Longitudinal oscillations of the sugar beet root crop body at vibrational digging up from soil

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ABSTRACT: The longitudinal vibrations theory of a continuous elastic body with one fixed extremity is designed. The Ostrogradskii–Hamilton principle of a stationary operation is applied. By Ritz's method the Ritz's equation of frequencies for viewed oscillatory process is obtained. In particular analytical expressions for definition of the first and second fundamental frequencies of body oscillations and forced oscillations amplitude of its any cross-section are obtained.

Keywords: longitudinal oscillations; forced oscillations; root crop; elastic body; Ostrogradskii-Hamilton principle; vibrational digging up

In given article on basis, introduced in Babakov (1968) common vibration theory of direct rods of a variable cross-section, oscillations of a continuous elastic body with one fixed extremity are examined. An example of such body can be the sugar beet root crop located in soil and, that is very essential, the soil enclosing a root crop also is an elastic medium.

Let's consider a case when vibrating motions will be affixed to the specified body in longitudinal-vertical plane (to this position the example will answer when to a root crop which is in not destroyed soil, force from two sides from a vibration digging out end-effector will be affixed at its extraction from soil).

Objectives

To develop substantive provisions of the theory of a of root crops vibration digging up from soil.

Procedure of research

At execution of introduced research, the basic variational principles of mechanics and a vibration theory of the rods having one fixed extremity are used.

OUTCOMES OF RESEARCH

To research of holonomic systems oscillations with the infinite number of freedoms degree apply a principle of Ostrogradskii-Hamilton stationary operation (1). In the theory longitudinal, torsional and transverse oscillations of Ostrogradskii-Hamilton direct rods functionals are applied which in the most common shape have such aspect (1):

$$S = \int_{t_1}^{t_2}\int_{0}^{l} L\left(t, x, y, \frac{\partial y}{\partial t}, \frac{\partial y}{\partial x}, \frac{\partial^2 y}{\partial t^2}, \frac{\partial^2 y}{\partial t \partial x}, \frac{\partial^2 y}{\partial x^2}\right) dx\, dt \quad (1)$$

where: $L = (T - Ï)$ – Lagrangian,
T – system kinetic energy,
$Ï$ – system potential energy.

The principle of Ostrogradskii-Hamilton for research of the continuous elastic body longitudinal oscillations occuring under vertical disturbing force operation which varies under such sort harmonic law

$$Q_{zb.}(t) = H \sin \omega t \quad (2)$$

where: H – forced oscillations amplitude,
ω – forced oscillations frequency.

As it is visible from the composed model (Fig. 1), continuous elastic body – the root crop having the cone-shaped shape (which apex angle is equal 2γ, and the upper is little bit above a surface soil level), is modelled as a variable cross-section rod with fixed lower extremity (point O). In a barycentre which is marked out by a point C, force $\bar{G}$ – a body weight is affixed. Its general length – h. Body (root crop) link with soil is defined by a soil common response $\bar{R}_x$ which is located along an axis x.

The mentioned above disturbing force $\bar{Q}_{zb.}$ is affixed to a body at once from its two sides, therefore

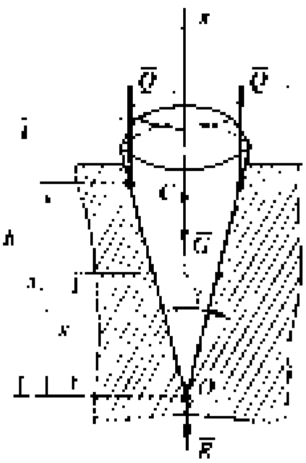

Fig. 1. The model of the forces operating on a root crop at the moment of capture by a vibration digging out end-effector

on the scheme it presented by two components $\overline{Q}_{zb.1}$ and $\overline{Q}_{zb.\,2}$. These forces are affixed apart x_1 from coordinates origin (point *O*) and they give rise to oscillations of a body (root crop) in a longitudinal – vertical plane which destroy its links with soil and create for last conditions of extraction.

Let's make up Ostrogradskii-Hamilton functional *S* for vibration process which is considered. With this purpose we shall introduce necessary labels:
$F(x)$ – body cross-sectional area in any point which is apart x from the lower extremity (m^2); E – Young's modulus for a body material (N/m^2); $y\ (x,\ t)$ – any body cross-section longitudinal bias in an instant t (m); $Q\ (x,\ t)$ – longitudinal exterior loading intensity, directional along a body axis (N/m); $\mu\ (x)$ – body running mass (kg/m).

According to (1) Ostrogradskii-Hamilton functional for longitudinal direct rods oscillations looks like:

$$S = \frac{1}{2}\int_{t_1}^{t_2}\int_{0}^{h}\left[\mu(x)\left(\frac{\partial y}{\partial t}\right)^2 - EF(x)\left(\frac{\partial y}{\partial x}\right)^2 + Q(x,t)\,y\right]dx\,dt \quad (3)$$

Let's find magnitude expressions which are included in a functional (3). Taking into account that the body has the shape of a cone, we discover that its cross-section area $F(x)$ in a point being on the arbitrary distance x from a point *0* will be equal

$$F(x) = \pi x^2\ \mathrm{tg}^2\,\gamma \quad (4)$$

It is obvious that body running mass can be defined with the help of such expression

$$\mu(x) = \rho \times \pi x^2\ \mathrm{tg}^2\,\gamma \quad (5)$$

where: ρ – body density (kg/m^3).

As the magnitude $Q\ (x,\ t)$ which is included in a functional (3), is distributed load intensity which is measured in (N/m), the disturbing force should have dimensionality of loading intensity. With the help of the first order impulsive function $\sigma_1(x)$ (1) it is possible to determine concentrated load intensity and thus to include concentrated forces as a component of the loading distributed on length.

So, if $Q_{zb.}(t)$ – the concentrated disturbing force which is affixed in a point x_1 and is measured in Newton function

$$Q_{zb.}\,(x,\,t) = Q_{zb.}\,(t)\ \sigma_1(x - x_1) \quad (6)$$

has dimensionality (N/m) and expresses concentrated load intensity in a point x_1.

Function $\sigma_i(x - x_1)$ is equated to null for everything x, except for $x = x_1$, where it becomes infinite.

Let the disturbing force operating under the law (2) is affixed on a body apart x_1 from a reference point (a point *0* on Fig. 1). Then according to (6) it is possible to write

$$Q_{zb.}\,(x,\,t) = H\sin\omega t\sigma_1(x - x_1) \quad (7)$$

As the continuous elastic body is interlinked to soil which also is an elastic medium at an operation on it of a disturbing force (2) there is a soil reaction force to body migration at its oscillations. This force also influences process of body natural oscillations in soil is especial in the beginning of oscillatory process while its links with a soil are not dislocated yet.

It is obvious that resistance soil force (for all body) is a distributed load on a contacting area of a body with soil and therefore its intensity is definable as soil reaction force to migration of body length unity.

Let c – coefficient of soil elastic deformation referred to a contacting area which is measured century (N/m^2). We shall consider that soil enclosing a body under a disturbing force operation $H\sin\omega t$ realizes forced oscillations behind the same harmonic law with amplitude which is defined by soil elastic properties. Then soil intensity $P\ (x,\ t)$ reaction to body migration to a point x will be equated

$$P\ (x,\,t) = 2\,\pi\,c\,x\ \mathrm{tg}\,\gamma\,\sin\omega t\ (N/\mathrm{m}) \quad (8)$$

Thus we shall have such relation for a longitudinal exterior loading:

$$Q\ (x,\,t) = Q_{zb.}\,(x,\,t) - P\ (x,\,t)$$

Taking into account expressions (4), (5), (7) and (8) the Ostrogradskii-Hamilton functional (3) will get such aspect:

$$S = \frac{1}{2}\int_{t_1}^{t_2}\int_{0}^{h}\Big\{\rho\pi\,x^2\,\mathrm{tg}^2\,\gamma\left(\frac{\partial y}{\partial t}\right)^2 - E\,\pi\,x^2\,\mathrm{tg}^2\,\gamma\left(\frac{\partial y}{\partial x}\right)^2 + \\ + [H\sin\omega t\ \sigma_1(x - x_1) - 2\pi\,c\,x\ \mathrm{tg}\,\gamma\,\sin\omega t] \times \\ \times y\,(x,\,t)\Big\}\,dx\,dt \quad (9)$$

For a body in soil natural shapes and frequency in pich determination the Ritz's method (1) is usable. According to this method we shall search for body harmonic longitudinal oscillations in such aspect:

$$y(x, t) = \varphi(x) \sin(pt + \alpha) \qquad (10)$$

where: $\varphi(x)$ – natural shape of principal oscillations,
p – fundamental frequency of principal oscillations.

As natural shapes and fundamental frequencies are interlinked to system free oscillations, it is necessary to select that part which features of system free oscillations in a functional (9). It is obvious that it will be a functional of such aspect

$$S_1 = \frac{1}{2}\int_{t_1}^{t_2}\int_0^h\left[\rho\pi\, x^2\, \mathrm{tg}^2\, \gamma \left(\frac{\partial y}{\partial t}\right)^2 - E\,\pi\, x^2\, \mathrm{tg}^2\, \gamma \left(\frac{\partial y}{\partial x}\right)^2\right] dx\, dt \qquad (11)$$

Substituting expression (10) in a functional (11) we shall receive:

$$S_1 = \frac{1}{2}\int_{t_1}^{t_2}\int_0^h\Big\{\rho\pi\, x^2\, \mathrm{tg}^2\, \gamma\, \varphi^2(x) p^2 \cos^2(pt + \alpha) - E\,\pi\, x^2\, \mathrm{tg}^2\, \gamma\, [\varphi'(x)]^2 \sin^2(pt + \alpha)\Big\} dx\, dt \qquad (12)$$

Integrating expression (12) on t in limits of one period

$T = \dfrac{2\pi}{p}$, we shall have:

$$S_2 = \frac{\pi}{2p}\int_0^h\Big\{\rho\pi\, x^2\, \mathrm{tg}^2\, \gamma\, \varphi^2(x) p^2 - E\,\pi\, x^2\, \mathrm{tg}^2\, \gamma\, [\varphi'(x)]^2\Big\} dx \qquad (13)$$

It agrees the Ritz's method a functional (13) value are considered on plurality of functions linear combinations, that is the expressions which are looking so:

$$\varphi(x) = \sum_{i=1}^{n} \alpha_i \psi_i(x) \qquad (14)$$

where: α_i – parameters which variations we obtain the necessary class of admissible functions,
$\psi_i(x)$ – basis functions which are specially selected and are known functions, satisfying to problem geometrical boundary conditions.

Thus, substituting expression (14) in expression (13), after the relevant transformations we shall receive:

$$S_2 = \frac{\pi}{2p}\int_0^h\Big[\rho\pi\, x^2\, \mathrm{tg}^2\, \gamma\, p^2 \sum_{i,k=1}^{n} \psi_i(x)\psi_k(x)\alpha_i\alpha_k - E\,\pi\, x^2\, \mathrm{tg}^2\, \gamma \sum_{i,k=1}^{n} \psi'_i(x)\, \psi'_k(x)\alpha_i\alpha_k\Big] dx \qquad (15)$$

Let's introduce such labels for further:

$$\int_0^h \rho\pi\, x^2\, \mathrm{tg}^2\, \gamma\, \psi_i(x)\psi_k(x)\, dx = T_{ik}$$

$$\int_0^h E\pi\, x^2\, \mathrm{tg}^2\, \gamma\, \psi'_i(x)\psi'_k(x)\, dx = U_{ik} \qquad (16)$$

$(i, k = 1, 2, ..., n)$

Substituting (16) in (15), we shall receive a functional as function from parameters $\alpha_1, \alpha_2, ..., \alpha_n$:

$$S_2(\alpha_1, \alpha_2, ..., \alpha_n) = \frac{\pi}{2p} p^2 \sum_{i,k=1}^{n} T_{ik}\alpha_i\alpha_k - \frac{\pi}{2p}\sum_{i,k=1}^{n} U_{ik}\alpha_i\alpha_k \qquad (17)$$

We examine on an extremum the functional (17). For this purpose differentiate with respect expression (17) to α_1, $(i = 1, 2, ..., n)$ also we shall equate to null obtained partial derivatives. In outcome we shall receive the linear homogeneous system equations to unknowns $(\alpha_1, \alpha_2, ..., \alpha_n)$ from which, in turn, we discover the equation of Ritz's frequencies for longitudinal oscillations of a continuous elastic body, fixed in soil:

$$\begin{vmatrix} U_{11} - p^2T_{11} & U_{12} - p^2T_{12} & \dots & U_{1n} - p^2T_{1n} \\ U_{21} - p^2T_{21} & U_{22} - p^2T_{22} & \dots & U_{2n} - p^2T_{2n} \\ \dots & \dots & \dots & \dots \\ U_{n1} - p^2T_{n1} & U_{n2} - p^2T_{n2} & \dots & U_{nn} - p^2T_{nn} \end{vmatrix} = 0 \qquad (18)$$

In practice, as a rule, only more often first and second the lowest frequencies are defined which influence most significantly a process which is considered.

Therefore definable are the first and second frequencies of viewed body natural oscillations.

For definition of the first and second frequency the equation (18) will get such aspect:

$$\begin{vmatrix} U_{11} - p^2T_{11} & U_{12} - p^2T_{12} \\ U_{21} - p^2T_{21} & U_{22} - p^2T_{22} \end{vmatrix} = 0 \qquad (19)$$

As a result of the given equation solution we obtain expressions for a determination of the first (basic) frequency value:

$$p_1 = \frac{0.662422}{h}\sqrt{\frac{E}{\rho}} \qquad (20)$$

and the second frequency:

$$p_2 = \frac{27.931592}{h}\sqrt{\frac{E}{\rho}} \qquad (21)$$

Let's calculate the first and second frequency value for a continuous elastic body which example can be a root crop of a sugar beet with the following parameters (2): $h = 250$ (mm); $E = 18.4 \times 10^6$ (N/m^2); $\rho = 1{,}300$ (kg/m^3). As evaluations result we shall receive:

$$p_1 = \frac{0.662422}{250 \times 10^{-3}} \sqrt{\frac{18.4 \times 10^6}{1{,}300}} = 315 \qquad (s^{-1})$$

$$p_2 = \frac{27.931592}{250 \times 10^{-3}} \sqrt{\frac{18.4 \times 10^6}{1{,}300}} = 13{,}292 \quad (s^{-1})$$

Let's transfer further to forced oscillations research of a continuous elastic body. Cleanly forced oscillations will occur according to the law

$$y\,(x,\,t) = \varphi(x) \sin \omega t \tag{22}$$

where: $\varphi(x)$ – the shape of forced oscillations.

For definition of body forced oscillations shape we shall substitute expression (22) in a functional (9), we shall receive the following functional:

$$S_3 = \frac{1}{2} \int_{t1}^{t_2} \int_{0}^{h} \Big\{ \rho\pi\, x^2 \operatorname{tg}^2 \gamma \omega^2\, \varphi^2\,(x) \cos^2 \omega t - E\,\pi\, x^2 \operatorname{tg}^2 \gamma\, [\varphi'(x)]^2 \sin^2 \omega t + [H\sigma_1\,(x - x_1) - 2\pi c x \operatorname{tg} \gamma]\, \varphi(x) \sin^2 \omega t \Big\}\, dx\, dt \tag{23}$$

Integrating expression (23) on t in limits of one period

$T = \dfrac{2\pi}{\omega}$, we shall have:

$$S_4 = \frac{\pi}{2\omega} \int_{0}^{h} \Big\{ \rho\pi\, x^2 \operatorname{tg}^2 \gamma \varphi^2\,(x)\, \omega^2 - E\,\pi\, x^2 \operatorname{tg}^2 \gamma\, [\varphi'(x)]^2 + H\sigma_1\,(x - x_1)\, \varphi(x) - 2\pi c x \operatorname{tg} \gamma\, \varphi(x) \Big\}\, dx \tag{24}$$

It agreement to the Ritz's method we shall consider a functional (24) value on linear combinations plurality of the following aspect

$$\varphi(x) = \alpha\psi(x) \tag{25}$$

where: α – parameter which variation we obtain of admissible functions class,
$\psi(x)$ – basis function.

Substituting expression (25) in a functional (24), we shall receive:

$$S_4 = \frac{\pi}{2\omega} \int_{0}^{h} \Big\{ \rho\pi\, x^2 \operatorname{tg}^2 \gamma \alpha^2 \psi^2\,(x)\, \omega^2 - E\,\pi\, x^2 \operatorname{tg}^2 \gamma \alpha^2\, [\psi'(x)]^2 + H\sigma_1\,(x - x_1)\, \alpha\psi(x) - 2\pi c x \operatorname{tg} \gamma \alpha \psi(x) \Big\}\, dx \tag{26}$$

Let's inject such labels:

$$\int_{0}^{h} \rho\pi\, x^2 \operatorname{tg}^2 \gamma\, \psi^2\,(x)\, dx = T \tag{27}$$

$$\int_{0}^{h} E\pi\, x^2 \operatorname{tg}^2 \gamma\, [\psi'\,(x)]^2\, dx = U \tag{28}$$

$$\int_{0}^{h} [H\sigma_1\,(x - x_1)\psi(x) - 2\pi c x \operatorname{tg} \gamma \psi(x)]\, dx = L \tag{29}$$

Substituting expressions (27)–(29) in (26), we shall have

$$S_4\,(\alpha) = \frac{\pi}{2\omega}\,(\omega^2 T \alpha^2 - U\alpha^2 + L\alpha) \tag{30}$$

So, on functions (25) plurality functional (26) turns to function from an explanatory variable α, looking like (30).

Necessary requirement of a functional (30) stationarity (that is extremum existence) are equality to null of its first variation, namely:

$$\frac{\partial S_4}{\partial \alpha}\, \delta\alpha = 0 \tag{31}$$

whence we obtain the following equation:

$$2\omega^2 T\alpha - 2U\alpha + L = 0 \tag{32}$$

from which it is discovered a necessary parameter value α. It will be equated:

$$\alpha = \frac{L}{2(U - \omega^2 T)} \tag{33}$$

Let's accept for basis function $\psi\,(t)$ the shape of fixed cross-section rod forced longitudinal oscillations with one rigidly fixed extremity which originate under an longitudinal harmonic force operation of the frequency ω affixed in a point $x = x_1$.

According to (1) shape of evocative rod forced oscillations has such aspect:

$$\psi(x) = D_1 \sin ax \qquad \text{at } x \le x_1 \tag{34}$$

$$\psi(x) = D_2 \cos a\,(h - x) \qquad \text{at } x > x_1 \tag{35}$$

where

$$D_1 = -\frac{1}{aEF} \times \frac{\cos a\,(h - x_1)}{\cos ah} \tag{36}$$

$$D_2 = -\frac{1}{aEF} \times \frac{\sin a\, x_1}{\cos ah} \tag{37}$$

$$a = \omega \sqrt{\frac{\mu}{EF}} \tag{38}$$

where: μ – rod running mass,
F – rod cross-sectional area,
E – Young's modulus for a rod material,
h – rod length,
ω – frequency of rod forced oscillations.

Having calculated parameters T, U also L it agrees expressions (27), (28) and (29), we shall receive a necessary parameter value α according to expression (33) at which the functional (26) will have a steady-state value:

$$\alpha = \frac{-HD_1 \sin ax_1 + HD_2 [\cos a\,(h - x_1) - 1] - 2D_1 \pi c \operatorname{tg}\gamma \left(\frac{\sin ax_1}{a^2} - \frac{x_1 \cos a\, x_1}{a}\right) - 2D_2 \pi c \operatorname{tg}\gamma \left[\frac{x_1}{a} \sin a\,(h - x_1) - \frac{1}{a^2}\cos a\,(h - x_1) + \frac{1}{a^2}\right]}{2E\pi \operatorname{tg}^2\gamma \left[D_1^2\left(\frac{a^2 x_1^3}{6} + \frac{x_1^2 a \sin 2ax_1}{4} + \frac{x_1 \cos 2\,ax_1}{4} - \frac{\sin 2\,ax_1}{8a}\right) - D_2^2\left(\frac{a^2 (x_1^3 - h^3)}{6} + \frac{x_1^2 a \sin(2ah - 2ax_1)}{4} + \frac{h}{4} - \frac{x_1 \cos(2ah - 2ax_1)}{4} - \frac{\sin(2ah - 2ax_1)}{8a}\right)\right] - 2\omega^2 \rho \pi \operatorname{tg}^2\gamma \left[D_1^2\left(\frac{x_1^3}{6} - \frac{x_1^2 \sin 2\,a\,x_1^2}{4a} - \frac{x_1 \cos\,2\,a\,x_1}{4a^2} + \frac{\sin 2\,a\,x_1}{8a^3}\right) + D_2^2\left(\frac{h^3 - x_1^3}{6} + \frac{x_1^2 \sin(2ah - 2ax_1)}{4a} + \frac{h}{4a^2} - \frac{x_1 \cos(2ah - 2ax_1)}{4a^2} - \frac{\sin(2ah - 2ax_1)}{8a^3}\right)\right]} \quad (39)$$

Taking into account expressions (25), (34) and (35), we shall receive expressions for the forced oscillations shape of a continuous elastic body, fixed in soil. They have such aspect:

$$\varphi(x) = \alpha D_1 \sin a\,x, \qquad \text{at } x \le x_1$$
$$\varphi(x) = \alpha D_2 \cos a\,(h - x), \qquad \text{at } x > x_1 \quad (40)$$

where: α – determined according to (39).

Having substituted expressions (40) in (22), we shall finally receive the law of continuous elastic body forced oscillations, fixed in soil:

$$y(x, t) = D_1\, \alpha \sin ax \sin \omega t, \qquad \text{at } x \le x_1$$
$$y(x, t) = D_2\, \alpha \cos a\,(h - x) \sin \omega t, \qquad \text{at } x > x_1 \quad (41)$$

By results of forced oscillations theoretical researches of a continuous elastic body fixed in soil concrete calculation of specified oscillations amplitude is carried out.

For an example we use a sugar beet root crop with the following parameters: length h, cone angle γ, Young's modulus E, density ρ, coefficient of a soil elastic deformation c. Let's accept their, according to (2), equal:

$h = 250 \times 10^{-3}$ (m); $\gamma = 14°$;

$E = 18.4 \times 10^6$ (N/m³);

$\rho = 1{,}300$ (kg/m³); $c = 1 \times 10^5$ (N/m²)

Disturbing force amplitude H it is selected in limits 100...600 (N). Disturbing force frequency ω, according to (2), we shall accept equal ω = 20.00 (Hz).

Calculation is carried out in program MathCAD with the purpose of definition of amplitude dependence of forced longitudinal oscillations of a root crop body from change of a disturbing force in a gamut 100...600 (N) for different body cross-sections.

Given calculation results in the graph, reduced on Fig. 2.

As it is visible from the reduced graph, with magnification of disturbing force magnitude the amplitude of continuous elastic body longitudinal forced oscillations increases under the linear law.

And with a distance of a root crop body cross-sectional area from an coordinates origin *0* the amplitude also increases. So, at $x = 0.07$ (m) the amplitude is in limits 1.7 ... 2.3 (mm), at $x = 0.1$ (m) – in boundaries 2.3 ... 3.5 (mm), at $x = 0.12$ (m) – in limits 2.8 ... 3.9 (mm), at $x = 0.15$ (m) (capture point) – in limits 3.2 ... 4.8 (mm).

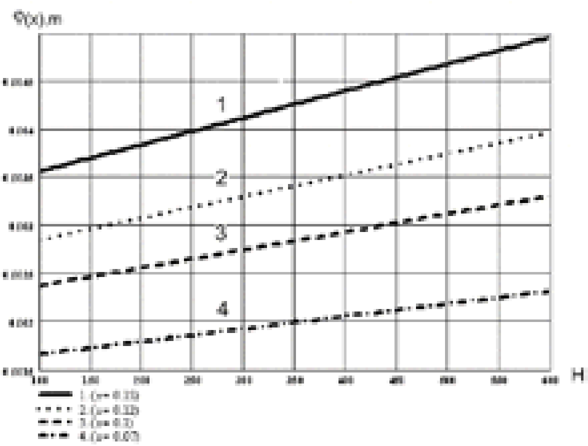

Fig. 2. Amplitude dependence of forced longitudinal oscillations of a root crop body on value of a disturbing force

CONCLUSION

Thus, on a foundation of the Ostrogradskii-Hamilton variational principle application the equations for an evaluation of fundamental frequencies of any order of a root crop body longitudinal oscillations, fixed in soil are obtained. So, analytical forms for a determination of the first and second fundamental frequency, and also expression for a determination of forced oscillations amplitude of any root crop cross-section concerning the equilibrium position are obtained. Introduced analytical researches enable studies of a root crop with soil links destruction process at its vibration digging up.

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Received for publication April 30, 2005
Accepted after corrections May 25, 2005

Podélné kmitání bulvy cukrové řepy při vibračním vyorávání

ABSTRAKT: V příspěvku je navržena teorie podélného kmitání spojitého elastického tělesa, která využívá Ostrogradskii-Hamiltonova principu. Za použití Ritzovy netody byly získány Ritzovy rovnice frekvencí oscilačního procesu. Po důkladném analytickém výpočtu byla získána první a druhá základní frekvence oscilace tělesa a silová oscilační amplituda pro jakýkoliv příčný průřez tohoto tělesa.

Klíčová slova: podélné oscilace; silové oscilace; řepná bulva; elastické těleso; Ostrogradskii-Hamiltonův princip; vibrační vyorávání

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