ON THE ALLOCATIVE PERFORMANCE OF ROTATING SAVINGS AND CREDIT ASSOCIATIONS

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by

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Abstract

This paper examines the allocative performance of rotating savings and credit associations (roscas), a financial institution which is observed world-wide. We develop a model in which individuals save for an indivisible good and study roscas which distribute funds using random allocation and bidding. The allocations achieved by the two types of rosca are compared with that achieved by a credit market and with efficient allocations more generally. We find that neither type of rosca is efficient and that agents are better off with a credit market than a bidding rosca. Nonetheless, a random rosca may sometimes yield a higher level of *ex ante* expected utility to prospective participants than would a credit market.

I. Introduction

Rotating savings and credit associations (roscas) are a widespread institution for financial intermediation. They are found all over the world, particularly in developing countries, and have heretofore received scant attention from economists.[1] This paper and its companion piece (Besley, Coate and Loury (1992)) constitute a first attempt to analyze their economic role and performance.

Roscas come in two main forms. The first type allocates funds randomly. In a *random rosca*, members commit to putting a fixed sum of money into a "pot" for each period of rosca's life. Lots are drawn and the pot is randomly allocated to one of the members. In the next period, the process repeats itself, except that the previous winner is excluded from the draw. The process continues, with every past winner excluded, until each member of the rosca has received the pot once. At this point, the rosca is either disbanded or begins over again. Individuals may also form a *bidding rosca* in which the pot is allocated via a bidding procedure. The individual who receives the pot in the present period does so by bidding the most in the form of a pledge of higher future contributions to the rosca or one time side payments to other rosca members. In a bidding rosca, individuals may still only receive the pot once — the bidding process merely establishes priority.

The extensive informal literature on the subject takes the view that roscas are primarily institutions whose role is to facilitate "saving up" to purchase indivisible goods. In Besley, Coate and Loury (1992) we explained how, in a world with an indivisible good, a group of individuals without access to credit markets could improve their welfare by

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[1]Roscas travel under a large number of different names. For example, they are called *Chit Funds* in India, *Susu* in West Africa, *Kye* in Korea. Bouman (1977) reports that 60% of the population in Addis Ababa belongs to a rosca. Radhakrishnan *et al.* (1975), reports that in 1967, there were 12491 registered chit funds in Kerala state in India alone. The classic anthropological studies are by Ardener (1964) and Geertz (1962). Further references to the literature on Roscas can be found in our companion paper.

forming a rosca. Roscas permit the mobilization of savings that would lie idle under autarkic saving and thus take advantage of *gains from intertemporal trade*. That paper also compared the allocations achieved by the two different types of rosca finding that, with homogeneous individuals, a random rosca produces a higher level of expected utility for participants than a bidding rosca (under a plausible restriction on preferences). The *ex ante* desirability of randomization stems from the non—convexity created by the indivisible good.

Given that a group of individuals can get together to form a rosca[2], they could potentially allocate funds in other ways, such as by organizing an informal credit market. Comparing roscas with a competitive credit markets is also interesting because the latter is the usual economists' benchmark of an efficient way to allocate resources. Thus to understand why roscas are sometimes chosen, we propose characterizing the full set of allocations that are feasible for the group. This places roscas in a broader context. As evidenced by their world—wide popularity, roscas are a simple and easily organized method of mobilizing savings. It is important to know how far these simple institutions go towards realizing the maximal possible gains from trade. Do they produce efficient allocations or does their simple structure impose a cost? In what ways do the allocations they produce differ from that which would result from formation of an informal credit market? Are bidding roscas more like a market than random roscas? Does the randomization inherent in a random rosca give it an advantage over a market? These more abstract and theoretically challenging questions about the allocations achieved using roscas are the subject of this paper. Answering them should give insights into both the strengths and

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[2]The typical scenario for a Rosca is a group of individuals who work in the same office block or belong to the same community. Social enforcement is important in explaining why individuals honor their commitment to participate. We are not concerned with enforcement problems here, which are discussed in our companion paper. We shall ask questions about what a group might achieve for a given membership, assuming that there is sufficient social enforcement power for any of the allocations that we describe to be implemented. This seems like a reasonable first step in studying these issues.

weaknesses of roscas as institutions for financial intermediation and, along with appreciating their simplicity, may go some way towards explaining why they are so widely observed in practice.

One of our main findings is that roscas do not, in general, produce efficient allocations. Their simple structure allows insufficient flexibility in the rate of accumulation of the indivisible good. We also find that bidding roscas are Pareto dominated by credit markets. Nonetheless, the element of chance offered by random roscas is still of value when compared with credit markets. Indeed, we present an example in which an *ex post* efficient credit market allocation is dominated (under the *ex ante* expected utility criterion) by a random rosca.

The remainder of the paper is organized as follows. Section II describes the model which provides the framework for our analysis. Section III then describes the allocations achieved by the two types of rosca and a credit market. Section IV develops properties of efficient and optimal allocations. Section V uses these results to assess the allocative performance of roscas and section VI concludes.

II. The Model

The model is the same as in our companion paper. A group of individuals would each like to own an indivisible durable consumption good. The group is assumed to have no access to an external credit market. Each individual in the group lives for a length of time T, receiving an exogenous flow of income over her lifetime of $y > 0$. Individuals have identical intertemporally additive preferences. Each individual's instantaneous utility depends on non–durable consumption, c, and on whether or not she enjoys the services of the durable. The durable does not depreciate and can be purchased at a given cost of B. Once purchased it yields a constant flow of services for the remainder of an individual's life. We also assume that the durable's services are not fungible across individuals; a consumer must own it to benefit from its services.

There is no discounting, which precludes any motive for saving or borrowing apart from the desire to acquire the durable. An individual's instantaneous utility with non–durable consumption c is $v(c)+\xi$ if he owns the durable, and $v(c)$ otherwise. We assume that $v(\cdot)$ is twice continuously differentiable, strictly increasing and strictly concave on $(0,\infty)$. We also assume that $v(c) \to -\infty$ as $c \to 0$ and $v(c) \to +\infty$ as $c \to \infty$. Abusing notation slightly we will write $v(\lambda,c) \equiv v(c)+\lambda\xi$, for $0\leq\lambda\leq1$, as instantaneous expected utility when c is non–durable consumption and λ is the probability of owning the durable.

We depart from our companion paper, by adopting the fiction that the group consists of a continuum of individuals. This is convenient since it allows the fraction of group members holding the indivisible good at any point in time to be treated as a continuous variable. This greatly simplifies the task of characterizing efficient allocations without affecting the economic logic. The character of the results would be maintained in the analytically more cumbersome finite case. We assume, without loss of generality, that the group's members are uniformly distributed over the unit interval and we index different individuals with numbers $\alpha\in[0,1]$.

A *consumption bundle* for an individual in the group may be described by a pair $<s,c(\cdot)>$, where $s \in [0,T]$ denotes the date of receipt of the durable good, and $c:[0,T] \to \mathbb{R}_+$ gives the rate of consumption of the non–durable at each date. An *allocation* is then a set of consumption bundles, one for each individual, and may be represented by a pair of functions $<s(\cdot),c(\cdot,\cdot)>$, such that $s:[0,1] \to [0,T]$ and $c:[0,1]\times[0,T] \to \mathbb{R}_+$. The function $s(\cdot)$, hereafter referred to as the *assignment function*, tells us the dates at which different individuals receive the durable.[3] By relabelling individuals as required, we may assume with no loss of generality that individuals with lower index numbers receive the durable earlier. Thus we may suppose that $s(\cdot)$ is non–decreasing on $[0,1]$. The second component

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[3]Throughout we restrict attention to allocations in which all the group's members receive the durable at some time during their lives. This requires that the value of owning the durable, ξ, be sufficiently large.

of an allocation gives us the consumption path $\{c(\alpha,\tau):\tau\epsilon[0,T]\}$ of each individual α.[4]

Under the allocation $<s,c>$, individual α enjoys utility:

$$(2.1) \qquad u(\alpha;<s,c>) \equiv \int_0^T v(c(\alpha,\tau))d\tau + \xi(T-s(\alpha)).$$

To be feasible for the group, the allocation must consume no more resources than its members have available over any time interval. To make this precise, for any assignment function $s(\cdot)$ and any date $\tau\epsilon[0,T]$, we define $N(\tau;s)$ to be the fraction of the group's members which has received the durable by time τ with assignment function $s(\cdot)$.[5] Then an allocation $<s,c>$ is *feasible* if and only if, for all $\tau\epsilon[0,T]$:

$$(2.2) \qquad \int_0^\tau (y - \int_0^1 c(\alpha,x)d\alpha)dx \geq N(\tau;s)B.$$

The left hand side of (2.2) denotes aggregate group saving at τ and the right hand side denotes aggregate investment. Hence, an allocation is feasible if aggregate investment never exceeds aggregate saving. In the sequel we will let F denote the set of feasible allocations.

III. Allocations In Roscas and a Credit Market

Our aim is to understand allocations in roscas in the context of the full set of allocations which are feasible for the group. This section begins by describing the allocations achieved by both types of rosca and, for purposes of comparison, that which can be achieved were the group to form a credit market.

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[4]The function $c:[0,1]\times[0,T] \to \mathbb{R}_+$ is required to be such that $c(\cdot,\tau)$ and $c(\alpha,\cdot)$ are (Lebesgue) integrable for all (α,τ).

[5]Thus $N(\tau;s)$ is the measure of the set $s^{-1}([0,\tau])\equiv\{\alpha\epsilon[0,1]\,|\,s(\alpha)\leq\tau\}$.

III.1 Random Rosca

Before explaining how a random rosca operates in our model, we briefly review the workings of a finite membered rosca.[6] Consider, then, a random rosca of length t with n members. Contributions will optimally be set so that the pot available to each winning member is equal to the cost of the indivisible good, B. This precludes the necessity of saving outside the rosca. The rosca will meet at the uniformly spaced meeting dates $\{t/n, 2t/n,...,t\}$, and at each meeting every member will contribute the sum B/n. A different individual is selected at each meeting to receive the pot of B, which allows him to buy the durable good. Prior to its initiation, a representative member of this rosca perceives his receipt date for the pot to be a random variable, $\tilde{\tau}$, with a uniform distribution on the set $\{t/n, 2t/n,...,t\}$. Each member will optimally save at the constant rate equal to B/t over the life of the rosca. Thus each member's lifetime utility is the random variable:

$$(3.1) \qquad W(t,\tilde{\tau}) \equiv t \cdot v(y - B/t) + (t - \tilde{\tau})\xi + (T - t) \cdot v(1,y).$$

The continuum case may be understood as the limit of this finite case as n approaches infinity.[7] As n grows, the rosca meets more and more frequently. In the limit it is meeting at each instant of time, and the receipt date of the pot, $\tilde{\tau}$, becomes a con-

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[6]For further detail on the operation of finite membered random and bidding roscas see our companion paper.

[7]Notice that the particular limit obtained depends upon the assumption that the spacing of meeting dates is uniform and the contribution at each meeting is constant. By having the meetings occur with different frequencies at different times during the life of the rosca, and by varying the rate of contribution across meeting dates, it may be shown that one can, in the limit, generate *every* feasible allocation <s,c> in which the consumption paths $c(\alpha,\tau)$ are constant in α, as the *ex post* outcome of a random rosca. (Moreover, one can in similar fashion generate every feasible allocation <s,c> in which the utility $u(\alpha;<s,c>)$ is constant in α, as the limiting outcome of a bidding rosca.) However, we do not attempt to exploit these fact in this paper. To do so would run contrary to the spirit of our analysis. The point of this exercise is to analyze the allocative performance of roscas as they operate in practice.

tinuous random variable uniformly distributed on the interval [0,t]. Thus the *ex ante* expected utility of a representative individual who joins the rosca of length t is given by the expectation of (3.1), taking $\tilde{\tau}$ to be uniform on [0,t]. Hence,

$$(3.2) \qquad W(t) = t \cdot v(1/2, y - B/t) + (T - t) \cdot v(1, y).$$

It seems natural to assume that the length of the rosca, t, will be chosen to maximize members' *ex ante* expected utility — W(t). Thus consider the problem of choosing t to maximize (3.2). We denote the solution by t_r and use W_r to represent the maximum expected utility. As in our companion paper, we exploit a simple way of writing down W_r. Defining $c = y - B/t$ to be the consumption rate during the rosca, we can view the problem as choosing c to maximize $T \cdot v(1,y) - B \cdot [\frac{v(1,y) - v(1/2,c)}{y - c}]$. Then, defining

$$(3.3) \qquad \mu(\lambda) \equiv \min_{0 \le c \le y} [\frac{v(1,y) - v(\lambda,c)}{y - c}], \text{ for } 0 \le \lambda \le 1,$$

we may write:

$$(3.4) \qquad W_r = T \cdot v(1,y) - B \cdot \mu(1/2).$$

The first term in (3.4) represents lifetime utility were the durable free, while the second is the minimized utility cost of saving up for the durable. This minimization trades off the benefit of a shorter accumulation period (or rosca length) against the benefit of higher consumption during this period (or smaller contributions). Letting $c^*(\lambda)$ denote the consumption level which solves (3.3), the optimal consumption rate in the random rosca is $c_r \equiv c^*(1/2)$. As established in our companion paper, the minimized cost $\mu(\cdot)$ is a decreasing, strictly concave function of λ, and the cost–minimizing consumption rate $c^*(\cdot)$ is an increasing function of λ. Both are twice–continuously differentiable on [0,1], where

they satisfy the identity $\mu(\lambda) \equiv v'(c^*(\lambda))$.

Let $<s_r, c_r>$ denote the allocation achieved with the optimal random rosca. By relabelling as required, we can assume that individual α receives the durable at meeting date αt_r. Thus the assignment function will be the linear function $s_r(\alpha) = \alpha t_r$. This implies that the fraction of the membership who have received the durable is increasing and linear over the interval of accumulation. All individuals have identical consumption paths which fall into two distinct phases. During the life of the rosca they consume at rate c_r and after it ends they consume at rate y. Thus, $c_r(\alpha, \tau) = c_r$, for $\tau \in [0, t_r]$; and, $c_r(\alpha, \tau) = y$, for $\tau \in (t_r, T]$. Notice that while group members have identical expected utilities, they enjoy different *ex post* utility levels.

III.2 Bidding Rosca

We turn next to the bidding rosca, where the order in which individuals receive the pot is determined by bidding. It is simplest to assume that the bidding takes place when the rosca is formed at time zero, and involves individuals committing to various contribution rates over the life of the rosca. Of the many auction protocols that might be imagined, all must result in individuals being indifferent between bid/receipt pairs, since individuals have identical preferences and complete information. Moreover, any efficient auction procedure must be structured so that total contributions committed through bids are just adequate to finance acquisition of the durable. These two requirements completely determine the outcome of the bidding procedure and it is unnecessary to commit to a particular auction protocol.[8]

If the bidding rosca is of length t, bidding determines the order of receipt over the interval [0,t]. Let $b(\alpha)$ denote the promised contribution of member α who, without loss of generality, can be assumed to receive the durable at date αt. A set of bids $\{b(\alpha) : \alpha \in [0,1]\}$

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[8]Our companion paper discusses how an ascending bid auction could implement this outcome.

constitutes an equilibrium if: (i) no individual could do better by out bidding another for his place in the queue; and (ii) contributions are sufficient to allow each member to acquire the durable upon receiving the pot.

Member α receiving the pot at date αt, will have non–durable consumption $c(\alpha)=y-b(\alpha)/t$ at each moment during the rosca's life. Thus we can characterize the bidding rosca in terms of the consumption rates $\{c(\alpha):\alpha\epsilon[0,1]\}$. Condition (ii) implies that individual α's equilibrium utility level is $t\cdot v(1-\alpha,c(\alpha)) + (T-t)\cdot v(1,y)$ in a bidding rosca of length t. Condition (i) implies, for all individuals α and some number x, that:

$$(3.5) \qquad\qquad v(1-\alpha,c(\alpha)) = x.$$

The number x represents the members' common average utility during the life of a bidding rosca of length t, in a bidding equilibrium.

Now define $\bar{c}$ to be the average non–durable consumption rate of members during the life of the rosca, i.e., $\bar{c}\equiv\int_0^1 c(\alpha)d\alpha$. Then condition (ii) is equivalent to:

$$(3.6) \qquad\qquad t\cdot(y-\bar{c}) = B.$$

Given the rosca's length t, the relations (3.5) and (3.6) uniquely determine members' non–durable consumption rates and their average utility over the life of the rosca. Equivalently, one could take as given the equilibrium average utility level for the duration of the rosca, x. Then (3.5) gives individuals' equilibrium consumption levels, $\{c(\alpha): \alpha\epsilon[0,1]\}$; and these, via (3.6) can be used to find the rosca's length, t.

As in the random rosca, it is natural to assume that the length of the bidding rosca is chosen to maximize the common utility level of its members. The foregoing discussion and (3.5) imply that this common welfare is $T\cdot v(1,y)-B\cdot\{[v(1,y)-x]/[y-\bar{c}]\}$. Now let $\hat{c}(\alpha,x)$ be the function satisfying $v(1-\alpha,\hat{c})\equiv x$, and define $\bar{c}(x)\equiv\int_0^1\hat{c}(\alpha,x)d\alpha$. Then, when the

equilibrium average utility during a bidding rosca is x, $\hat{c}(\alpha,x)$ is individual α's non–durable consumption rate during the rosca, and $B/[y-\bar{c}(x)]$ is the rosca's length. Denote by t_b and W_b, respectively, the duration and common utility level of the optimal bidding rosca. Then, we may write:

$$(3.7) \qquad\qquad W_b = T\cdot v(1,y) - B\cdot\mu_b,$$

where

$$(3.8) \qquad\qquad \mu_b \equiv \underset{x}{Min}\ [\frac{v(1,y) - x}{y-\bar{c}(x)}].$$

Letting x^* give the minimum in (3.8), then $t_b = B/[y-\bar{c}(x^*)]$ is the length of the optimal bidding rosca. Individual α's consumption rate during the life of the rosca will be $c_b(\alpha)\equiv\hat{c}(\alpha,x^*)$.

Lifetime utility expressed in (3.7) admits the same interpretation noted for the random rosca; it is the difference between lifetime utility if the durable were free, and the minimal cost of saving up. The latter, determined in (3.8), again trades–off higher welfare during the rosca versus faster acquisition of the durable.

Let $<s_b,c_b>$ denote the allocation generated by the optimal bidding rosca. As with the random rosca, the assignment function is linear, i.e. $s_b(\alpha) = \alpha t_b$. Unlike the random rosca, each individual receives a different consumption path under a bidding rosca. However the general pattern is similar with an accumulation phase followed by a phase in which members consume all of their incomes. Hence the allocation of non–durable consumption is described by $c_b(\alpha,\tau)=c_b(\alpha)$, for $\tau\epsilon[0,t_b]$ and $c_b(\alpha,\tau)=y$, for $\tau\epsilon(t_b,T]$.

III.3 A Credit Market

The final institution that we consider is a credit market. There are at least two reasons for including this in our study of roscas. First, informal credit markets are

widespread in developing countries and it clearly constitutes one option open to our group.[9]

Second, one is naturally curious to compare a credit market with a bidding rosca, both of which use bidding to determine the allocation of funds. We specify the behavior of such a market as follows. Let $r(\tau)$ denote the market interest rate at time τ. It is convenient to define $\delta(\tau)$ as the present value of a dollar at time τ, i.e. $\delta(\tau) \equiv \exp(-\int_0^\tau r(z)dz)$ and to think of the market as determining a sequence of present value prices $\{\delta(\tau):\tau\epsilon[0,T]\}$ at which the supply of and demand for loanable funds are equated. Hence, an individual who buys the durable good at time s pays $\delta(s)B$ for it. Given the price path, an individual α chooses a purchase time $s(\alpha)$ and a consumption path $\{c(\alpha,\tau):\tau\epsilon[0,T]\}$ to maximize utility; that is, he solves

(3.9)

$$\underset{\{c(\,\cdot\,),s\}}{\text{Max}} \int_0^T v(c(\tau))d\tau + \xi(T{-}s)$$

$$\text{subject to } \int_0^T \delta(\tau)c(\tau)d\tau + \delta(s)B \leq y \int_0^T \delta(\tau)d\tau, \ s \in [0,T].$$

We define a *market equilibrium* to be an allocation $<s_m,c_m>$ and a price path $\delta(\cdot)$ satisfying two conditions: first, $<s_m(\alpha),c_m(\alpha,\tau)>$ must be a solution to (3.9) for all $\alpha\epsilon[0,1]$, and second, at each date $t\epsilon[0,T]$,

(3.10) $$\int_0^t [y - \int_0^1 c_m(\alpha,\tau)d\alpha]d\tau = N(t;s_m)\cdot B.$$

Since individuals are identical, the first condition implies that, in equilibrium, all

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[9]It should be noted however that we model credit as being allocated in an idealized competitive market which may not characterize the reality of informal credit. To treat it otherwise would risk stacking the deck in our comparisons below. Moreover, there is no generally agreed upon model of how such informal markets function.

individuals are indifferent between durable purchase times. The second condition just says that savings equals investment at all points in time. We use W_m to denote the equilibrium level of utility enjoyed by members with a credit market. Below, we show that this can be written in a form analogous to (3.4), and (3.7).

Direct computation of equilibrium prices and the associated allocation is difficult, even in the simple case of logarithmic utility studied in section V. However, we are able to infer the existence and some of the properties of the credit market equilibrium in our model by using the fact that, with identical individuals, it must coincide with a Pareto efficient allocation which gives equal utility to every individual. Hence, describing properties of the credit market allocation must await consideration of efficient allocations more generally.

IV. Efficient and *Ex Ante* Optimal Allocations

The previous section described the allocations achieved by roscas and a credit market. We now turn to characterizing the group's "best" feasible allocations. Two welfare criteria are natural here. The first is *ex post* Pareto efficiency, or more simply, *efficiency*. We consider an allocation to have been *efficient* if there is no alternative feasible allocation which makes a non—negligible set of individuals strictly better off, while leaving all but a negligible set of individuals at least as well off.

The second criterion is defined in terms of *ex ante* expected utility. The allocation $<s,c>$ is better than $<s',c'>$, in this sense if $\int_0^1 u(\alpha;<s,c>)d\alpha > \int_0^1 u(\alpha;<s',c'>)d\alpha$. The thought experiment required is as follows: an individual will be assigned to any position in the queue for the durable with equal probability. We then ask which allocation would be best for *any* individual viewed from behind this "veil of ignorance". This allocation will thus be that which yields the highest level of expected utility. We call this the *ex ante optimal allocation*. It is obvious that this must correspond to a particular *ex post* efficient allocation.

Notice that, since all group members enjoy the same level of utility with a bidding

rosca and a credit market, the criteria of *ex post* utility and *ex ante* expected utility coincide when applied to allocations emerging from any one of these institutional forms. This is not the case for the random rosca. It is possible that an allocation generated by a random rosca might be Pareto dominated, and yet itself be preferred to some Pareto efficient allocation in terms of the *ex ante* optimality criterion. Indeed, we will present an example in which precisely this reversal occurs.

IV.1 Efficient Allocations

To characterize efficient allocations, we introduce weights $\theta(\alpha) > 0$ for each agent $\alpha \in [0,1]$, normalized so that $\int_0^1 \theta(\alpha) d\alpha = 1$. Define the set of all such weights: $\theta \equiv \{\theta : [0,1] \rightarrow \mathbb{R}_{++} \mid \theta$ integrable and $\int_0^1 \theta(\alpha) d\alpha = 1\}$. A standard result in the welfare economics of finite economies is that an efficient allocation maximizes a weighted sum of individuals' utilities. This result also holds for our continuum model as we show in:

Lemma 1: If $<s',c'>$ is efficient then there exists a set of weights in θ such that $\int_0^1 \theta(\alpha) u(\alpha; <s',c'>) d\alpha \geq \int_0^1 \theta(\alpha) u(\alpha; <s,c>) d\alpha$, for all feasible allocations, $<s,c>$.

Proof: See Appendix. □

To investigate the properties of efficient allocations we therefore study, for fixed $\theta \in \theta$, the problem:

$$(4.1) \qquad \max_{<s,c> \in F} W(\theta; <s,c>) \equiv \int_0^1 \theta(\alpha) u(\alpha; <s,c>) d\alpha,$$

for $u(\alpha; <s,c>)$ as defined in (2.1). Let $<s_\theta, c_\theta>$ denote an allocation which solves this problem, and $W_\theta \equiv W(\theta; <s_\theta, c_\theta>)$ denote the maximized value of the objective function.

Although (4.1) may appear to be a "non–standard" optimization problem, it can be solved explicitly under our simplifying assumptions on preferences. The solution is exhibited in Theorem 1 below. We develop the Theorem and discuss its implications at some length in what follows. The proof of this result exposes the nature of the resource allocation problem which any institution for financial intermediation that the group chooses must confront.

The first point to note about the efficiency problem is that it can be solved in two stages, loosely corresponding to static and dynamic efficiency. The first requires that any level of aggregate consumption be optimally allocated across group members, i.e. maximizes the weighted sum of instantaneous utility at each date, while the second determines the optimal acquisition path for the durable good.

Let $\bar{c}_\theta(\tau)$ denote aggregate consumption in period τ, i.e. $\bar{c}_\theta(\tau) \equiv \int_0^1 c_\theta(\alpha,\tau)d\alpha$. A brief inspection of the efficiency problem should convince the reader that this should be distributed among group members so as to maximize the weighted sum of utilities from consumption in period τ. To state this more precisely, define the problem

(4.2)

$$\underset{\chi(\cdot)}{\text{Max}} \int_0^1 \theta(\alpha)v(\chi(\alpha))d\alpha$$

$$\text{subject to } \int_0^1 \chi(\alpha)d\alpha = w$$

for all $w > 0$. Let $\chi_\theta(\cdot,w)$ denote the solution and let $V_\theta(w)$ denote the value of the objective function. Then individual α's consumption at time $\tau \in [0,T]$ is given by $c_\theta(\alpha,\tau) = \chi_\theta(\alpha,\bar{c}_\theta(\tau))$, and total weighted utility from non–durable consumption is given by $V_\theta(\bar{c}_\theta(\tau))$. It remains, therefore, to determine $s_\theta(\alpha)$ and $\bar{c}_\theta(\tau)$.

Note first that since preferences are strictly monotonic, the feasibility constraint that $\langle s,c \rangle \in F$ [see (2.2)] may without loss of generality be written as:

$$(4.3) \qquad \int_0^T [y - \int_0^1 c(\alpha,\tau)d\alpha]d\tau = N(\tau;s)B, \text{ for all } \tau\epsilon[0,T].$$

That is, all savings must be put immediately to use. Moreover, in the absence of discounting, if the flow of aggregate savings $\int_0^1[y-c(\alpha,\tau)]d\alpha$ equals zero at some date τ, then efficiency demands that it must also be zero at any later date $\tau' > \tau$. Otherwise, by simply moving the later savings forward in time one could assign some individuals an earlier receipt date for the durable without reducing anyone's utility from non–durable consumption. The foregoing implies that any assignment function solving (4.1) must be continuous, increasing, and satisfy s(0)=0. This in turn implies that such an assignment function is invertible, and that it is differentiable almost everywhere. We can use these facts to write $N(\tau;s) = s^{-1}(\tau)$, for $\tau\epsilon[0,s(1)]$, and $N(\tau;s) = 1$, for $\tau\epsilon(s(1),T]$. Substituting this expression for N(t;s) in (4.3) and differentiating with respect to τ, we find that for all $\tau\epsilon[0,s(1))$ we have $y-\bar{c}(\tau)=B/s'(s^{-1}(\tau))$, and for all $\tau\epsilon(s(1),T]$ we have $y-\bar{c}(\tau)=0$. Therefore, we may conclude that the following condition must be satisfied by any solution to (4.1):

$$(4.4) \qquad s'(\alpha) = B/[y-\bar{c}(s(\alpha))], \text{ for all } \alpha\epsilon[0,1); \text{ and, } \bar{c}(\tau)=y, \text{ for all } \tau\epsilon(s(1),T].$$

Equations (4.4) are the analogue of "production efficiency" in our model, i.e. there can be no outright waste of resources. Hence, for part two of the solution, we are interested in the problem of choosing functions $s_\theta(\alpha)$ and $\bar{c}_\theta(\tau)$ to maximize

$$(4.5) \qquad \int_0^T V_\theta(\bar{c}(\tau))d\tau + \xi T - \xi\int_0^1 \theta(\alpha)s(\alpha)d\alpha$$

subject to (4.4). Employing a notational convention analogous to that introduced earlier, we write $V_\theta(\lambda,c)$ to denote the quantity $V_\theta(c)+\lambda\xi$, and define the function $\mu_\theta(\cdot)$ as

follows:

$$(4.6) \qquad \mu_\theta(\lambda) \equiv \min_{0 \le \sigma \le y} \left[\frac{V_\theta(1,y) - V_\theta(\lambda,\sigma)}{y - \sigma} \right], \ 0 \le \lambda \le 1.$$

For each λ denote the solution of the minimization in (4.6) by $\sigma_\theta(\lambda)$. We now have:

Theorem 1: Let $<s,c>$ be an efficient allocation, and let $\theta \epsilon \Theta$ be the weights for which $<s,c>$ provides a solution in (4.1). Then the maximized value can be written in the form

$$W_\theta = T \cdot V_\theta(1,y) - B \cdot \int_0^1 \mu_\theta(1 - \int_x^1 \theta(z)dz)dx$$

and the assignment function satisfies:

$$s(\alpha) = B \cdot \int_0^\alpha [y - \sigma_\theta(1 - \int_x^1 \theta(z)dz)]^{-1}dx, \ \ \alpha \epsilon [0,1].$$

Moreover, for all $\alpha \epsilon [0,1]$, non–durable consumption obeys

$$c(\alpha,s(x)) = \chi_\theta(\alpha, \sigma_\theta(1 - \int_x^1 \theta(z)dz)), \text{ for } x \epsilon [0,1]; \text{ and } c(\alpha,\tau) = \chi_\theta(\alpha,y), \text{ for } \tau \epsilon (s(1),T).$$

Proof: In view of the discussion preceding the statement of the Theorem:

$$W_\theta \equiv \text{Max} \{ \int_0^{s(1)} V_\theta(\bar{c}(\tau))d\tau + (T-s(1)) \cdot V_\theta(y) + \xi T - \xi \int_0^1 \theta(\alpha)s(\alpha)d\alpha \}$$

subject to

$$s'(\alpha) = B/[y - \bar{c}(s(\alpha))], \ \ \alpha \epsilon [0,1).$$

Now employ the change of variables: $\tau = s(\alpha)$, $d\tau = s'(\alpha)d\alpha$, $\tau \epsilon [0,s(1)]$; note that $s(1) = \int_0^1 s'(\alpha)d\alpha$; and use (4.4) and the definition in (4.6) to get the following:

$$W_\theta = \text{Max} \{ \int_0^{s(1)} V_\theta(\bar{c}(\tau))d\tau + (T-s(1)) \cdot V_\theta(y) + \xi \cdot (T - \int_0^1 \theta(\alpha)s(\alpha)d\alpha) \}$$

$$= \text{Max} \{ T \cdot V_\theta(1,y) - \int_0^1 s'(\alpha) \cdot [V_\theta(1,y) - V_\theta(1 - \int_\alpha^1 \theta(z)dz, \bar{c}(s(\alpha)))]d\alpha \}$$

$$= T \cdot V_\theta(1,y) - \text{Min}\{B\int_0^1 \{[V_\theta(1,y) - V_\theta(1 - \int_\alpha^1 \theta(z)dz, \bar{c}(s(\alpha)))]/[y - \bar{c}(s(\alpha))]\}d\alpha\}$$

$$= T \cdot V_\theta(1,y) - B\int_0^1 \mu_\theta(1 - \int_\alpha^1 \theta(z)dz)d\alpha.$$

This proves (i). Note that the minimization above is pointwise, with respect to $\bar{c}(s(\alpha))$, at each $\alpha\in[0,1]$. So it implies that for $\alpha\in[0,1]$, $\bar{c}(s(\alpha))=\sigma_\theta(1-\int_\alpha^1\theta(z)dz)$. In view of this and (4.4) we conclude that $s(\alpha)=\int_0^\alpha s'(x)dx$ satisfies (ii). Now also from (4.4) we know that $\bar{c}(\tau)=y$, for $\tau > s(1)$, and we noted earlier in the text that $c(\alpha,t)=\chi_\theta(\alpha,\bar{c}(t))$, $\forall\alpha\in[0,1]$, $\forall t\in[0,T]$. Taken together, these prove (iii). □

As noted, Pareto efficiency requires two conditions beyond the absence of physical waste of resources. First, any given aggregate level of non-durable consumption should be allocated efficiently among individuals and second, it must optimally manage the intertemporal trade-off between aggregate consumption of the non-durable and faster diffusion of durable ownership. We discussed above how $V_\theta(\cdot)$ summarized the first of these stages. More needs to be said about the dynamic efficiency considerations — in particular, the relevance of the minimization conducted in (4.6).

To see this consider the expression for W_θ in Theorem 1. This welfare measure is the difference of two terms. The first, $T\cdot V_\theta(1,y)$, would be the maximal weighted utility sum if the durable were a free good. The second term is, therefore, the (utility equivalent) cost of acquiring the durable. It is this cost which is minimized in (4.6). It has two competing components: non-durable consumption foregone in the process of acquiring the durable (since $\bar{c}(s(\alpha)) < y$) and durable services foregone in allowing some non-durable consumption (since $s'(\alpha) < B/y$). During the small interval of time that the durable is being acquired by agents $\tilde{\alpha}\in(\alpha,\alpha+d\alpha)$, then the sum of these two components is approximately $[V_\theta(1,y)-V_\theta(1-\int_\alpha^1\theta(z)dz,\bar{c}(s(\alpha)))]$, while the duration of this time interval is $s'(\alpha)d\alpha=Bd\alpha/[y-\bar{c}(s(\alpha))]$. Efficient accumulation therefore means minimizing the product of these terms at each $\alpha\in[0,1]$. This is precisely the problem described by (4.6).

A geometric treatment of the minimization problem (4.6) may also be helpful (see Figure 1). The function $V_\theta(\cdot)$ is smooth, increasing and strictly concave because we have assumed that $v(\cdot)$ has these properties. Therefore, choosing σ to minimize the ratio $[V_\theta(1,y)-V_\theta(\lambda,\sigma)]/[y-\sigma] = [V_\theta(y)-V_\theta(\sigma)+(1-\lambda)\xi]/[y-\sigma]$ means finding that point $(\sigma,V_\theta(\sigma))$ on the graph of $V_\theta(\cdot)$ such that the straight line containing it, and containing the point $(y,V_\theta(y)+(1-\lambda)\xi)$, is tangent to the graph of $V_\theta(\cdot)$. Notice from the diagram that $\sigma_\theta(\lambda)$ must be increasing, rising to y as λ increases to 1.

This observation, together with Theorem 1, permits us to deduce some useful properties of efficient allocations:

Corollary 1: Let $<s,c>$ be an efficient allocation. Then

(i) the assignment function $s(\cdot)$ is increasing, strictly convex and satisfies

$$\lim_{\alpha \to 1} s'(\alpha) = +\infty, \text{ and}$$

(ii) for all $\alpha \in [0,1]$, $c(\alpha,\cdot)$ is increasing on the interval $[0,s(1)]$, and constant thereafter.

Proof: (i) In view of (4.4) and Theorem 1, we know that any efficient allocation $<s,c>$ satisfies $s'(\alpha) = B/[y-\sigma_\theta(1-\int_\alpha^1 \theta(z)dz)]$, for some $\theta \in \Theta$. As noted above $\sigma_\theta(\lambda)$ is increasing and approaches y as λ increases to 1. Hence, the result.

(ii) This follows immediately from Theorem 1 after noting that $\chi_\theta(\alpha,\cdot)$ is increasing and that $\sigma_\theta(\lambda)$ is increasing. $\square$

The properties of the assignment function imply that, in an efficient allocation, the fraction of the group who have received the durable by time τ is increasing and strictly concave. In addition, the rate of accumulation (the time derivative of $N(\tau;s)$) approaches zero as τ goes to $s(1)$.

Finally, it is worth drawing attention to the relationship between the characterization in Theorem 1 and the welfare expressions found for the random rosca in (3.4) and for the bidding rosca in (3.7). These expressions all take the same general form, allowing the intuitive interpretation that welfare is the hypothetical utility achieved when the durable is a free good, net of the implicit utility cost of acquiring the durable's services. This observation is the basis for the results in section V.

IV.2 The Optimal Allocation

The optimal allocation is that efficient allocation in which individuals are equally weighted, $\theta(\alpha) \equiv 1$, $\forall \alpha \in [0,1]$. Since individuals are identical and are assigned types randomly, this solution maximizes the *ex ante* expected utility of a representative group member. This is also clear from (4.1). Hence, noting that $1 \in \Theta$, we can write *ex ante* expected utility as $W(1;<s,c>)$, and the *optimal allocation* $<s_0,c_0>$ must satisfy: $W(1;<s_0,c_0>) \geq W(1;<s,c>)$, for all $<s,c> \epsilon F$.

Notice also that $V_1(w) \equiv v(w)$, and $\chi_1(\alpha,w) \equiv w$, for all (α,w). This implies that if all individuals have equal weights and utility is strictly concave, then aggregate non–durable consumption should be allocated equally among group members. It is also useful to note that $\mu_1(\lambda) \equiv \mu(\lambda)$, where $\mu(\cdot)$ is defined in (3.2), and that $\mu'(\gamma) = -\xi/[y - \sigma_1(\lambda)]$, using the Envelope Theorem. These facts, together with Theorem 1, yield:

Theorem 2: Let $<s_0,c_0>$ be the optimal allocation. Then, *ex ante* expected utility can be written in the form

$$W(1;<s_0,c_0>) \equiv W_0 = T \cdot v(1,y) - B \cdot \int_0^1 \mu(\alpha) d\alpha,$$

and the optimal assignment function satisfies

$$s_0(\alpha) = -\frac{B}{\xi} \cdot \int_0^\alpha \mu'(x) dx.$$

Moreover, for all $\alpha \epsilon [0,1]$, non–durable consumption obeys

$c_0(\alpha, s_0(x)) = y + \xi/\mu'(x)$ for $x\epsilon[0,1]$, and $c_0(\alpha, \tau) = y$ for $\tau\epsilon(s_0(1),T]$.

In addition to the properties outlined in Corollary 1, therefore, the optimal allocation also gives individuals identical consumption paths. This consumption path rises smoothly to y at the end of the accumulation phase. The fraction of the group with the durable is increasing and concave over the interval of accumulation.

It is helpful to note the relationship between the problem solved by the optimal allocation and that of a single individual accumulating a perfectly divisible good. Since individuals are identical in our model, if the durable good were perfectly divisible, then there would be no gains from trade and autarkic saving would be optimal.[10] It is precisely the indivisibility of the durable which creates the problem for the group. Nonetheless, the group may approximately replicate the situation under perfect divisibility, even in the presence of indivisibilities, by randomly assigning individuals to positions in the queue at the initial date. This is tantamount to granting each individual a "share" of the aggregate amount of the durable good available at any subsequent date.

Indeed, it can be shown that the optimal non−durable consumption path, $\bar{c}_0(\cdot)$, is precisely that attained by an individual accumulating a perfectly divisible durable good.[11]

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[10]Even with indivisibility the allocation problem reduces in this way if the durable's services were fungible across agents − if there were, e.g., a perfect rental market for its services. There are, of course, good (adverse selection/moral hazard) reasons why such trade in durable services might not obtain, especially in a LDC setting. Moreover, some reports on the use of roscas stress their role in financing personal expenditures (daughter's wedding, feast for fellow villagers, tin roof for house) which, though not producing a fungible asset, generate private consumption benefits lasting for some time that are not transferable to others.

[11]To be more precise, it can be shown that the optimal aggregate consumption path $\{\bar{c}_0(\tau):\tau\epsilon[0,T]\}$ solves the problem:

$$\underset{c(\,\cdot\,)}{\text{Max}} \int_0^T [v(c(\tau)) + \xi(T-\tau)K'(\tau)]\,d\tau$$

$$\text{subject to } B\cdot K'(\tau)=y-c(\tau); \quad K(0)=0; \quad K(T)=1; \quad 0\leq c(\tau)\leq y.$$

Here, the function $K(\tau)$ is to be interpreted as the stock of the divisible asset the individual holds at time τ.

In the absence of trade in durable services, financial intermediation provides the only means to overcome the limitations imposed on the group members by the constraint of indivisibility. In the optimal allocation it is completely overcome in the sense that *ex ante* expected utility is just the same as it would be under perfect divisibility.

The credit market and rosca allocations can be related to the efficient and optimal ones. The random rosca maximizes *ex ante* expected utility subject to the assignment function being linear. The bidding rosca imposes the constraint that utilities be equal, in addition to the constraint that the assignment function be linear. A credit market corresponds to the efficient allocation where life—time utilities are equal. These relationships are important in understanding the results of the next section.

V. The Allocative Performance of Roscas

Having characterized efficient and optimal allocations, we are now in a position to evaluate the allocative performance of roscas. We begin by discussing efficiency.

Proposition 1: The allocations achieved by bidding and random roscas are inefficient.

Proof: By Corollary 1(i) efficient allocations have *strictly convex* assignment functions, while the analysis of sections III.1 and III.2 showed that roscas, with their uniformly spaced meeting dates and constant contribution rates, lead to *linear* assignment functions. □

This proposition tells us that the simple structure of roscas does have a cost and identifies the nature of it. The convexity of efficient assignment functions is a consequence of the fact that, as the remaining horizon becomes shorter, the value of the durable good to a group member who acquires it diminishes, so the amount of current consumption foregone to finance diffusion of durable goods should also decline. Roscas, with their

uniformly spaced meeting dates and constant contribution rates, cannot achieve this subtle intertemporal shift in resource allocation. Their simple form therefore prevents the realization of maximal gains from trade.

Notwithstanding, the best random rosca does yield maximal *ex ante* expected utility to its members subject to the constraint of the assignment function being linear. Moreover, the best bidding rosca generates the highest *common* level of utility for its members, among all feasible allocations satisfying the linear assignment function requirement.

As noted in the introduction, our companion paper established that random roscas resulted in higher expected utility than bidding roscas; that is, $W_r > W_b$. Using the analysis of this paper, we can complete this ranking by noting that the Optimum will be best of all. This can be proved directly by using the fact, from Theorem 2, that the cost of saving up equals $\int_0^1 \mu(\alpha) d\alpha$. Using Jensen's inequality and the strict concavity of $\mu(\cdot)$, we obtain $\int_0^1 \mu(\alpha) d\alpha < \mu(1/2)$ to prove that the cost of saving up will be greater under a random rosca. The Optimum is better than a random rosca precisely because it can offer a non–linear assignment function.

The credit market allocation which is constrained by definition to provide group members with equal utilities, generates lower *ex ante* expected utility than the optimal allocation $<s_0, c_0>$.[12] In general, however, a credit market Pareto dominates a bidding rosca. To understand this, recall that, in addition to being constrained to provide individuals equal utilities, the bidding rosca is also constrained to have a linear assignment function. We summarize these observations in

Proposition 2: While not achieving the optimal allocation, a credit market is preferred to a

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[12]The failure of the market to achieve the ex–ante optimum parallels results in other literatures where indivisibilities are important. See, for example, the model of conscription in Bergstrom (1986), the location models of Mirrlees (1972) and Arnott and Riley (1977), the club membership model of Hillman and Swan (1983), and the labor market model of Rogerson (1988).

bidding rosca, i.e. $W_o > W_m > W_b$.

Proof: Since each individual's utility is constant in both $<s_m, c_m>$ and $<s_b, c_b>$, and since $<s_b, c_b>$ is Pareto inefficient while by the First Fundamental Theorem of Welfare Economics $<s_m, c_m>$ is efficient, we must have $W_m > W_b$. Moreover, the constancy of individuals' utility in a competitive equilibrium implies:

$$(5.1) \qquad \forall \theta \in \Theta: \ W_\theta \equiv W(\theta; <s_\theta, c_\theta>) \geq W(\theta; <s_m, c_m>) = W(\theta_m; <s_m, c_m>) \equiv W_m.$$

where θ_m are the weights associated with the competitive allocation. The inequality in (5.1) reflects the fact that $<s_\theta, c_\theta>$ maximizes the weighted sum of utilities with weights θ; the equality is due to the fact that the weighted average of a constant function does not depend on the weights. So $W_m = \underset{\theta \in \Theta}{Min} \{ \underset{<s, c> \in F}{Max} W(\theta; <s,c>)\}$! The competitive equilibrium solves an elegant mini–max problem. Thus, not only is $W_m < W_o$ (equality is impossible since then, by the strict concavity of $v(\cdot)$ and the fact that $c_m \neq c_o$, a strict convex combination of $<s_m, c_m>$ and $<s_o, c_o>$ would be feasible and would dominate $<s_o, c_o>$), but W_m is less than *any* maximized weighted sum of utilities. □

This proof demonstrates that the credit market equilibrium uses weights which minimize W_θ. This is key to our constructive demonstration, in Proposition 3 below, that there exist circumstances under which the credit market allocation is strictly dominated, in terms of *ex ante* expected utility, by the optimal random rosca. Hence, our final result on welfare comparisons shows that the "equal utility" constraint can be more of an impediment to generating *ex ante* welfare than the "linear assignment function" constraint. As already mentioned, W_r is the maximal *ex ante* welfare subject to having a linear assignment function, while W_m maximizes the same criterion subject to the constraint that

utilities are equal. The question naturally arises whether one can prove a general result on the relation of these values. One might have suspected that under some plausible conditions the competitive allocation would dominate the inefficient random rosca. However, this is not the case. What follows is an illustration of the fact that a simple institution of financial intermediation, allocating its funds by lot, can actually out perform a credit market.

Proposition 3: In the case of logarithmic utility, there exists a $\bar{\xi}$ such that for all $\xi > \bar{\xi}$, a random rosca dominates the credit market; i.e. $W_r > W_m$.

Proof: See the Appendix.

The technique of proof is indirect, since explicit representation of credit market allocations, even in the case of logarithmic utility, seems intractable. We use the fact, from the proof of Proposition 3, that the market gives the least maximized weighted utility sum, over all possible weights. We then construct a set of weights whose maximized utility sum is less than W_r, to infer the result. Intuitively the result may be understood by recognizing that, when ξ is very large, respecting the equal utility constraint means those receiving the durable early must get much lower non-durable consumption than those acquiring it late. This causes individuals' *marginal* utilities of income to diverge. However, since preferences are additive and there is no discounting, an ideal intertemporal path of consumption would equate marginal utilities of income through time, something which *is* achieved under a random rosca. The effect of increasing ξ is thus to increase this divergence in marginal utilities thereby lowering *ex ante* expected utility in a market. The magnitude of ξ does not, however, affect the utility cost of having a linear assignment function. Thus when ξ is sufficiently large the random rosca dominates.

VI. Conclusion

Given the world–wide prevalence of roscas, it is important to understand their economic role and performance. Following the large informal literature, we have sought their rationale in the fact that some goods are indivisible. This makes autarkic saving inefficient. Our companion paper spelled out how, in a world with an indivisible good, a group of individuals without access to credit markets could improve their welfare by forming a rosca and compared the allocations achieved by the two different types of rosca. It found that with homogeneous individuals, randomization is preferred to bidding as a method of allocating funds within roscas.

This paper completes the picture by considering roscas in the larger context of the set of feasible allocations which can be attained by a group of individuals. One important finding is that roscas do not, in general, produce efficient allocations. Their simple structure allows less flexibility in the rate of accumulation of the indivisible good than is necessary to achieve maximal gains from trade. A further finding is that bidding roscas are Pareto dominated by credit markets. This is not so surprising since both institutions use prices to allocate access to the indivisible good, but the credit market has greater flexibility. Nonetheless, the element of chance offered by random roscas is still of value. Credit market allocations may be dominated (under the *ex ante* expected utility criterion) by those produced by a random rosca. In light of the significantly greater complexity of a credit market, this is a noteworthy finding.

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Appendix

Proof of Lemma 1: Each feasible allocation $<s,c>\epsilon F$ gives rise to an allocation of utility $u(\alpha;<s,c>)$, $\alpha\epsilon[0,1]$. Let $U \equiv \{ u(\cdot;<s,c>) \mid <s,c>\epsilon F \}$. Given the convention of labelling agents according to their order of receipt of the durable, U constitutes the utility possibility set for our model. Under our assumptions U is a convex subset of the space of Lebesgue integrable functions on the unit interval, $L^1([0,1])$, with a non—empty interior, closed in the norm topology. Convexity is assured by the fact [see equation (4.4) in the text] that on the efficient frontier of F $s'(\alpha)$ is inversely proportional to the aggregate savings rate $[y - \int c(x,s(\alpha))dx]$. Thus the convex combination of two consumption allocations allows receipt dates for every agent which are less than the same convex combination of the corresponding assignment functions. It is obvious that the set is closed. Moreover, it will have an interior point in $L^1([0,1])$ by virtue of our assumption that the flow utility function $v(c)$ ranges from $-\infty$ to $+\infty$ as c varies over $(0,\infty)$.

Now an efficient allocation $<s',c'>\epsilon F$ generates an allocation of utility $u'\epsilon U$ satisfying (suppress dependence of u on $<s,c>$ hereafter): $u(\alpha) > u'(\alpha)$ a.e. if and only if $u\notin U$.

The Hahn—Banach Theorem implies a continuous linear functional $p:L^1([0,1]) \to R$ exists, such that $p(u') \geq p(u)$, $\forall u\epsilon U$. It is well known that the dual of $L^1([0,1])$ may be identified with the set of bounded, measurable functions on $[0,1]$. (See, e.g., Goffman and Pendrick (1965), Theorem 1, p.147). Therefore, there exists such a function, ϕ, satisfying:

$$p(u') = \int_0^1 u'(\alpha)\phi(\alpha)d\alpha \geq \int_0^1 u(\alpha)\phi(\alpha)d\alpha = p(u), \forall u\epsilon U.$$

Obviously $\phi(\cdot)$ must be non—negative, a.e. Moreover, if u' corresponds to an allocation in which all agents enjoy positive utility from the flow consumption good then, given any subset A of agents of measure strictly less than one, there is an alternative feasible allocation making all agents in A strictly better off, and all agents in $[0,1]\backslash A$ strictly worse off. Therefore ϕ must be strictly positive, a.e. The weights $\theta(\cdot)$ correspond to the function $\phi(\cdot)$ normalized to integrate to one. $\square$

Proof of Proposition 3: When $v(c)=\ln(c)$, simple but tedious calculation reveals that welfare under the random rosca is:

$$W_r = T[\ln(y)+\xi] - \frac{B}{y}[1+\chi(\xi/2)]$$

where the function $\chi(\cdot)$ is implicitly defined by $\chi(\xi)-\ln(1+\chi(\xi))\equiv\xi$, $\xi\geq 0$. Similarly, for the market we have

$$W_m = T[\ln(y)+\xi+\int_0^1 \theta_m(\alpha)\ln(\theta_m(\alpha))d\alpha] - \frac{B}{y}\int_0^1 [1+\chi(\xi\int_x^1 \theta_m(z)dz)]dx,$$

for some $\theta_m\epsilon\Theta$. We know from the proof of Proposition 3 that $W_m = \min_{\theta\epsilon\theta} W_\theta$. Hence, $W_r > W_m$ if and only if $\exists\theta\epsilon\theta$ such that $W_r > W_\theta$. The proof constructs some weights for which this is so. First we need:

Lemma 2: Let $f(\cdot)$ be an increasing, strictly concave function satisfying $f(0) = 0$, and let $g(\cdot)$ be a function on $[0,1]$, strictly decreasing satisfying $g(1) = 0$ and $g(0) = 1$. Then

$$\int_0^1 f(g(x))dx > f(1)\int_0^1 g(x)dx.$$

Proof: Let $\tilde{x}$ be a random variable which is uniformly distributed on [0,1]. Define $\tilde{y} = g(\tilde{x})$, and $\tilde{z} = 1$, if $\tilde{x} \le \int_0^1 g(x)dx$, and $\tilde{z} = 0$, if $\tilde{x} > \int_0^1 g(x)dx$. Then

$$E(\tilde{y}) = \int_0^1 g(x)dx = E(\tilde{z}),$$

where $E(\cdot)$ denotes the expectations operator. Moreover, $\tilde{z}$ is riskier than $\tilde{y}$ in the sense of second order stochastic dominance. Therefore, since $f(\cdot)$ is strictly concave:

$$E(f(\tilde{y})) = \int_0^1 f(g(x))dx > E(f(\tilde{z})) = f(1)\int_0^1 g(x)dx.$$

This proves the lemma. □

This lemma implies that

$$\int_0^1 \chi(\xi\int_x^1 \theta(z)dz)dx > \chi(\xi)\int_0^1 (\int_x^1 \theta(z)dz)dx.$$

This in turn implies that

$$W_\theta < T[\ln(y)+\xi+\int_0^1 \theta(x)\ln(\theta(x))dx] - \frac{B}{y}[1+\left[\int_0^1 x\theta(x)dx\right]\cdot\chi(\xi)],$$

where we have also used the fact that $\int_0^1 \int_x^1 \theta(z)dzdx = \int_0^1 x\theta(x)dx$.

Hence, a sufficient condition for $W_r > W_m$, is that:

$$\exists \theta\in\theta \text{ such that } -\frac{B}{y}\chi(\xi/2) > \int_0^1 \theta(x)\ln(\theta(x))dx - \frac{B}{y}\chi(\xi)\int_0^1 x\theta(x)dx.$$

Now define $E(\bar{\theta}) \equiv \left[\min_{\theta\in\theta}\int_0^1 \theta(x)\ln(\theta(x))dx \text{ s.t. } \int_0^1 x\theta(x)dx=\bar{\theta}\right]$. Then $W_r > W_m$ if:

$$\exists\bar{\theta}\in(0,1) \text{ such that } (\frac{B}{Ty})\cdot[\bar{\theta}\chi(\xi)-\chi(\xi/2)] > E(\bar{\theta}).$$

Let $\frac{B}{Ty}\equiv\gamma\in(0,1)$, and consider the problem: $\max_{\bar{\theta}\in[0,1]}\{\gamma\chi(\xi)\bar{\theta} - E(\bar{\theta})\}\equiv\Omega^*$. We conclude

that it is sufficient for $W_r > W_m$ that $\Omega^* > \gamma\chi(\xi/2)$.

Lemma 3: (i) $E(\overline{\theta})$ is strictly convex, and $E(\overline{\theta}) \geq E(\frac{1}{2})$, $\forall \overline{\theta} \in [0,1]$; and, (ii) if $E'(\overline{\theta}) = \lambda$, then $E(\overline{\theta}) = \lambda\overline{\theta} + \ln(\lambda(e^\lambda - 1)^{-1})$.

Proof: Define the Lagrangean

$$L = \int_0^1 \theta(x)\ln(\theta(x))dx + \lambda[\overline{\theta} - \int_0^1 x\theta(x)dx] + \mu[1 - \int_0^1 \theta(x)dx].$$

The first order condition with respect to $\theta(x)$ is: $\ln(\theta(x)) + 1 - \lambda x - \mu = 0$, $x \in [0,1]$. Inverting and integrating this condition, using the constraint, yields: $e^{\mu-1}\left[\dfrac{e^\lambda - 1}{\lambda}\right] = 1$. Solving this for μ, substituting into the first order condition, multiplying by $\theta(x)$ and integrating yields (ii). To prove (i) observe that integrating the first order condition, after inverting and multiplying through by x, and using the above derived expression for μ yields:

$$\int_0^1 x\theta(x)dx \equiv \overline{\theta} = e^{\mu-1}\int_0^1 xe^{\lambda x}dx = \left[\dfrac{\lambda e^\lambda - (e^\lambda - 1)}{\lambda^2}\right] \Big/ \left[\dfrac{e^\lambda - 1}{\lambda}\right] = e^\lambda/(e^\lambda - 1) - \lambda^{-1} \equiv \phi(\lambda).$$

It is straightforward now to see that $\phi(\lambda) \to 0$ as $\lambda \to -\infty$; $\phi(\lambda) \to 1$ as $\lambda \to +\infty$ and $\phi(\lambda) \to 1/2$ as $\lambda \to 0$. Part (i) is now proved by noting that $E'(\overline{\theta}) = \phi^{-1}(\overline{\theta})$, from the envelope condition. □

This result and simple calculation reveals: $\Omega^* > \gamma\chi(\xi/2)$ if and only if $\dfrac{e^{\gamma\chi(\xi)} - 1}{\gamma\chi(\xi)} > e^{\gamma\chi(\xi/2)}$. Note also that, from the definition of $\chi(\cdot)$, that $\chi'(\xi) = (1 + \chi(\xi))/\chi(\xi)) > 1$. Thus $\chi(\xi) > \chi(\xi/2) + \xi/2$. Moreover, $(e^z - 1)/z = \int_0^1 e^{zx}dx$ is a strictly increasing function. So:

$$\frac{e^{\gamma\chi(\xi)} - 1}{\gamma\chi(\xi)} > \frac{e^{\gamma\chi(\frac{\xi}{2})}e^{\gamma(\frac{\xi}{2})} - 1}{\gamma\chi(\xi/2) + \gamma\xi/2} = e^{\gamma\chi(\frac{\xi}{2})}\left[\frac{e^{\gamma\xi/2}}{\gamma\chi(\xi/2) + \gamma\xi/2}\right] - [\gamma(\chi(\xi/2) + \xi/2)]^{-1}.$$

The first of the two terms on the right hand side grows unboundedly as $\xi \to \infty$. Moreover, for a sufficiently large ξ, the second term vanishes. Hence, for large enough ξ, $[e^{\gamma\chi(\xi)} - 1]/\gamma\chi(\xi) > e^{\gamma\chi(\xi/2)}$ and $W_r > W_m$. (Note that $\overline{\xi}$, the critical value of ξ, depends only on $\gamma \equiv B/Ty$.) □
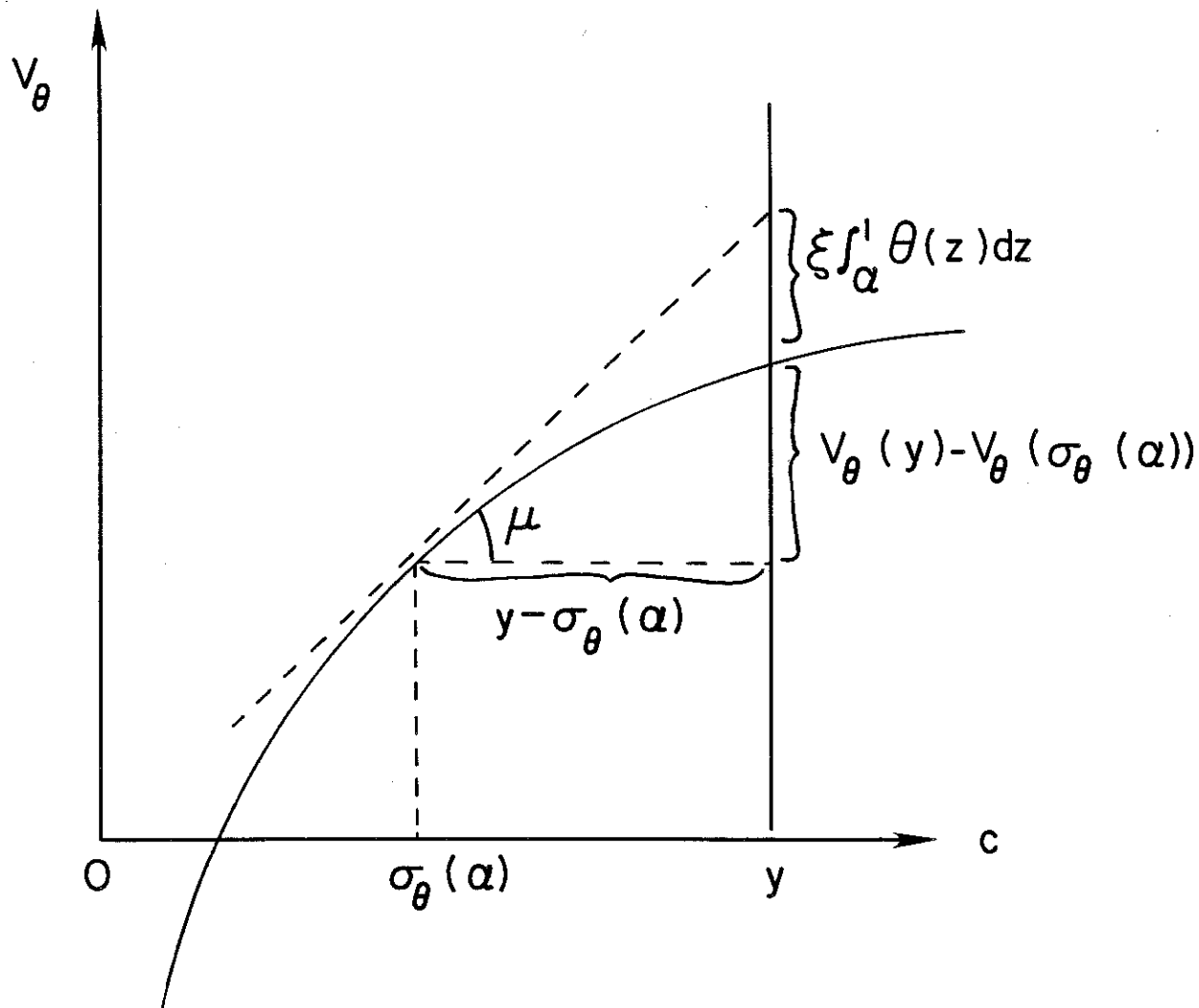
Figure 1

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