Design of hoppers using spreadsheet

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Abstract

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This paper presents a spreadsheet add-in for the design of mass flow conical and wedge hoppers. The Jenike's hopper design charts for mass flow were curve fitted. The relationships obtained were used together with other relevant expressions to develop an add-in tool for the determination of the pertinent hopper design parameters (exit size, mass flow rate, semi-included angle, flow factor, and critical applied stress) in the Microsoft Excel environment. The add-in was tested with experimental data, and results obtained were in agreement with those obtained in the literature.

Keywords: hopper design; curve fitting; Jenike's hopper design charts; MS Excel add-in

The need for effective handling of large quantity and variety of food materials produced industrially in powdered form necessitated the introduction of hoppers for material handling. The complexity surrounding hopper design is mostly due to the material flow ability, which depends principally on the mechanical behaviour of the stored material, semi-included angle, flow factor and the critical applied stress.

Powder properties significantly affect powder behaviour during storage, handling and processing (Peleg 1978; Rhodes 1990; Knowlton 1994; Stasiak, Molenda 2004). Therefore, powder flow property measurement is very important in handling and processing operations, such as flow from hoppers, transportation, mixing, compression, and packaging. Jenike (1994) identified the properties that affect material flow in storage vessels and their measurement techniques. He also applied measured property data to two-dimensional stress analysis to develop charts and a mathematical model for determining the minimum hopper angle and hopper opening exit size for material flow from conical and wedge-shaped hoppers (Fig. 1). The geometrical characteristics of the hopper affect the rate of flow of the material out of the hopper (Bidgwater, Scott 1983; Griffith 1991; Holdich 2002; Fitzpatrick et al. 2004). Jenike and Johanson (1969) explained the various ways that grains might move during emptying, and the flow patterns that are developed.

The Jenike's hopper design methodology involves: the experimental determination of the wall friction angle and effective angle of internal friction; the determination of the semi-included angle and flow factor (from the already known wall friction angle), and the effective angle of internal friction with the aid of the Jenike's charts, which correlates the semi-included angle (θ), flow factor (*ff*), wall friction angle (δ_w) and the effective angle of internal friction (δ) as illustrated in Fig. 2, where the light dashed lines illustrate how to obtain the pertinent parameters (George 2001); the determination of the critical applied stress from a plot of the mass flow function (MFF) and inverse of the flow factor (*ff*), MFF = 1/*ff* (Fig. 3); and the calculation of the minimum hopper angle and hopper exit size (George 2001; Fitzpatrick et al. 2004). The main shortcoming of the Jenike's hopper design procedure is that parameter determination with the aid of the Jenike's charts and the graphs of MFF = 1/*ff* is always error prone, tedious and time consuming.

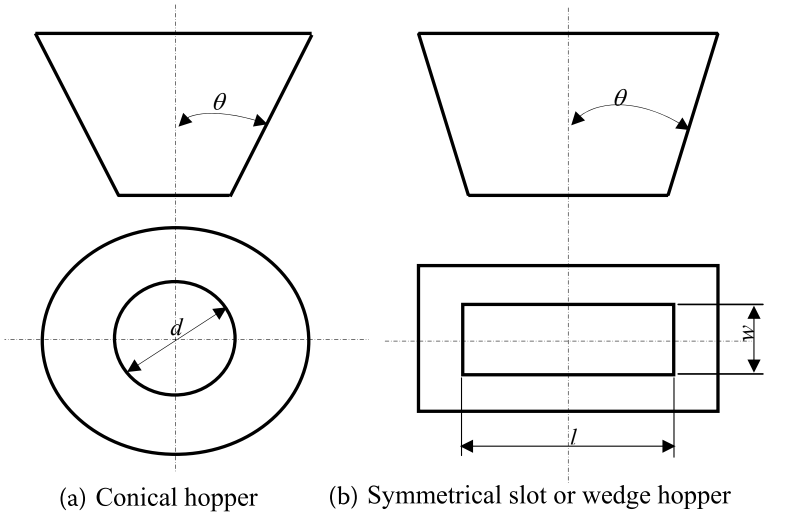

Fig. 1. Conical and wedge hoppers showing the semi-included angle (θ), diameter (*d*), slot width (*w*), and length (*l*)

The trend in contemporary engineering practice is the application of computer technology to design processes that have well established procedure such as the Jenike's hopper design procedure. Spreadsheet solution of such design problems is relatively simple and straightforward, especially when the solution algorithm can be appended to the spreadsheet. The MS Excel environment allows the appendage of computer programmes written in Visual Basic for Applications (VBA), called MS Excel add-in tools (Liengme 2000).

The purpose of this paper is, therefore, to design the conical and wedge hoppers more efficiently and automatically by replacing the Jenike's charts with algebraic polynomials, and applying relevant relationships, including the Lagrange interpolation scheme, to determine the relevant design parameters using the MS Excel spreadsheet with an add-in.

MATERIAL AND METHODS

The MS Excel curve fitting tool is a popular tool for automatic curve fitting by computer (Liengme 2000; Mustafa 2000). It was used to curve fit the Jenike's charts. The correlations for the semi-included angle (θ_i) and flow factor (ff_i) as functions of the wall friction angle (δ_w) at specified effective angle of internal friction (δ_i) are listed in Tables 1 and 2, respectively.

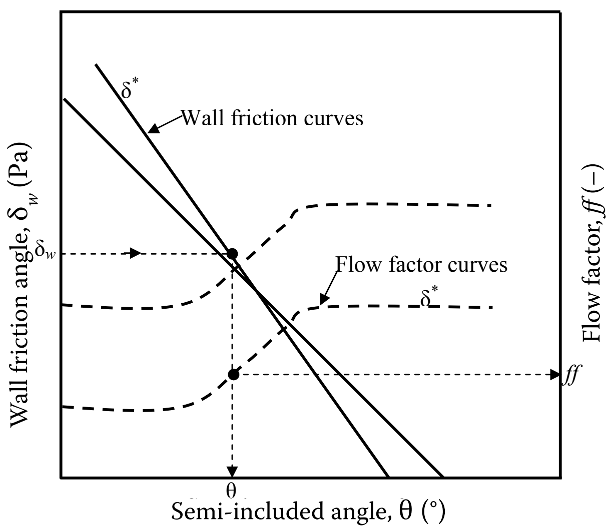

Fig. 2. An illustration of the Jenike's hopper design chart for the determination of the wall friction angle (δ_w) and flow factor (*ff*)

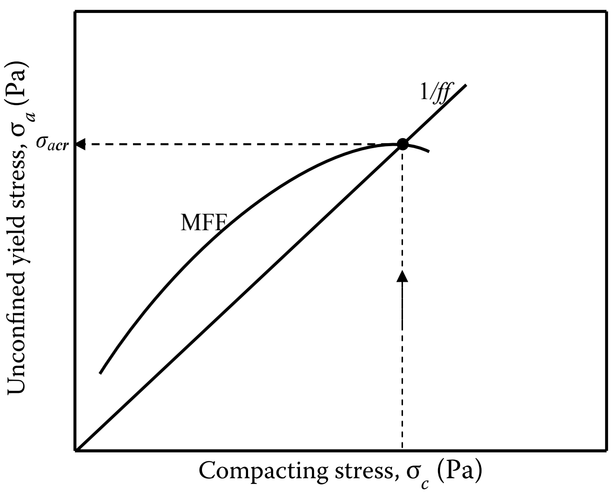

Fig. 3. Graphical solution for the critical applied stress (σ_{acr}), which depends on the flow factor (*ff*) and mass flow function (MFF)

Table 1. Semi-included angle as function of wall friction angle

Hopper type	i	Correlation (θ_i [°])	at δ_i (°)	R^2
Conical	0	$-0.0331\delta_w^2 - 0.6781\delta_w + 52.663$	30	0.9981
	1	$-0.0122\delta_w^2 - 0.9024\delta_w + 47.814$	40	0.9990
	2	$-0.0027\delta_w^2 - 1.0962\delta_w + 46.10$	50	0.9975
	3	$-0.0033\delta_w^2 - 0.9695\delta_w + 43.343$	60	0.9996
Wedge	0	$0.0023\delta_w^2 - 1.5646\delta_w + 64.68$	30	0.9997
	1	$0.0005\delta_w^2 - 1.4802\delta_w + 63.414$	40	0.9996
	2	$0.0004\delta_w^2 - 1.3416\delta_w + 59.643$	50	0.9998
	3	$-0.00003\delta_w^2 - 1.2155\delta_w + 56.767$	60	0.9997

The wall friction angle (δ_w) and effective angle of internal friction (δ) are obtained experimentally. To obtain the semi-included angle (θ) and flow factor (*ff*) we substitute δ_w into the correlations in Tables 1 and 2, and interpolate for θ and *ff* with δ serving as the interpolation point. Thus, using the Langrage polynomial interpolation scheme (Chapra, Canale 2002; Oko 2008), we get the following numerical schemes for the semi-included angle and flow factor as functions of the wall friction angle and effective angle of internal friction:

$$\theta(\delta_w,\delta) = \sum_{i=0}^{3} \theta_i \left(\prod_{\substack{j=0 \\ j\neq i}}^{3} \frac{\delta-\delta_j}{\delta_i-\delta_j} \right)_i \quad (1)$$

and

$$ff(\delta_w,\delta) = \sum_{i=0}^{3} ff_i \left(\prod_{\substack{j=0 \\ j\neq i}}^{3} \frac{\delta-\delta_j}{\delta_i-\delta_j} \right)_i \quad (2)$$

Experimentally, one obtains the unconfined yield stresses or applied stresses (σ_{ai}) that correspond to some compacting stresses (σ_{ci}); where $i = 0, 1, \ldots, n-1$ are the numbers of experiments conducted. The critical applied stress, which corresponds to the unconfined yield stress at the point of the intersection of the MFF and 1/*ff* curves (Fig. 3), is determined by the following expression:

$$\Delta\sigma_{ai} = \sigma_{ai} - \frac{\sigma_{ci}}{ff}\,;\ i = 0, 1,\ldots,n-1 \quad (3)$$

where:

$\Delta\sigma_{ai}$ – difference between MFF and 1/*ff* at the data points; the point at which $\Delta\sigma_a = 0$ yields the desired critical applied stress, $\sigma_c = \sigma_{ci}$

By using the Lagrange polynomial interpolation as follows:

$$\sigma_c(\Delta\sigma_a = 0) = \sum_{i=0}^{n-1} \sigma_{ci} \left(\prod_{\substack{j=0 \\ j\neq i}}^{n-1} \frac{-\Delta\sigma_{aj}}{(\Delta\sigma_{ai} - \Delta\sigma_{aj})} \right)_i \quad (4)$$

Table 2. Flow factor as function of wall friction angle

Hopper type	i	Correlation (ff_i [–])	at δ_i (°)	R^2
Conical	0	$-0.00031\delta_w^2 - 0.0065\delta_w + 2.0707$	30	0.9960
	1	$-0.0001\delta_w^2 - 0.005\delta_w + 1.6251$	40	0.9984
	2	$-0.0004\delta_w^2 - 0.0065\delta_w + 1.4573$	50	0.9975
	3	$-0.00003\delta_w^2 - 0.0056\delta_w + 1.3474$	60	0.9989
Wedge	0	$-0.00009\delta_w^2 - 0.0059\delta_w + 2.0648$	30	0.9984
	1	$0.002\delta_w^2 - 0.0112\delta_w + 1.6083$	40	0.9871
	2	$-0.000002\delta_w^3 + 0.0002\delta_w^2 - 0.009\delta_w + 1.3027$	50	0.9878
	3	$-0.000003\delta_w^3 + 0.0003\delta_w^2 - 0.0104\delta_w + 1.2058$	60	0.9938

Table 3. Experimental data from George (2001), (Pa)

S/No.	Measurement	Value	
1	Shear stress internal friction	σ_{ci}	σ_{ai}
	$i = 0$	2,400	970
	$i = 1$	2,000	910
	$i = 2$	1,600	850
	$i = 3$	1,300	780
2	Wall friction (δ_w)	Normal shearstress (Δv)	Shear stress (Δu)
	$i = 0$	2,000	689
	$i = 1$	3,000	1,030
3	Effective angle of internal friction (δ)	Rise (Δy)	Run (Δx)
		1,000	1,730

we obtain the critical applied stress (σ_{acr}):

$$\sigma_{acr} = \frac{\sigma_c(\Delta\sigma_a = 0)}{ff} \quad (5)$$

The outlet dimension (diameter [d] for conical hopper and width [w] for wedge hopper) depends upon the semi-included angle (θ [°]), material bulk density (ρ [kg/m^3]), critical applied stress (σ_{acr} [Pa]), and acceleration due to gravity (g [m/s^2]): $d = f_d(\theta, \rho, \sigma_{acr}, g)$ and $w = f_w(\theta, \rho, \sigma_{acr}, g)$. Once the outlet width of the wedge hopper is determined, its length (l), which is always greater than three times the outlet width ($l > 3w$), is readily chosen. The exact expressions for the outlet diameter and width of the hoppers can be found in Jenike (1994), George (2001) and Fitzpatrick et al. (2004). The mass flow rate ($\dot{m}$) also depends upon the semi-included angle, bulk density and acceleration due to gravity, and is given by the Johanson equation (Johanson 1966; Johanson, Kleysteuber 1969; Hsiau et al. 2001).

The hopper design is carried out using the following algorithm:

start

input data

i. experimentally determined wall friction angle and effective angle of internal friction;

ii. experimentally determined compacting stresses and corresponding unconfined yield stresses or applied stresses;

iii. bulk density, acceleration due to gravity;

compute (using the relevant relationships for the design parameters: θ, ff, σ_{acr}, d or w and l);

output data (output the design parameters): semi-included angle; flow factor; critical applied stress; discharge hopper diameter, or discharge hopper width and length; and exit mass flow rate;

stop.

Following the computational algorithm presented, a computer program was developed in MS Excel VBA environment as an add-in.

RESULTS AND DISCUSSION

The MS Excel add-in was tested with the input data from measurements provided by George (2001) for the conical hopper in respect of the compacting and applied shear stresses, wall friction, effective angle of internal friction and at a bulk density of the powdered material of ρ = 1,300 (kg/m^3) (Table 3). The data in Table 3 for the compacting and applied stresses and the correlations in Table 2

Table 4. Conical hopper design parameters

S/No.	Quantity	Units	Value	
			Excel add-in	George (2001)
1	semi-included angle	°	27.83	28.000
2	flow Factor	–	1.83529	1.840
3	critical applied stress (σ_{acr})	Pa	836.9751	830.000
4	discharge diameter of hopper	m	0.16175	0.161
5	exit mass flow rate from hopper	kg/s	23.85734	23.86

Table 5. Wedge hopper design parameters

S/No.	Quantity	Units	Excel add-in value
1	semi-included angle	°	35.783
2	flow Factor	–	1.920
3	critical applied stress (σ_{acr})	Pa	859.577
4	discharge width of hopper	m	0.0808
5	discharge length of hopper	m	0.242
6	exit mass flow rate from hopper	kg/s	18.886

are used to obtain the critical applied stress. The data for the wall friction in Table 3 are curve fitted, and the slope of the resulting curve is used to determine the angle of wall friction (δ_w) where $\tan\delta_w = dv/du \approx \Delta v/\Delta u$; and the data for the effective angle of internal friction are similarly used to determine the effective angle of internal friction (δ) where $\tan\delta = dy/dx \approx \Delta y/\Delta x$.

The design parameters for the conical hopper that the MS Excel add-in produced are tabulated in Table 4 along with those obtained by George (2001), using the same input data. It is seen that the largest percentage relative error of 0.84% occurred in the computed value for the critical applied stress (σ_{acr}). This, granted that the results by George (2001) are error free, may be attributed to roundoff errors in the computational process. But this error is quite acceptable for hopper applications. Table 5 shows the design parameters for the wedge hopper based on the same input data used for the conical hopper. Here, direct comparison cannot be made as George (2001) did not consider the wedge hopper. However, one observes that all the parameters increased except the equivalent discharge diameter (d_e = 0.121 m) and the mass flow rate of the wedge hopper, which are less than those of the conical hopper. Therefore, the results obtained confirm that the replacement of the Jenike's chart by algebraic correlations aids in computerizing the hopper design process, which is the trend in contemporary engineering practice.

CONCLUSION

This work has eliminated the manual application of the Jenike's hopper design charts for mass flow as well as the graphical determination of the critical applied stress. Algebraic polynomial correlations have been generated to replace the Jenike's hopper design charts, and a numerical scheme is used to determine the critical applied stress. A spreadsheet add-in tool in MS Excel environment has been developed for the automatic design of the conical and wedge hoppers based on the Jenike's hopper design scheme. Apart from the stand-alone application of this spreadsheet add-in tool, it can also be integrated into a larger plant design software for improved productivity. Of course, the add-in is also a veritable tool for the effective teaching of engineering students how to design hoppers.

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