perspective center through the image point (x_i, y_i) is evaluated. Then, a parametric equation $\Xi(l)$ of that ray in object space is formed. The next step is to locate the point P_l on $\Xi(l)$ that is the closest to the 3-D polyline $S = \{\vec{s}_i\}$. This point is always coupled with a point P_t^j on one of the line segments $\{\vec{s}_j\}$ to which it is the closest. This procedure also establishes an association between every image space location and its corresponding segment in the object space. That association is in fact temporary as it applies only to the current iteration. Nonetheless, it enables us to utilize the general model developed for parametric curves and to apply it locally using the parametric representation of the determined line segment $\{\vec{s}_j\}$. In particular, the parameter associated with P_t^j along with the direction vector of the segment $\{\vec{s}_j\}$ are used to form a set of equations (7). Finally, based on the entire set of image space observations system (9) is formed to iteratively refine the EOPs.

Before concluding let us bring up one crucial point that was deliberately left out in the previous discussion. The question is how many image locations should be chosen to yield a sufficient and robust solution? Further recall that the more image locations are used, the more time consuming the EO estimation task becomes. Essentially, that is since the closest point minimization operator must be reapplied for each newly introduced image space location. Also, having more image space 'measurements' increases the number of observation equations along with the dimension of the associated parameter space. In principle, however, only three non-redundant coordinates and tangents measurements are required for the recovery task. Hence, from the efficiency point of view we would be interested to reduce the number of such measurements to a minimum as long as robust solution for EOPs can still be achieved. Unfortunately, reducing the number of image space measurements reveals a new, somehow unexpected problem. This problem along with the proposed strategy to overcome it are presented in the next section.

4 HIERARCHICAL DETERMINATION OF EXTERIOR ORIENTATION

In some cases the iterative algorithm just outlined suffers from the following problem. While it usually converges monotonically to local minimum (within the six dimensional parameter space), it may or may not converge into the desired global minimum. A convergence to local minimum is more likely when relatively complex curves that are not sampled densely enough are involved. A promising direction to reduce computational costs and, at the same time, to prevent the solution scheme from converging into local minimum is to introduce a coarse-to-fine strategy. The motivation behind this strategy is two-fold. First, the more smoothed the curve is the fewer number of points are necessary to reliably capture its shape. Consequently, having fewer points would reduce the number of observation equations along with the number of the parameters in the associated adjustment scheme. Second, as has been also experienced by other researchers [1], smooth features are more likely to yield a close to convex minimization function. Hence, the convergence into local rather than global minimum, in this case, is less likely.

In the proposed approach the scale-space representation is obtained by convolving the curves in both domains with the Gaussian kernel. Our one and only goal here is to show that even that trivial and not necessarily invariant simplification scheme provides encouraging results for typical nearly vertical imaging conditions and, therefore, gives an effective solution for the previously described problem. Applying a discretized mask of one-dimensional Gaussian kernel independently to each component of the 3-D sequence as well as applying its scaled version to the associated 2-D sequence generates a set of corresponding signals at varying levels of detail. Given this multi-scale representation the hierarchical recovery proceeds iteratively by determining the EOPs from corresponding features at coarse level of the scale space and using their estimated values as approximations at the next finest level. Obviously, as our "smoothing" scheme is **not** projectively invariant, the estimated orientation parameters will vary as we move throughout the scale-space. Fortunately, however, the trend of the change in the EOPs, as has been experienced in a number of near vertical imagery scenarios, is consistent and robustly headed towards the correct recovery.

5 EXPERIMENTS AND RESULTS

This section aims at testing the feasibility and the performance of the proposed algorithms for image orientation through a series of experiments using both synthetic as well as real data.

5.1 Experiments with Analytical Curves

To begin, it is important to establish a reference configuration with respect to which all the upcoming scenarios will be analyzed. The most natural choice is to simulate a photogrammetric resection from three symmetrically distributed and error free control points (Figure 1(a)). The image coordinates corresponding to the points A, B, and C (bottom plot of Figure 1(a)) are generated by projecting the 3-D data into the image plane (using a perfectly vertical projection), followed by a perturbation of the projected image data with white noise with $\pm 10\mu m$ standard deviation. From the generated 3-D to 2-D correspondences the exterior orientation parameters are estimated. This point-based configuration is compared with the line-based (also minimum) configuration plotted in Figure 1(b). Here, three straight lines passing through the points A, B, and C are used as error-free control. As in the point-based configuration exactly the same projected 2-D coordinates are used to set up the extended collinearity equations (2). Further, at each of the projected points the computed (from the coordinates of the corresponding image points) tangent direction along with its dispersion are used to form the tangent equation (6). The estimation results are summarized in Table 1.

	X_0 (km)	Y_0 (km)	Z_0 (km)	ω (deg)	φ (deg)	κ (deg)
Approximate EOPs	1.0	1.0	1.0	20	20	20
Estimated EOPs [configuration 1(a)]	0.00026 (±2.6E-4)	0.00002 (±2.6E-4)	1.51993 (±7.6E-5)	-0.00108 (±0.0076)	0.0080 (±0.0076)	0.0031 (±0.0029)
Estimated EOPs [configuration 1(b)]	0.00026 (±2.8E-4)	0.00002 (±2.8E-4)	1.51993 (±1.1E-4)	-0.00108 (±0.0088)	0.0080 (±0.0088)	0.0031 (±0.0023)
Estimated EOPs [configuration 1(c)]	-0.00011 (±2.4E-4)	-0.000064 (±2.4E-4)	1.52001 (±6.8E-5)	0.00197 (±0.0078)	-0.00386 (±0.0078)	0.0031 (±0.0021)

Table 1: Comparison between point-based and line-based space resection.

From the above table two important observations can be made: First, both configurations are robust with respect to the initial approximations of the orientation parameters. Second, though the point-based approach yields a slightly better precision than the line-based counterpart this difference is not significant, and for practical purposes we can safely conclude that, using the above three straight lines as control, gives satisfying and robust results. Also recall that in line photogrammetry there is basically no limitation on the number of observations made along a straight line, essentially as no corresponding control points have to be associated with such image space measurements. For example, taking five points along each straight line (Figure 1(c)) and employing the same approximations generates more accurate results. The last observation is not surprising though. Note that although any additional point (beyond the first two) along a straight line does not carry additional information required to reduce the overall deficiency of the problem, it does increase the robustness of the indirectly fitted image line coefficients. That in turn increases the overall precision of the estimated orientation parameters.

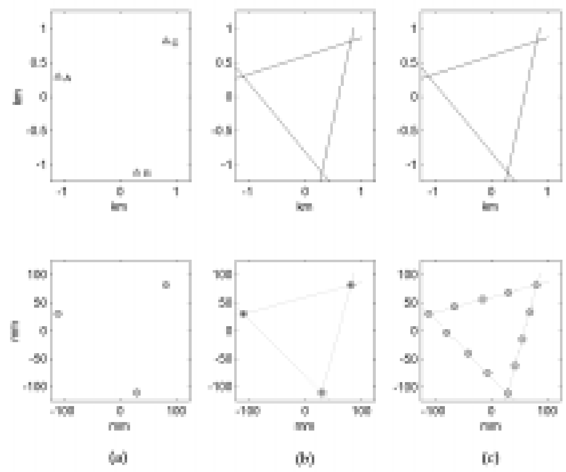

Figure 1: Basic configurations for point-based (a) and line-based (b), (c) space resection.

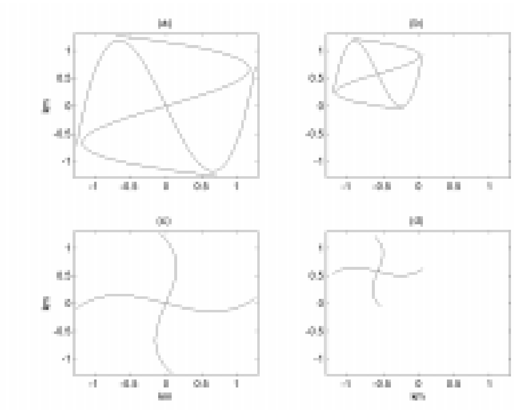

Figure 2: Additional control configurations.

Before arriving at the general principle affecting the precision of the estimated orientation parameters, let us simulate a few additional control configurations depicted in Figure 2. As before, a perfectly vertical projection is applied. Then, five equally distributed image coordinate measurements along the projection of each control feature are used. The quality of a configuration as it pertains to the estimation of the orientation parameters is measured by comparing the determinant of the covariance matrix of each configuration with that of configuration 1(b). The results of that comparison are summarized in Table 2.

	Conf. 2(a)	Conf. 2(b)	Conf. 2(c)	Conf. 2(d)
det(X)/det(1(b))	0.024	14.257	4.487	1157.5

Table 2: Precision comparison under different spatial configurations.

Table 2 clearly shows that the precision of the recovered orientation parameters primarily depends on both the spatial and the directional distributions of the involved control features. More image measurements (coordinates and tangents) associated with a diversity of well distributed tangent directions yield more precisely estimated parameters.

5.2 Experiments with Free-Form Curves

Our data set is comprised of a small-scale (1:40,000) aerial image covering the area of Zihron Yaakov (ISRAEL) along with a number of major and secondary roads in that area represented as 3-D polylines. The 2-D point sequences corresponding to the 3-D roads have been digitized from the image with an estimated accuracy of ± 1.0 pixel ($\pm 30 \mu m$).

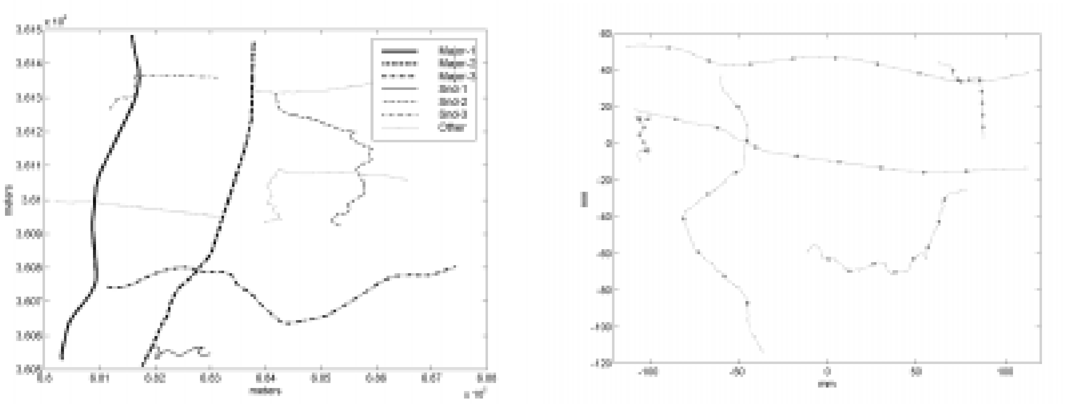

Figure 3: Road network (left) and eight equally spaced points introduced along each delineated road in the image (right).

The EOPs are estimated from the configuration given in Figure 3 (right) with eight equally spaced points introduced along each delineated image space feature. Introducing a pair of extended collinearity equations (2) for each image location yields the following results for the orientation parameters.

X0 [m]	Y0 [m]	Z0 [m]	ω [deg]	φ [deg]	κ [deg]	# of Iterations
3097.5 ± 2.4	9880.7 ± 2.6	6516.7 ± 0.48	1.687 ± 0.020	0.171 ± 0.018	-86.32 ± 0.005	**8**

Table 3: Exterior orientation parameters for the configuration shown in Figure 3.

The obtained results confirm the feasibility of the proposed method in precisely recovering the EOPs. To elaborate more, especially on the results of the proposed hierarchical approach an interested reader is referred to Zalmanson [10].

6 CONCLUSIONS

A novel method for determining the exterior orientation parameters from a correspondence between a set of space curves represented parametrically or naturally and its partial 2-D projection is developed and successfully tested. The beauty of the proposed method is that the underlying principle is not tied exclusively with the perspective transformation governing the image formation process. In fact, it is general enough to be employed for any other linear or non-linear functional relationship between measurable quantities in two arbitrary domains. Furthermore, similar ideas can be used to extend the proposed model or any others governed by different geometric transformation from linear features to surfaces. The proposed method can be seen as a building block for a variety of applications in diverse fields. For example, it can be employed in applications integrating information coming from different sources (map conflation, change detection, medical image analysis, surface matching, etc.). It can aid a variety of model based object recognition tasks by employing the implicitly available curve and surface matching techniques. Finally, it can serve as an indispensable tool in various military applications for target location and recognition.

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