Journal of Monetary Economics 13 (1984) 165-181. North-Holland

TESTING THE INTERPRETATION OF INDICES IN A MACROECONOMIC INDEX MODEL*

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This paper discusses and contrasts different approaches to the construction of dynamic models containing unobservables. It is argued that some analysis is easy to carry out using Dynamic Factor Analysis (DFA), a frequency domain technique, while other analysis is easier to carry out in the context of a Dynamic Multiple Indicator-Multiple Cause Model (DYMIMIC), a time domain technique. An example is presented in which six macroeconomic variables are found to be connected by two common factors using DFA. The interpretation of these factors as 'anticipated' and 'unanticipated' aggregate demand is then tested using the DYMIMIC model. This interpretation is strongly rejected.

1. Introduction

Unobservable index or dynamic factor models were introduced into economics through the work of Sargent and Sims (1977) and Geweke (1975). The model asserts that all of the dynamic interrelations between a vector of economic variables can be explained by a relatively few number of common, unobservable factors or indices. Since their introduction these models have generated substantial disagreement concerning their usefulness in analyzing economic time series.

On the one hand, most would agree that these models may be useful for short-run forecasting. This agreement stems from viewing the models as constrained multiple time series models. As the number of estimated parameters in unconstrained multiple time series models is generally quite large relative to sample size, parameter estimates, and to some extent forecasts, are imprecise. A factor structure is one way of constraining the number of parameters to be estimated. Other constraints, or the use of priors as in Litterman (1980), may be more appropriate.

*The financial support of the National Science Foundation (SES-82-08037) and the Harvard Graduate Society is gratefully acknowledged. We thank Benjamin Friedman, Robert King, Charles Plosser, an anonymous referee, and the participants of seminars at Harvard and the University of Rochester for useful comments on an earlier draft.

On the other hand, index models were introduced not as forecasting models, but rather as ways of interpreting dynamic interrelationships between economic variables. It is the interpretation of the models and of the factors themselves which has generated the disagreement. Often the unobservable factors are given names like 'nominal aggregate demand' or 'permanent income'. Some argue that these are reasonable interpretations of the factors, while others [see Klein (1977), Nerlove (1981), or Rothenberg (1981)] assert that the factors are merely convenient summaries of some features of the data and cannot be given economic interpretations. Similar disagreement arises when psychologists attempt to interpret factors from intelligence test data as 'quantitative ability' or 'mechanical aptitude'.

This disagreement arises, at least in part, because there is no model generating the factors other than unobservable univariate time series models. The factors are identified because we observe some indicators of the factors. Causes of the factors are not modelled. Engle and Watson (1980, 1981) introduce a model, which they call a dynamic multiple indicator–multiple cause model (DYMIMIC) in which causes can be explicitly incorporated into the unobservable index model.[1] The introduction of causal variables not only aids in interpreting the factors, the inclusion of these additional variables makes parameter estimates more precise and tests more powerful.

The estimation strategy suggested by Sargent and Sims (SS) and others[2] and the strategy proposed by Engle and Watson (EW) are quite different. In the SS strategy, frequency domain methods are used and lag distributions are unrestricted. This allows methods used in standard factor analysis to be easily adapted for dynamic factor analysis. The methods used by EW are time domain methods based on the state space model, widely used in engineering. Their method allows (in fact requires) tightly parameterized lag distributions, which are difficult to incorporate into the frequency domain methods. Of course anything that can be accomplished in the time domain can be accomplished in the frequency domain and vice-versa. Some analysis, however, is easier in the time domain; some is easier in the frequency domain.

The purpose of this paper is to demonstrate how these methods may be used as complements. Preliminary analysis is carried out using the methods of SS and Geweke and Singleton (1981) and then the methods proposed by EW are used to (hopefully) sharpen the estimates and carry out tests involving causes of the factors. These tests, we argue below, are important aids for interpreting the factors. As a vehicle for demonstrating this point we focus on the model analyzed by Singleton (1980b) using frequency domain procedures. Section 2 briefly describes the statistical model, while section 3 describes the economic

[1] In the terminology of Sargent and Sims the model is a mixed observable–unobservable index model.

[2] See Geweke (1975, 1977), Geweke and Singleton (1981), and Litterman and Sargent (1979).

model and the hypothesis to be tested. Section 4 presents the estimation and hypothesis testing results. The final section contains some concluding remarks.

2. The statistical model

The DYMIMIC model is a dynamic version of the static MIMIC model described in Zellner (1970) and Goldberger (1972, 1977). In its general form the model can be written as

$$A(B)Y_t = C(B)F_t + D(B)Z_t + \varepsilon_t, \tag{2.1}$$

$$G(B)F_t = H(B)Z_t + u_t, \tag{2.2}$$

where Y_t is a $p \times 1$ vector of observed variables, F_t is a $j \times 1$ vector of unobserved variables, Z_t is a $k \times 1$ vector of observed weakly exogenous variables, and ε_t and u_t are disturbance terms. The matrices $A(B)$, $C(B)$, $D(B)$, $G(B)$, and $H(B)$ are matrix polynomials in the backshift operator, B. The vector Y_t serves as an indicator of the unobservable F_t, and we call eq. (2.1) the indicator equation. The vector Z_t 'drives' the F_t's and is uncorrelated with current and future values of the disturbances ε_t and u_t. We call the Z_t's causes, and eq. (2.2) the causal equation.

The SS index models are special cases of (2.1) and (2.2). Their unobservable index model sets $Z_t = 0$, for all t, so that only the indicators are observed. Their observable index model sets $u_t = 0$, for all t, so that F_t is an exact linear combination of observable variables.

Clearly, the parameters of (2.1) and (2.2) are not identified without further restrictions. In the unobservable index model $A(B)$ is diagonal, ε_t and u_t are uncorrelated at all leads and lags, and the elements of ε_t are uncorrelated at all leads and lags, i.e.,

$$\mathrm{E}(\varepsilon_{it} u_{mt-s}) = 0, \qquad \forall s, \quad i = 1, \dots, p, \quad m = 1, \dots, j,$$

and

$$\mathrm{E}(\varepsilon_{it}, \varepsilon_{jt-s}) = 0, \qquad \forall s, \quad \text{if} \quad i \neq j.$$

These assumptions imply that all of the dynamic interaction between the elements of Y_t can be explained by the unobservable factors. If the number of factors is small relative to the number of indicators, this places testable restrictions on the data. More precisely, this implies that the spectral density matrix of Y, which is rank p, can be represented as the sum of a rank j matrix and a diagonal matrix. Additional restrictions or normalizations must be placed on the 'factor loadings', $C(B)$, the transition parameters, $G(B)$, and covariance matrix of u_t to completely identify the model. Conditions for identifiability are given in Geweke and Singleton (1981).

The model can be estimated, and the restrictions tested using a complex arithmetic generalization of standard factor analysis. In standard factor analysis a $p \times p$ covariance matrix is 'factored' into a rank j matrix plus a diagonal matrix. In dynamic factor analysis the $p \times p$ spectral density matrix is factored into a matrix of rank j plus a diagonal matrix at each frequency. When there are no constraints across frequencies (i.e., when lag distributions are unrestricted) the estimation and testing can be carried out independently across frequency bands. As non-overlapping frequency band estimates are asymptotically independent, an overall test of the restrictions is easily obtained by simply adding up the test statistics (which are χ^2 random variables) from each frequency band. When there are constraints across frequency bands estimation and testing become more difficult.

In the time domain, estimating unrestricted or loosely parameterized models is difficult as a large number of parameters must be estimated using a non-linear maximum likelihood procedure. Tightly parameterized models are reasonably easy to estimate, using for instance the EM procedure suggested in Watson and Engle (1983). The method also allows the addition of weakly exogenous variables, Z_t, into the model, so that the general DYMIMIC model can be estimated.[3] As this is the method used to estimate the model discussed below, we briefly describe it. (For additional details, see the paper by Watson and Engle.)

If we could observe the factors, F_t, then it would be straightforward to estimate the parameters of the model. Conversely, if the parameters of the model were known then standard signal extraction methods could be used to estimate the factors. (A Kalman filter and smoother could be used, for instance.) In the EM algorithm these two simple procedures are used in tandem. From an initial guess of the parameters, the factors are estimated. The estimates of the factors, their variances, and the observed data are combined using standard regression formulae to obtain new estimates of the parameters. The procedure is repeated until convergence. It can be shown [see Dempster, Laird and Rubin (1977)] that the resulting parameter estimates satisfy the first-order conditions for maximization of the likelihood function.

A practical research strategy for specifying, estimating, and testing index models can now be proposed. Frequency domain methods can be used to determine the number of factors, as these tests are easily carried out in the frequency domain, and are much more difficult to carry out in the time domain. The time domain methods can then be used to estimate tightly parameterized versions of the model, and tests concerning the causes of the factors can be carried out. In the next two sections we apply this strategy, by applying time domain methods to a model analyzed by Singleton (1980b) using frequency domain methods.

[3]Care must be taken in introducing the same variable into the measurement and causal equations. This may introduce identification problems.

3. The economic model

The Singleton paper analyzes the relationship between monthly observations on four interest rate variables, yields on three-month, six-month, one-year and five-year U.S. government securities, and two real variables, the civilian unemployment rate and the real value of manufacturers' shipments. None of the variables are seasonally adjusted, and the sample period runs from July 1959 to May 1971.[4] Singleton finds two interesting results: two common factors explain the interrelationships in the six variables, and only one factor is necessary to explain the real variables and their relationship with the interest rate variables. One interpretation of these results, suggested by Singleton, is that the data are generated by a model similar to the one proposed by Lucas (1973). That is, one of the factors corresponds to 'anticipated aggregate demand'. This interpretation of the factors can be easily tested using the DYMIMIC model.

Following Singleton we will derive a version of the Lucas model consistent with the results of his analysis. Let R_t be the 4×1 vector of interest rates and Y_t be the 2×1 vector of real variables. Then the Lucas interpretation suggests

$$R_t = \alpha(B)(n_t - \hat{n}_t) + \gamma(B)\hat{n}_t + \varepsilon_{Rt}, \tag{3.1}$$

$$Y_t = \beta(B)(n_t - \hat{n}_t) + \varepsilon_{Yt}, \tag{3.2}$$

where n_t is nominal aggregate demand and $\hat{n}_t$ is anticipated aggregate demand.[5] The elements of the disturbance vector $\varepsilon_t' = (\varepsilon_{Rt}', \varepsilon_{Yt}')$ are mutually uncorrelated, but they may be autocorrelated. These disturbances, presumably, represent factors unique to each of the indicator variables unrelated to the business cycle.

The model is given additional content by assuming a mechanism which generates the expectation series $\hat{n}_t$. Suppose that $\hat{n}_t$ is a minimum linear mean square error forecast of n_t formed at time $t-1$ using an information set $\tilde{\mathscr{I}}_{t-1}$. This information set is composed of current and lagged values of x, a vector of economic variables. That is,

$$\tilde{\mathscr{I}}_{t-1} = (\tilde{x}_{t-1}, \tilde{x}_{t-2}, \dots),$$

and

$$\hat{n}_t = \tilde{x}_{t-1}'\tilde{\Psi}(B),$$

[4]Singleton's analysis included June 1971; however, the data which was graciously supplied by Singleton did not include June 1971.

[5]We purposely are using these terms without first defining them in terms of observable variables. If they can be defined exactly in terms of observables, then there is no need for latent variable models to assess the validity of the model. In this discussion we remain agnostic as to whether the latent variable framework is the best method for testing the model. We do believe that if a latent variable approach is taken, the DYMIMIC model is useful.

where $\tilde{\Psi}(B)$ forms the projection of n_t onto $\tilde{\mathscr{I}}_{t-1}$. Since $\hat{n}_t$ is an optimal linear forecast, the forecast error $(n_t - \hat{n}_t)$ is uncorrelated with any linear combination of variables in $\tilde{\mathscr{I}}_{t-1}$. In particular it is uncorrelated with $\hat{n}_t$ and lagged values of $\hat{n}_t$. Under certain situations $(n_t - \hat{n}_t)$ may be uncorrelated with its own lagged values, but this need not be true in general. Of course $(n_t - \hat{n}_t)$ must be white noise if its own lagged values are in $\tilde{\mathscr{I}}_{t-1}$, but informational lags may preclude this. Alternatively if we allow for measurement error it may be the case that the exact values of n_t and $(n_t - \hat{n}_t)$ are never observed. Since $(n_t - \hat{n}_t)$ may be serially correlated we assume that it has an autoregressive representation

$$\phi_{11}(B)(n_t - \hat{n}_t) = \eta_{1t}, \tag{3.3}$$

where η_{1t} is white noise. By construction η_{1t} is uncorrelated with linear combinations of variables in $\tilde{\mathscr{I}}_{t-1}$ and with lagged values of $(n_t - \hat{n}_t)$.

In the econometric specification of the model we want to allow for the possibility that we have not included all of the relevant variables in $\tilde{\mathscr{I}}_{t-1}$. In particular, suppose that we only incorporate information in $\mathscr{I}_{t-1}$ which is a subset of the true information set $\tilde{\mathscr{I}}_{t-1}$. If we let $(x_{t-1}, x_{t-2}, \ldots)$ denote the information in $\mathscr{I}_{t-1}$ which is useful in predicting n_t, then

$$\hat{n}_t = x'_{t-1}\Psi(B) + \xi_t, \tag{3.4}$$

where $\Psi(B)$ forms the projection of n_t onto $\mathscr{I}_{t-1}$. The error term, ξ_t, represents that part of n_t that can be predicted from the large information set $\tilde{\mathscr{I}}_{t-1}$ but not from our smaller set $\mathscr{I}_{t-1}$. Since ξ_t cannot be predicted by $\mathscr{I}_{t-1}$ it is uncorrelated with $(x_{t-1}, x_{t-2}, \ldots)$. Furthermore, since it can be perfectly predicted by $\tilde{\mathscr{I}}_{t-1}$, ξ_t is uncorrelated with current and future values of $(n_t - \hat{n}_t)$. [This follows since ξ_t is a linear combination of variables in $\tilde{\mathscr{I}}_{t-1}$, and we showed above that $(n_t - \hat{n}_t)$ is uncorrelated with current and lagged variables in $\tilde{\mathscr{I}}_{t-1}$.] Finally, we note that nothing prevents ξ_t from being serially correlated or correlated with lagged values of $(n_t - \hat{n}_t)$. Projecting ξ_t onto its lagged values and lagged values of $(n_t - \hat{n}_t)$ yields

$$\phi_{22}(B)\xi_t = \phi_{21}(B)(n_{t-1} - \hat{n}_{t-1}) + \eta_{2t}, \tag{3.5}$$

with η_{2t} white noise. Combining (3.4) and (3.5), we have

$$\phi_{22}(B)\hat{n}_t = \phi_{21}(n_{t-1} - \hat{n}_{t-1}) + x'_{t-1}\lambda(B) + \eta_{2t}, \tag{3.6}$$

with

$$\lambda(B) = \Psi(B)\phi_{22}(B).$$

Notice that η_{2t} is uncorrelated by construction with all past, present and future values of η_{1t}.

The model (3.1), (3.2), (3.3) and (3.6) is a DYMIMIC model. It can be transformed into an unobservable index model by expressing x_{t-1} in terms of its Wold representation, i.e., replacing x_{t-1} by unobservable 'noise'. The advantage of the DYMIMIC model is that this substitution is unnecessary.

The hypothesis of interest is now clear. If the two factors generating the data are $\hat{n}_t$ and $(n_t - \hat{n}_t)$, then x_{t-1} should enter the equation for the second factor and should not enter the equation for the first factor. While many variables may be useful for forecasting n_t, a primary concern of many models of the business cycle has been the causal influence of the money supply. In the next section we present test results using lagged values of the rate of change of the money supply for x_{t-1}.

4. Empirical analysis

The first step in the empirical analysis is the estimation of a two-factor unobservable index model using the time domain method discussed in section 2. This model is interesting in its own right and will serve as a benchmark for carrying out the tests described in the last section. Prior to estimating the model all variables were regressed on a constant term and a linear trend.[6] Seasonal dummy variables were included in the regressions for the real variables. The residuals from these regressions are used in the analysis reported below. The model estimated was of the form

$$\theta_R(B)R_t = \alpha(B)F_{1t} + \gamma(B)F_{2t} + \varepsilon_{Rt}, \tag{4.1}$$

$$\theta_Y(B)Y_t = \beta(B)F_{1t} + \varepsilon_{Yt}, \tag{4.2}$$

$$\phi_{11}(B)F_{1t} = \eta_{1t}, \tag{4.3}$$

$$\tilde{\phi}_{22}(B)F_{2t} = \tilde{\phi}_{21}(B)F_{1t-1} + \tilde{\eta}_{2t}. \tag{4.4}$$

In terms of (3.1)–(3.4),

$$F_{1t} = (n_t - \hat{n}_t),$$

$$F_{2t} = \hat{n}_t.$$

[6] The data were detrended to make these results comparable to the results in Singleton (1980b). Recent evidence [see Nelson and Plosser (1982)], suggests that it might be better to model the non-stationary behavior of the data in terms of integrated processes. One approach in the context of the present model is to view one of the factors as non-stationary and one as stationary. This approach will be investigated in a subsequent paper.

and (4.4) is eq. (3.4) with x_{t-1} 'solved out', so that $\tilde{\phi}_{22}$, $\tilde{\phi}_{21}$, and $\tilde{\eta}_{2t}$ are functions of $\phi_{22}, \phi_{21}, \eta_{2t}$ and the parameters of the process generating x_t. The matrices $\theta_R(B)$ and $\theta_Y(B)$ are diagonal, and all disturbance terms are uncorrelated white noises.

A fairly rich parameterization was chosen. Each interest rate equation included six lags of its own rate and current and three lagged values of each factor. The equations for the real variables included own lags one through six and eleven through thirteen. Current and three lagged values of the first factor were also included. The factors were assumed to be generated by AR(3) processes.

This rich parameterization may yield a model with unidentified parameters. To see this, note that the model can be rewritten as

$$\theta_{R_i}(B) R_{it} = \omega_{1i}(B)\eta_{1t} + \omega_{2i}(B)\tilde{\eta}_{2t} + \varepsilon_{R_i t}, \qquad i = 1,\ldots,4,$$

$$\theta_{Y_j}(B) Y_{jt} = \omega_{3j}(B)\eta_{1t} + \varepsilon_{Y_j t}, \qquad j = 1,2,$$

Table 1

Indicator equations for model 1.

	NTBL3	*NTBL6*	*NTB1Y*	*NTB5Y*	*RSHP*	*UNEM*
F^1	−0.160	−0.148	−0.066	−0.016	1.892	0.021
F^1-1	0.101	0.037	−0.096	−0.143	−1.315	−0.037
F^1-2	0.115	0.154	0.241	0.177	0.537	−0.027
F^1-3	−0.013	0.010	−0.033	0.021	0.632	−0.002
F^2	0.173	0.199	0.268	0.227	—	—
F^2-1	−0.007	0.032	−0.083	−0.064	—	—
F^2-2	−0.025	−0.040	−0.035	−0.024	—	—
F^2-3	−0.007	−0.012	0.046	−0.001	—	—
OWN − 1	0.671	0.426	0.652	0.503	0.568	0.617
OWN − 2	−0.075	0.072	−0.041	−0.003	−0.504	0.114
OWN − 3	0.316	0.200	0.182	0.215	0.326	−0.123
OWN − 4	−0.169	0.043	−0.058	0.067	−0.382	0.059
OWN − 5	0.061	0.008	0.064	−0.051	0.037	−0.126
OWN − 6	−0.091	−0.080	−0.020	0.075	−0.036	−0.114
OWN − 11	—	—	—	—	0.120	0.048
OWN − 12	—	—	—	—	−0.083	0.121
OWN − 13	—	—	—	—	−0.114	−0.125
OWN variance	0.009	0.009	0.001	0.010	25.07	0.035
% *F1*	47.71	45.28	40.97	24.99	86.95	90.35
% *F2*	49.06	52.54	58.99	70.25	—	—
% *OWN*	3.23	2.18	0.04	4.76	13.05	9.65

where

$$\omega_{1i}(B) = \alpha_i(B)/\phi_{11}(B) + \gamma_i(B)\phi_{21}(B)B/\phi_{22}(B)\phi_{11}(B),$$

$$\omega_{2i}(B) = \gamma_i(B)/\phi_{22}(B), \qquad i = 1,\ldots,4,$$

and

$$\omega_{3j} = \beta_j(B)/\phi_{11}(B), \qquad j = 1,2.$$

While the $\omega(B)$'s are identified, it is quite possible that the rich parameterization may allow arbitrary common factors in the numerators and denominators of the $\omega(B)$ polynomials. This would cause severe problems in quadratic hill-climbing maximization methods as the hessian would be singular. This is not a problem with the EM algorithm, which will converge to some point on the ridge of the likelihood function. This will also not affect the hypothesis tests carried out below, as the parameters of interest — the coefficients on x_{t-1} — are identified, and likelihood ratio tests concerning these parameters will have the usual asymptotic distribution. Common factors in $\theta_{R_i}(B)$, $\omega_{1i}(B)$, and $\omega_{2i}(B)$ or $\theta_{Y_j}(B)$ and $\omega_{3j}(B)$ do not cause identification problems as ε_{R_i} and ε_{Y_j} are white noise. For efficiency reasons, of course, one would want to impose the common factor restriction if it is valid.

The results for the indicator equations are reported in table 1, and the results for the causal equations are reported in the first column of table 2. The last three rows in table 1 present the percentage of the variance of each of the independent variables explained by the various disturbances. So for instance,

Table 2

Causal equations.

	Model 1		Model 2		Model 3		Model 4	
	F^1	F^2	F^1	F^2	F^1	F^2	F^1	F^2
$F^1 - 1$	1.242	0.655	1.215	0.625	1.127	0.603	1.131	0.637
$F^1 - 2$	0.209	−1.021	0.263	−1.010	0.324	−0.978	0.327	−1.010
$F^1 - 3$	−0.473	0.301	−0.500	0.319	−0.475	0.308	−0.482	0.309
$F^2 - 1$		0.719		0.725		0.716		0.732
$F^2 - 2$		0.067		0.051		0.047		0.043
$F^2 - 3$		−0.082		−0.074		−0.067		−0.071
$m - 1$[a]				7.579	−25.852	5.727	−25.106	
$m - 2$				17.323	86.665	26.044	83.113	
$m - 3$				7.690	40.077	9.095	40.408	
$m - 4$				14.556	49.937	15.162	49.737	
$m - 12$				23.026	19.944	22.210	17.633	
Log likelihood	613.89		615.86		625.40		623.82	

[a] $m = \log MI - \log MI_{-1}$.

48% of the variance in three-month bill rates is explained by the innovation in the first factor, 49% by the second factor, and only 3% by its 'unique' or own disturbance. The small size of the uniquenesses suggests that the two-factor model fits the data quite well.

As a check on the specification of the model the one-step-ahead forecast errors were calculated using the Kalman filter. For a correctly specified model these errors (normalized by their theoretical standard deviations) should have zero mean and be serially uncorrelated. No serial correlation was apparent and the forecast error means ranged from -0.040 to 0.040. The innovations showed no evidence of misspecification.

The complicated dynamics and the identification problem mentioned above make the parameter estimates difficult to interpret. The dynamic behavior of the model is compactly characterized by the impulse response functions shown in figs. 1–6. These trace out the changes in the independent variables induced by a one-standard-deviation shock to the factors and own disturbance terms. A positive one-standard-deviation shock to the first factor causes an instanta-

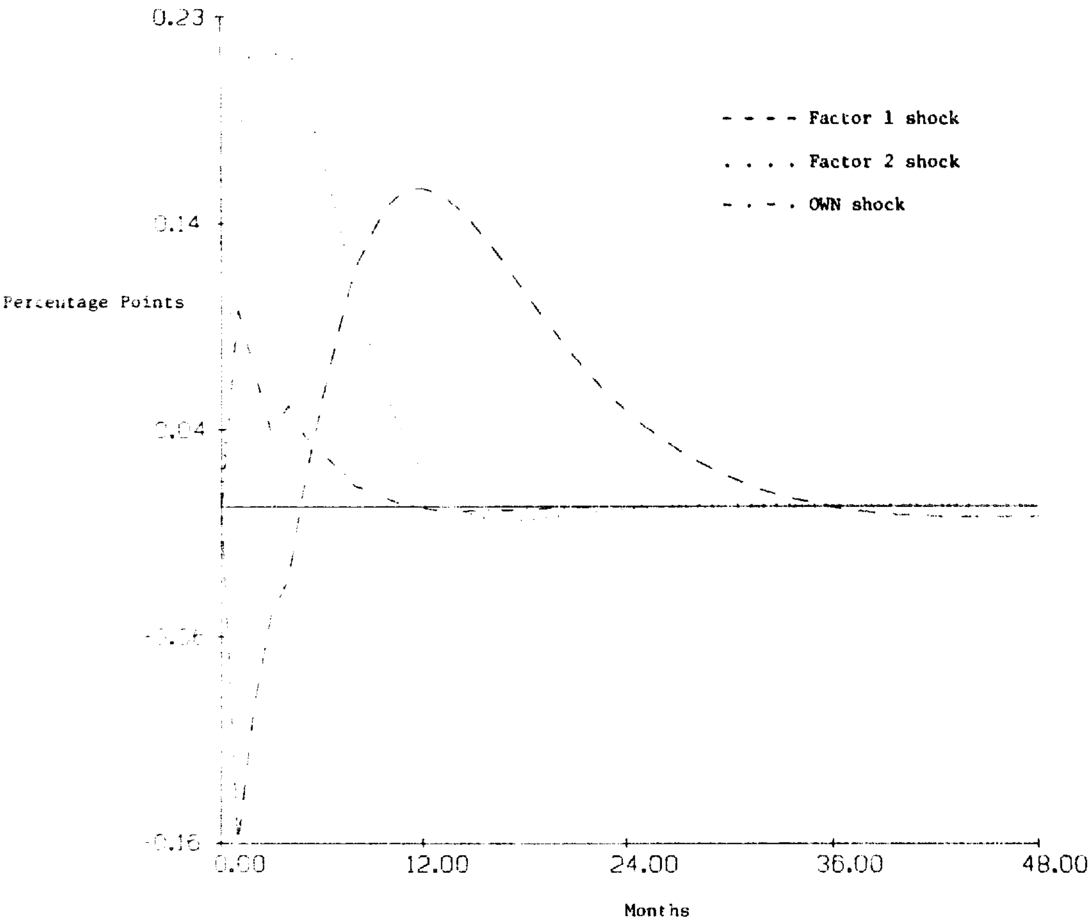

Fig. 1. Response of three-month bill rate.

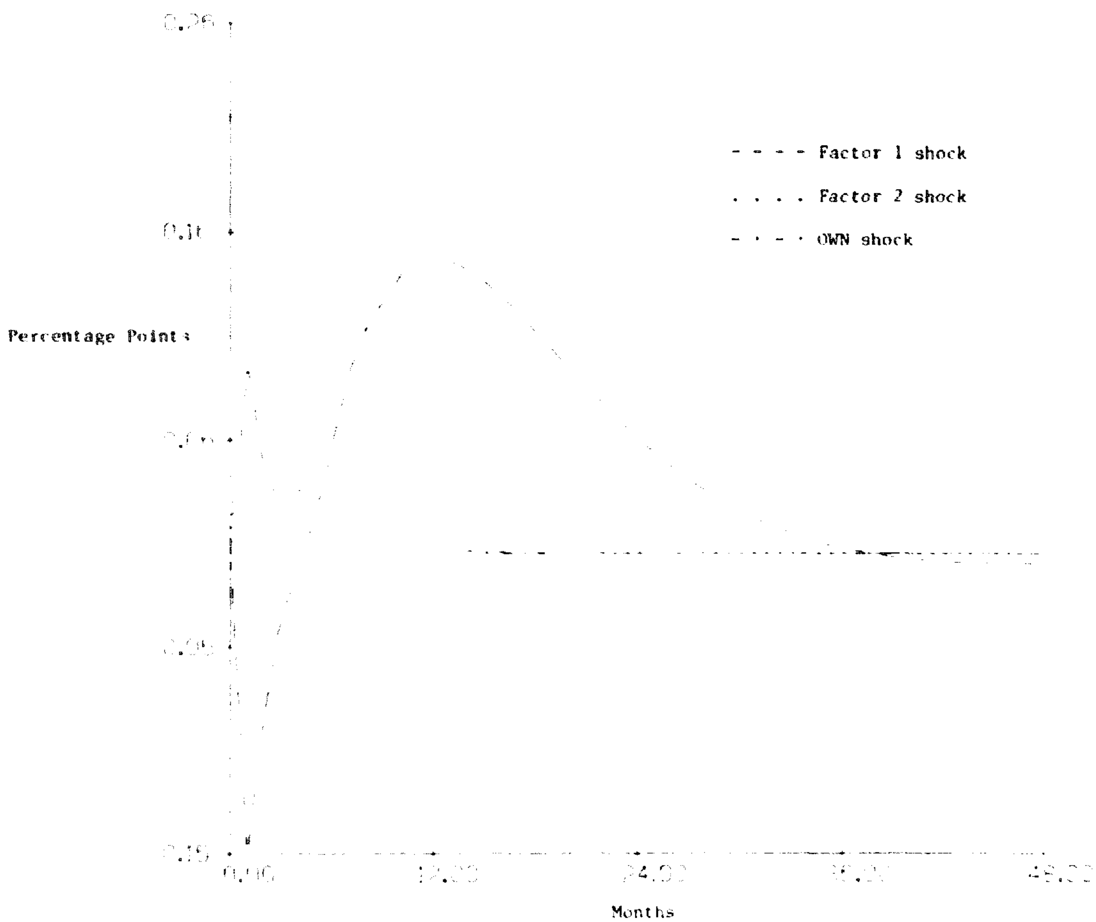

Fig. 2. Response of six-month bill rate.

neous decline in interest rates followed by a sharp increase, and finally a gradual return to trend. The pattern of the responses for all interest rates are very similar. The response is attenuated as the maturity increases. Three-month rates, for example, fall 16 basis points, then sharply rise to 15 basis points above trend after one year, and return to their trend level after three years. Five-year rates, on the other hand, fall only 2 basis points, rise and peak at 8 basis points above trend after sixteen months, and are still 2 basis points above trend after three years. Real activity increases in response to the shock, peaking after two quarters (the unemployment rate falls by 0.26 percentage points) and returning to trend after three years.

A positive one standard deviation shock to the second factor leads to an instantaneous increase of about 25 basis points in all interest rates. Rates then return smoothly to their trend values. Rates of shorter maturities return more rapidly.

These estimated impulse response functions are at odds with a slight modification of a linearized expectations model of the term structure. Suppose,

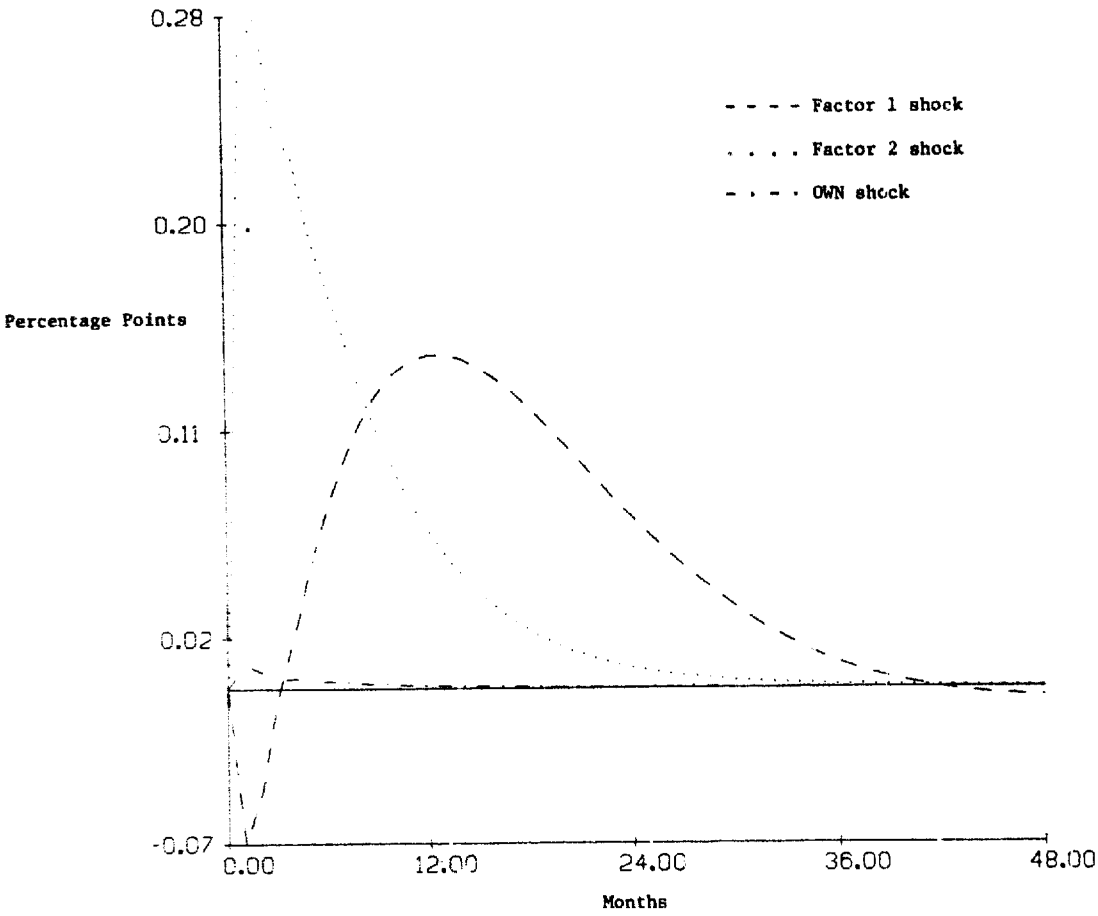

Fig. 3. Response of one-year bond rate.

for example, that

$$\bar{R}_t^m = (1/m) \sum_{i=0}^{m-1} \mathrm{E}_t(\bar{R}_{t+i}^1),$$

where

$$\bar{R}_t^j = R_t^j - e_t^j,$$

R_t^j being the yield on a j-period security and e_t^j being the yield specific, unique disturbance. It is then possible to deduce the impulse response of an m-period security from that of a one-period security. The response of the m-period security yield at time t is simply the average of the responses of the one-period yield from time t to $t+m-1$. The responses of six-month, one-year, and five-year yields should then be (forward) smoothed versions of the response of three-month yields, where the amount of smoothing increases with maturity. So for example, the response of the six-month rate, a_t^6, can be

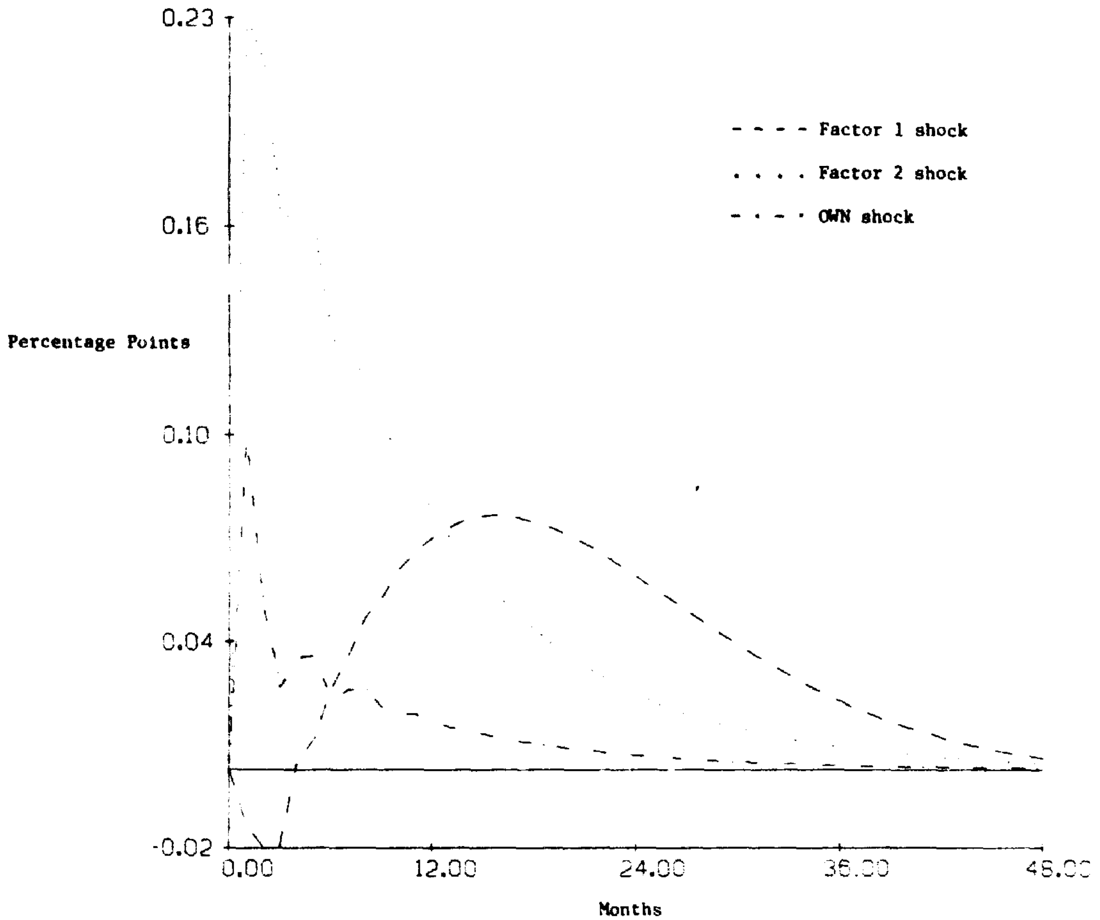

Fig. 4. Response of five-year bond rate.

calculated from the response of the three-month rate, a_t^3, as

$$a_t^6 = \tfrac{1}{2}\left(a_t^3 + a_{t+3}^3\right).$$

Similarly,

$$a_t^{12} = \tfrac{1}{4}\left(a_t^3 + a_{t+3}^3 + a_{t+6}^3 + a_{t+9}^3\right), \quad \text{etc.} \tag{4.5}$$

The impulse responses estimated by model 1 are larger and more persistent than suggested by this term structure model. An example of this is shown in fig. 7 where we compare the responses of one-year yields estimated from model 1 with a set of responses constructed from (4.5). While we have not carried out a formal statistical test our estimates are consistent with the volatility tests presented in Shiller (1979) and Singleton (1980a).

The next step of the empirical analysis involves testing the interpretation of the factors. As discussed in the last section the 'anticipated and unanticipated aggregate demand' interpretation can be tested by including lagged values of

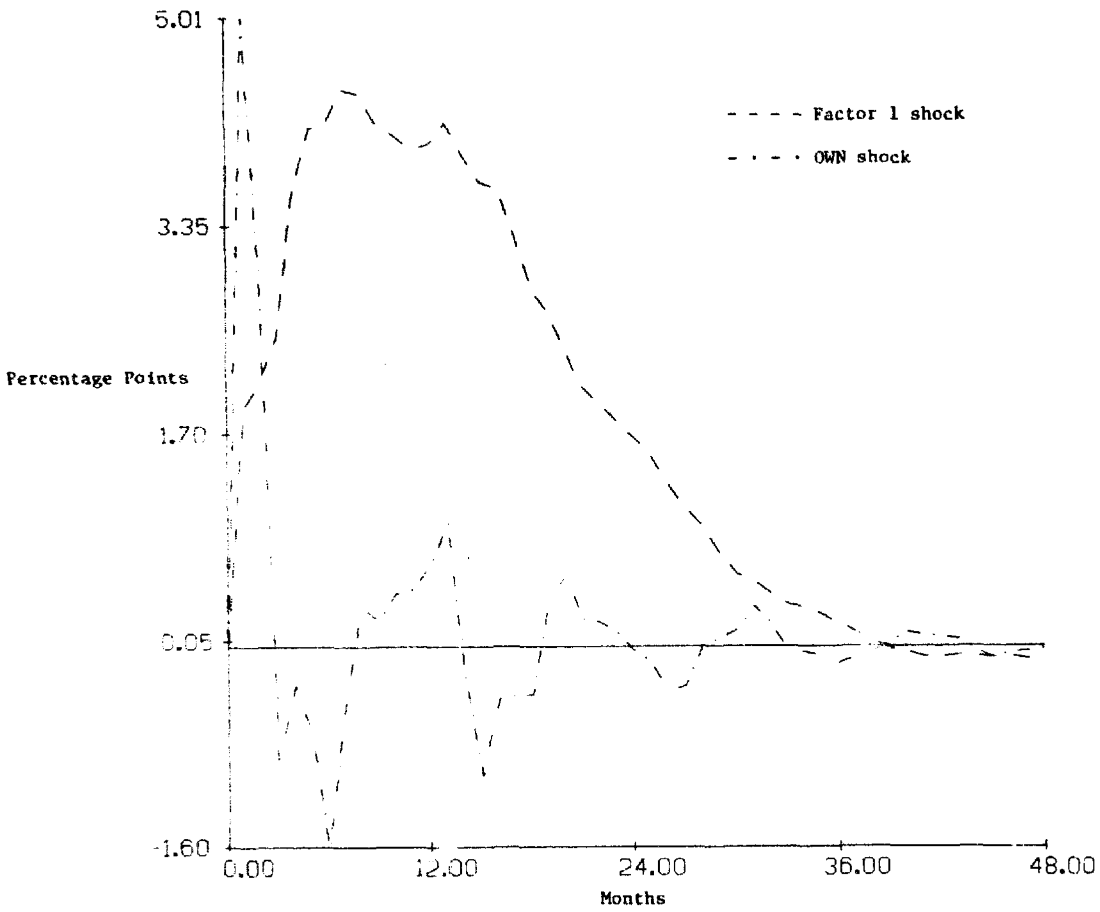

Fig. 5. Response of manufacturers' shipments.

the money supply in the equations for the factors. The change in the logarithm of seasonally unadjusted *M1* was used in the analysis. The data were first regressed on a constant, linear trend, and seasonal dummy variables. The residuals from these regressions were in the analysis below. The second through fourth columns of table 2 report the results for the causal equations. (The results for the indicator equations are not reported. In all models discussed they are very similar.) In the second column we present results using lags 1–4, and 12 of the money supply to explain the second factor only. The likelihood ratio statistic of 3.94 indicates that we cannot reject the hypothesis that the rate of change of *M1* does not cause the second factor. In model 3 the same lags of money are included in the equations for the first and the second factor. The likelihood ratio statistic of model 3 vs. model 2 is 19.08, far above the critical value for a χ^2_5 random variable [χ^2_5 (0.99) = 15.1]. For completeness, model 4 includes lags of money only in the equation for the first factor. Comparing models 3 and 4 it can be seen that the point estimates are reasonably close and the value of the log likelihood function falls only slightly.

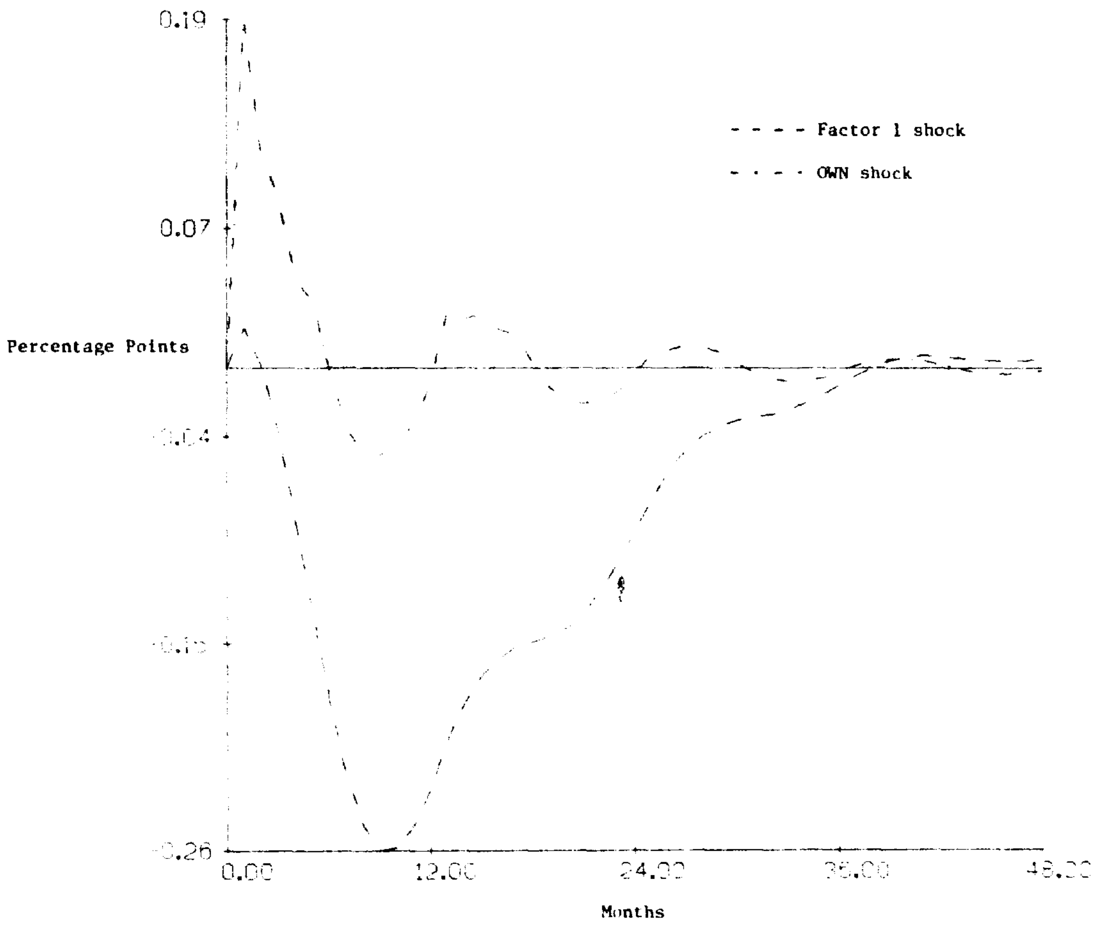

Fig. 6. Response of civilian unemployment rate.

These results are exactly opposite the predictions of the model presented in the last section. Lagged rates of growth of the money supply cause factor 1 (and hence output) and do not cause factor 2. The hypothesis that the first factor corresponds to an 'unanticipated' variable (if 'anticipations' are rational) is strongly rejected.

One possible explanation for part of these findings is that we have erroneously included too much information in the information set. Money supply figures are revised for a few months after their initial publication. These revisions reflect information gathered by the Federal Reserve from non-member banks. The data that we used included these revisions, which could only be observed after a few months delay. To guard against this, we decided to err on the side of omission. Models 2–4 were re-estimated using lags 4–7 and 12 of the rate of change of *M1*. The results of the hypothesis tests were unchanged.[7]

[7]The values of the log likelihood were
Model 2: $L = 616.96$, Model 3: $L = 623.49$, Model 4: $L = 620.96$.

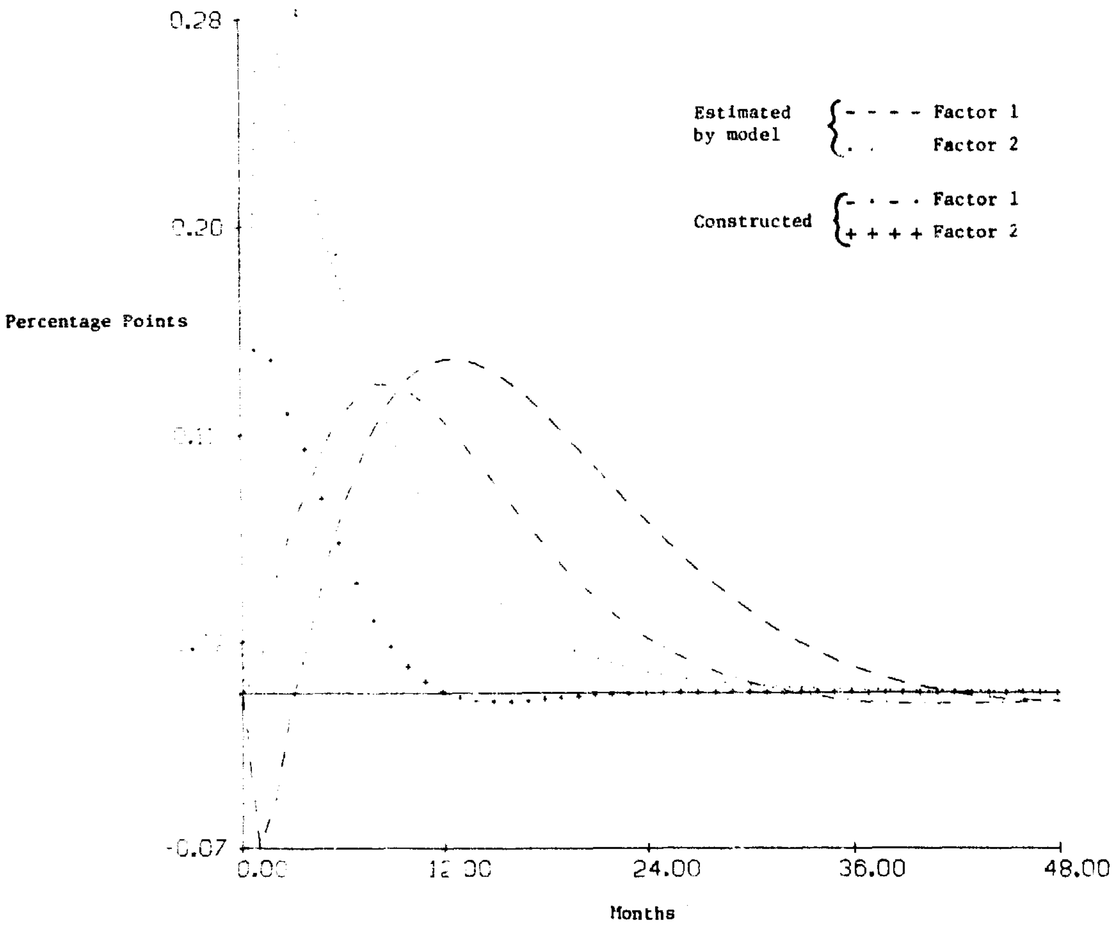

Fig. 7. Responses of one-year rates.

5. Conclusions

The results of the last section indicate that practical time domain methods exist for estimating fairly large, richly parameterized models with unobserved factors. The method proposed has the advantage over previous methods in that causes of the unobserved factors can be directly incorporated in the model. These causes may be extremely useful in interpreting the factors.

One interpretation of the two factors often found in macroeconomic index models is that they represent anticipated and unanticipated aggregate demand. We have shown that rational 'anticipations' impose restrictions on the processes generating the factors that are easily tested. These restrictions are strongly rejected using a data set consisting of four nominal interest rate variables and two real variables.

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