Expected Uncertain Utility and Subjective Sources†

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Abstract

Source preference is the assertion that between two prospects yielding the same distribution of monetary rewards, decision makers may have a strict preference for one over the other. Evidence on the home bias reveals source preference strong enough to preclude investors from taking advantage of diversification opportunities. We show that the EUU model provides a suitable framework for analyzing source preference and the home bias by providing a rich class of sources. We characterize EUU decision makers' risk attitudes within each source, relate these attitudes to uncertainty aversion, source preference and violations of the independence axiom.

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1. Introduction

An agent considers bets on the Democratic vote share in the next election, on the year-end value of the S&P500 or on the roll of a die. Each bet yields 1$ if some specified event occurs and 0 otherwise. The election outcomes are *a source* if the agent's betting preference is coherent when comparing only election related bets. That is, the agent assigns probability ${}^{k}\!/_{n}$ to the election event D if for any partition of the state space into election-events $D_1, \ldots, D_n$ such that she is indifferent between betting on D_i and D_j for all i, j, she is also indifferent between betting on D and $\bigcup_{i=1}^{k} D_i$. We can define the subjective probability of each S&P event or of the outcome of a die analogously if her preference is similarly coherent for bets on the S&P500 or for the outcomes of the die. In this paper, we analyze agents who may have a coherent betting preference for all election-related bets, for all S&P related bets and for all bets on the outcome of die, yet fail to have a coherent betting preference for bets that depend on multiple sources.

Sources, in our model, are *subjective* in the sense that they are derived from the decision-makers prior, which in turn, is derived from her ranking of Savage-acts over monetary prizes. Hence, we do not assume that the agent has coherent preferences of exogenously given collections of events; rather, we show that any agent whose preferences admit an *expected uncertain utility* representation (Gul and Pesendorfer (2012)) and have multiple subjective sources. Our main result (Theorem 1 and Corollary 1) characterizes agent's utility functions for acts that depend on a single source. We provide three applications of our main result. First, we relate the source utility functions to Allais-style (Allais (1953)) evidence, that is, experimental evidence that finds violations of the independence axiom. Second, we analyze the agent's betting preference when she compares bets from different sources. We identify a measure of the uncertainty of a source and relate this measure to the agent's betting behavior. Third, we consider a stylized investment problem in which the agent combines assets from different sources and relate the results to the home bias.

Keynes (1921), Ellsberg's (1961) and Schmeidler (1989) provide thought experiments suggesting that, between two events, agents prefer to bet on the event with a "more certain" or "better known probability" even when the two events have the same probabilities. These thought experiments motivated experimental work on *source preference* (Fox and Tversky

(1995)) that confirmed, quantified and refined the original thought experiments. Two main conclusions emerge from this research: first, source preference is observed not only when choosing between completely vague and probabilistically described sources (Chipman (1960), Curley and Yates (1989)) but also when choosing between sources with somewhat more or somewhat less vague specification or between sources about which agents consider themselves to be more or less informed (Heath and Tversky (1991)). Second, source preference tends to get reversed when the probability of winning becomes sufficiently small. When choosing between bets that have long-odds, decision makers prefer vagueness (Curley and Yates (1989)) or sources about which they are less informed (Heath and Tversky (1991)).[1] In our model, every source is identified by a polynomial that can be used to classify sources as more or less uncertain. We give conditions under which agents prefer bets on events belonging to less uncertain sources when the probability of winning is large but reverse this preference when the probability of winning is small.

Within a source, EUU agents are probabilistically sophisticated (Machina and Schmeidler (1992)) and have utility functions like those studied in the non-expected utility theory literature. For a subclass of EUU agents, these source utilities are rank dependent expected utility (Quiggin (1982)) with an inverted-S probability transformation function in many sources.[2] For the general case, we show that there is a source with quadratic utility, as introduced by Machina (1982), and axiomatized by Chew, Epstein and Segal (1991).

In the typical Allais-style experiment subjects are given objective lotteries, that is, acts that depend on a roulette wheel, on the draws of a card from a deck, or on some other objective randomization device.[3] In our interpretation, the objective randomization devices in Allais-style experiments are a source like any other; there is no presumption that objective randomization devices yield the least uncertain source or that all objective randomization devices yield the same source. Indeed, Heath and Tversky (1991) provide experimental evidence that randomization devices are not perceived to be the least uncertain source. Specifically, they show that subjects tend to prefer familiar "subjective"

[1] See, also, Abdellaoui, Baillon, Placido and Wakker (2010) who provide a method for identifying sources and estimating source-specific lottery preferences.

[2] Kahnemann and Tversky (1979), Tversky and Kahneman (1992), Camerer and Ho, (1994), Wu and Gonzalez, (1996), Prelec (1998) and Abdellaoui (1998) provide evidence in favor of the inverted S-shape.

[3] Of course, some experiments simply state probabilities without specifying a randomization device. We interpret this as a situation where one of many possible randomization devices will be used to determine payoffs but the agent does not know which.

bets over equally likely bets on the outcome of a randomization device if these bets have a high probability of success. More broadly, how a subject perceives a source is an empirical question that cannot be decided based on ex ante views about its nature.

In early research related to the home bias, Grubel (1968) documents the potential welfare gains from holding an internationally diversified portfolio and Lease, Lewellen and Schlarbaum (1974) show that many individuals hold portfolios that are inadequately diversified even within the US market. French and Poterba (1991) quantify the welfare loss associated with international underdiversification and dismiss institutional constraints and transactions costs as possible explanations. They suggest a novel preference model, related to the observations of Heath and Tversky (1991), is needed for its explanation. In section 5, we show how our theory accommodates the home bias. We consider an agent with a risky endowment and show that for any asset that depends on an uncertain source there is a range of prices at which the agent is unwilling to trade (buy or sell) the asset. A related paper by Epstein and Wang (1994) examines asset pricing with maxmin expected utility (MEU) maximizers and establishes conditions under which an asset has a non-trivial no-trade price interval.[4] Their conditions require consumption to be constant over a set of states for which the asset's return varies. Our result does not require a similar assumption; it applies even if the state space is the unit interval and the agent's consumption is strictly increasing in the state.

The theoretical literature contains relatively few multi-source models. Klibanoff, Marinacci and Mukerji (2005) and Ergin and Gul (2009) consider preferences that permit two distinct sources. These papers focus on the relationship between the two-source model and compound lotteries. Chew and Sagi (2008) define sources in a general Savage-style model which they call small worlds. They give conditions under which a lottery preference characterizes the agent's behavior in any source and provide examples of preferences with multiple sources. Nau (2006) provides a more general notion of source and source-dependent risk attitude that permits two such sources with state-dependent preferences.

[4] Epstein (2001) and Epstein and Miao (2003) formulate a two-country general equilibrium model with maxmin expected utility agents and analyze the home bias in this setting.

2. The Utility Function

The agent has a preference over Savage acts that yield a monetary prize in every state. The prize is in the nondegenerate interval $X = [l, m]$; Ω is the state space and $\mathcal{F} = \{f : \Omega \to X\}$ are the Savage acts. To simplify the notation, we write x for the constant act that yields x in every state. We assume the agent satisfies Axioms 1-6 in Gul and Pesendorfer (2012) and, therefore, an expected uncertain utility (EUU) function $W : \mathcal{F} \to I\!R$ represents her preference. The function W has two parameters; a prior μ and an interval utility u.

The prior is a countably additive, complete and non-atomic probability measure μ on some $\sigma-$algebra $\mathcal{E}_\mu$ of subsets of Ω. The $\sigma-$algebra $\mathcal{E}_\mu$ are the events that the agent considers least uncertain. Let $\mathcal{F}_\mu$ denote the $\mathcal{E}_\mu-$measurable acts. For any (possibly non-measurable) act f, let $[f]_1 \in \mathcal{F}_\mu$ be its maximal measurable minorant and let $[f]_2 \in \mathcal{F}_\mu$ be its minimal measurable majorant.[5] We refer to $[f] = ([f]_1, [f]_2)$ as the *envelope* of f.

An interval utility assigns a value to each prize interval; that is, $u : I \to I\!R$ where $I = \{(x, y) \,|\, l \leq x \leq y \leq m\}$; u is continuous and satisfies $u(x, y) > u(x', y')$ if $x > x'$ and $y > y'$. The agent evaluates each act according to the expected interval utility of its envelope, that is, W has the form

$$W(f) = \int u[f] d\mu \tag{1}$$

where $u[f] = u([f]_1, [f]_2)$. We write $W = (\mu, u)$ for the EUU function W with parameters μ, u. The utility index $v_u : X \to I\!R$ is defined as $v_u(x) := u(x, x)$ and, for all $(x, y) \in I$, the parameter σ_u^{xy} is defined to be the unique $\sigma \in [0, 1]$ that satisfies

$$v_u(\sigma x + (1 - \sigma) y) = u(x, y)$$

In Gul and Pesendorfer (2012), we show that the parameter σ_u^{xy} measures the uncertainty aversion of EUU utilities.[6] Specifically, $W = (\mu, u)$ *is more uncertainty averse than* $\bar{W} = (\bar{\mu}, \bar{u})$ if v_u is a positive affine transformation of $v_{\bar{u}}$ and if $\sigma_u^{xy} \geq \sigma_{\bar{u}}^{xy}$.

[5] The act $[f]_1$ is the maximal measurable minorant of f iff $[f]_1$ is $\mathcal{E}_\mu$ measurable, $[f]_1 \leq f$ and if $[f]_1 \geq g$ a.s. for every $\mathcal{E}_\mu$ measurable g such that $g \leq f$. The minimal measurable majorant $[f]_2$ is defined analogously. Lemma 1.2.2 in van der Waart and Wellner (1996) shows that every act $f \in \mathcal{F}$ has an envelope.

[6] See Gul and Pesendorfer, Corollary 1, for a characterization of uncertainty aversion.

A bet is a binary act that yields y if $\omega \in A$ and $x < y$ if $\omega \in A^c := \Omega \backslash A$; we write yAx for the bet on A with prizes x and y. In general, the ranking of bets may depend on the prizes; that is $W(xAy) > W(xBy)$ but $W(x'By') > W(x'Ay')$ for some $x < y, x' < y'$.[7] To simplify the exposition of some of the results in this paper, we rule out prize-dependent betting preferences by adding the following axiom:

Axiom 7: *If $x < y, x' < y'$, then $W(yAx) \geq W(yBx)$ implies $W(y'Ax') \geq W(y'Bx')$.*

Axiom 7 yields the following Lemma[8]:

Lemma 1: *If $W = (\mu, u)$ satisfies Axiom 7, then $u(x, y) = \alpha v(x) + (1 - \alpha)v(y)$ for some strictly increasing and continuous utility index $v : X \to I\!R$.*

We call W that satisfy Axiom 7, *separable symmetric EUU* (SSEUU) and write $W = (\mu, \alpha, v)$ for the SSEUU with parameters μ, α, v. Note that the $W = (\mu, \alpha, v)$ is more uncertainty averse than $W = (\bar{\mu}, \bar{\alpha}, \bar{v})$ if $\alpha \geq \bar{\alpha}$ and v is a concave transformation of $\bar{v}$. In the special case satisfying $\sigma_u^{xy} = 1$ for all $x < y$, we have $u(x, y) = v_u(x)$ for all $(x, y) \in I$ and, therefore, $W = (\mu, u)$ is an SSEUU with $\alpha = 1$; that is, $W = (\mu, 1, v_u)$.

3. Sources and Source Utility

A *source* is a collection of events and a probability measure that represents the agent's betting preference over those events. For example, for $W = (\mu, u)$, the pair $(\mathcal{E}_\mu, \mu)$ is a source because μ represents the agent's betting preference over elements of $\mathcal{E}_\mu$. In this section, we identify all sources of EUU agents and characterize their utility function over acts that are measurable with respect to a source.

A class of events $\mathcal{A}$ is a λ-system if it contains Ω and is closed under the formation of complements and of countable disjoint unions. The pair $(\mathcal{A}, \pi)$ is a λ-prior if $\mathcal{A}$ is a $\lambda-$system and if π is a countably additive, non-atomic probability measure on $\mathcal{A}$. The distinction between a source and a proper source below mirrors the distinction between a $\lambda-$prior and prior.

[7] As we show in Gul and Pesendorfer (2012) this prize-dependence is necessary to address variations of the Ellsberg experiment analyzed by Machina (2011) and L'Haridon and Placido (2010).

[8] Equivalently, instead of adding Axiom 7, we could replace Axiom 4 of Theorem 1 in Gul and Pesendorfer (2012) with Axiom 4$'$: If $x < y, x' < y'$, then $yAx \succeq yBx$ implies $y'Ax' \succeq y'Bx'$ for all A, B. The original Axiom 4 applies only to ideal sets; i.e., sets in $\mathcal{E}_\mu$.

Definition: *The λ-prior (prior) $(\mathcal{A}, \pi)$ is a source (proper source) for μ if for every u, $x < y$ and $W = (\mu, u)$,*

(i) $W(yAx) \geq W(yBx)$ if and only if $\pi(A) \geq \pi(B)$ for $A, B \in \mathcal{A}$ and

(ii) yA_nx converges pointwise to yAx, $A_n \in \mathcal{A}$ implies $W(yAx) = \lim W(yA_nx)$

When the choice of μ is clear, we will omit the phrase 'for μ' and refer to the λ-prior or prior as a source or proper source. Zhang (2002) provides the following example of an urn experiment that suggest that a decision-maker may have coherent betting preferences over collections of sets and yet, it may not be possible to extend these preferences, in a coherent manner, to any algebra containing those events: a single ball is drawn from an urn with two balls. The balls are either red, white, green or blue. It is known that exactly one ball is red or white and exactly one ball is green or blue. Therefore, it is plausible that the agent perceives the events $A = \{red, white\}$ and $B = \{green, blue\}$ as equally likely. It is also known that exactly one ball is either red or green and exactly one ball is white or blue so that the agent plausibly perceives the events $C = \{red, green\}$ and $D = \{white, blue\}$ as equally likely. It is equally plausible that the agent would be indifferent between bets on any two of the four events A, B, C and D. By contrast, an agent who exhibits the typical pattern in the Ellsberg paradox is likely to prefer betting on A over betting on $F = \{red, blue\}$. Note that $F = (A \cap C) \cup (B \cap D)$ and, thus, F can be obtained as the union of intersections of events in $\{A, B, C, D\}$. This suggests that the agent could plausibly perceive A, B, C and D to be part of a source that does not contain F and, therefore, this source cannot be a σ-algebra.

Call a source for μ a Zhang-source (or an improper source) if betting preferences on $(\mathcal{A}, \pi)$ cannot be extended to any σ-algebra that contains $(\mathcal{A}, \pi)$; that is, the source $(\mathcal{A}, \pi)$ is a Zhang-source if there is an event F in the smallest σ-algebra that contains $\mathcal{A}$ such that for some $W = (\mu, u)$ and all $x < y$, we have (1) $W(yFx) = W(yF^cx)$ and (2) $W(yAx) > W(yFx)$ whenever $A \in \mathcal{A}$ and $\pi(A) = 1/2$. Lemma A1, in the appendix, establishes the existence of a rich set of Zhang-sources.[9]

[9] More formally, Lemma 2 below shows that every source is identified with a polynomial. Lemma A1 shows that every polynomial other than the identity has a Zhang-source associated with it.

Note that sources are independent of the interval utility u; that is, a source for μ ensures coherent betting preferences for any W with prior μ. It can be shown that if we exclude the SSEUU $(\mu, 1/2, v_u)$, then *every* $W = (\mu, u')$ has a coherent betting preferences on the λ-system $\mathcal{A}$, whenever *any* $W = (\mu, u)$ has a coherent betting prefence on $\mathcal{A}$. Thus, sources and and proper sources depend only on the prior and, for "generic" EUU decision makers, are independent of the specification of the interval utility.

For any prior μ, define the inner probability μ_* as follows: $\mu_*(A) = \sup_{\substack{E \in \mathcal{E}_\mu \\ E \subset A}} \mu(E)$. For the bet yAx, the EUU formula (1) yields:

$$W(xAy) = \mu_*(A)u(x,x) + \mu_*(A^c)u(y,y) + (1 - \mu_*(A) - \mu_*(A^c))\, u(x,y) \tag{2}$$

Let $\mathcal{Z}$ be the set of all sequences $a = (a_1, a_2, \ldots)$ such that $a_i \in [0,1]$ and $\sum_{i=1}^{\infty} a_i = 1$. A function $\gamma : [0,1] \to [0,1]$ is a polynomial if there is a sequence $a \in \mathcal{Z}$ such that

$$\gamma(t) = \sum_{i=1}^{\infty} a_i \cdot t^i$$

Let Γ be the set of all polynomials and δ^n be the polynomial t^n.

Let $(\mathcal{A}, \pi)$ be a λ-prior. Then, we say that $\gamma \in \Gamma$ is its source-polynomial if

$$\mu_*(A) = \gamma(\pi(A)) \tag{3}$$

for all $A \in \mathcal{A}$. Lemma 2, below, shows that every source has a source-polynomial and, conversely, for every polynomial, there is a corresponding proper source.

Lemma 2: *(i) A λ-prior is a source if and only if it has a source-polynomial; (ii) Every polynomial is the source-polynomial of some proper source.*

Lemma 2, shows that the set of polynomials generated is the same whether we consider sources or proper sources. Equations (2) and (3) establish that the source polynomial γ together with u are sufficient for describing the decision-makers betting preferences at any source; that is, γ and u are enough to compute $W(yAx)$. Hence, for each u, we get exactly the same set of betting behaviors whether we considers sources or proper sources.

Let $(\mathcal{A}, \pi)$ be and let $\beta = \pi(A)$ for $A \in \mathcal{A}$. To evaluate the utility of a bet yAx, we substitute (3) into expression (2) to obtain

$$W(yAz) = \gamma^{\pi}(\beta)u(y,y) + \gamma^{\pi}(1-\beta)u(x,x) + (1-\gamma^{\pi}(\beta)-\gamma^{\pi}(1-\beta))\, u(x,y) \tag{4}$$

Thus, fixing the probability β, the polynomial γ of the source determines the weights on $u(x,x), u(y,y)$ and $u(x,y)$. For example, for $\gamma = \delta^n$, (4) simplifies to

$$W(yAz) = \beta^n u(x,x) + (1-\beta)^n u(y,y) + (1 - \beta^n - (1-\beta)^n) u(x,y) \tag{5}$$

As n increases the weight on $u(x,y)$ increases while the weights on $u(x,x)$ and $u(y,y)$ decrease.

For any source let $\mathcal{F}_{\mathcal{A}}$ denote the collection of $\mathcal{A}-$measurable acts. We call $\mathcal{F}_{\mathcal{A}}$ source-acts or the acts in source $(\mathcal{A}, \pi)$. For any source-act $f \in \mathcal{F}_{\mathcal{A}}$, let $G^f(x) = \pi(\{f \leq x\})$. Then, G^f is the cumulative of f. Let $\mathcal{L}$ be the set of a cumulative distribution functions with support X. We write $F \gg G$ to denote stochastic dominance, that is, $F \leq G$ and $F \neq G$; we write $F_n \Rightarrow F$ to denote convergence in distribution, that is, $\lim_n F_n(x) = F(x)$ at every continuity point of F. A function $V : \mathcal{L} \to I\!R$ is a lottery utility if $V(F) > V(G)$ whenever $F \gg G$ and $\lim V(F_n) = V(F)$ whenever $F_n \Rightarrow F$.

Theorem 1, below, shows that utilities over source acts depend only on their lottery. The lottery utility of a source depends only on its source-polynomial and the interval utility:

Theorem 1: *Let $W = (\mu, u)$ and let $(\mathcal{A}, \pi)$ be a source with polynomial γ. There is a lottery utility V^u_γ such that for all $f, g \in \mathcal{F}_{\mathcal{A}}$,*

$$W(f) \geq W(g) \text{ if and only if } V^u_\gamma(G^f) \geq V^u_\gamma(G^g)$$

In Appendix A, we refine Theorem 1 (Theorem 1A) by adding an explicit formula for the lottery utility V^u_γ; Corollary 1, below, provides this formula for the special case of SSEUUs. We illustrate the general case with the following two examples.

Example 1: The polynomial of the *ideal source*, $(\mathcal{E}_\mu, \mu)$, is δ^1 and $V^u_{\delta^1}$ is expected utility with utility index v_u.

Example 2: A source with polynomial δ^2 has quadratic lottery utility (Machina (1982), Chew et al. (1991)): for $w = (w_1, w_2) \in X^2$, let $x_w = \min\{w_1, w_2\}$, $y_w = \max\{w_1, w_2\}$ so that $(x_w, y_w) \in I$. Then, $V^u_\gamma(F) = \int_{w_1 \in X} \int_{w_2 \in X} u(x_w, y_w) dF(w_1) dF(w_2)$. Thus, the quadratic utility of F is the expected value of $u(x_w, y_w)$ where $w = (w_1, w_2)$ is the random variable obtained by taking two independent draws from F.

Rank-dependent expected utility (RDEU, Quiggin (1982)) is a well-known class of lottery utilities commonly used in experimental studies that examine violations of the independence axiom (see, for example, the survey by Starmer (2000)). Rank dependent utility has two parameters, a *probability transformation function (PTF)* and a von Neumann-Morgenstern utility index. A PTF is a continuous, increasing bijection $\tau : [0,1] \to [0,1]$. Let T be the set of all PTFs. For $F \in \mathcal{L}$ and $\tau \in T$, let $F^\tau(x) = 1 - \tau(1 - F(x))$ for all $x \in X$. The lottery utility V is a rank-dependent expected utility if there is $\tau \in T$ and a continuous, strictly increasing function $v : X \to I\!R$ such that

$$V(F) = \int v dF^\tau \tag{6}$$

We let $V^{\tau v}$ denote the RDEU with utility index v and PTF τ.

Note that the polynomials are subset of PTFs. For any $\gamma \in \Gamma$, let $\hat{\gamma}(t) = 1 - \gamma(1-t)$ and note that $\hat{\gamma}$ is also a PTF. The following Corollary (to Theorem 1A) shows that the SSEUU $W = (\mu, \alpha, v)$ is a RDEU with utility index v in every source; the sources' polynomial together with the parameter α determine the RDEU's PTF.

Corollary 1: *Let $W = (\mu, \alpha, v)$. Then, $V^u_\gamma = V^{\tau v}$ for $\tau = \alpha\gamma + (1-\alpha)\hat{\gamma}$.*

Corollary 1 characterizes the model Abdellaoui, Baillon, Placido and Wakker (2012) use to identify sources and to estimate source specific lottery preferences. Their estimates reveal that typical sources have PTFs with an inverted S-shape. Kahnemann and Tversky (1979), Tversky and Kahneman (1992), Camerer and Ho, (1994), Wu and Gonzalez, (1996), Prelec (1998) and Abdellaoui (1998) provide earlier evidence in favor of PTFs with an inverted S-shape. More specifically, PTFs that provide a good fit are concave on $[0, t^o]$, convex on $[t^o, 1]$ for some $t^o \in (0, 1/2)$ and cross the $45°$ line between 0 and 1/2. PTFs of the form

$$\tau = \alpha\gamma + (1-\alpha)\hat{\gamma} \tag{7}$$

have these properties whenever $1/2 < \alpha < 1$ and γ places enough weight on higher order polynomials. More precisely, let $\gamma = \sum a_i \delta^i$ and $1/2 < \alpha < 1$. Then, γ has the above mentioned properties if

$$a_1 + (1-\alpha)\sum_{i=2}^{\infty} i \cdot a_i > 1$$

For example, if $\gamma = a_1\delta^1 + (1-a_1)\delta^i$, the PTF in equation (7) has the desired properties if $(1-\alpha)i > 1$.

RDEUs with an inverted-S PTF are not globally risk averse. Specifically, such decision makers may be risk loving for a binary gamble with small odds of success. We conclude this section by characterizing EUU utilities that are always risk averse. The EUU $W = (\mu, u)$ is *risk averse* at source $(\mathcal{A}, \pi)$ if $V^u_\gamma(F) \geq V^u_\gamma(G)$ whenever G is a mean preserving spread of F. Theorem 2, below, relates risk aversion at every source to Schmeidler (1989)'s notion of uncertainty aversion and to the functional form of u.

Theorem 2: *For any $W = (\mu, u)$ the following conditions are equivalent:*

(i) W is risk averse at every source;

(ii) $W(g) \geq W(f)$ and $\alpha \in [0,1]$ implies $W(\alpha f + (1-\alpha)g) \geq W(f)$;

(iii) v_u is concave and $\sigma^{xy}_u = 1$ for all $(x, y) \in I$.

The parameter σ^{xy} measures comparative uncertainty aversion and $\sigma^{xy} = 1$ corresponds to maximal uncertainty aversion.[10] Theorem 2 shows that for EUU decision makers, risk and uncertainty aversion are linked; agents who are risk averse in every source must be maximally uncertainty averse.

Though formally identical to Schmeidler's notion of uncertainty aversion, (ii) differs from Schmeidler's definition since acts yield monetary prizes in our model whereas they

[10] See Gul and Pesendorfer (2012) for a discussion of how our comparative measure of uncertainty is related the Epstein (1999)'s comparative measure. Since we work in a Savage setting, the comparative notion of Ghirardato and Marinacci (2003) does not apply. Nonetheless, a plausible adaptation of their definition to our setting would imply that greater σ^{xy} implies more ambiguity aversion according to their definition.

yield lotteries in Schmeidler's model. As a consequence (ii) implies that the utility index v must be concave while Schmeidler's definition has no such implication.[11]

4. Source Preference

In this section, we analyze how EUU agents choose between bets on events from distinct sources. First, we introduce a comparative measure of the uncertainty of a source. Next, we use this measure to analyze how EUU agents rank bets with equal odds from distinct sources.

For a fixed prior μ, we say that event A dominates event B if for every $W = (\mu, u)$ and $y > x$, $W(yAx) \succeq W(yBx)$. Hence, A dominates B if every EUU decision-maker with prior μ prefers betting on A to betting on B. If neither A nor B dominate the other, we say A and B are comparable. Event B is *more uncertain than* event A if (i) A and B are comparable and (ii) $W(yAx) \succeq W(yBx)$ for $W = (u, \mu)$ implies $\bar{W}(yAx) \succeq \bar{W}(yBx)$ for $\bar{W} = (\bar{u}, \mu)$ whenever $\bar{W}$ is more uncertainty averse than W. We can use this measure to compare the uncertainty of sources:

Definition: *Source $(\mathcal{A}, \pi)$ is more uncertain than source $(\mathcal{A}^o, \pi^o)$ if $A \in \mathcal{A}, B \in \mathcal{A}^o$ and $\pi(A) = \pi^o(B)$ imply A is more uncertain than B.*

Lemma 3, below, shows that its polynomial measures the uncertainty of a source. We write $\gamma^\pi < \gamma^{\pi^o}$ to mean $\gamma^\pi(t) < \gamma^{\pi^o}(t)$ for all $t \in (0, 1)$.

Lemma 3: *Source $(\mathcal{A}, \pi)$ is more uncertain than source $(\mathcal{A}^o, \pi^o)$ if and only if $\gamma^\pi < \gamma^{\pi^o}$.*

The simplest kind of source preference is an unequivocal preference for less uncertain sources. The Ellsberg two-urn paradox provides empirical evidence for this ranking. Subjects are told that both urns contain 100 balls, either red or black. Urn 1 has exactly 50 red and 50 black balls while no additional information about the composition of urn 2 is given. Subjects are asked to rank B_1, R_1, B_2, R_2 where C_i is a bet that delivers $x > 0$ dollars if a ball of color $C \in \{B, R\}$ is drawn from urn $i \in \{1, 2\}$. Subjects tend to be indifferent between B_1 and R_1 and between B_2 and R_2 but strictly prefer the first two bets to the

[11] Schmeidler's definition implies that the capacity must be convex. As we demonstrate in Gul and Pesendorfer (2012), SSEUU maximizers are Choquet expected utility maximizers (Choquet (1953-54)) with a capacity that is convex if and only if $\alpha = 1$.

last two. This is source preference in its purest form: when choosing among "identical" 50-50 percent chance prospects, decision makers prefer the "less uncertain" urn.[12]

Definition: *The utility W is averse to uncertain sources if $\gamma \geq \gamma^o$ implies $V^u_\gamma \geq V^u_{\gamma^o}$.*

Aversion to uncertain sources is a simple property that matches Ellsberg original insight. Subsequent experimental research, however, has consistently yielded a more equivocal attitude towards more uncertain sources. Curley and Yates (1989) confirm the original preference for less uncertain sources when the odds of winning are large but document the reverse preference when the odds of winning are small. We call this pattern of behavior *uncertainty loving at poor odds.* Heath and Tversky (1991) and Abdellaoui et al. (2010) identify a formally identical pattern: subjects prefer betting on the issue about which they are more knowledgeable when the (common) odds are winning are favorable but prefer betting on the issue about which they are less knowledgeable when the odds of winning are unfavorable.[13]

Consider any sequence of polynomials, γ^n, such that $\lim_n \gamma^n(t) = 0$ for all $t < 1$. In that case, the source associated with γ^n become arbitrarily uncertain as n approaches infinity and we say that γ^n *converges to maximal uncertainty.*[14] Let $\bar{z}$ denote the degenerate lottery that yields z for sure.

Definition: *The utility $W = (\mu, u)$ is uncertainty loving at poor odds if for every $\gamma, x < y$, there is $r \in (0,1)$ such that for γ^n converging to maximal uncertainty,*

$$\lim_{n\to\infty} V^u_{\gamma^n}(\beta\bar{y} + (1-\beta)\bar{x}) \geq V^u_\gamma(\beta\bar{y} + (1-\beta)\bar{x})$$

if and only if $\beta \leq r$.

[12] With the aid of a procedure that Machina (2009) calls "orthogonal representation" even the Ellsberg single-urn paradox has been interpreted as relative uncertainty aversion. Machina (2009) credits Anscombe and Aumann (1963) with the idea of orthogonal representation.

[13] Curley-Yates and Heath-Tversky differ in how they identify more or less uncertain environments. In Curley and Yates decision makers identify one source as subjectively more uncertain when its description is more "vague." In Heath and Tversky more uncertain environments are environments about which decision makers perceive themselves to be less knowledgeable.

[14] Let $x(F) = \sup\{x \in X \mid F(x) = 0\}$ and let $y(F) = \inf\{x \in X \mid F(y) = 1\}$ that is, $x(F), y(F)$ are the minimal and maximal elements in the support of F. It can be shown that for any sequence γ^n converging to maximal uncertainty $\lim V^u_{\gamma^n}(F) = u(x(F), y(F))$.

The definition above compares the same bet on equally likely events in two distinct sources. It requires there to be a threshold r such that the agent prefers the very uncertain source if and only if the odds of winning are smaller than r.

Theorem 3: *The utility $W = (\mu, u)$ is*

(i) averse to uncertain sources if and only if $\sigma_u^{xy} = 1$ for all $(x, y) \in I$;

(ii) uncertainty loving at poor odds if and only if $0 < \sigma_u^{xy} < 1$ for all $(x, y) \in I$.

To illustrate part (ii) of Theorem 3, let $X = [0, m]$ and consider the SSEUU with $v(x) = x$ and $\alpha = 3/4$. Let F be the lottery that yields $x > 0$ with probability p and 0 with probability $1 - p$. Consider a source $(\mathcal{A}, \pi)$ with polynomial $\gamma^\pi = \delta^n$. Then, the utility of F in this source is $V^{\tau v}(F)$ where $\tau = {}^3\!/_4 \cdot p^n + {}^1\!/_4 \cdot (1 - (1 - p)^n)$ and, thus,

$$V^{\tau v}(F) = x \cdot \left({}^3\!/_4 \cdot p^n + {}^1\!/_4 \cdot (1 - (1 - p)^n)\right) \tag{8}$$

If $p = {}^1\!/_2$ and hence F offers an equal chance of winning (the prize x) and losing (the prize 0), the right hand side of equation (8) is decreasing in n and therefore the agent prefers less uncertain environments. However, if p is small (for example $p = .1$) and hence the lottery F offers a small chance of winning, the right hand side of equation (8) is increasing in n and, therefore, the decision maker prefers more uncertain environments.

5. The Home Bias

The home bias refers to the tendency of investors not to allocate *any* fraction of their assets outside their domestic market. Since foreign markets offer diversification benefits, the home bias refers to a particularly strong form of a source preference.

Our analysis of the home bias considers the following setting. An investor is endowed with an act f and considers buying or selling a security 1_A that yields 1\$ in the event A and zero otherwise. The asset has price c and, therefore, after trading a units the investor ends up with $g_a = f + a \cdot (1_A - c)$. In a standard setting where the investor is an expected utility maximizer there is a unique price c^* for which the optimal a is zero. At any price other than c^* this agent will either buy or sell the asset. This observation makes it clear why the home bias is a puzzle: it is unlikely that a significant fraction of investors would, in equilibrium, choose not to trade in any particular asset.

We assume that the agent is risk averse with respect to every source and therefore W is the SSEUU $(\mu, 1, v)$ with v concave.[15] A source is *non-hedging* if for every A in that source such that $0 < \pi(A) < 1$, there is an interval of prices such that the agent does not trade in the asset 1_A if her endowment is f.

Definition: *The source* $(\mathcal{A}, \pi)$ *is non-hedging if* $0 < \pi(A) < 1$ *implies there are* $c_* < c^*$ *such that* $W(f) \geq W(f + a \cdot (1_A - c))$ *whenever* $f + a \cdot (1_A - c) \in \mathcal{F}$ *and* $c \in [c_*, c^*]$.

Theorem 4 below shows that if f is in the ideal source, $\mathcal{F}_\mu$, then every other source is non-hedging.

Theorem 4: *Let* $W = (\mu, 1, v)$ *with* v *concave and let* $f \in \mathcal{F}_\mu$. *If* $(\mathcal{A}, \pi)$ *is a source with* $\gamma^\pi \neq \delta^1$, *then* $(\mathcal{A}, \pi)$ *is non-hedging.*

Theorem 4 shows that within EUU theory, there is a rich class of non-hedging sources and therefore the model offers a suitable framework for analyzing the home bias. Results similar to Theorem 4 have appeared in the literature. Dow and Werlang (1992) show that in a static model with one risky and one riskless asset and maxmin preferences, there is a set of asset prices that support the optimal choice of a riskless portfolio. Hence, their non-hedging relies on the equilibrium consumption being riskless.

Epstein and Wang (1994) analyze price-indeterminacy in a dynamic economy with maxmin expected utility preferences. Translated to our terminology, an asset price is indeterminate if the asset is in a non-hedging source for the representative household. They show that, in a dynamic model, non-hedging can occur even when the equilibrium positions are not riskless; they only require that the returns to the relevant assets are non-measurable with respect to consumption.[16] In our model, non-measurability is satisfied by definition; that is, if two acts belong to distinct sources they are always mutually non-measurable. Hence, EUU theory relates non-hedging (i.e., price indeterminacy) to source preference and can generate non-hedging even in a static setting.

[15] Theorem 5 in Gul and Pesendorfer (2012) shows that an SSEUU with $\alpha = 1$ is a Choquet expected utility with a belief function as a capacity.

[16] See Epstein and Wang (1994), p. 305.

6. Conclusion

We analyzed sources and the source preference of EUU maximizers. We show that every EUU admits a large number of sources, each characterized by a polynomial that measures the uncertainty of the source. When the utility index is separable, EUU agents are rank dependent expected utility maximizers in every source and each source gives rise to a probability transformation function. We provide conditions under which such RDEU have S-shaped probability transformation function, a specification often used to address Allais-style evidence.

We show that EUU agents may avoid trading assets in uncertain sources and, therefore, EUU theory offers a framework for analyzing the home bias. A better understanding of when the home bias may or may not occur requires a complete analysis of multi-source preferences; that is, a characterization of how arbitrary EUU functions behave on multi-source portfolios. Theorem 1 provides this type of characterization for single-source act. An analogous characterization for two and multi-source acts is left for future research.

7. Appendix

7.1 Preliminaries

Let μ be a prior. Recall that its inner probability μ_* is defined as follows: $\mu_*(A) = \sup_{\substack{E\in\mathcal{E}_\mu \\ E\subset A}} \mu(E)$. For any $A \subset \Omega$, the *core* of A is a set $E \subset A$ such that $\mu(E) = \mu_*(A)$. It is easy to verify that every A has a core and that this core is unique up to a set of μ-measure 0.

Let $E \in \mathcal{E}_\mu$, $N = \{1, \ldots, n\}$ and $\{A_i\}_{i\in N}$ be a finite partition of E. Let $\mathcal{N}$ be the set of all nonempty subsets of N and for $J \in \mathcal{N}$, let $\mathcal{N}(J) = \{L \in \mathcal{N} \,|\, L \subset J\}$. Let $A^J = \bigcup_{i\in J} A_i$, let C^J be the core of A^J and let $C^N = E$. The *ideal split* $\{E_*^J\}_{J\in\mathcal{N}} \subset \mathcal{E}_\mu$ of $\{A_i\}_{i\in N}$ is inductively defined as follows: $E_*^{\{i\}} := C^{\{i\}}$ for all $i \in N$; for J such that $|J| > 1$,

$$E_*^J := C^J \backslash \left(\bigcup_{\substack{L\in N(J) \\ L\neq J}} E_*^L \right)$$

Note that $\{E_*^J\}$ is a partition of E that satisfies $\bigcup_{L\in\mathcal{N}(J)} E_*^L \subset A^J$ for all $J \in \mathcal{N}$ and $\mu_*(A^J) = \mu(C^J) = \sum_{L\in\mathcal{N}(J)} \mu(E_*^L)$.

Let $\mathcal{F}^o$ denote the set of simple acts. For any act $f \in \mathcal{F}^o$ with range $\{x_1, \ldots, x_n\}$ let $\{E_*^J(f)\}$ be the ideal split of $\{f^{-1}(x_i)\}$. Then, $\mathbf{f} \in \mathbf{F}_\mu$ such that

$$\mathbf{f}(\omega) = (\min_{i\in J} x_i, \max_{i\in J} x_i) \tag{A1.1}$$

for $\omega \in E_*^J(f)$ is the envelope of f. (This is Lemma A1 in Gul and Pesendorfer (2012).) For $W = (\mu, u)$ (A1.1) implies that

$$W(f) = \sum_{J\in\mathcal{N}} \mu(E_*^J) u(\min_{i\in J} x_i, \max_{i\in J} x_i) \tag{A1.2}$$

If $f = x_2 A x_1$ then $\mu(E_*^{\{2\}}) = \mu_*(A), \mu(E_*^{\{1\}}) = \mu_*(A^c)$ and, therefore, for $x_2 > x_1$,

$$W(f) = \mu_*(A)u(x_2, x_2) + \mu_*(A^c)u(x_1, x_1) + (1 - \mu_*(A) - \mu_*(A^c))u(x_1, x_2) \tag{A1.3}$$

A set D is diffuse for μ if $\mu_*(D) = \mu_*(D^c) = 0$. Let $\mathcal{D}$ be the set of all diffuse sets for μ. If the continuum hypothesis holds, then, for any μ there is a pairwise disjoint collection $\{D_1, D_2, \ldots\}$ such that $D_i \in \mathcal{D}$ and $\bigcup_{i\geq 1} D_i = \Omega$. (This is an immediate consequence of Lemma A2 in Gul and Pesendorfer (2012).)

7.2 Proof of Lemma 1

For $D \in \mathcal{D}, y > x, f = yDx$ (A1.3) implies $W(yDx) = u(x, y)$. Since u is continuous with $u(x, x) \leq u(x, y) \leq u(y, y)$ and μ is non-atomic and hence convex-valued it follows that there is E such that

$$W(yEx) = \mu(E)u(y, y) + (1 - \mu(E))u(x, x) = W(yDx) = u(x, y)$$

Set $\alpha = 1 - \mu(E)$. Then, Axiom 7 implies that $\alpha u(x, x) + (1 - \alpha)u(y, y) = u(x, y)$ for all $x < y$ and, since v_u is strictly increasing, Lemma 1 follows. $\square$

7.3 Proof of Lemma 2

Part (i): Let $W = (\mu, u)$ with $u(x, y) = x$ for all $(x, y) \in I$. Then, applying (A1.3), we get $W(yAx) = y\mu_*(A) + x(1 - \mu_*(A))$. It follows that for any $\lambda-$source $(\mathcal{A}, \pi)$ and $y > x$,

$\pi(A) \geq \pi(B)$ if and only if $\mu_*(A) \geq \mu_*(B)$. Setting $\gamma(t) = \mu_*(A)$ for some A such that $\pi(A) = t$ ensures that $\gamma : [0,1] \to [0,1]$ is well-defined.

To show that γ is a polynomial let $0 < t = {}^k\!/_n < 1$ be a rational number. For any integer $r \geq 1$, consider a partition $\{A_i\}_{i \in N_r} \subset \mathcal{A}$ of Ω such that $N_r = \{1, \ldots, nr\}$ and $\pi(A_i) = 1/nr$ for all $i \in N_r$. Since $\pi(\bigcup_{i \in J} A_i) = \pi(\bigcup_{i \in L} A_i)$ whenever $|L| = |J|$ it follows that $\mu_*(\bigcup_{i \in J} A_i) = \mu_*(\bigcup_{i \in L} A_i)$ whenever $|L| = |J|$ and therefore

$$\mu_* \left(\bigcup_J A_i \right) = \gamma(t) \tag{A2}$$

for any $J \subset N_r$ with $|J| = rk$. Let $\{E_*^J\}$ be an ideal split of the partition $\{A_i\}_{i \in N_r}$. Since $\pi(\bigcup_{i \in J} A_i) = \pi(\bigcup_{i \in L} A_i)$ whenever $|L| = |J|$ a straightforward inductive argument shows that $\mu(E_*^J) = \mu(E_*^L)$ whenever $|J| = |L|$. Hence, we can define

$$a(j, nr) = \binom{nr}{j} \mu(E_*^J)$$

for J such that $|J| = j$. Since $\{E_*^J\}$ is a partition of Ω,

$$\sum_{j=1}^{nr} a(j, nr) = 1$$

Let $K_r = \{1, \ldots, kr\}$. Equation (A2) implies,

$$\begin{aligned} \gamma(t) = \mu_*(\bigcup_{i \in K_r} A_i) &= \sum_{\substack{L \subset K_r \\ L \neq \emptyset}} \mu(E_*^L) \\ &= \sum_{i=1}^{kr} \sum_{\substack{L \subset K_r \\ |L| = i}} a(i, nr) \binom{nr}{i}^{-1} \\ &= \sum_{i=1}^{kr} a(i, nr) \binom{kr}{i} \binom{nr}{i}^{-1}. \end{aligned}$$

Let $a_i(r) = a(i, nr)$ and $b_i(r) = \binom{kr}{i}\binom{nr}{i}^{-1}$. Note that $\lim_{r \to \infty} b_i(r) = ({}^k\!/_n)^i$. To see this, observe that $b_i(r)$ is the probability of drawing i red balls in i tries, without replacement, from an urn that has nr balls, kr of which are red, while $({}^k\!/_n)^i$ is the corresponding probability when the draws are made with replacement. As r goes to infinity, the two probabilities become the same.

Since the sequence (in r) $\{a_i(r)\}$ lies in a compact set for each i, it has a convergent subsequence. With a diagonal argument, we can find a subsequence such that $\{a_i(r_j)\}$ converges for all i. Without loss of generality, assume r_j is $r = 1, 2, \ldots,$ itself and let $a_i = \lim_{r\to\infty} a_i(r)$. Hence,

$$\gamma(t) = \sum_{i=1}^{\infty} a_i t^i \tag{A3}$$

as desired. Since $(\mathcal{A}, \pi)$ is a source, γ is continuous. Hence, equation (A3) holds for irrational t's as well.

To prove the converse, suppose for some $\lambda-$prior $(\mathcal{A}, \pi)$ and polynomial γ,

$$\mu_*(A) = \gamma(\pi(A))$$

for all $A \in \mathcal{A}$. Then,

$$\begin{aligned} W(yAx) &= \gamma(\pi(A))u(y,y) + [1 - \gamma(\pi(A)) - \gamma(\pi(A^c))]u(x,y) + \gamma(\pi(A^c))u(x,x) \\ &= \gamma(\pi(A))u(y,y) + [1 - \gamma(\pi(A)) - \gamma(1-\pi(A))]u(x,y) + \gamma(1-\pi(A))u(x,x) \\ &= \gamma(\pi(A))[u(y,y) - u(x,y)] + [1 - \gamma(1-\pi(A))][u(x,y) - u(x,x)] + u(x,x). \end{aligned}$$

Since this expression is continuous and strictly increasing in $\pi(A)$ for every interval utility u and $y > x$, we conclude that π is a source.

Part (ii): Fix a prior μ and polynomial, γ such that $\gamma(t) = \sum a_i t^i$ for some $a \in \mathcal{Z}$. We will construct a prior, $(\mathcal{E}_\pi, \pi)$, such that $\mu_*(A) = \gamma(\pi(A))$ for all $A \in \mathcal{E}_\pi$. Since every prior is convex valued, it follows that for every cumulative F, there is $g \in \mathcal{F}_\mu$ such that $G^g_\mu = F$. Choose a countable set $Y = \{z_n\} \subset X$ such that $z_i < z_{i+1}$ for all i and let $g_0 \in \mathcal{F}^o_\mu$ be such that $\mu(g_0^{-1}(z_i)) = a_i$. As we noted in section 7.1, there exists a collection of pairwise disjoint diffuse sets $\{D_1, D_2, \ldots\}$ such that $\bigcup_i D_i = \Omega$. For every $i = 1, 2, \ldots$, choose $g_i \in \mathcal{F}_\mu$ such that g_i is uniformly distributed on $[l, m]$ and the sequence $g_0, g_1, \ldots$ is a sequence of independent random variables. Define $g \in \mathcal{F}$ such that

$$g(\omega) = \begin{cases} g_i(\omega) & \text{if } \omega \in D_i \cap g_0^{-1}(z_j) \text{ for } i < j \\ g_j(\omega) & \text{if } \omega \in \bigcup_{i \geq j} D_i \cap g_0^{-1}(z_j) \end{cases}$$

Let ν be the unique measure on the Borel sets of X such that $\nu[l, x] = \frac{x-l}{m-l}$ for all $x \in X$. Let $\mathcal{E}^g$ be the σ-algebra generated by g so that for any $A \in \mathcal{E}^g$, there is a Borel set $B \subset X$ such that $A = g^{-1}(B)$. For $A = g^{-1}(B) \in \mathcal{E}^g$, let $\pi(g^{-1}(B)) = \nu(B)$. Finally, let $(\mathcal{E}_{\pi^g}, \pi^g)$ be the completion of $(\mathcal{E}^g, \pi)$, as defined in Billingsley (1995), page 49.[17] We claim that π^g is a prior and that

$$\mu_*(A) = \gamma(\pi^g(A)) \tag{A4}$$

for every $A \in \mathcal{E}_{\pi^g}$. Since g is onto, π is a probability measure. Since ν is nonatomic, so are π and π^g. Hence, π^g is a prior.

Next, we prove (A4) for $A \in \mathcal{E}^g$. Let $A = g^{-1}(B)$ and let $E_i^A = g_0^{-1}(z_i) \cap \bigcap_{j=1}^{i} g_j^{-1}(B)$. Clearly, $E_i^A \in \mathcal{E}_\mu$ and $E_i^A \subset A$ and therefore, $\bigcup_{i\geq 1} E_i^A \subset A$. Also, the sets E_i^A are pairwise disjoint and therefore,

$$\mu(\bigcup_{i\geq 1} E_i^A) = \sum_{i\geq 1} \mu(E_i^A) = \sum_{i\geq 1} a_i \cdot (\nu(B))^i = \gamma(\pi(A)).$$

Hence, $\mu_*(A) \geq \gamma(\pi(A))$. Suppose $E \in \mathcal{E}_\mu, \mu(E) > 0$ and $E \not\subset \bigcup_{i\geq 1} E_i^A$. Let $E' = E \backslash \left(\bigcup_{i\geq 1} E_i^A\right)$ and let i be such that $E'' := E' \cap g_0^{-1}(z_i) \neq \emptyset$. It follows that $E'' \subset g_0^{-1}(z_i) \cap (E_i^A)^c$. This, in turn, implies that $E'' \cap g_j^{-1}(B^c) \cap D_j \neq \emptyset$ for some $j \leq i$ and, therefore, $E \not\subset g^{-1}(B)$. We conclude that $\mu_*(A) \leq \mu(\bigcup_{i\geq 1} E_i^A) = \gamma(\pi(A))$ as desired.

It is easy to verify that for any $A \in \mathcal{E}_{\pi^g}$, there is $A_1 \subset A \subset A_2$ such that $A_1, A_2 \in \mathcal{E}^g$ and $\pi(A_1) = \pi(A_2)$. Therefore, since (A4) holds for all $A \in \mathcal{E}^g$, it must also hold for all $A \in \mathcal{E}_{\pi^g}$. Then, part (i) of this lemma ensures that $(\mathcal{E}_{\pi^g}, \pi^g)$ is a proper source.

□

Lemma A1: *For every prior μ, there is a Zhang-source $(\mathcal{A}, \pi)$.*

Proof: Let D_1, D_2 be a partition of Ω into two diffuse sets. Choose four independent, uniformly distributed random variables, $f_i, i = 1, 2, 3, 4$ on $(\mathcal{E}_\mu, \mu)$ with support X. Let

$$g_1(\omega) = \begin{cases} f_1(\omega) & \text{if } \omega \in D_1 \\ f_2(\omega) & \text{if } \omega \in D_2 \end{cases}$$

[17] $\mathcal{E}_{\pi^g}$ consists of the sets A for which there are $B, C \in \mathcal{E}^g$ such that $\pi(C) = 0$ and the symmetric difference of A and B is contained in C. The the completion of π is the unique probability measure π^g, defined on $\mathcal{E}_{\pi^g}$, that agrees with π on $\mathcal{E}^g$.

and let

$$g_2(\omega) = \begin{cases} f_3(\omega) & \text{if } \omega \in D_1 \\ f_4(\omega) & \text{if } \omega \in D_2 \end{cases}$$

Let ν be the unique measure on the Borel sets of X such that $\nu[l, x] = \frac{x-l}{m-l}$ for all $x \in X$. For $i = 1, 2$, let $(\mathcal{E}_{\pi^{g_i}}, \pi^{g_i})$ be the proper source constructed from g_i as described in the proof of Lemma 2(ii). It is easy to verify that the source-polynomial of both of these sources is δ^2. To simplify the notation, we write $(\mathcal{E}_i, \pi_i)$ instead of $(\mathcal{E}_{\pi^{g_i}}, \pi^{g_i})$.

Let $\mathcal{E}_1' = \{(B \cup C) \cap F \,|\, B \in \mathcal{E}_1, C, F \in \mathcal{E}_2, \pi_2(F) = 0, \pi_2(C) = 1\}$. We claim that $\mathcal{E}_1'$ is a σ-algebra. To see this, first note that $\Omega = (\Omega \cup \emptyset) \cap \Omega \in \mathcal{E}_1^1$. Then, for $A \in \mathcal{E}_1'$, call $(B \cup C) \cap F$ a π_1'-*breakdown* of A if $A = (B \cup C) \cap F$, $B \in \mathcal{E}_1$, $C, F \in \mathcal{E}_2$, $\pi_2(C) = 0$ and $\pi_2(F) = 1$. Let $(B \cup C) \cap F$ be a π_1'-breakdown of A. Then, $A^c = (B^c \cup F^c) \cap (C^c \cup F^c)$. Since $\pi_2(C) = 0$ and $\pi_2(F) = 1$, we have $\pi_2(C^c) = 1$ and $\pi_2(F^c) = 0$; since π_2 is complete and $\pi_2(C^c) = 1$, we have $(C^c \cup F^c) \in \mathcal{E}_2$ and $\pi_2(C^c \cup F^c) = 1$. Therefore, $(B^c \cup F^c) \cap (C^c \cup F^c)$ is a π_1'-*breakdown* of A^c and hence $A^c \in \mathcal{E}_1'$.

To conclude the proof that $\mathcal{E}_1'$ is a σ-algebra, we will prove that countable unions of elements of $\mathcal{E}_1'$ are in $\mathcal{E}_1'$. Let $(B^i \cup C^i) \cap F^i$ be a π_1'-breakdown of A^i and let $A = \bigcup_{i=1}^\infty A^i$. Then, let $B = \bigcup_i B^i$, $C = \bigcup_i C^i$, $G = \bigcap_i F^i$ and $H = \bigcup_i F^i$. Clearly, $B \in \mathcal{E}_1$ and $C, G, H \in \mathcal{E}_2$, $\pi_2(G) = \pi_2(H) = 1$ and $(B \cup C) \cap G \subset A \subset (B \cup C) \cap H$. Hence, there exists F satisfying $G \subset F \subset H$ such that $A = (B \cup C) \cap F$. Since π_2 is complete, $F \in \mathcal{E}_2$ and $(B \cup C) \cap F$ is a π_1'-*breakdown* of A^c. Hence, $A \in \mathcal{E}_1'$.

Extend π_1 to $\mathcal{E}_1'$ by letting $\pi_1(A) = \pi_1(B)$ given any π_1'-breakdown $(B \cup C) \cap F$ of A. Let π_1' denote this extension. To prove that π' is well-defined, we make the following observation: given any π_1'-breakdown, $(B \cup C) \cap F$ of A,

$$\mu_*(B \cap F) = \mu_*(B) = \mu_*(B \cup C) = \mu_*(A) \tag{A5}$$

To see why this is the case, note that $\pi_2(F) = 1$ implies $\mu_*(F) = 1$ and since μ is complete, $F \in \mathcal{E}$ and $\mu(F) = 1$. It follows that $\mu_*(B \cap F) = \mu_*(B)$. Since $\pi_2(C) = 0$, we have $\pi_2(C^c) = 1$ and therefore, $\mu_*(C^c) = \mu(C^c) = 1$. Hence, there exists $E \in \mathcal{E}$ such that $C \subset E$ and $\mu(E) = 0$. It follows that $E' \subset B \cup C$ and $E' \in \mathcal{E}$ implies $E' \cap E^c \subset B \cup F$. Therefore, $\mu_*(B) = \mu_*(B \cup C)$. Since $B \cap F \subset A \subset B \cup C$, the last equality implies $\mu_*(A) = \mu_*(B \cup C)$.

Let $(B \cup C) \cap F = (\hat{B} \cup \hat{C}) \cap \hat{F}$ be two π_1'-breakdowns of $A \in \mathcal{E}_1'$. It follows from equation (A5) that $\mu_*(B) = \mu_*(\hat{B})$ and hence π_1' is well-defined. Let $A^i \in \mathcal{E}_1'$ be a pairwise disjoint sequence. Then, let $(B^i \cup C^i) \cap F^i$ be a π_1'-breakdown of A^i and let $A = \bigcup_{i=1}^{\infty} A^i$. Define, $B = \bigcup_i B^i$ and $C = \bigcup_i C^i$. We established above (while proving that $\mathcal{E}_1'$ is closed under countable unions) that there is $F \in \mathcal{E}_2$ such that $\pi_2(F) = 1$ and $(B \cup C) \cap F$ is a π_1'-breakdown of A^i. Then, $\pi_1'(A) = \pi_1(B) = \sum_i \pi_1(B^i) = \sum_i \pi_1'(A^i)$ as desired.

Equation (A5) also establishes that $\mu_*(A) = (\pi_1'(A))^2$ for all $A \in \mathcal{E}_1'$. Then, Lemma 2 ensures that $(\mathcal{E}_1', \pi_1')$ is a source with source-polynomial δ^2. Define $\mathcal{E}_2', \pi_2'$ and π_2'-breakdown analogously by reversing the roles of $\mathcal{E}_1, \pi_1$ and $\mathcal{E}_2, \pi_2$. We claim that (1) $[A_i \in \mathcal{E}_i$ for $i = 1, 2,\ \pi_1'(A_1) \cdot \pi_2'(A_2) > 0]$ implies $A_1 \cap A_2 \neq \emptyset$ and (2) $A \in \mathcal{E}_i', \pi_i'(A) = 0$ for some $i = 1, 2$ implies $[A \in \mathcal{E}_1' \cap \mathcal{E}_2'$ and $\pi_1'(A) = \pi_2'(A) = 0]$.

Let $(B_i \cup C_i) \cap F_i$ be a π_i'-breakdown of A_i. Assertion (1) follows easily from the way $\mathcal{E}_1, \mathcal{E}_2$ are constructed. In particular, it follows from the fact that the functions f_1, f_2, f_3, f_4 are independent. To prove (2), without loss of generality, let $i = 1$ and $(B \cup C) \cap F$ be a π_1'-breakdown. If $\pi_1'(B) = 0$ then, since π_1 is complete, $B \cap F \in \mathcal{E}_1'$. Similarly, since π_2 is complete, $\pi_2(C \cap F) = 0$. Then, $[(C \cap F) \cup (B \cap F)] \cap \Omega$ is a π_2'-breakdown of A and $\pi_2'(A) = \pi_2(C \cap F) = 0$.

Let $\mathcal{A} = \mathcal{E}_1' \cup \mathcal{E}_2'$ and let $\pi(A) = \pi_i(A)$ if $A \in \mathcal{E}_i'$. First, we will show that π is well-defined; that is, if $A \in \mathcal{E}_1' \cap \mathcal{E}_2'$, then $\pi_1'(A) = \pi_2'(A)$. We will consider three cases: (a) $\pi_1'(A) = 0$ and (b) $\pi_1'(A) > 0$. For (a), note that (2) above establishes the desired result. For (b), note that (2) implies $\pi_2'(A) > 0$. If $\pi_1'(A) \cdot \pi_2(A) = 1$, there is nothing to prove. Otherwise, assume without loss of generality that $\pi_1'(A) < 1$. Then, by (1) above, $A^c \cap A \neq \emptyset$, a contradiction.

We will show that $\mathcal{A}$ is a λ-system and that π is a λ-prior. Since $\mathcal{E}_1', \mathcal{E}_2'$ are σ-algebras, (i) $A \in \mathcal{A}$ implies $A^c \in \mathcal{A}$ and (ii) $\Omega \in \mathcal{A}$ and $\pi(\Omega) = \pi_1'(\Omega) = 1$. Hence, to complete the proof, we need only show that if $A^i \in \mathcal{A}$ for all $i = 1, \ldots$ and $A^i \cap A^j = \emptyset$ for $i \neq j$, then $\bigcup A^i \in \mathcal{A}$ and $\pi(A) = \sum_i \pi(A^i)$.

Let $N_1 = \{i \,|\, A^i \in \mathcal{E}_1'\}$ and $N_2 = \{i \,|\, A^i \notin \mathcal{E}_2'\}$. Since the sets A^i are pairwise disjoint, (1) and (2) above imply that if $N_2 \neq \emptyset$, then $i \in N_1$ implies $\pi_j'(A^i) = 0$. Invoking (2) again ensures that we must have either $A^i \in \mathcal{E}_1'$ for all i or $A^i \in \mathcal{E}_2'$ for all i. Without loss of generality, assume the former. Since $\mathcal{E}'_1$ is a σ-algebra, $A \in \mathcal{E}'_1 \subset \mathcal{A}$. Moreover, $\pi(\bigcup_i A^i) = \pi'_1(\bigcup_j A^i) = \sum_i \pi'_1(A^i) = \sum_i \pi(A^i)$ as desired.

We have shown that $(\mathcal{A}, \pi)$ is a λ-prior. Note that $\mu_*(A) = (\pi(A))^2$ and hence, by Lemma 2, $(\mathcal{A}, \pi)$ is a source with source-polynomial δ^2.

To complete the proof, let $C = [l, (m+l)/2]$, $A = g_1^{-1}(C), B = g_2^{-1}(C)$ and note that $\pi(A) = \pi(B) = 1/2$ and therefore $\mu_*(A) = \mu_*(B) = 1/4$. Let $F = [A \cap B] \cup [A^c \cap B^c]$ and note that the smallest σ-algebra that contains $\mathcal{A}$ contains F. Then, we have

$$\mu_*(F) = \mu_*(F^c) = 2\mu\left(\bigcap_{i=1}^{4} f_i^{-1}(C)\right) = 2\left(\frac{1}{2}\right)^4 = \frac{1}{8}$$

Hence, if π were to be extended to a σ-algebra containing $\mathcal{A}$, $\pi(F)$ would have to be $1/2$ but $\mu_*(F) \neq (1/2)^2$ and therefore this extension cannot be a source. □

7.4 Theorem 1A

For all $w = (w_1, \ldots, w_n) \in X^n$, let $y_w = \max_i w_i$ and $x_w = \min_i w_i$. Then, let

$$\nabla_1^u(F) = \int u(x,x)dF(x)$$

and for $n = 2, \ldots$ let

$$\nabla_n^u(F) = \int_{w_1 \in X} \cdots \int_{w_n \in X} u(x_w, y_w) dF(w_n) \cdots dF(w_1)$$

Thus, $\nabla_n^u(F)$ is the expected value of $u(x_w, y_w)$ where $w = (w_1, \ldots, w_n)$ is the random variable obtained by taking n independent draws according to F.

Theorem 1A: *Let $W = (\mu, u)$, let $(\mathcal{A}, \pi)$ be a $\mu-$source with polynomial $\gamma = \sum_{i=1}^{\infty} a_i \delta^i$. Then, for all $f, g \in \mathcal{F}_{\mathcal{A}}$, $W(f) \geq W(g)$ if and only if $V_u^\gamma(G_\pi^f) \geq V_u^\gamma(G_\pi^g)$ where*

$$V_\gamma^u(F) = \sum_{i=1}^{\infty} a_i \cdot \nabla_i^u(F)$$

for all $F \in \mathcal{L}$.

Proof: In step 1 we prove the theorem for sources such that $\gamma^\pi = \delta^n$ for some n and all *simple acts* in $\mathcal{F}_{\mathcal{A}}^o$. In step 2, we extend the result in step 1 to all simple acts in any source. In step 3, we extend the the step 2 result to all acts.

Step 1: Let $f \in \mathcal{F}^o_{\mathcal{A}}$ be a simple act with the set of possible outcomes $Y = \{x_1, \ldots, x_k\}$; let $K := \{1, \ldots, k\}$ and let $\{A_1, \ldots, A_k\}$ be such that $f^{-1}(x_i) = A_i$. Let $\{E^J_*\}_{J \in \mathcal{N}(K)}$ be the ideal split of $\{A_1, \ldots, A_k\}$. Since $\gamma^\pi(\pi(A)) = \mu_*(A)$, a straightforward inductive argument implies

$$\mu(E^J_*) = \sum_{\substack{L \subset J \\ L \neq \emptyset}} (-1)^{|J|-|L|} \gamma^\pi \left(\pi \left(\bigcup_{i \in L} A_i \right) \right) \tag{A6}$$

Since $\gamma^\pi = \delta^n$ this, in turn, implies

$$\mu(E^J_*) = \sum_{\substack{L \subset J \\ L \neq \emptyset}} (-1)^{|J|-|L|} \left(\pi \left(\bigcup_{i \in L} A_i \right) \right)^n \tag{A7}$$

Define the function $\iota : Y \to K$ such that $x_{\iota(y)} = y$ for all $y \in Y$. For $J \subset \mathcal{N}(K)$ let $Z^J = \{x_i\}_{i \in J}$ and $p^n(Z^J) = \{(z_1, \ldots, z_n) \in X^n \mid \bigcup_{i=1}^n \{z_i\} = Z^J\}$. Hence, $p^n(Z^J)$ is the set of $z \in X^n$ such that for each $y \in Z^J$, there is some i such that $z_i = y$. Equation $(A6)$ implies

$$\mu(E^J_*) = \sum_{z \in p^n(Z^J)} \pi(A_{\iota(z_1)}) \cdot \pi(A_{\iota(z_2)}) \cdots \pi(A_{\iota(z_n)}) \tag{A8}$$

The right-hand side of $(A8)$ is the probability that n independent draws from a random variable with cumulative distribution G^f_π will yield the outcome $z = (z_1, \ldots, z_n)$. Given this realization, the interval act $\mathcal{F}$ will yield the interval (x_z, y_z).

Therefore, (A1.2) implies that

$$V^u_{\delta^n}(f) = \sum_{z \in Y^n} u(x_z, y_z) \pi(A_{\iota(z_1)}) \cdots \pi(A_{\iota(z_n)}) = \int \ldots \int u(x_z, y_z) dG^f_\pi(z_n) \ldots dG^f_\pi(z_1)$$

as desired.

Step 2: Equation (A6) implies that $\mu(E^J_*)$ is linear in γ^π and hence,

$$V^u_\gamma(G^f_\pi) = \sum_{i=1}^\infty a_i V^u_{\delta^i}(G^f_\pi) = \sum_{i=1}^\infty a_i \nabla^u_i$$

whenever $\gamma = \sum_{i=1}^\infty a_i \delta^i$.

Step 3: Note that the restriction of W to $\mathcal{F}_{\mathcal{A}}$ is continuous in the topology of uniform convergence. The function $V : \mathcal{F}_\pi \to I\!R$ such that

$$V(f) = \int \dots \int u(x_z, y_z) dG_\pi^f(z_n) \dots dG_\pi^f(z_1)$$

is also continuous in the topology of uniform convergence. By step 1, W and V agree on all simple acts and the set of all simple acts is dense in $\mathcal{F}_{\mathcal{A}}$. Hence, V is the restriction of W to $\mathcal{F}_{\mathcal{A}}$. □

7.5 Proof of Corollary 1

For u such that $u(x, y) = \alpha v_u(x) + (1 - \alpha) v_u(y)$, we have,

$$\begin{aligned}
\nabla_n^u(F) &= \int \dots \int \alpha v_u(x_z) + (1 - \alpha) v(y_z) dF(z_n) \dots dF(z_1) \\
&= \alpha \int v_u d\hat{F}^n(x) + (1 - \alpha) \int v_u dF^n(x)
\end{aligned}$$

where $\hat{F}^n = 1 - (1 - F)^n$ Then,

$$\begin{aligned}
V_\gamma^u(F) &= \sum_{i=1}^{\infty} a_i \nabla_i^u(F) \\
&= \alpha \int v_u d\left[\sum_{i=1}^{\infty} a_i \hat{F}^i\right] + (1 - \alpha) \int v_u d\left[\sum_{i=1}^{\infty} a_i F^i\right] \\
&= \alpha \int v_u d\gamma(F) + (1 - \alpha) \int v_u d\hat{\gamma}(F) \\
&= \int v_u d[\alpha\gamma + (1 - \alpha)\hat{\gamma}](F)
\end{aligned}$$

as desired. □

7.6 Proof of Theorems 2-4 and Lemma 3

Lemma A2: *Let F be the lottery that yields y with probability α and $x < y$ with probability $1 - \alpha$. Then, for all $\alpha \in (0, 1)$, $\lim_{n\to\infty} \nabla_n^u(F) = u(x, y)$.*

Proof: It follows from the definition of ∇_n^u that

$$\nabla_n^u(F) = \alpha^n u(x, x) + (1 - \alpha)^n u(y, y) + (1 - \alpha^n - (1 - \alpha)^n) u(x, y)$$

Taking the limit of the right-hand side of the equation above yields the desired result. $\square$

Proof of Theorem 2: First, we will show that (i) implies (iii). Suppose $\sigma_u^{xy} < 1$ for some $x < y$. Then, choose α such that $u(z,z) < u(x,y)$ for $z = \alpha y + (1-\alpha)x$. Then, Lemma A2 implies $\nabla_n^u(F) > \nabla_n^u(\bar{z})$ for F that yields y with probability α and x with probability $1-\alpha$ and n sufficiently large. Hence, W is not risk averse with respect to every source. If v_u is not concave then W is not risk averse for $f \in \mathcal{F}_\mu$.

Next, we will prove (iii) implies (ii). Note that if $\sigma_u^{xy} = 1$ for all $x < y$, then $u(x,y) = v_u(x)$ for all $(x,y) \in I$. Hence, $W(f) = \int v_u[f]_1 d\mu$. Therefore, $W(f)$ is the inner integral of $v_u \circ f$ (Zhang (2002)). Since $\mathcal{E}_\mu$ is a σ-algebra, we have $\mu_*(E \cup B) = \mu(E) + \mu_*(B)$ for every $B \subset E^c, E \in \mathcal{E}_\mu$. Hence, by Theorem 2.3 in Zhang (2002), $W(f)$ is the Choquet integral of $v_u \circ f$ with respect to the capacity μ_*. Choose $E_A, E_B \in \mathcal{E}_\mu$ such that $E_A \subset A$, $E_B \subset B$, $\mu_*(A) = \mu(E_A)$ and $\mu_*(A) = \mu(E_B)$. Hence, $\mu_*(A) + \mu_*(B) = \mu(E_A) + \mu(E_B) = \mu(E_A \cap E_B) + \mu(E_A \cup E_B) \leq \mu_*(A \cap B) + \mu_*(A \cup B)$. That is, μ_* is convex.

Suppose $W(g) \geq W(f)$ and let $h(\omega) = v_u^{-1}(\alpha v_u(f(\omega)) + (1-\alpha)v_u(g(\omega)))$ for all $\omega \in \Omega$. Then, the characterization of uncertainty aversion in Schmeidler (1989) (i.e., the proposition on page 582) ensures that

$$W(h) = \int v_u \circ h \, d\mu_* = \int \alpha v_u \circ f + (1-\alpha) v_u \circ g \, d\mu_* \geq W(f)$$

where the integrals above are Choquet integrals. The concavity of v_u ensures that $W(\alpha f + (1-\alpha)g) \geq W(h)$ and therefore, $W(\alpha f + (1-\alpha)g) \geq W(f)$ as desired.

Finally, to prove (ii) implies (i), consider any proper source $(\mathcal{A}, \pi)$. Then, $f, g \in \mathcal{F}_\mathcal{A}$ implies $\alpha f + (1-\alpha)g \in \mathcal{F}_\mathcal{A}$. Theorem 5 of Ergin and Gul (2009) and (ii) implies V_γ^u restricted to simple acts in $\mathcal{F}_\mathcal{A}$ is risk averse. Then, the continuity of V_γ^u ensures that it is risk averse on $\mathcal{F}_\mathcal{A}$. By Lemma 2 (ii), for every γ, there is a proper source with source-polynomial γ and therefore V_γ^u is risk averse for all γ and hence for every source (by Lemma 2(i)). $\square$

Proof of Lemma 3: Theorem 3 in Gul and Pesendorfer (2012) shows that A is more uncertain than B if and only if $\mu_*(A) < \mu_*(B)$ and $\mu_*(A^c) < \mu_*(B^c)$. Then, applying Lemma 1 yields the the desired result. $\square$

Proof of Theorem 3: First, we prove (i). For any $\alpha \in [0,1]$ choose $A \in \mathcal{E}_\pi$ and $B \in \mathcal{E}_{\pi^o}$ such that $\alpha = \pi(A) = \pi^o(B)$. Choose $y > x$. If $\sigma_u^{xy} = 1$, then

$$W(yAx) = \gamma(\alpha)u(y,y) + (1-\gamma(\alpha))u(x,x)$$
$$W(yBx) = \gamma^o(\alpha)u(y,y) + (1-\gamma^o(\alpha))u(x,x)$$

Hence, $W(yAx) \geq W(yBx)$ if and only if $\gamma(\alpha) \geq \gamma^o(\alpha)$.

Next, let $\gamma = \delta^1$, $\gamma^o = \delta^2$. Suppose $\sigma_u^{xy} < 1$ for some $x < y$. Without loss of generality, (i.e., if necessary, by applying a positive affine transformation to u) let $u(y,y) = 1$, $u(x,x) = 0$ and $u(z,z) > u(x,x)$ for $z = \sigma_u^{xy}x + (1-\sigma_u^{xy})y$. choose $A \in \mathcal{E}_\pi$ and $B \in \mathcal{E}_{\pi^o}$ such that $\alpha = \pi(A) = \pi^o(B)$. Manipulating $W(yAx) = \gamma(\alpha) + (1-\gamma(\alpha)-\gamma(1-\alpha))u(x,x)$ and the similar expression of $W(yBx)$ establishes that $W(yAx) < W(yBx)$ if and only if

$$\frac{\alpha}{1-\alpha} < \frac{u(z,z)}{1-u(z,z)}$$

Hence, choosing $\alpha < u(z,z)$ reveals that the decision-maker does not prefer the less uncertain source π to the more uncertain π^o.

For part (ii) fix x, y and let $z = \sigma_u^{xy}x + (1-\sigma_u^{xy})y$. Let r be such that $V_\gamma^u(r\bar{y} + (1-r)\bar{x}) = u(z,z)$. By Lemma A2, $\lim V_{\gamma^n}^u(\alpha\bar{y} + (1-\alpha)\bar{x}) = u(z,z)$ for all α and since $V_\gamma^u(\alpha\bar{y} + (1-\alpha)\bar{x})$ is strictly increasing in α

$$\lim V_{\gamma^n}^u(\alpha\bar{y} + (1-\alpha)\bar{x}) < V_\gamma^u(\alpha\bar{y} + (1-\alpha)\bar{x})$$

if and only if $\alpha < r$. To prove necessity, let $V_\gamma^u(r\bar{y} + (1-r)\bar{x}) = u(z,z)$ and note that $u(x,x) < u(z,z) < u(y,y)$ since $r \in (0,1)$. By Lemma A2, $\lim V_{\gamma^n}^u(\alpha\bar{y} + (1-\alpha)\bar{x}) = u(x,y)$ for all α. Therefore, $u(z,z) = u(x,y)$. It follows that $u(x,x) < u(x,y) < u(y,y)$ and hence $0 < \sigma_u^{xy} < 1$. $\square$

Proof of Theorem 4: Note that $W(f + a \cdot (1_A - c)) = \int v_u[f + a \cdot (1_A - c)]_1 d\mu$. Lemma 1.2.2 in van der Waart and Wellner (1996) establishes that for any $f \in \mathcal{F}_\mu$ and $g \in \mathcal{F}$,

$$([f+g]_1, [f+g]_2) = (f + [g]_1, f + [g]_2)$$

Therefore, there are sets $E_1, E_2 \in \mathcal{E}_\mu$ such that

$$[f + a \cdot (1_A - c)]_1 = f - ac + aE_1 0$$

if $a > 0$, while, for $a < 0$,

$$[f + a(1_A - c)]_1 = f - ac + a(E_1 \cup E_2)0$$

Since v_u is concave (and continuous), it is also absolutely continuous and hence there is a nonincreasing function v_u' such that $v_u(y) = \int_{\beta_*}^{y} v_u'(z)dz + v_u(\beta_*)$. Therefore, the display equations above imply that any c such that

$$\int_{E_1 \cup E_2} v_u'(f)d\mu \geq c \int_{\Omega} v_u'(f)d\mu \geq \int_{E_1} v_u'(f)d\mu$$

is a price at which the investor with utility function W would neither buy nor sell 1_A. If $A \in \mathcal{A}$, then $\mu(E_1) = \gamma(\pi(A))$ and $\mu(E_2) = 1 - \gamma(\pi(A)) - \gamma(\pi(A^c))$. Clearly, the interval of non-trade prices is non-trivial if $\mu(E_2) > 0$. Note that

$$\mu(E_2) = 1 - \gamma(\pi(A)) - \gamma(\pi(A^c)) > 0$$

for all $\gamma \neq \delta^1$ and for all A with $1 > \pi(A) > 0$. $\square$

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